SOCIAL JUSTICE AND THE DEVELOPMENT OF COMBINATORIAL AND PROBABILISTIC REASONING IN YOUTH AND ADULT EDUCATION

JUSTIÇA SOCIAL E O DESENVOLVIMENTO DOS RACIOCÍNIOS COMBINATÓRIO E PROBABILÍSTICO NA EDUCAÇÃO DE JOVENS E ADULTOS

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ABSTRACT

The present paper aims to discuss characteristics of Youth and Adult Education in Brazil and, specifically, how combinatorial and probabilistic reasoning may be developed as means of social justice. We argue that gradual teaching of combinatorial and probabilistic concepts, use of different representations and strategies of problem solving regarding to these areas of mathematics, and also perception of the relations between them, are essential to the development of these forms of reasoning and must occur since early schooling years. Therefore, Youth and Adult Education students have the right to be in contact with combinatorics and probability, in order to be able to solve school and everyday problems that involve the knowledge of non-deterministic situations and the search of possibilities.

Keywords: Youth and Adult Education; Combinatorics; Probability; Combinatorial Reasoning; Probabilistic Reasoning.

RESUMO

O presente artigo visa discutir as características da Educação de Jovens e Adultos (EJA) no Brasil e, mais especificamente, como os raciocínios combinatório e probabilístico podem ser desenvolvidos como meios de justiça social. Nós defendemos que o ensino gradual de conceitos da Combinatória e da Probabilidade, o uso de diferentes representações e estratégias na resolução de problemas dessas áreas da Matemática, bem como a percepção das relações existentes entre elas, são essenciais ao desenvolvimento de tais formas de raciocínio e, portanto, devem ser explorados desde o início da escolarização. Desta maneira, o contato formal com a Combinatória e com a Probabilidade em sala de aula é direito dos estudantes da EJA, para que estes estejam aptos a resolver problemas – escolares e do cotidiano – que envolvam conhecimentos acerca de situações não determinísticas e o levantamento de possibilidades.

Palavras-chave: EJA; Combinatória; Probabilidade; Raciocínio Combinatório; Raciocínio Probabilístico.

1. Youth and adult education in Brazil

Brazilian Youth and Adult Education, as a teaching modality that offers Elementary, Middle and High School to students that did not attend school in the expected age, arose from different movements and proposals that intended to provide schooling to people originally excluded from the educational system in regular modality – directed to children and teenagers. Despite the intention of this modality to be a temporary one – until all young people and adults had proper schooling – this mode is still necessary, especially in certain parts of Brazil. According to recent census (o Globo, 2017), Brazil still has about 12 million of illiterates, which corresponds to about 7% of the population 15-year-old and older. The percentages of illiterates is about 15% in the Northeast of Brazil and about 8% in the North – the nation's poorest regions – and this justifies permanence of Youth and Adult Education in the country – and is a call for social justice.

Since 1996 the Law of Directives and Basis for National Education – Lei de Diretrizes e Bases da Educação Nacional, LDB, nº 9394 – (Brasil, 1996) institutes this teaching modality, guaranteeing unpaid basic education (Elementary, Middle and High School) for young people and adults. Moreover, LDB (Brasil, 1996) ensures that the education to be provided must be appropriate to young people and adult students, that is, this teaching modality has to take into account the characteristics and needs of its public in order to provide the formal knowledge they need and are looking for.

In this sense, Youth and Adult Education operates as a mechanism of social justice whereas it endeavors to give to its students the same opportunities as the ones that students of regular schooling modality have – such as professional opportunities and the chance to continue their schooling after High School – considering, however, specificities of schooling necessary to those starting school at a later age or returning to school after some time.

This educational law (Brasil, 1996, free translation) claims that:

> Art. 37 – Youth and Adult Education is destined to those who have not had access or continuity to Elementary, Middle and High School at the proper age.
>
> § 1º Schooling systems will ensure free and appropriate schooling opportunities, having in mind the characteristics, interests, life and work conditions of its students that could not attend school at the expected age.
>
> § 2º The Public Power will enable and stimulate workers' access and permanence in school, through integrated and complementary actions.

