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Counting with Alice

Contando com Alice

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Abstract

There are many mathematical references in Lewis Carroll's two tales for children: *Alice's Adventures in Wonderland* (1865) and *Through the Looking-Glass* (1872). Many critics suggested that Carroll inserted hidden meanings in those passages. We rather consider them as part of the story's setting and narrative. Yet, those passages may be interpreted and used as convenient to illustrate mathematical ideas. In this paper, we consider two passages from the *Alice* tales that relate to arithmetic, and we discuss them in relation to issues of personal identity, mathematical certainty, the role of notations and the processes of composition and decomposition in mental calculation. Hence, we show how literary texts can be used to convey ideas related to mathematics, mathematical culture and mathematical education. We conclude on the importance of mathematical writings as literary texts.

Keywords: Lewis Carroll. Alice. Identity. Arithmetic. Notation.

Resumo

Existem muitas referências matemáticas nos dois contos de Lewis Carroll para crianças: *As Aventuras de Alice no País das Maravilhas* (1865) e *Através do Espelho* (1872). Muitos estudiosos sugeriram que Carroll inseriu significados ocultos nessas passagens. Nós preferimos considerá-los como parte do cenário e da narrativa da história. No entanto, essas passagens podem ser convenientemente interpretadas e usadas para ilustrar idéias matemáticas. Neste artigo, consideramos duas passagens dos contos de *Alice* relacionadas à aritmética e as discutimos em relação a questões de identidade pessoal, certeza matemática, o papel das notações e os processos de composição e decomposição no cálculo mental. Assim, mostramos como os textos literários podem ser usados para transmitir ideias relacionadas à matemática, cultura matemática e educação matemática. Concluímos sobre a importância dos escritos matemáticos como textos literários.

Palavras-chaves: Lewis Carroll. Alice. Identidade. Aritmética. Notação.

1. Introduction

Lewis Carroll (1832-1898), the well-known children's author was a mathematician. His real name was Charles L. Dodgson and he worked as mathematical lecturer most of his adult life at the University of Oxford. He published extensively on various mathematical subjects such as algebra, geometry, logic, voting theory and recreational mathematics (Wilson & Moktefi 2019). Yet, Carroll is celebrated today for his literary writings, notably his two tales for children: *Alice's Adventures in Wonderland* (1865) and *Through the Looking-Glass* (1872) (hereafter: the *Alice* tales).

It is naturally not surprising that Carroll's literary writings contain a multitude of references to mathematical subjects since writers commonly get inspiration from their actual lives and experiences (Heath 1974; Gardner 2001). Yet, it does not make Carroll's fictions mathematical treatises. We would not say, for instance, that Antoine de Saint-Exupéry's celebrated tale *The Little Prince* (1943) is a manual of aviation on the ground that its author, an experienced aviator, included an aviator and his plane in the tale. Saint-Exupéry's aviation and Carroll's mathematics are not the objects of their tales. This does not undermine their role in the setting of the stories and their delight. Peter Alexander argued that "if Lewis Carroll had not been a logician as well as an artist the "Alice" books would have been much less convincing and aesthetically satisfying than they are" (Alexander 1944, p. 551).

However, many mathematical commentators have made higher claims and argued that Carroll was parodying the mathematics of his time. Consequently, a careful reader who collects the clues disseminated in the *Alice* tales should not fail to discover insights on Victorian mathematics or even anticipations of more recent mathematical ideas. Such interpretations have been recently discussed by Francine Abeles who, justifiably, asks for more caution in reading those mathematical passages. In particular, she offers a set of "historiographic and mathematical criteria" before attributing a given idea to Carroll, including an inquiry as to whether Carroll's familiarity with the idea is attested in his 'serious' mathematical works (Abeles 2017).

