INTEGRATION OF MATHEMATICS TEACHING WITH LEARNING OBJECTS IN CLASSICAL AND PROFESSIONAL HIGH SCHOOL EDUCATION

INTEGRAÇÃO DO ENSINO DE MATEMÁTICA COM OBJETOS DE APRENDIZAGEM NA EDUCAÇÃO SECUNDÁRIA GERAL E PROFISSIONAL

João Bosco Laudares

jblaudares@terra.com.br

Pontifical Catholic University of Minas Gerais

Lana Paula Ricotta Nery

lanaricotta@hotmail.com

Utramig - Education Foundation for Labor of Minas Gerais

Tânia Fernandes Bogutchi

bogutchi@pucminas.br

Pontifical Catholic University of Minas Gerais

Yara Patrícia Barral de Queiroz Guimarães

yara.matematica@cefetmg.br

Federal Center for Technological Education of Minas Gerais

ABSTRACT

This article presents results of a research aimed at creating didactic material for the mathematics teaching-learning process in the high school and professional-technical education, which was developed within the period of a professional master's program. This research is the result of a cooperative work involving two study and research groups linked to both the respective master's program and institutions of integrated technical and high school education. All products were from masters' degree students and professors in the master's degree program, who work in institutions of basic and professional education. Two products are highlighted: a "Repository of Learning Objects for Secondary and Professional Education (ROAMEP)" and a book (in press), which presents the productions and studies in the areas of mathematics and professional education. The Learning Objects (LOs) address most of the mathematics content in high school education and mathematics applications in professional technical courses. The LOs were created in the digital format, and some of them make the use of the dynamic geometry software whereas others involve activities without the use of digital technologies. In order to integrate mathematics education and professional education, the activities for technical courses address knowledge on technology including electricity, mechanics, and chemistry, involving questions of mathematical models and introduction to differential and integral calculus in addition to the knowledge required for secondary education.

Keywords: Mathematics education. Professional education. Mathematics Teaching. Learning objects. High school education.

RESUMO

Este artigo apresenta resultados de uma pesquisa que teve como objetivo criar material didático para o processo ensino e aprendizagem de Matemática do ensino médio e técnico profissional, no âmbito de um Mestrado Profissional. É resultado de um trabalho cooperativo entre dois grupos de estudo e pesquisa vinculados ao respectivo mestrado e a instituições de ensino técnico integrado ao médio. Todos os produtos foram de mestres egressos e docentes do mestrado referido, que atuam na educação básica e instituições de educação profissional. Destacam-se dois produtos: um "Repositório de Objetos de Aprendizagem para o Ensino Médio e Educação Profissional (ROAMEP)", e um livro, no prelo, que apresenta produções e estudos na área de educação matemática e educação profissional. Os OA abordam a maioria do conteúdo de Matemática do ensino médio e aplicações de cursos técnicos profissionais. Foram criados no formato digital, sendo que uns fazem uso de *softwares* da geometria dinâmica e, outros apresentam atividades sem uso das tecnologias digitais. Para integrar a educação matemática com a educação profissional, as atividades, além de conhecimentos do ensino médio, abordam saberes da tecnologia: eletricidade, mecânica, química envolvendo questões de modelos matemáticos e introdução ao cálculo diferencial e integral para cursos técnicos.

Palavras-chave: Educação matemática. Educação profissional. Ensino de Matemática. Objetos de aprendizagem. Ensino médio.

1. Introduction

Teaching of mathematics has undergone several changes for some time since mathematics education has become an area of study and research for many teachers. In the period 1996-2014, a great growth was observed in the number of master (245%) and PhD (242%) degrees in the Education area, and 157 (30%) of the 525 professional master's programs in 2014 were on Teaching. Thus, the number of quality publications has increased continuously and the contribution of journals of professional programs (Teaching area) ranked *Qualis* A1 to B3 increased more than 300% in this period (CGEE, 2016).

This great interest in knowing better what is involved in the teaching and learning of mathematics indicates a general search to improve the educational practice in classrooms. The classroom came to be seen in another way, as a space to produce knowledge, which is reflected in the learning achieved by the students.

The Learning Objects (LOs) presented in this article are the result of a cooperation between two research groups. One is linked to a master's degree program and the other to an integrated general and technical secondary institution. This article presents the products developed by both professors in this master's program and former graduate students, who are currently teachers in the basic and professional education. The focus of this research was the creation of didactic material for the teaching-learning process of mathematics for general and technical education.

Some LOs constructed during this cooperation are highlighted. A Repository of Learning Objects for Secondary and Professional Education (ROAMEP; for Repositório de Objetos de Aprendizagem de Matemática para o Ensino Médio e Profissional; repositorioroamep.wix.com.br/site) with the LOs and a book (in press), which presents the productions and studies carried out in the field of mathematics and professional education, including a synthesis of the LOs.

The book is divided into two parts. The first comprises four chapters and addresses the design and development of an LO, with a chapter on the creation of the ROAMEP, the history of the computer (since most LOs use computer technology as a tool), and the project results that led

to organize the book. The second part comprises eleven chapters with the LOs created during this period, and most of the contents followed in high school were also included. Twenty products from the dissertations presented to the master program were explored in these eleven chapters. They were organized by subject, in tune with the National Common Base Curriculum (NCBC), the curriculum currently followed by the Brazilian secondary schools. Chart 1 presents the LOs that compose the ROAMEP and items of the book mentioned above.

Chart 1: Learning objects (LOs) found in chapters of the book (in press) and Repository of Learning Objects for the Secondary and Professional Education (ROAMEP)

Titles of LOs
Representation phenomenon models using functions
Exponential and Logarithmic functions with applications in Secondary and Technical Education
Exploring Trigonometry in the Simple Harmonic Model
Sine, cosine, and their inverse sine and cosine arcs in the electro-electronic area
Activities for meaningful teaching of trigonometry
Combinatorial analysis (with problem solving)
Exploring the properties of the Pascal triangle
Statistical measures in a tourism course
Statistics and Probability
Solving problems in physical phenomena (electro-electronic area)
Systems of linear algebraic equations (application in electrical circuits)
Flat and solid geometry (applications in technical mechanical design)
Activities for representation of conic sections
Analytical geometry for the study of straight lines and conical shapes
Uncomplicating complex numbers
Limits and Continuity
Introduction to the study of real function limits
Study on the behavior of functions through analysis of their derivatives
Exploration of convergence in topics of differential, integral, and numerical calculation
Virtual Environment for Learning of Mathematical Logic (VELML)

Source: Material prepared by authors and organizers.