The law recognizes that students of Youth and Adult Education have experienced different situations during their lifetime – in everyday and in work activities. Thus, they have passed through experiences that children and teenagers usually never did. Because of that, these students' needs and expectations regarding formal schooling are different from students' prospects of regular schooling.

Sometimes, unfortunately, Youth and Adult Education is seen as merely compensatory and is not differentiated from regular schooling. In order to change this misconception,

it is indispensable that we have in mind the reality of this modality and the characteristics, potentials and expectations of its students.

In this direction, Oliveira (1999) highlights that these students have three common characteristics that must be taken into account when we look at the development of Youth and Adult Education and the possibilities this teaching modality opens up to the public it is meant for. These characteristics of Youth and Adult Education regard to the three following conditions: 1) the students are not children; 2) the students were excluded from regular school and 3) the students belong to a certain cultural group.

The first characteristic pointed out by Oliveira (1999) emphasizes that because of the rich and distinct experiences the adults have, these students need to be seen by the school as a different public than the regular modality one. "When school hosts this student, it needs to remember that, in general, this institution is structured and organized to welcome regular schooling public" (Lima, 2018, p. 42, free translation). It is, thus, needed that the school adapts, in order to treat these students differently than the way children are treated.

The second condition, according to Oliveira (1999), is regarding to the background of students of Youth and Adult Education. Not only coming from different contexts of life, when compared to children and teenagers that attend school in regular basic education, the young and adult student that seek for this teaching modality to begin or to retake their schooling is, originally, a student excluded from the regular system of schooling. These students are, then, inserted in a school where "curricula, teaching programs and methods were originally designed for children and teenagers who walk the path of schooling on a regular basis" (Oliveira, 1999, p. 61, free translation). In this sense, curricula, programs and methods have to be adapted to the adult student that is starting school for the first time or returning after some period away from school.

The third and last condition considers the fact that students of Youth and Adult Education belong to a population with low purchasing power and are, generally, active in the world of work. These characteristics also need to be taken into account when planning and implementing appropriate educational plans for this public. The students of this teaching modality:

> do not belong to the dominant social group or to the group characteristically object of general educational practices. The problem here regards to cultural homogeneity and heterogeneity, to confrontation between different cultures and to the relation between cultures and, also, differences in abilities and intellectual performance of subjects (Oliveira, 1999, p. 62-63, free translation).

This causes the emergence of prejudice concerning Youth and Adult Education and the belief that its students do not own the same learning capacities and intellectual skills as the regular schooling ones. This idea, if not remedied by the offering of a teaching that allows the development of these students and provides equal opportunities to those of students of regular schooling, is a view that constitutes a barrier to the promotion of basic education to young people and adult students and to the support of social justice.

The adequate observance of these three conditions pointed out by Oliveira (1999) – the condition of non-child, the condition of excluded from regular school and the condition of belonging to specific cultural groups – is necessary to provide social justice in Youth and Adult Education. In this way, the students in this modality of education will be provided with proper treatment, recognizing their present conditions and their prior experiences and also considering their potentialities.

This discourse is also supported by the Resolution #1 CNE/CEB (Brasil, 2000), which established the National Curricular Guidelines for Youth and Adult Education (years before the publication of specific curricular documents, which are presented and discussed later in the present paper) and emphasized that this teaching modality must be based on the principles of equity, difference and proportionality while contextualizing and making use of such curricular guidelines, as well as proposing a suitable pedagogical model for its students. These principles are explained in the passage that follows (Brasil, 2000, Art. 5):

> I – regarding to equity, the specific distribution of curricular components in order to provide an equal level of instruction and restore equal rights and opportunities due to the right to education;
> II –regarding difference, the identification and recognition of the inherent and inseparable otherness of young people and adults in their formative process, of the valorization of the merit of each one and the development of their knowledge and values;
> III – regarding proportionality, appropriate disposition and allocation of curricular components to the specific needs of Youth and Adult Education, with spaces and times in which pedagogical practices assure their students a formative identity common to the other participants in basic schooling.

This way, it is possible to notice that while reinforcing the need for adequacy of teaching in Youth and Adult Education, considering the characteristics of its public, the main objective is to promote social justice to young people and adults that will put them at the same level of rights and opportunities as other students of equivalent levels of schooling.