The enthusiasm of commentators to 'decode' the *Alice* tales is not proper to mathematicians. Several interpretations, including many eccentric ones, have been offered to make sense of Carroll's masterpieces. Derek Hudson appropriately wrote that "Carroll has been the victim of misplaced ingenuity from critics who have taken not only themselves but the *Alice* books far too seriously" (Hudson 1958, p. 31). Mathematical critics are no exception. There is no doubt that Carroll inserted many mathematical references in his literary works. Yet, commentators have often exaggerated the import of those allusions. Part of their motivation was to ensure the continuity between the mathematician and the writer:

> So it would seem with these mathematical allusions in his writing that Lewis Carroll was less a split personality of Charles Dodgson than one might first suppose. Through Lewis Carroll, the mathematician Charles Dodgson comes through while the humourist and entertainer is able to run rampant without the inhibition of embarrassing his position. (Willerding 1960, p. 218)

In this paper, we take a more humble position. There is no need to fill a gap between the mathematician and the writer because there is no such gap. The illusion that such a gap exists springs from popular misconceptions as to the incompatibility of two cultures: the literary and the mathematical. Carroll suffered from, and at the same time, fed such misconceptions (Moktefi 2019). We believe that there is no need to search for hidden mathematical meanings in the *Alice* tales to enhance the mathematical standing of Carroll. We rather encourage appealing to *Alice*'s mathematics, whether sound or unsound, as suitable to illustrate mathematical ideas as we offer to proceed in the two passages bellow.

2. Four times five

The first passage takes place when Alice is lost in Wonderland and goes through several transformations that make her question who she is (or who she became). It goes as follows:

> "Dear, dear! How queer everything is to-day! And yesterday things went on just as usual. I wonder if I've been changed in the night? Let me think: *was* I the same when I got up this morning? I almost think I can remember feeling a little different. But if

> I'm not the same, the next question is 'Who in the world am I?' Ah, *that's* the great puzzle!" And she began thinking over all the children she knew that were of the same age as herself, to see if she could have been changed for any of them.
>
> "I'm sure I'm not Ada," she said, "for her hair goes in such long ringlets, and mine doesn't go in ringlets at all; and I'm sure I can't be Mabel, for I know all sorts of things, and she, oh, she knows such a very little! Besides, *she's* she, and *I'm* I, and — oh dear, how puzzling it all is! I'll try if I know all the things I used to know. Let me see: four times five is twelve, and four times six is thirteen, and four times seven is — oh dear! I shall never get to twenty at that rate! However, the Multiplication-Table doesn't signify: let's try Geography. London is the capital of Paris, and Paris is the capital of Rome, and Rome — no, *that's* all wrong, I'm certain! I must have been changed for Mabel! (Gardner 2001, pp. 22-23)

Several commentators addressed this passage. In particular, Alexander Taylor offered an interpretation of it that would make sense of the multiplication Alice undertook (Taylor 1952, p. 47). Taylor argues that 4 times 5 is 12, as Alice reported, if one considers number base 18. Indeed:

$$4 \text{ x } 5 = 20 = 1 \text{ x } 18 + 2 = 12 \text{ [in base 18]}$$

If we increase the base by 3 at each step, Alice's calculations prove true:

$$4 \text{ x } 6 = 24 = 1 \text{ x } 21 + 3 = 13 \text{ [in base 21]}$$

$$4 \text{ x } 7 = 28 = 1 \text{ x } 24 + 4 = 14 \text{ [in base 24]}$$

$$\ldots$$

$$4 \text{ x } 12 = 48 = 1 \text{ x } 39 + 9 = 19 \text{ [in base 39]}$$

Interestingly, the next step in the process is not 20. Indeed:

$$4 \text{ x } 13 = 52 = 1 \text{ x } 42 + 10 = 1(10) \text{ [in base 42]}$$

It is this 'mathematical fact' that, Taylor says, Dodgson wanted to introduce in this passage from Alice. It is unclear why this fact would be of interest and whether Carroll really intended it. Taylor concedes the absence of evidence regarding Carroll's intent, but contends that "it can hardly be a coincidence; nor could he invent such a problem in a kind of day-dream, without knowing what he was doing" (Taylor 1952, p. 47). Of course, one may challenge the claim that there is a problem to be recognized. That the calculations found in that passage admit of a 'mathematical explanation' does not make them a mathematical problem in the first place. It suffices to think of the wonderful creatures one often detects on the walls of a cave or in a cloudy sky. Such visions are created in the mind of the viewer rather than being the work of a mysterious artist. As it happens, other visions are possible. Martin Gardner argues that Alice does not reach 20 because multiplication tables traditionally stop at 12. Hence, if we continue Alice's calculations, the last line she would have memorised would be 4 times 12 which make 19. Thus, Alice does not reach 20 (Gardner 2001, p. 23).