The authors of LOs sought applications of the contents selected from the high school content, along with teachers in the technical area of various courses, including electronics, mechanics, mechatronics, buildings, biomedical equipment. Thus, by using the constructed objects, it is possible to have a good knowledge about mathematical applications in various technical areas, as well as to learn their content.

Then, the concepts of learning object/digital technology, investigation activity, and didactic sequence, which gave theoretical basis to the construction of activities in LOs, will be presented.

2. Theoretical approach

The LOs are also known as educational products because they involve contents addressed in classrooms.

According to Wiley (2000), an LO is a digital entity that comprises various components, which must be made available virtually. For a long time, this definition left doubts about the need or not of an LO having a computer (or computer resource) as a tool. Then, the Institute of Electrical and Electronics Engineers (IEEE, 2004) presented the definition that allows to classify the LO

used in learning as digital or non-digital entities. In addition, they have the characteristic of being reused.

In this sense, LO can be understood as didactic tools that contribute to the teaching-learning process. This didactic alternative, which was designed to be executed with or without the aid of a computer program, can be used by the teacher to introduce a subject into the classroom as well as by the students to complement their studies in an autodidactic way.

For example, the LO, which has a computer as a tool, may be based on a computer program to answer or prepare questions about a content. There are several applications (such as GeoGebra, Winplot, Maxima, Mathematica, Excel, ExerLearning, and others) which are a great support for teachers.

On the other hand, a learning object may not require a computer to be used. If an activity is designed in such a way as to lead students to construct concepts and help them formalise and define a content, then such activity is also an LO.

Various LOs have been prepared in the form of investigative activities (Ponte, Brocardo, and Oliveira, 2006). According to these authors, the contents of such LOs are organised to enable students to construct meanings.

Everywhere, services that use this tool are available. Various mobile and tablet apps (such as App from banks, delivery products, and transportation vehicles) make everyday life easier. Grossi, Borja, and Moraes (2018, p.211) highlight the importance that social standards are determined, (re) organised, and redefined for the current society.

Several activities were developed and published to be used through software. Many experience reports, in which computers are used as a tool, have been presented in scientific events. They indicate how much these tools have been used for didactic-methodological ways.

Grossi, Borja, and Moraes (2018, p.212) also address the need for education to incorporate "digital technologies and their potential (e.g., the use of social networks and their applications) as learning objects in the pedagogical practices of various curricular subjects". In their study, these authors concluded that "appropriation of digital technologies available in social networks [...] allows the use of various didactic applications that can influence and reconfigure the teaching-learning process" (p.223).

Borba and Penteado (2007) showed what the use of information technology could represent when they stated that "[...] (informatics) opens possibilities for changes within knowledge itself, and resonance involving a given pedagogy, media, and a view of knowledge is possible" (p.43). However, it must be understood that a computer will not facilitate learning if students are not instigated.

Teachers, who want to elaborate an activity using digital technologies, should be careful not to underuse the equipment, when they value programs that provide the answers for their students (who do not have to search for the solutions). This gives a false sense of understanding, but gaps will appear. In this sense, Borba and Penteado (2007) also highlighted that

> [...] with the arrival of this new computer actor, we try to overcome old practices. Such practice is also in harmony with a view of knowledge construction, which privileges the process (not the product/result) in the classroom, and an epistemological stance, which understands knowledge as having a subject-dependent component (p.44).

The subject, who is the protagonist of the action, is there open to what is proposed, and willing to build when provoked. So, learning happens.

> Where the environment neither creates problems nor presents new requirements, intellect development is neither motivated nor stimulated with new goals, adolescent thinking does not develop all potentialities, and does not reach the higher forms, or reaches them with extreme delay (Vygotsky, 2018, p.171).

Elaboration of an LO requires a detailed structural planning. For this reason, some authors of LOs involved in the study chose to use the didactic sequence as theoretical support, defined according Zabala (2010) by "[…] a set of ordered, structured, and articulated activities to achieve certain educational objectives, whose beginning and end are known by both teachers and students" (p.18).

To plan a class using the didactic sequence (as a reference guide), the teacher must first introduce the subject to be addressed. This is called by Zabala (2010) as a moment of presentation by the teacher (through a dialogue with his/her students) of a problem situation in relation to a theme. At this time, different points of view are compared, and hypotheses are established. While the proposed activity is introduced, and the execution stages are explained, knowledge is constructed to reach generalization. Exercises to establish concepts and solve problems are at the discretion of the teacher and working in the LOs is an excellent alternative to fix content.

When starting the research that resulted in the LOs, a theoretical framework of reduced production was outlined for this teaching material. Some initiatives have been found, such as the Virtual Education Network (RIVED, in which most products are focused on basic mathematics of elementary education); LabVir repository (developed by researchers at the University of São Paulo and directed to the areas of Physics and Chemistry); Virtual Laboratory of UNIJUÍ, RS (which has various LOs without specific thematic structure); and International Bank for Educational Objects (BIOE, created by the Brazilian Ministry of Education and Culture to locate, catalogue, and make available digital educational objects already produced).

However, given the specificity, structure, and scope required for the ROAMEP (such as creation of a teaching materials network, with all mathematics content, for an unprecedented thematic production of questions, with application in the professional area of high school education), productions favouring collaboration to build the repository were not found. Given the interdisciplinary proposal of ROAMEP for integration between mathematics and professional education, an incentive is expected for the development of teaching material from the integration results.

The literature was reviewed mainly within the conceptual scope of educational objects for media creation, in the research fields of educational computing and virtual environments (which consider the technology used in classroom, blended, or virtual courses as a tool and support for mathematics teaching), thus privileging cooperative learning in group.

In the LOs mentioned here (and presented below), the authors evaluated the learning by discussion in the classroom. Then, the students presented their opinions on the didactic strategy adopted, lists of fixation exercises, and subsequent written evaluation.

3. Methodology

Regarding the methodological, experimental, and analytical principles, all LOs that compose the ROAMEP were idealized, structured, and created and then tested, strictly observing the criteria of scientific methodology. They are products of qualitative empirical research developed by master's students in the field of education. The methodology was developed in three stages, with identity and integration in structuring moments of the investigation, as follows: (1) idealization and creation of a specific design for the activities necessary to achieve

the pedagogical objectives, as proposed for the LOs; (2) application of LOs to research subjects who studied in technical professional schools; (3) analysis of the results, followed by a possible reformulation for validation of LOs, and storage at the ROAMEP.