In the following section, we turn our attention specifically to mathematics education and how it is expected to be in Youth and Adult Education. The expectations towards mathematics learning in this school modality also relate to social justice.

2. Teaching mathematics in youth and adult education

Mathematical knowledge provides support to most of the social relations that occur within society. However, when it comes about teaching mathematics in Youth and Adult Education, it is important to realize that, given the previously mentioned characteristics of students of this teaching modality, young and adult students do not go to school "only looking for the acquisition of instruments for immediate use in daily life, because, in fact, part of these more frequently used in everyday notions and skills, they already dominate reasonably" (Fonseca, 2007, p. 51, free translation).

Thus, more than teaching different mathematical concepts, as if young and adult students knew nothing about them, the formal teaching of mathematics should provide the extension of existing knowledge, while laying foundations for assimilation of new mathematical and formalized knowledge because, in fact, "even if they are illiterate, individuals have built methods to relate to the social demands of literacy, which have been learned throughout several occasions of the cultural lives in which they operate" (Fonseca & Simões, 2014, p. 519). In other words, the teaching of mathematics in this teaching modality should aim to promote situations "of systematization, re-elaboration and/or extension of some concepts" (Fonseca, 2007, p. 51, free translation), as well as cognitive development, social mobility and citizenship.

While extracurricular experiences of young people and adults influence their contact with mathematics, the relationship of these students regarding formal schooling tends to be utilitarian, that is, mathematics is seen as a useful tool to everyday situations. However, this relation goes beyond the utilitarian dimension, as these students also demand an explicitness of the utility of this knowledge (Fonseca, 2007).

This increases the importance of valuing and managing previous knowledge and experiences of these students, which can serve as a support for production of meaning of the mathematical knowledge communicated in classroom. In this sense, Januário, Freitas & Lima (2014, p. 538, free translation), point out that the students of Youth and Adult Education:

> Usually differentiate themselves from other groups of students as presenting a more marked commitment to their learning and, therefore, almost always present greater needs to know the reasons why they should learn this or that content.

It is worth pointing out, also, that there is a great difference between everyday mathematical practices and school practices, since these practices approach different ways of building mathematical knowledge. Consequently, especially in Youth and Adult Education, given the wealth of previous knowledge possessed by its public of students, the teaching and learning situations of mathematics will constitute "a set of tensions between the argumentative line of everyday practices […] and a set of criteria structured in a body of knowledge organized under the aegis of deductive logic" (Fonseca, 2007, p. 29, free translation). In this situation, therefore, occurs "the contact and the conflict of formal and informal cultures of mathematics" (Januário, 2012, p. 72, free translation), which may result in new ways of constructing mathematical knowledge. In this sense, it is essential to recognize and to value previous knowledge, in order to allow the students to use their own solutions to develop new (and formalized) mathematical knowledge.

This kind of approach to mathematics teaching has the potential to satisfy the demand regarding the adequacy of Youth and Adult Education to the characteristics and needs of its students. Thus, it is necessary to provide contact and systematization of different concepts, amplifying and formalizing mathematical knowledge – such as the one related to combinatorics and to probability.

3. Combinatorial and probabilistic reasoning

Among the mathematical knowledge to be acquired and/or increased at school by young and adult students, we focus, in the present paper, on the knowledge referred to combinatorics and to probability. These two areas of mathematics deal with the study of non-deterministic situations, that is, problems in which one deals with the examination, understanding and/or counting of different possibilities.

Combinatorics is the area of mathematics that studies discrete sets and the different configurations that can be obtained from the realization of certain transformations referring to the composition of the elements of these sets. Morgado, Pitombeira de Carvalho, Pinto de Carvalho & Fernandez (1991) also affirm that one of the most commonly problems treated by combinatorics is the enumeration or classification of the subsets of a finite set, satisfying certain pre-established conditions.

Probability is "the branch of mathematics that creates, develops and, in general, researches models that can be used to study random experiments or phenomena" (Morgado *et al.*, 1991, p. 119, free translation). The knowledge of such models constitutes a mathematical tool to measure uncertainty and to understand situations that are part of the scientific, professional and social fields nowadays.