Gardner's explanation has the advantage of simplicity. Also, it does not require Alice's calculations to be sound. Yet, one may still wonder why one tries to make sense of what Taylor himself named a source of 'nonsense' (Taylor 1952, p. 46). We should keep in mind that Alice is expressing her confusion, in that passage, after the transformations she went through (Figure 1). She does not know who she is anymore. To figure out who she is, she appeals to her memory and finds out that she does not know anymore the facts that she used to know. If her calculations were right, the confusion would have vanished and the narrative lost. It is, hence, appropriate that her mathematical, and for the matter also geographical, lessons got tangled. Incidentally, neither Taylor nor Gardner has offered an explanation to the geographical fact that followed in

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the passage above. What circumstance would make London the capital of Paris, and Paris the capital of Rome?

Figure 1. "Who in the world am I?"

Source: Wikimedia Commons

It is difficult to say if Carroll ever had anything specific in mind while writing the above passage. But that should not preclude us from appreciating the mathematical nonsense it holds. And certainly, that should not prevent us from quoting it as appropriate when it serves our purposes, whether mathematical or not. As indicated above, the passage nicely illustrates the philosophical problem of personal identity, both synchronic and diachronic (Olson 2019). Indeed, on the one hand, we see Alice trying to figure out if she differs from Mabel at once; and on the other hand, Alice wonders if she is the same person she used to be. Interestingly, in both queries, Alice appeals to her knowledge and memory to ensure her discontinuity with Mabel and her continuity with the person she used to be. John Lock's popular theory of personal identity precisely emphasises the need for this psychological continuity: "You only know that you are the same person as yesterday because your memories today are much the same as those of yesterday" (Teichman & Evans 1999, p. 30). An obvious weakness of this view is that memory can fail us. We might remember what actually never happened. Also, we might forget what actually did happen.

The situation gets more complicated when we consider memories of a multitude of events or facts that change over time and are not continuous themselves. It is very fitting that Alice tested her memory by considering elementary facts she learned at school and that were unlikely to change overnight, unlike herself. Liberal education that was promoted in Carroll's time precisely favoured such 'permanent' subjects over 'progressive' ones, a distinction that was made by philosopher William Whewell (1845). The former subjects (which included basic arithmetic and Euclidean geometry) were to be mastered first by students before proceeding to the latter subjects (which included natural sciences). This division faced strong opposition all along the Victorian period with the calls for more technical teaching and the introduction of Science courses. Carroll was rather a champion of liberal education. In his writings, he praised the certainty and permanence of mathematical facts, in comparison with the continuous changes that occur in the natural sciences:

> It may well be doubted whether, in all the range of Science, there is any field so fascinating to the explorer – so rich in hidden treasures – so fruitful in delightful surprises – as that of Pure Mathematics. The charm lies chiefly, I think, in the absolute *certainty* of its results: for that is what, beyond almost all mental treasures, the human intellect craves for […] Most other Sciences are in a state of constant flux – the precious truths of one generation being smiled at as paradoxes by the second generation, and contemptuously swept away as childish nonsense by the third. If you would see a specimen of the rapidity of this process of decomposition, take Biology for a sample: quote, to any distinguished Biologist you happen to meet, some book published thirty years ago, and observe his pitying smile!
>
> But neither thirty years, nor thirty centuries, affect the clearness, or the charm, of Geometrical truths. (Dodgson 1890, pp. xv-xvi.)