In this article, presentation of the LOs is limited to scientific dissemination. Detailing of theoretical references or presentation of application results was not the purpose of this article, since these aspects were fully covered in the masters' theses of the LO creators. Most of them are in the area of professional education (Federal Institutes, Federal Centres for Technological Education, and Technical Schools), where they teach.

4. Repository of learning objects for the secondary and professional education (ROAMEP)

The Brazilian and several world academic institutions have followed a movement of awareness, dissemination, and opening to scientific information, as well as the creation of free- or open-access repositories.

Thus, a virtual repository with easy access to students and teachers was created to organize and store all LOs of this project. This allowed us to use this teaching material as an auxiliary tool to complement the classes by teachers and as support for the study by students.

The repository was built on the *Wix* platform, which allows flexibility in its development and maintenance. There, the LOs were structured in a sequence of contents according to the curricula of secondary and professional education. They are available in the *PDF* or website formats, thus making consultation and study easier. Some LOs available in the ROAMEP are presented as follows.

5. Learning object on exponential and logarithmic functions with application in the Physics, Chemistry, and Biology areas.

This LO comprises activities that explore the geometric and graphic interpretation of these functions. Many students present great difficulty in understanding the concepts of exponential and logarithmic functions and the relationship between them. Thus, this LO addresses many applications in the high school and technical education providing a better assimilation and development of work capacity in this content.

According to the PCN+ (2002), "the richness of situations involving functions allows the teaching to be permeated by examples of the daily life and graphic forms that the media and other knowledge areas use to describe phenomena of dependence between quantities". Regarding exponential and logarithmic functions,

> they are used to describe the variation of two quantities in which growth of the independent variable is very fast, being used in knowledge areas such as financial mathematics, population growth, sound intensity, pH of solutions, and others (PCN+, 2002, p.121).

This LO comprises sequential didactic activities with the exponential and logarithmic functions from the basic functions $y = a^x$ and $y = \log_a x$. Their geometric and graphical interpretation, horizontal and vertical translations, and resolution of exponential and logarithmic equations and inequalities are explored.

In the initial sequence of LO activities, the concept of exponential and logarithmic functions is graphically explored. Subsequently, the LO shows interactions between mathematics and other

disciplines, such as physics, chemistry, and biology, with graphic interpretation of models in specific problems in these fields.

The content of LO activities was constructed using the GeoGebra software and structured based on the Java language. The internet pages were created by the *ExerLearning* software in *html*. The entire LO is self-explanatory, and following the instructions is enough to run it. The keyboard and mouse are used in the sequence of LO´s screens.

The LO comprises interactive activities, which are distributed in 4 topics: (1) Graphic Study of the Exponential Function, Translations and Graphic Resolution of Exponential Equations and Inequalities; (2) Graphic Study of the Logarithmic Function, Translations and Graphic Resolution of Logarithmic Equations and Inequalities; (3) Simultaneous Study of Exponential and Logarithmic Functions to Identify Inversion Relationships and Possible Symmetries; and (4) Technical Applications of Exponential and Logarithmic Functions in Natural Phenomena. Practically all of them have a dynamic approach with the use of GeoGebra. They comprise questionnaires (multiple-choice questions with five-alternative answers) and discursive questions.

The four topics are displayed on the LO home screen. The four sets of activities can be accessed and selected with the mouse and there are a series of images related to natural phenomena.

Only a few activities of this LO have been chosen to show its structure, and some activities related to topics 1 and 4 are presented here.

Activity 1 presents the graphic study of the exponential function to identify its properties and constitution. In it, translating graphs, interpreting graphs, and solving exponential equations and inequalities is possible. Figure 1 shows the construction of a graph of the base-2 exponential function after insertion by the student (in the green column) of random data of a domain belonging to the real interval [-3,3]. The values in the image set are obtained in the blue column.

Figure 1: Base-2 Exponential Function.
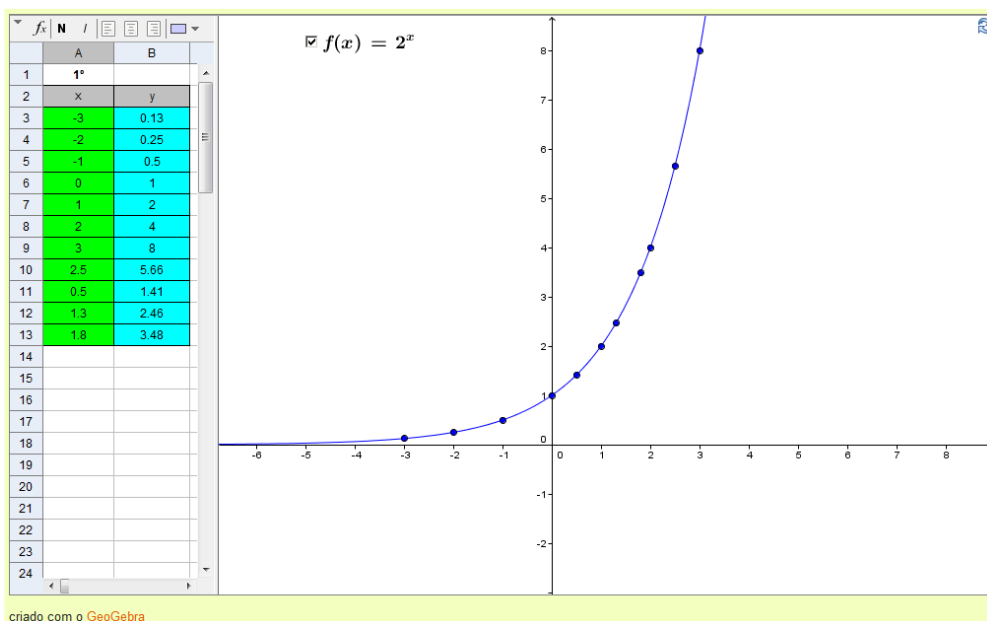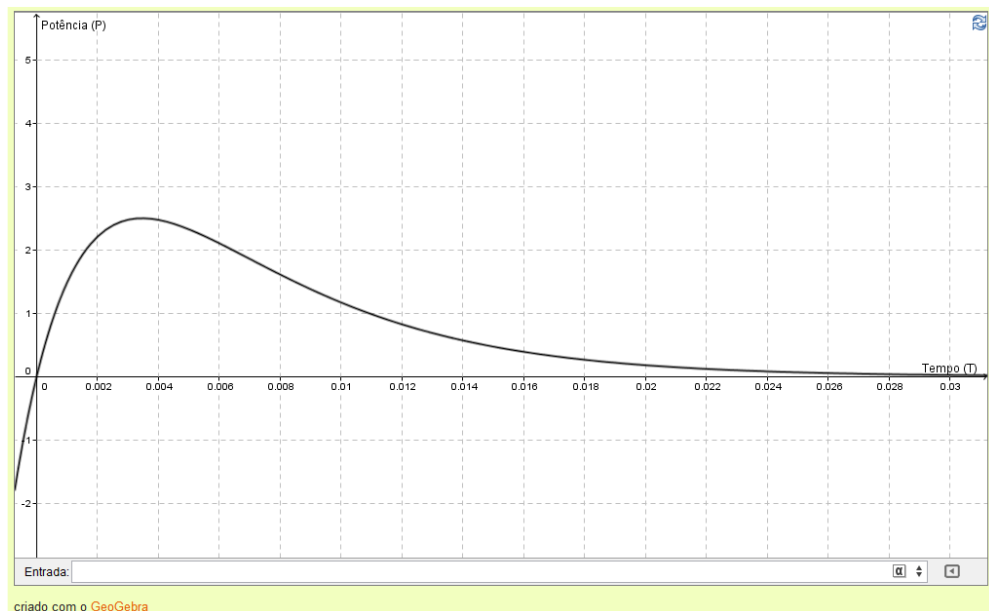