Thus, combinatorial and probabilistic reasoning are ways of thinking that allow us to relate sets of elements, to think about proportions and to understand random events of different natures. Such reasoning is thinking apparatus for logical-mathematical reasoning and is essential for a broad understanding of the world, including social and political spheres. In consequence, the study of combinatorics and probability in Youth and Adult Education is an important means of social justice for young people and adult students, since it can play the role of inserting them into social discussions (and enabling them to make decisions) of which they usually are on the sidelines. Therefore, it is important to emphasize the importance of the development of such reasoning in Youth and Adult Education, so students of this teaching modality can be able to understand non-deterministic situations related to the survey of existing possibilities and to solve problems present in everyday life, and also in the school environment, that are of a combinatorial and/or a probabilistic nature.

It is important to note that, in the light of different authors (Piaget & Inhelder, 1951 *apud* Navarro-Pelayo; Batanero & Godino, 1996; Santos, 2015), combinatorial reasoning and probabilistic reasoning are related, given that combinatorial reasoning allows the understanding of random experiments (from the most elementary to the most elaborated ones) which is necessary to the construction of knowledge concerning probability. In this sense, Batanero, Godino & Navarro-Pelayo (1996), emphasize that "combinatorics is not only a support in the calculation of probabilities, but there is a close interrelation between the idea of a compound experiment based on a discrete sample space and combinatorial operations" (p. 23, free translation). On the other hand, probabilistic concepts, such as sample space, for example, constitute important tools for solving combinatorial problems.

It is also possible to observe approximations between such areas of mathematics by looking at the symbolic representations used while solving both combinatorial and probabilistic problems. For example, the use of the tree diagram to point out the possibilities in combinatorial problems, or to support the construction and analysis of sample spaces when it comes about probabilistic problems.

From this, we defend a teaching that also promotes the articulation and communication of ideas between these areas of mathematics, given their similarities (related to the survey of possibilities) and differences for the broad development of both forms of reasoning.

In addition, we emphasize that authors such as Fischbein (1975), Borba (2016) and Campos & Carvalho (2016) argue that the development of such reasoning does not occur in a short period of time, from one-off teaching of concepts and formulas to solve problems regarding combinatorics and probability. As a result, these authors recommend the encouragement of the development of both reasoning from the earliest years of schooling. In this sense, combinatorial and probabilistic concepts, as well as different problem solving strategies, symbolic representations and the relations between these two important areas of mathematics should be focus of mathematics teaching throughout Elementary, Middle and High School.

Despite these recommendations, the teaching of combinatorics and probability has been prioritized only in the last stage of basic schooling for many years, tending to be taught only in High School and being characterized by the excessive use of formulas. Such attitude, therefore, does not support the development of combinatorial and probabilistic reasoning because it makes it difficult to explore the characteristics of the problems and relations between them. This situation is even worse when it concerns Youth and Adult Education, given that in this teaching modality combinatorics and probability seem to receive less attention in curricula, which leads the students of this teaching modality to present even greater difficulties when solving combinatorial and probabilistic problems – revealing gaps regarding these forms of reasoning.

4. Curricula concerning combinatorics and probability

In this section we discuss the guidelines present in official curricular documents for Youth and Adult Education regarding combinatorics and probability (Lima & Borba, 2017).

The Curricular Proposal for Youth and Adult Education (Brasil, 2001) referring to Elementary School emphasizes that concerning teaching Mathematics in Youth and Adult Education:

> mathematical activity must be oriented towards integrating, in a balanced way, its formative role (regarding the development of fundamental intellectual capacities for the structuring of thinking and logical reasoning) and its functional role (regarding the applications in practical life and the resolution of problems of diverse fields of activity) (p. 99-100, free translation).

In this document, combinatorics and probability do not have much prominence. Combinatorics is presented as a consequence of one of the meanings attributed to multiplication (only the teaching of *Cartesian Product* situations is defended – the specific type of combinatorial problem that can be solved by the use of a direct multiplication of the numbers mentioned in word problems); and probability appears as a tool to the treatment of statistical data, aiming the realization of predictions from identification of events' characteristics. One will not find more specific orientations concerning activities with concepts of these areas of mathematics in this first stage of schooling.