Alice's transformations in Wonderland contrast with this permanence of mathematics. Her confusion contrasts with the clearness of mathematical truths. When she tests her mathematical knowledge to determine her identity and gets the calculations wrong, we just see what mathematics would have been if mathematical truths were to change, like Alice, overnight.

3. I lost count

The second passage takes place when Alice is asked by the White and Red Queens different questions to test her arithmetical skills (**Figure 2**). We specifically consider additions:

> "Can you do Addition?" the White Queen asked. "What's one and one and one and one and one and one and one and one and one and one?"
>
> "I don't know," said Alice. "I lost count."
>
> "She ca'n't do Addition," the Red Queen interrupted […]
>
> "Can *you* do sums?" Alice said, turning suddenly on the White Queen, for she didn't like being found fault with so much.
>
> The Queen gasped and shut her eyes: "I can do Addition," she said, "if you give me time — but I ca'n't do Subtraction under *any* circumstances!" (p. 265-266)

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Figure 2. "Can you do Addition?"

Source: Wikimedia Commons

Here, Alice is asked to sum:

1+1+1+1+1+1+1+1+1+1

This might seem easy at first, but Alice failed to count how many '1's she had to sum. This difficulty illustrates the role of notations in mathematics and the place of composition and decomposition in mental calculations. To understand this claim, let us first look at Roman numbers: the first numbers I, II, III are easy to grasp in a glance because they are highly iconic. For instance, the 3 marks in 'III' represent the number 3 precisely because they *are* 3. As such, the sign 'III' has the property it represents. This visual immediacy is lost when we appeal to symbolic notations such as '3' to convey the idea it represents. This difference is even more evident when we make use of diagrams. Think of Euler circles to represent sets. If we wish to represent the fact that a set S is included in a set P, it suffices to draw a circle S inside a circle P to convey the idea of inclusion. This visual aid is lost when we express this relation symbolically: $S \subset P$. The symbol '$\subset$' merely represents the relation of inclusion, while the circles *have* that relation (Moktefi 2015a). The situation gets complicated and the visual aid is lost when the number of circles increases, for instance when we handle complicated problems involving a high number of sets (Moktefi & Edwards 2011).

Similarly, the sign 'IIIIII' is less satisfactory to represent the idea of '6' because it does not immediately convey its idea and the risk of error increases. In that case, it is suitable to conventionally introduce a notation that would abbreviate the sign. For instance, in roman numbers, we put 'V' to stand for 'IIIII', and hence we simply write 'VI' instead of 'IIIIII'. The notation 'VI' is simpler and more immediate to grasp, on the condition that one learns the above convention. The same remarks can be made of position notations which we are using in modern mathematics: the number 56 for instead is merely an abbreviation of: 5 x 10 + 6. To read the sign '56' correctly, one needs to know the position notation convention and to decompose appropriately the number 56.

Composition and decomposition are known to be essential in elementary arithmetic (Leinhardt, Putnam, & Hattrup 1992). For instance if one wishes to mentally sum: 56 + 23, one may easily proceed as follows:

$$56 + 23 = (50+6) + (20+3) = (50+20) + (6+3) = 70 + 9 = 79$$

In this process, we decompose first the given numbers in accordance with the position notation, and then we recompose their components to obtain the appropriate result.

In the above passage, Alice faced a 'decomposed' sum and was asked to compose it:

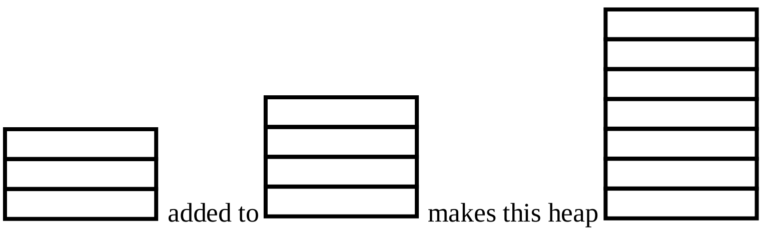
$$1+1+1+1+1+1+1+1+1+1$$

This calculation requires time as the White Queen observed. But Alice could have also benefited from the use of a pen and pencil, to prevent her from losing count. Also, the introduction of intermediary signs, as we did for roman numbers, could have helped Alice in her calculations. First instance, by replacing each pair '1+1' by '2'. She then obtains the sum:

$$2+2+2+2+2$$

This is easier to compose. She can now add those numbers directly or introduce additional signs. This process is known to teachers who have to count an endless number of students' exam copies. It is often very convenient to group every 10 copies together into a single pile, and then simply count the number of piles.