Source: Material prepared by the author.

In continuation, questions involving understanding of the graph so obtained are presented, allowing theoretical analysis of the exponential function. By selecting the alternative, the

student can know immediately whether the answer is right by checking the signalling at the end of the question. Trying again until achieving the correct answer is possible, to deepen the analysis of results. The activity also presents the base ½ exponential function separately, and both base-½ and -2 functions simultaneously.

Activity 4 presents technical applications for exponential and logarithmic functions (such as radioactive deterioration, transient power behaviour in an RC circuit, population growth, ^{14}C archaeological dating, and RC circuit II), with the purpose of developing and relating the applicability of phenomena and behaviours that can be represented by exponential and logarithmic functions.

Figure 2 shows an application (with verification) of the transient power behaviour represented by the exponential function $P(t) = 10(e^{-200t} - e^{-400t})$. After typing this function in the input field (computer screen), the graph of function $P(t)$ and the questionnaire relative to the graph will appear.

Figure 2: RC circuit - Translated exponential function $P(t) = 10(e^{-200t} - e^{-400t})$ for $t \geq 0$.

Source: Material prepared by the author

6. Learning object for Trigonometry in the Electro-Electronic area with application in the study of signals

This LO comprises activities to explore the simple harmonic model with conceptual, algebraic, and graphical analyses of trigonometric functions. These analyses lead to the study and mathematical contextualization of the alternating signal from a classical physical model. Applicability of trigonometry is recognized. However, it is often taught disconnected from possible applications. These aspects were pointed out in the PCN+ (BRASIL, 2002), where, with a long-time investment in the algebraic calculation of identities and equations and to the detriment of priority aspects (such as calculations of inaccessible distances, trigonometry in the circle, trigonometric functions, and analysis of their graphs).

Eight activities were created to compose this LO. They form a didactic sequence that proposes motivational challenges to understand it and concretize the trigonometric mathematical

concepts. They allow students to make considerations and relationships with other disciplines and contents studied in the high school (e.g., physics) and technical education, and with situations experienced in everyday life.

These activities were built for the use of concrete materials as well as GeoGebra software, with the development of dynamic computational applications (*applets*). These *applets* are easy to execute and following the internal instructions and using the mouse is necessary. They can be accessed by any device where the software is installed or manipulated in HTML format through internet browsers.

The activities that compose this LO initially explore the concept of function, using graphical models for analysis of variables and domain and image sets. Then the systematic and partial construction of the graph (representing the function generated) is made from the classical model of alternating signal generator using the equation $v = V_{max} \sin \theta$ (which represents the behaviour of the model of alternating signal generation), according to the analysis of this model and its comparison with the representations of the trigonometric functions: sine and cosine. Then, the interaction, visualization, comparison, and systematization of mathematical concepts are investigated with using the representative model and the computational resources (*applets*) with view to interact with other disciplines. Thus, according to Zabala (2010, p.76), "the purpose of the proposed activities is to improve the significance in learning the conceptual contents".

Some of the activities of this LO were selected to verify its structure, including those related to the use of these computational resources (*applets*). In activity 4, the use of applets 1 and 2 allows to visualise the representation of the sine and cosine functions from the angular variations in a more attractive approach. In these *applets*, values for the sine and cosine functions are given for various angles in the trigonometric circle. A point A can be manipulated (using the mouse), and thus it is possible to verify both variations in angles and values for these functions in the four quadrants (through projection in the Cartesian axes *x* and *y*).

Activity 6 presents *applet* 3, which allows to associate sine function with the model of generation of this signal (movement of a coil under the action of a magnetic field).