On the other hand, the Curricular Proposal for Youth and Adult Education intended for Middle School (Brasil, 2002) emphasizes that the teaching of mathematics, in this stage, "should aim the development of concepts and procedures related to numerical, geometric, algebraic, metric competence, reasoning involving proportionality, as well as combinatorial, statistical and probabilistic reasoning" (p. 20). Thus, it is given greater visibility to combinatorics and probability in this stage of schooling. This document also argues that different concepts and procedures should be introduced, in order to support the development of different reasoning, including the combinatorial and probabilistic ones.

Finally, the National Curricular Parameters for High School – PCN + (Brasil, 2002)[1] point out that in this stage of completion of basic education, mathematics should

[1] As there is no specific curricular document directed to this stage of schooling in Youth and Adult Education, the document for High School in regular education is adopted as the official orientation.

provide the development of skills and abilities that lead students "to understand and interpret situations, to make appropriate use of specific languages, to argue, to analyze and evaluate, to draw their own conclusions, to make decisions, to generalize and to many other actions necessary for their formation" (Brasil, 2002, p. 111, free translation).

The teaching of mathematics, in this broad sense of formation, is pointed out as having, among its objects of study, "finite sets of data, [...] which give rise to very distinct procedures from those of other subjects, regarding how quantifications are made from the use of combinatorial counting processes, frequencies and statistical measures and, also, probabilities" (Brasil, 2002, p. 126, free translation).

This document conveys attention to the importance of the development of combinatorial reasoning, stating that it is a tool that allows "to decide on the most adequate way to organize numbers or information in order to be able to count possible cases" (Brasil, 2002, free translation), which also acts as a support for the calculation of probabilities and, consequently, for the development of probabilistic reasoning. It is also emphasized that it is important not to use excessive formulas in the teaching of combinatorial concepts:

> formulas must be a consequence of combinatorial reasoning developed from the solution of diverse problems and should have the function of simplifying calculations when the amount of data is too large. These contents should have greater space and commitment of work in High School, keeping close the perspective of applied problem solving to avoid excessive and sterile theorization (Brasil, 2002, p. 127).

These documents (Brasil, 2001; 2002a; 2002b) defend the teaching of different combinatorial and probabilistic concepts throughout the period of basic schooling, but give more evidence to them when it comes to High School. These documents also describe the importance of exploring different problem solving strategies and the use of varied symbolic representations.

Due to previous discussion, the curricular guidelines for Youth and Adult Education supports, to some extent, the teaching of combinatorics and probability for the development of both reasoning in favor of formal and social formation of students of this teaching modality. However, such kind of instruction seems to gain less evidence and have less effect concerning Youth and Adult Education.

Finally, we emphasize that the articulated work between these areas of mathematics – given their similarities and specificities – can contribute to the development of both combinatorial and probabilistic reasoning, as pointed out by Lima (2018), discussed in the following section.

5. Research on combinatorial and probabilistic reasoning of young people and adults

Studies carried out with students of Youth and Adult Education have brought up very unsatisfactory performance regarding combinatorial and probabilistic reasoning (Lima, 2010; Lima, 2018), while authors argue that the teaching of combinatorics and probability can and should be introduced since the first years of schooling, taking into account the development of students' reasoning throughout basic education (Fischbein, 1975; Borba, 2016; Campos & Carvalho, 2016).

Rita Lima (2010) performed her master's study with 150 students of Youth and Adult Education of different stages of schooling (equivalent to all basic education). In this study she investigated the understanding of multiplicative problems, focusing on the different types of combinatorial problems according to Borba (2010): *Cartesian Product*, *combination*, *arrangement* and *permutation*[2].

This author observed that students of Youth and Adult Education presented great difficulties in solving the combinatorial problems proposed. Among these problems, the best performance was observed regarding to *Cartesian Product* situations, while the lower performance was related to *combination* problems. Similar results were also found in other studies carried out with students of Youth and Adult Education and also of regular schooling (Pessoa, 2009; Borba; Rocha & Azevedo, 2015; Lima, 2018).

In this study, Lima (2010) also emphasizes the importance of schooling and specific formal education for the improvement of students' performance: "we realized that the teaching of combinatorics is a fundamental element in the development of combinatorial reasoning, as with the passing of the school years, performance improves regarding this reasoning" (p. 127, free translation).