If Alice's failure illustrates the benefit of composition, one should not underestimate the importance of decomposition for mathematical understanding. For instance, a child might learn by rote *that* '3+4 = 7', yet not understand *why* 3+4=7. The decomposition of numbers to units, as seen when children learn to sum with counters, explains the essence of the addition process. Interestingly, this very situation occurs in H. G. Well's novel *Joan and Peter* (1918), where the teacher, Miss Mills, is delivering an arithmetic lesson:

> Then Miss Mills taught Peter to add and subtract and multiply and divide. She had once heard some lectures upon teaching arithmetic by graphic methods that had pleased her very much. They had seemed so clear. The lecturer had suggested that for a time easy sums might be shown in the concrete as well as in the figures. You would first of all draw your operation or express it by wood blocks, and then you would present it in figures. You would draw an addition of 3 to 4, thus:
>
> 
>
>
> And then when your pupil had counted it and verified it you would write it down:
>
> 3 + 4 = 7.
>
> (Wells 1918, p. 120)

We see how decomposition dissects the calculation to make the result intelligible to the pupil. This insight has its own cost since it requires lengthy developments that are unsuitable for a more experienced pupil. For the latter, it is more suitable to learn some simple rules that would rather shorten the process. For instance, pupils are taught to simply add '0' at the right of a number when it is multiplied by 10. Such rules permit economy of time and effort in calculations.

The interplay between composition and decomposition was known to Carroll. It is evidently found later in his logic diagrams, which differ from the Euler circles mentioned earlier. Carroll rather proceeds as follows: suppose we have the premises of an argument involving a number n of terms. We first decompose the information contained in those premises into 2^n compartments corresponding to all the combinations of those n terms. Then, we recompose that information to exhibit the relation that connects the terms that we wish to save in the conclusion (Abeles 2007; Moktefi 2013). Not only a pupil who uses Carroll's diagrams manages to calculate the conclusion of an argument, but additionally, she should understand *why* that is the conclusion.

Since Alice's problem resulted from the over-decomposition of the sum that was offered to her and the impossibility to access writing tools to record the sum, this passage also reminds us of the importance of notations and paper tools as scientific instruments that extend our cognition in mathematical practice (Dutilh Novaes 2012; Moktefi 2017). Carroll paid a particular attention to issues of notation in his works and often pleaded for the introduction of better notations (Abeles 2019). He notably promoted a symbolic approach to logic that was novel in his time, despite the opposition of his colleagues (Marion & Moktefi 2014). Carroll justified the appeal to symbolic notation by the ease it offers for logical calculations:

> Think of some complicated algebraical problem, which, if worked out with x, y, z, would require the construction of several intricate simultaneous equations, ending in an affected quadratic. Then imagine the misery of having to solve it in *words* only, and being forbidden the use of *symbols*. This will give you a very fair idea of the difference, in solving a Syllogism or Sorites, between the use of *Symbolic* Logic, and of *Formal* Logic as taught in the ordinary text-books. (Bartley 1986, p. 47)

Carroll's notations have not been adopted by his cotemporaries and followers, yet his philosophy of notation deserves to be revisited.

4. Conclusion

In the above sections, we considered two passages from *Alice* and offered an interpretation to illustrate some subjects that are related to mathematical culture and education. Naturally, other settings may motivate different readings and interpretations. As such literary texts can serve as instruments for mathematical teaching and learning.