Figure 3: Applet 3 - Model of Alternating Signal Generation - Representative Function
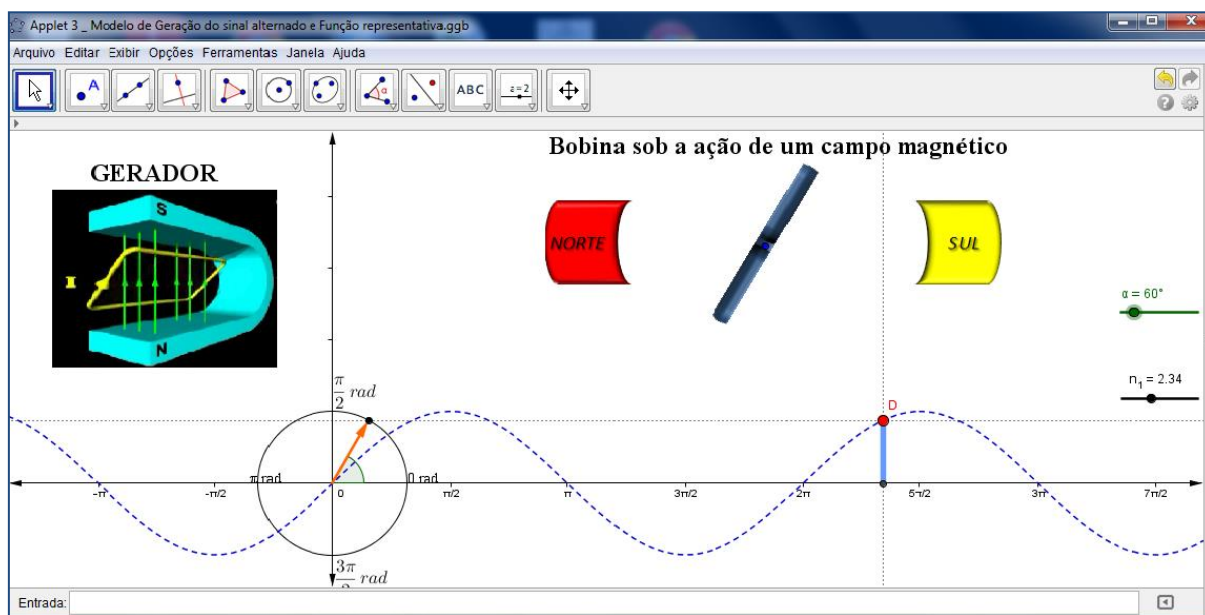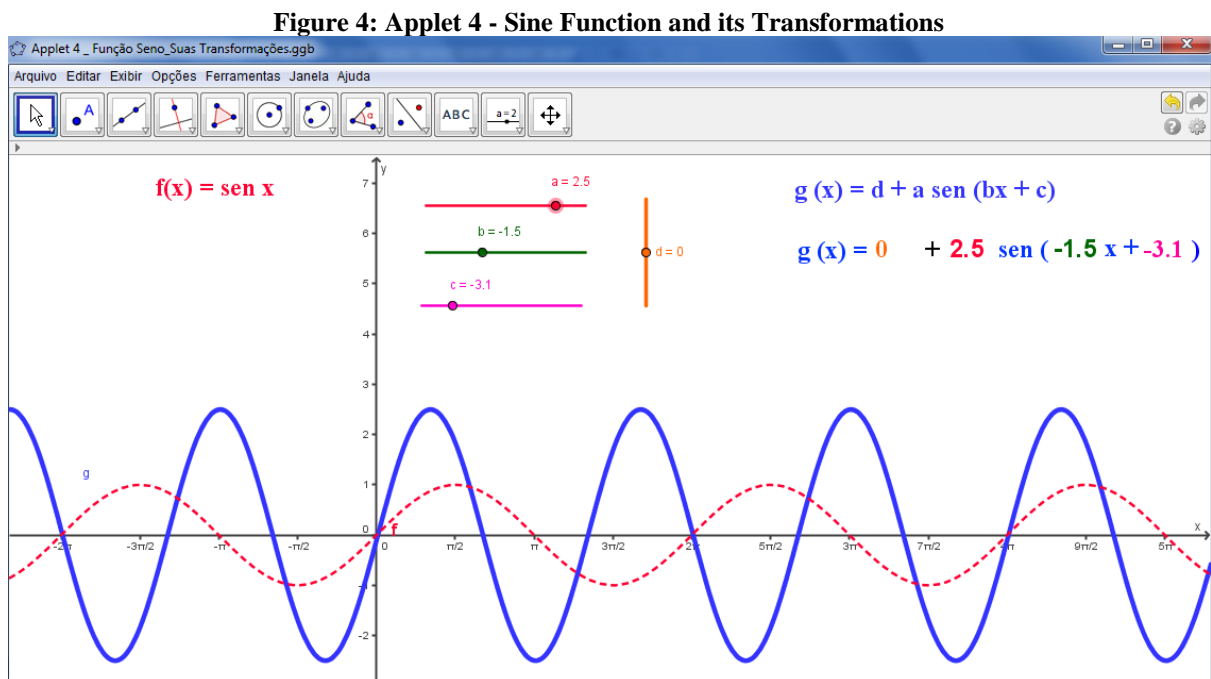

Source: Material prepared by the author.

Figure 3 shows that this movement is obtained by acting on the sliders of the GeoGebra software. This allows to observe the alternation in the generated signal and create (or mathematically represent) the projections on the ordinate axis.

In this activity, verifications are requested regarding behaviour of the generated signal and sinusoidal function if (and when) the projections are positive or negative. In addition, the concepts of amplitude and periodicity characteristics of the generated alternating signal are checked and resumed.

In activity 7 (*applet* 4), comparison between the simplified sine function [f(x) = a.senx] and general equation [g(x) = d + a. sen (b.x + c)] is presented with all variables: amplitude (with variation in parameter *a*), period (with variation in parameter *b*), and lags [with variation in parameters *c* (corresponding to longitudinal or horizontal displacements) and *d* (corresponding to vertical displacements)].

Figure 4 shows *applet* 4. Manipulation of the sinusoidal function is a way to visualise that sine function represents the conceptual model of alternating signal generator. In addition, it is possible to check the conditions under which the translated function g(x) can be compared with the simple function f(x).

Figure 4: Applet 4 - Sine Function and its Transformations

Source: Material prepared by the author.

7. Learning object about complex numbers applied to the electrotechnology area.

This LO (entitled "Uncomplicating Complex Numbers") addresses a content that is received by most students as complicated. In fact, most students find it difficult to understand situations in which this content is applied; on the other hand, students in technical courses (who have the subject of Electrical Circuits as part of their curriculum) can see and experience this application.

Spinelli (2009) stated about it that:

the great indignation of Secondary School students (and many teachers as well, when they study complex numbers) refers to the fact that this type of number is not suitable for any need in their daily life. After all,

complex numbers neither express score results nor represent quantities. In addition, it does not make much sense to order them; on the other hand, using them as a basic encoding element is strange at least. Thus, according to only the uses that students know, a complex number would not deserve to receive the title of "number" (p.4).

The LO was planned with focus on classes of the high school and technical education and followed the National Curricular Parameters (PCN) for High School Education. Thus, although this LO is an application in electrical circuits, it is an activity that can be applied to any high school student, contributing much for them to understand its content (especially in non-technical activities).

The first two activities give focus on the teaching of complex numbers from the conceptual point of view, without application. The other activities provide application of operations in the analysis of electrical circuits.

The definitions, concepts, properties, and geometric and algebraic representations were considered when this LO was prepared. It is composed of six activities that form a didactic sequence in a computerised environment, involving the study of complex numbers with applications in alternating current circuit analysis.

Figure 5 shows the initial screen of the LO. As it is self-explanatory, the user is expected not to have much trouble to follow the commands.