Similarly, Ewellen Lima (2018) observed very low performance in the resolution of combinatorial and probabilistic problems from students of Youth and Adult Education in stages of schooling equivalent to Elementary, Middle and High School. In this study the author investigated the performance of 24 Youth and Adult Education students while solving combinatorial problems regarding *Cartesian product*, *combination*, *arrangement* and *permutation* (Borba, 2010) and probabilistic problems approaching the four cognitive demands pointed out by Bryant & Nunes (2012): 1) *understanding randomness*; 2) *working out the sample space*; 3) *comparing and quantifying probabilities* and 4) *understanding correlation*[3].

Regarding this study, Lima (2018) pointed out that the performance of the students of Youth and Adult Education was directly influenced by their level of schooling. However, this effect seems to have been related to schooling, in general, not being, necessarily, a consequence of specific teaching of combinatorics and probability.

Concerning combinatorial problems, problems of *Cartesian product* were the ones in which the participants obtained better performance, while in *combination* problems the lowest performance was observed. While solving probabilistic problems, performance in the problems of *sample space* was closely related to the corresponding types of

[2] These combinatorial problems differ from each other depending on the choices to be made and the ordering to be considered during their resolution (BORBA, 2010). In *Cartesian product* problems one must choose one element from each of the two or more sets involved in the problem and the orders of these elements do not constitute distinct possibilities. In the other combinatorial problems (*combination*, *arrangement* and *permutation*) there is a unique set from which elements must be chosen. In *combination* and *arrangement* problems, the choice consists in some of the elements of the set. In the *combination* problems the orders do not constitute new possibilities, but they do in the *arrangement* problems. Finally, in *permutation* problems, all the elements of the set must be used for the construction of possibilities and the ordering of these elements will constitute the different possibilities.

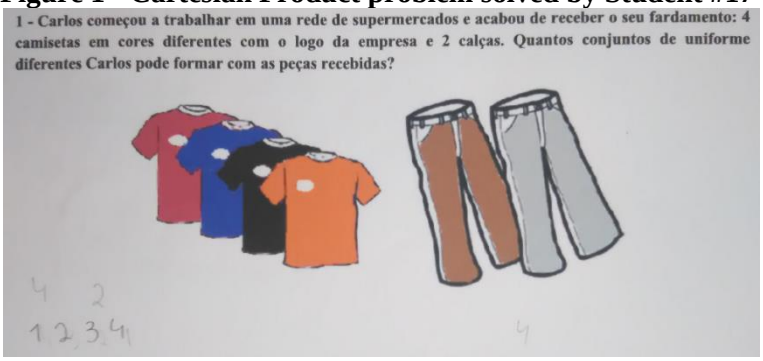
[3] The first cognitive demand pointed out by Bryant & Nunes (2012) refers to the understanding of the non-deterministic character of random situations; the second demand refers to the survey and understanding of the set composed of all the possible events related to a given situation; the third cognitive demand is related to the quantification and comparison of probabilities, as well as the different representations of such quantification; and the fourth cognitive demand pointed out by these authors depends on the understanding of the three previous demands, since it refers to the understanding of the existence of relations between different events.

combinatorial problems it was associated with. Of the other types of probabilistic problems, the *correlation* ones obtained better performance, while most difficulties were presented in the problems of *comparison of different probabilities*, since the proportional character of these problems was often not taken into account by the participants.

The main difficulties faced by the participants in solving the combinatorial problems were based on the understanding of the characteristics of the problems presented (regarding the choices to be made and the role of ordering) and on the exhaustion of possibilities. In general, while solving the probabilistic problems, the participants had difficulties in justifying their answers, which revealed a superficial understanding of probability.

The main types of mistakes observed in this study are illustrated in Figures 1, 2 and 3.