In addition, one should not forget that mathematical texts themselves can be seen as literary works, in the sense that they are written to convey ideas to a reader. This literary dress can contribute to the reception of the mathematical ideas. Carroll certainly took the issue very seriously and worked on writing in such a way as to be understood by a wider audience. For instance, in 1885, he published a *Tangled Tale*, a collection of stories that were previously published in a periodical. Each story contains one or more mathematical problem that the readers were asked to solve. Carroll explained his intention as follows:

> The writer's intention was to embody in each knot (like the medicine so dexterously, but ineffectually, concealed in the jam of our childhood) one or more mathematical questions – in Arithmetic, Algebra, or Geometry, as the case might be – for the

> amusement, and possible edification, of the fair readers of that Magazine. (Carroll 1885, non-paginated preface)

Another example of this literary dress of mathematical problems is given by Carroll's contributions to the journal *Mind*. He published there in the early 1890s two papers on hypotheticals, a subject that occupied his mind in those years (Moktefi & Abeles 2016). Both papers were written in the form of a dialogue to exhibit the conflicting opinions among logicians on the subject. For instance, the first paper: "A logical paradox" was mainly a dialogue between two uncles as to the presence of a barber in his shop. Carroll commented that the "paradox, of which the foregoing paper is an ornamental presentment, is, I have reason to believe, a very real difficulty in the Theory of Hypotheticals" (Carroll 1894, p. 438). This ornamental function certainly contributed to the popularity of the paper which was widely discussed by Carroll's contemporary logicians and their immediate followers (Moktefi 2007).

More generally, Carroll's logical writings were written in such a way as to be understood by a wide audience. Carroll believed in the social utility of logic, notably to overcome religious difficulties, and worked on its promotion (Richards 2015; Moktefi 2015b). Although Carroll's logical ideas did not get the attention he hoped for, his style of exposition certainly secured a wider readership, with his examples being quoted in modern textbooks. Logician Hugh MacColl, who was himself a novelist, reviewed Carroll's *Symbolic Logic* in 1896 and commented as follows:

> Before offering any detailed criticism of Lewis Carroll's methods we may state certain favourable points which his book undoubtedly possesses. It is well arranged, its expositions are lucid, it has an excellent stock of examples – many of them worked out, and not a few witty and amusing; and its arguments, even when wrong, are always acute and well worth weighing. (MacColl 1896, p. 520)

Interestingly, when he wrote these lines, MacColl himself abandoned the study of logic for about 13 years and devoted himself to literary activities. Yet, MacColl later confessed that the reading of Carroll's work encouraged him to come back to the study of logic, and it is known that he produced since some of his most important writings (Abeles & Moktefi 2011). This offers a nice illustration of how Carroll's prose helped to disseminate his work and seduce his readers, including among those who did not adopt his ideas.

Carroll's most ambitious attempt to present mathematical ideas in a literary form is certainly found in *Euclid and his Modern Rivals*, first published in 1879, then extended and revised in 1885. The book was a beautiful review of the rival manuals that were offered by modern authors to replace Euclid's *Elements* as the standard textbook for geometrical teaching in schools and colleges (Montoito & Garnica 2015). Carroll wrote the book in the form of a drama in four acts, taking place in Hell. A judge reviews each of Euclid's manuals and eventually resolves that none can compete with Euclid's work. Carroll explains in the preface:

> I have not thought it necessary to maintain throughout the gravity of style which scientific writers usually affect, and which has somehow come to be regarded as an 'inseparable accident' of scientific teaching [...] Nevertheless it will, I trust, be found that I have permitted myself a glimpse of the comic side of things only at fitting seasons, when the tired reader might well crave a moment's breathing-space, and not on any occasion where it could endanger the continuity of a line of argument. (Dodgson 1885, p. x)

The last sentence is interesting in that it shows how Carroll did not intend the literary dress to confuse the serious arguments he was making in the book. In this sense, literature *in* mathematics differs from mathematics *in* literature. Indeed, in the latter, as we show in our discussion of the two *Alice* passages above, the mathematics are an integral part of the fiction, contribute to its narrative and need not to be assessed separately in search for hidden meanings.

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