Figure 5: Copy of the initial screen of the LO "Uncomplicating Complex Numbers".
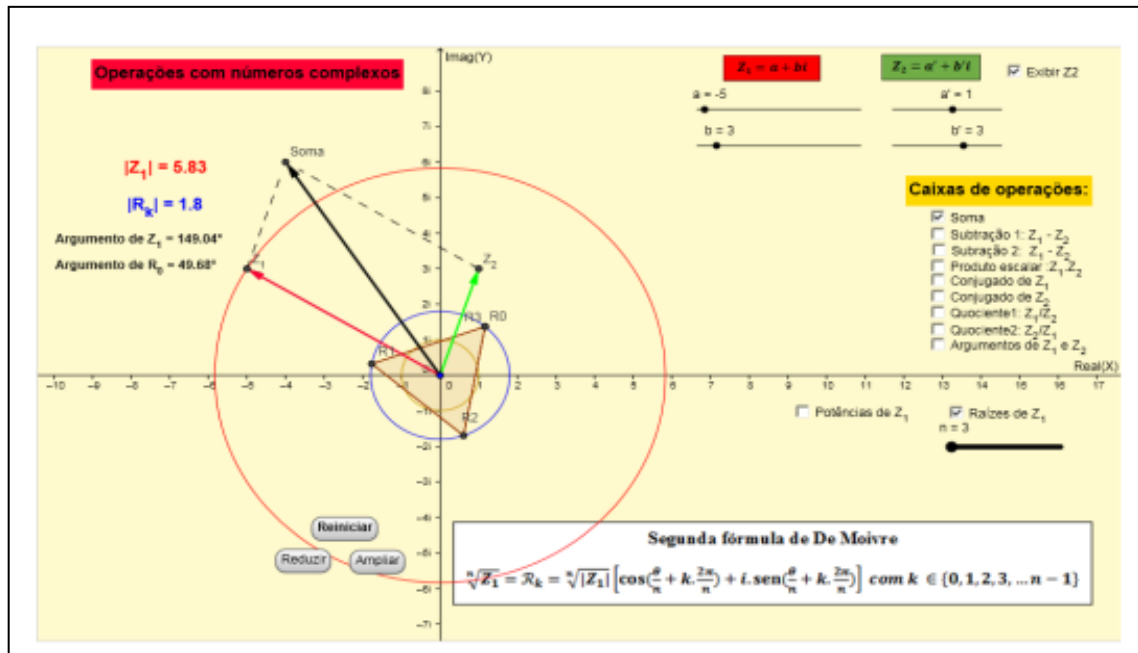

Source: Material prepared by the author of the LO.

This activity has links with theoretical support for every need found there. Development of autonomy is one of the author's objectives, since the students may need the teacher's guidance to learn the content, but they can also follow the commands alone. Thus, the teacher can use the LO to either complement his/her classes or introduce this content.

When the students answer the questions in the questionnaire, they receive a feedback with didactic guidance to be followed before starting the next question if they have opted for a wrong answer.

Figure 6: Computerized environment of the LO (2nd activity / 2nd questionnaire).
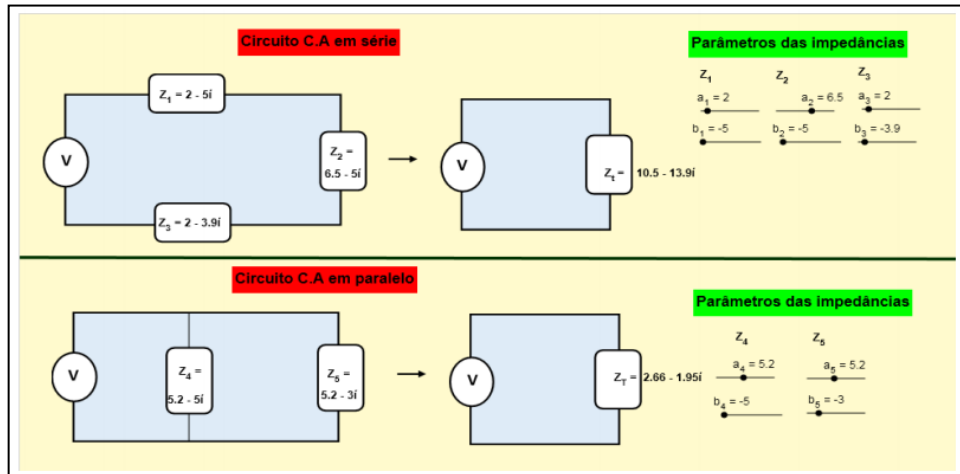

Source: Material prepared by the author of the LO.

The options of answers to the questions allow several possible simulations to check the true or false alternative depending on the request. Here, students can observe various real situations that may arise during experimentation.

The third activity is presented below to illustrate the application of complex numbers in electrical circuits. The LO informs the proposal of this activity in its book of activities:

> Each element Z in the circuit is represented by a complex number. This condition (together with the purposive omission of the equivalence relation) gives conditions to the students to answer a questionnaire on the addition and subtraction operations involving complex numbers (Pinto, 2015, p.22).

Figure 7 shows the screen of the computerised environment in the third activity:

Figure 7: Computerised environment in the third activity.

Source: Material prepared by the author of the LO.

Students of the professional education may have less difficulty in "visualising and experiencing" this application in the discipline of electrical circuits, as they receive a basis for what they will learn in practice.

This LO is an example that can be used by both teachers who teach the content of complex numbers and those who teach the discipline of electrical circuit in the technical area and want to review its content.

8. Learning Objects for Notions on Differential and Integral Calculus: the numerical view on change rates and calculation of area under curves.

The present LO was applied in undergraduate courses for students of professional (technological) education. This LO, whose focus was coincident, was elaborated from activities of different authors. Then, an LO for Notions on Differential and Integral Calculus was organised from the intersection of the general objectives with dissertation products of four authors. Chart 2 shows the educational products that compose this LO:

Chart 2: Activities that formed the LO for Notions about Differential and Integral Calculus

Activity titles	Participation in the LO
Limit and continuity	Introduction to the study on limits and continuity
Introduction to the study on limits of real function	Obtaining the limit in some real functions
Exploring convergence in topics of differential, integral, and numerical calculus	Study on change rates
Study on the behaviour of functions by analysis of their derivatives	Study on the growth and decay of real functions by derivative of these functions
Exploring convergence in topics of differential, integral, and numerical calculus	Calculation of area under curves by Riemann Sums

Source: Material prepared by authors of the article.

This LO was enriched, since it involved investigative activities of different authors, with distinct approaches on different topics of calculus. A single LO was organized and prepared

after passing through various important notions that compose the discipline of Calculus I in higher education institutions.

This is also accessible to students in the professional secondary education, in which they need to have notions about this content for application in technical subjects (including technical courses in electronic, mechatronics, and biomedical equipment). Therefore, it is present in the ROAMEP.

The concepts discussed were presented only after the activities were applied, enabling this knowledge to be constructed through mathematical investigation using the GeoGebra software.