Figure 1 - Cartesian Product problem solved by Student #17



Source: Lima (2018)

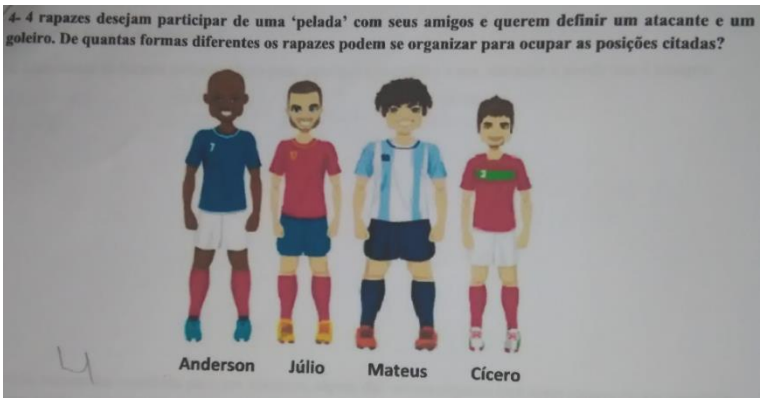
Even though the *Cartesian Product* problem was the one in which best performance was observed (LIMA, 2018), some students had difficulties solving it. Difficulties were observed even among students attending the last year of High School in the Youth and Adult Education, as is the case of Student #17 (Figure #1).

The Cartesian Product problem posed was: "Carlos just started working in a supermarket and received his uniforms: four shirts in different colors with the company logo and two trousers. How many different complete uniforms can Carlos form with the clothing parts he received?"

While solving this problem, Student #17 indicated the options orally, having used each shirt only once while choosing the trousers to complement the possible uniforms, according to his personal taste. In this manner, the student named four possibilities – half of the total number of possibilities. The student, as other students, demonstrated difficulties regarding the choice of elements of the sets, not realizing that each trouser could be combined with both shirts and did not find the right solution for the problem.

In Figure #2 we present the *arrangement* problem, in which was asked: "Four boys want to play football with their friends and need to choose who will be the goalkeeper and who will be a striker. In how many different ways can the boys be chosen to occupy these positions?"

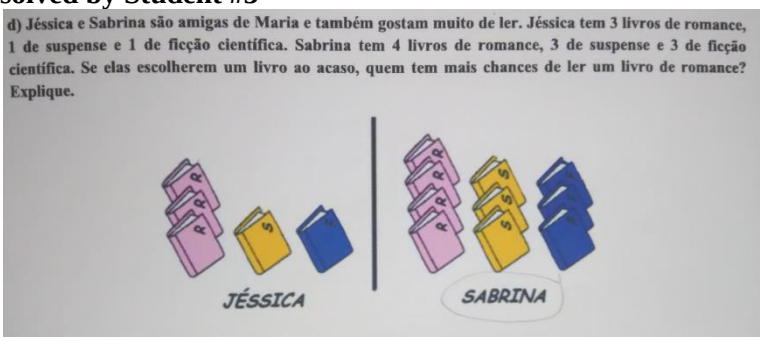
Figure 2 - Arrangement problem solved by Student #13



Source: Lima (2018)

As mentioned previously, participants in the study of Lima (2018) also presented difficulties regarding ordering when solving combinatorial problems. Student #13, a 9th grade student, showed this type of difficulty solving the *arrangement* problem (Figure #2). When indicating possibilities orally, the student considered the following pairs: Anderson and Júlio; Mateus and Cícero; Cícero and Anderson; Anderson and Mateus. Thus, the student indicated four distinct pairs, but did not make clear the positions that would be occupied by each boy, and, consequently, did not consider the possibility of changing the positions of boys in the same pair – a change that would form a different possibility. In this way, Student #13 wasn't able to find all the possibilities regarding this problem – a difficulty also demonstrated by other students.

Figure 3 - Comparison of probabilities problem regarding the permutation problem solved by Student #3



Source: Lima (2018)

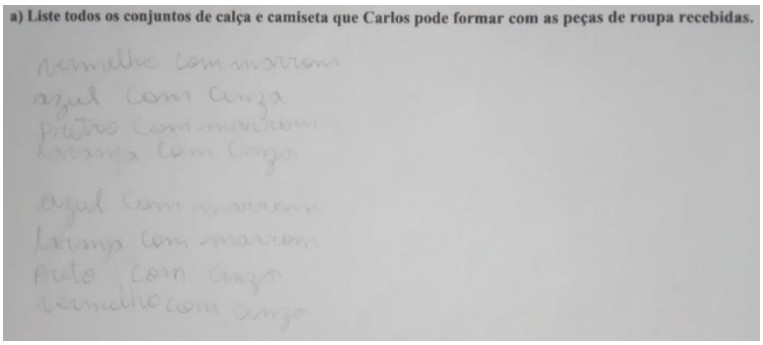
Concerning probability, as mentioned previously, the lowest performance referred to the *comparison of probabilities* problems. One problem posed (Figure #3) was: "Jéssica and Sabrina are Maria's friends and are also into reading. Jessica has three novel books, one thriller book and one science fiction book. Sabrina has four novel books, three thriller books and three science fiction books. If they choose, each one, a random book, who is more likely to read a novel book? Explain your answer".

Student #3, a 5th grade student, stated that Sabrina would have a better chance of choosing a novel book, justifying his response by saying: "Sabrina has more novel books than Jessica. She has one more book". Therefore, this, and other students, based their judgments only on the absolute number of novel books of each woman, not taking into account proportionality of the sample spaces, in order to compare the probabilities in question.

Moreover, this study points out that the articulation between these types of problems – and consequently, between combinatorial and probabilistic reasoning – is a promising teaching strategy that can promote students' reflection about the properties of the different types of problems and the relations between them. This proposed articulation can, thus, promote the development of both forms of reasoning.

In Figure 4 is presented a revisit of the *Cartesian product* problem, from the construction of sample space (one of the articulations proposed in the study of Lima, 2018). The problem requested the list of all the possibilities of combining four shirts with two trousers.

Figure 4 - Sample space problem regarding the Cartesian Product problem solved by Student #17



Source: Lima (2018)

In revisiting the previously solved *Cartesian Product* problem (Figure #1), Student #17 was able to indicate all the possibilities required. The student did this by solving the correspondent problem of *sample space*, which asked: "List all sets of trousers and shirts that Carlos can form from the clothing parts he received". By using written listing and reassessing the possibilities considered earlier, the student was able to correctly apply choosing and ordering in the problem and find all the possibilities.

The different types of articulation between combinatorial and probabilistic problems proposed in the study of Lima (2018), showed a potential contribution to both forms of reasoning of young people and adults. The problems posed also took into account appropriate contexts for young and adult students. The contexts were familiar and applicable to real life situations.

Thereby, the results of these studies point out the need of specific formal teaching of combinatorial and probabilistic concepts from the beginning to the end of basic schooling, also in Youth and Adult Education. This teaching should aim the contact with various combinatorial and probabilistic problems (including the articulation between them), so the students of this teaching modality can understand the characteristics and properties of such problems and develop their combinatorial and probabilistic reasoning. Consequently, the development of such reasoning is relevant for logical-mathematical reasoning itself and for the inclusion of these students in the discussion of a variety of social activities.

6. Final considerations

In the paper here presented we discuss the role that Youth and Adult Education plays regarding social justice. This teaching modality aims to provide basic education to those who did not have formation at the proper age. In this sense, schooling in this teaching modality must provide not only new knowledge but, also opportunities for young and adults students – as the same prospects that regular schooling students have.

Focusing on the teaching of mathematics, we highlight the importance of this area of knowledge for life in society, given that mathematical knowledge is present (and essential) in various spheres of life, both social and professional. In consequence, the enlargement of such knowledge (having, as a bottom line, the previous knowledge that young people and adults already have) is important to increase the inclusion of students of the Youth and Adult Education into society.

In particular, we have focused on two important mathematical reasoning: combinatorial and probabilistic. Such reasoning refers to the understanding of possibilities and nondeterministic situations. The knowledge about combinatorics and probability enables people to solve problems present in both classroom and in everyday situations. Although it is defended that the development of such reasoning must takes place in formal education throughout the distinct modalities and years of schooling, recent research has evidenced great difficulties of students of Youth and Adult Education concerning the resolution of combinatorial and probabilistic problems.

Given the importance of both reasoning to the performance inside and out of school and for the inclusion in activities and discussions in social life, the lack regarding such reasoning that is shown by Youth and Adult Education students is something that deserves attention. In seeking to awaken discussion and encourage actions that look for the development of combinatorial and probabilistic reasoning of Youth and Adult Education Brazilian students, we emphasize how these forms of reasoning may be aimed at in schools, as means for social justice, that is, opportunities for all.

7. References

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