At this time, the activities prepared for the study of change rates will be presented. They precede the activity on the study of growth and decay of functions with the use of derivatives. The last LO activity addresses calculation of the area under curves using Riemann Sums. Here, learning occurred through experimentation, verbalization, generalization, and abstraction, after the students performed experiments during activities guided by the GeoGebra software. They discussed the subject by raising hypotheses, verbalizing the content, generalising and abstracting with formalisation of concepts obtained through mathematical investigation (Ponte, 2006).

These activities were applied to the subject of Calculus I in regular classes of mechanical engineering courses, mathematics bachelor's degrees, and information systems of both private institution and professional (technical) courses of higher education.

The third activity in this LO led the students to understand the concept of change rates after they constructed tables and observed the trend of values for Δx e Δy, as to conclude about the result for $\dfrac{\Delta y}{\Delta x}$.

Figure 8 shows the given function $C(x) = 5000 + 10x + 0,05x^2$ and the table to be filled as part of the activity on change rates.

Figure 8: View of the statement in a question about change rates.

c) Considere $\Delta x = x - 100$ e $\Delta y = y - 6500$, sendo x e y cada um dos pontos da tabela e observando os dados obtidos com o *VCN*, preencha a tabela abaixo com alguns valores sugeridos:

Δx	5	4	3	2	1	$\to 0$
Δy						
$\frac{\Delta y}{\Delta x}$						$\to$ ___

d) Então, quando $\Delta x \to 0$, para qual valor $\frac{\Delta y}{\Delta x}$ está tendendo? _____

e) Logo, a taxa de variação instantânea da função $C(x) = 5000 + 10x + 0,05x^2$ no ponto $x = 100$ é igual a _____

f) Para obter a taxa média, subtraia as imagens correspondentes de 105 e 100, e divida o resultado por $105 - 100$. Qual é o resultado encontrado? Escreva como podemos interpretar esse valor. _____

Source: Material prepared by the author of activities.

Some values were suggested in this table, but the students were also invited to test other values to better see the results obtained. In this activity, the VNC (Virtual Numeric Calculus) program was used. It was also created and developed by professors in the Departments of Mathematics

and Computing at the same institution where the given master program is developed. However, this program may be substituted for another (e.g., MS Excel).

In this activity (calculation of cost in US$ to produce x units of a certain product), students need to think of values for x that can produce the requested values for Δx e Δy when function C is considered. After the table is filled, the purpose of the following questions is to collaborate in the construction of student's thought until they can achieve the objective of the activity.

The objective of the last activity in the LO is to contribute to the learning of calculation of definite integrals, considering convergence of the upper and lower limits of the areas in the n partitions under a curve. It has been applied under the same conditions as mentioned above.

This LO has had the support of the GeoGebra software, as the numerous experiments (and their respective views) were possible due to the dynamic resource that this software provides. Regarding the function $f(x) = \dfrac{sen(x+2)}{1+x^2}$, in the interval $\left[-2, 1\right]$, the students were invited to perform partitions and obtain the area under the graph using the Lower and Upper Sums command of the GeoGebra software. Figure 9 shows an example (with 19 partitions) of the result from the Lower Sums:

**Figure 9: Copy of the GeoGebra screen for the lower Riemann sum (with 19 partitions).**
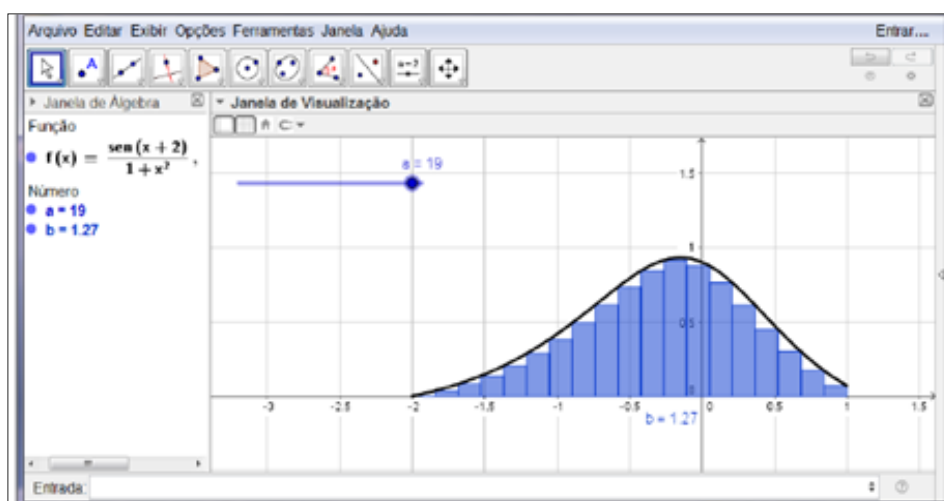
Source: Material prepared by the author of the activity.

Students were asked to make various attempts to change the number of partitions, recording the results so obtained in a table. They were then asked to repeat the same experiments using the "Upper Riemann Sum" command just as they have used the previous command.

Figure 10 shows a copy of the GeoGebra software screen after both commands were used.

Figure 10: Copy of the GeoGebra software screen with the "LowerRiemannSum" and "UpperRiemannSum" commands (both with 19 partitions)
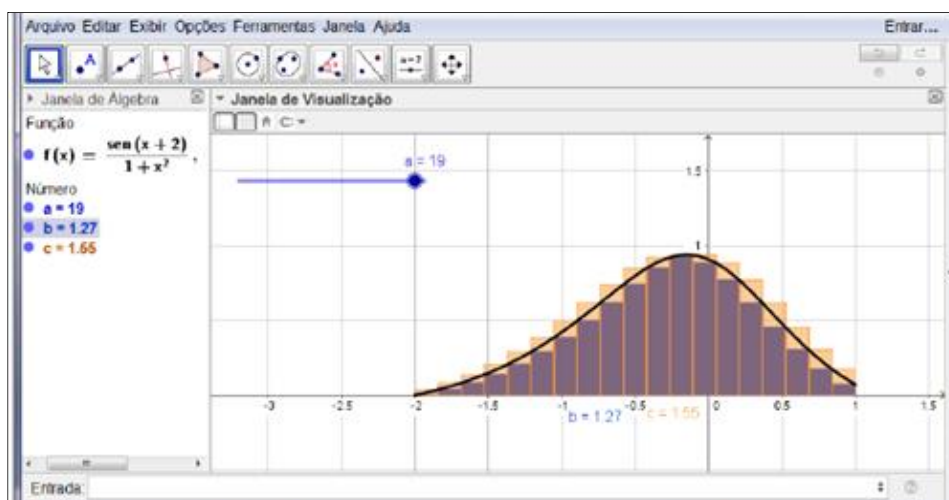
Source: Material prepared by the author of the activity.

Students were suggested to use different colours in the commands (a darker colour in "LowerRiemannSum" than in the upper sums) for better visualization. Thus, students easily concluded that the ideal result is the arithmetic mean of the values obtained with the upper and lower sums.

Other examples of functions were also used, and new experiments were performed. During a written evaluation of the learning at the end of the application activities, the researcher was surprised by the phrase of a student who told how all this was worth: "Teacher, the tables and graphs we constructed in the GeoGebra software came right here to my mind".

9. Learning object for Boolean algebra and mathematical simplification models

This LO is designated by the acronym VELML (for Virtual Environment for Learning of Mathematical Logic) and has a website format. It was developed due to the need for more simple and friendly auxiliary tools for teaching and learning Boolean Algebra, with simplifications and application in the technical areas of electro-electronics.

Currently, the use of electronic artefacts (such as computers, cell phones, digital televisions, the internet, and others) has indicated that the study of the binary system, mathematical and propositional logic, and Boolean algebra is fundamental for the electronic systems (as well as sciences and technologies) to evolve.

In Boolean algebra, each function is represented by a symbol of a logical block, which performs the function according to its truth table. Boolean variables are represented by letters assuming only two values ("0" and "1", for studies in the area of electronics).

According to Daghlian (1995):

> Graphical representation of Boolean functions is done by means of symbols (known as logic blocks or gates), which are normalized by international standards. Logical gates are the bases of logic circuits, whose purpose is to combine different Boolean quantities, in order to perform a certain function. Each logic gate can have different input lines, but only one output line [...]. (p.154).

The LO contains articles for searching for mathematical models, mathematical logic, and Boolean algebra activities. The purpose of activities that compose the LO is that students

can establish and learn Boolean functions, apply theorems, properties, and mathematical identities to simplify expressions and, thus, minimize digital electronic circuits. The eight activities in this LO are divided into three blocks, which serve three situations in the given Boolean model: (1) mathematisation of the actual situation; (2) simplification of expression, generated from the real situation; and (3) application of the model to generate a digital electronic circuit from the actual situation proposed.

Figure 11 shows the initial screen of this LO, highlighting the Menu of Basic Boolean Functions, such as the AND, OR, and NOT functions.

Figure 11: Basic Boolean Functions menu

Source: Material prepared by the author.

When one of these functions (e.g., "OR function") is chosen, another screen appears, where the concept of the given function (Truth Table), Boolean expression, and logic circuit are presented. Students are invited to act on inputs A and B of the circuit, comparing the outputs (by means of changes in the binary values) with the Truth Table (by means of an LED on or off). The purpose of these activities is to establish knowledge.

While continuing these activities, forms of simplification of expressions are presented by means of Boolean algebra or by Karnaugh (Karnaugh Map). When applying the Boolean simplification model, it is possible to compare methods (as well as evaluation activities) to stimulate students to develop their own projects using mathematical methods.

Figure 12: Activity 8 - Result of a simplified circuit.
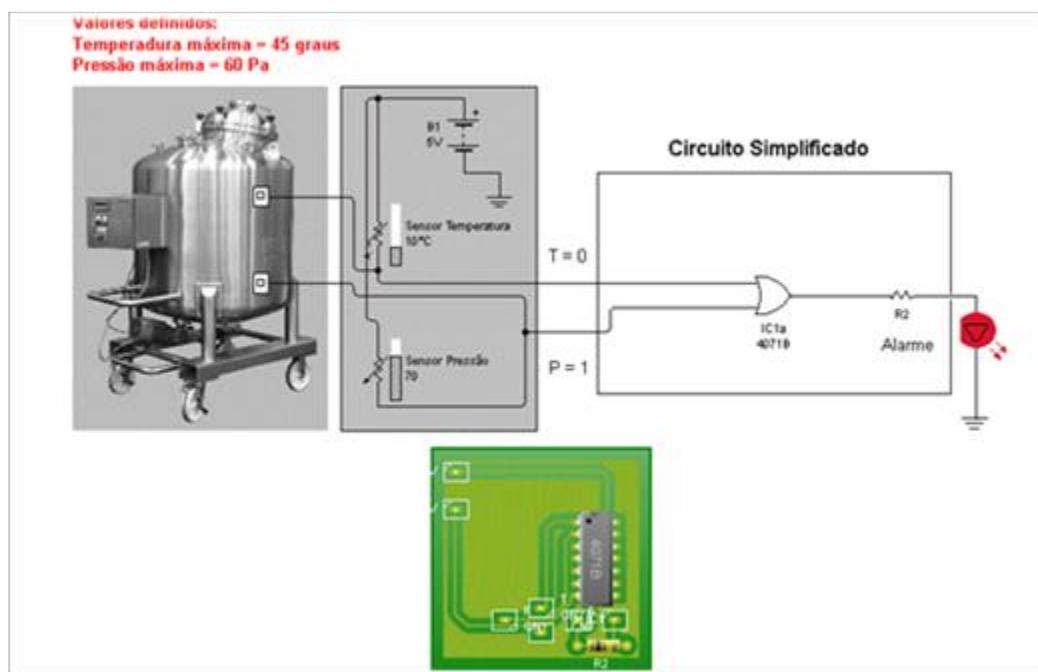
Source: Material prepared by the author.

Activity 8 presents a problem to show a real situation (figure 12) and stimulate the students to seek applications of simplification mathematical models. In this activity, other activities are included (and all proposed content is used) at each menu option to aid in project development. The project is about an electric circuit in a system for industrial control of chemical reactions that activates a remote alarm whenever one of the variables (such as temperature or pressure) exceeds the maximum values defined.

This LO has a simple access being directed to students of high school education, especially professional courses in the area of electro-electronics (including electronics and computer science). These LOs are products of integration between people who work in the mathematical and professional education, as well as other LOs in the ROAMEP.

10. Final considerations

Both the launch of a book (currently in press; for publication soon) as well as the ROAMEP (already with possible access) are products created by a team of masters and doctors (with production in the area of mathematics and professional education) to support the activities of both mathematics teachers and students in the general and professional (technical) high school education.

As the learning objects are products of a professional graduate program and resulted from the work of two groups for research and studies, they were already tested by their authors as well as analysed in dissertation defenses (and activities of members of groups mentioned above).

In short, the ROAMEP is an original production for integration between mathematical and professional education. It is an open virtual product, which may be enhanced with other products in the area of mathematics teaching for both primary and general and professional high school education.

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