

# The Teaching Proposal of Probability in a Brazilian Elementary School Textbook Collection<sup>1</sup>

## A Proposta de Ensino de Probabilidade em uma Coleção Brasileira de Livros Didáticos dos Anos Iniciais do Ensino Fundamental

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### Abstract

We present in this article the analysis of the probability teaching proposal of a textbook collection used in the initial years of Brazilian elementary school, approved by the National Program of Textbooks (PNLD) in 2016. This work is related to the study group of Didactics of the Mathematics (DDMat). As a theoretical and methodological contribution, we used the anthropological theory of didactic, developed by Yves Chevallard and collaborators, that allows us to model and analyze the choices regarding mathematics and didactics present in the teaching of probability in the collection adopted. By the production and analysis of data, we noticed that some notions regarding probability were proposed from the volume referring to the second year of elementary school, in activities about possibilities. The formalization of the probability concept was presented in the last volume of the collection with activities associated with the idea of fraction, ratio and percentage.

**Key-words:** Probability, Brazilian textbooks, primary school, didactical organization, mathematical organization.

### Resumo

<sup>1</sup> Work constituted from the version published in the annals of the II Latin American Symposium of Didactics of Mathematics in 2018, in Jarinu - SP/Brazil.

Nesse artigo apresentamos a análise da proposta de ensino de probabilidade em uma coleção de livros didáticos dos anos iniciais do ensino fundamental, aprovada no Programa Nacional do Livro Didático (PNLD) de 2016. Este trabalho é vinculado ao Grupo de Estudos em Didática da Matemática (DDMat). Como aporte teórico e metodológico, utilizamos a teoria antropológica do didático, desenvolvida por Yves Chevallard e colaboradores, que nos permite modelar e analisar as escolhas referentes à matemática e à didática presentes no ensino da probabilidade na coleção adotada. Com a produção e análise de dados, notamos que algumas noções relativas à probabilidade foram propostas desde o volume referente ao segundo ano do ensino fundamental, em atividades de possibilidades. A formalização do conceito de probabilidade é apresentada no último volume da coleção, com atividades associadas à ideia de fração, razão e porcentagem.

**Palavras-chave:** Probabilidade, livros didáticos brasileiros, anos iniciais do ensino fundamental, organização matemática, organização didática.

## 1. Introduction

Probability is fundamental to understand events and random phenomena that permeate our daily lives, (GAL, 2005). Bryant and Nunes (2012, p.9) state that:

Our understanding of the probability of uncertain outcomes plays an extremely important part of our lives. We depend on it to decide about the medical treatment that we should follow, the insurance that we need, the car that we buy, and the precautions that we should take to protect our families and our homes. All these and many other decisions depend on our knowledge of possible events that might happen and on our understanding of how likely these different events are.

Given this importance, to answer the research question - How the teaching of probability throughout compulsory education is proposed<sup>2</sup> in collections of textbooks by the same author? -we analyzed the collections *Ápis* (Mathematical Literacy) and *Ápis* (Mathematics) that make up the collection destined to the primary school, *Project Teláris*, destined to the elementary school, and *Mathematics - Contexts and Applications*, concerning high school education in theses of a Master degree. The Thesis developed by the first author (VERBISCK, 2019).

All these collections were approved by the National Program of Didactic Books (PNLD) in 2016, 2017 and 2018, respectively, and all of them were written by Luiz Roberto Dante. In Brazil, this program (PNLD) is responsible to evaluate and distribute textbooks in public schools. Every year, the Government selects and evaluates book collections from one stage of basic school levels, approving some of them, in a three-year cycle.

The collections we used were chosen by inferring that in textbooks of the same authorship there would be (or could be) continuity of the proposal of the study of

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<sup>2</sup> In Brazil, the compulsory education is composed by the Primary school (for children between six and ten years old), the Middle school (for children between eleven and fourteen years old) and the High school (for teenagers between fifteen and seventeen years old).

probability. Also, our study of previous researches has identified the absence of longitudinal analyzes on the topic "the teaching of probability", which brought more legitimacy to our study. In this text, we will restrict ourselves to study the proposal of teaching of probability in the collection of textbooks from the Primary school, approved in the 2016 PNLD.

As a theoretical and methodological reference, we have chosen the Anthropological Theory of the Didactic (ATD), developed by Yves Chevallard (1998) and collaborators. We dedicated a section of this work for a brief presentation of the main elements that we will be used for the analysis of probability content.

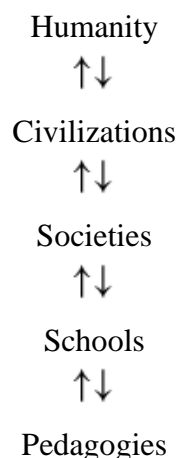
Thus, this article has been divided into four main topics. Firstly, we present some elements of the ATD and then we bring the production and analysis of the textbook collection, showing what was identified, through the modeling of the mathematical organization and the didactic organization of the activities and discourses related to the study of probability. The results are presented in two parts: the first covers the years of mathematical literacy (1st to 3rd years), and the second includes the last two years (4th and 5th years). We conclude this text with some considerations about probability in the textbook analyzed, based on the theoretical and methodological input adopted.

## 2. Some elements of the Anthropological Theory of the Didactic (ATD)

The ATD, proposed by Yves Chevallard (1998), has been used in several studies in Mathematics Education to present aspects of the knowledge in the mathematical activity. Casabò (2018, p. 4001) states that "From the perspective of the ATD, the aim of didactics as a science is to elucidate the mechanisms by which, in a given society, knowledge is diffused within institutions and among persons".

In this work, we focus on the praxeological analysis of a proposal of probability teaching in a Brazilian elementary school textbooks collection. To facilitate this analysis, we consider a *Scale of levels of didactic codeterminacy*, Chevallard (2002). "The higher levels of the scale correspond to the conditions and constraints related to the general way of organizing teaching and learning processes" (CASABÒ, 2018, p. 4003), as we can see in Figure 1:

**Figure 1:** Scale levels of didactic codeterminacy





Disciplines ⇌ Sectors ⇌ Domains ⇌ Themes ⇌ Questions

Source: Casabó(2018, p. 4005)

At higher levels, we found *Humanity* (in which all of us, human beings, were included); *Civilization*, that “in a deliberately minimalist way, we will understand here, under this name, a set of complex praxeologies presented and mobilized in a set of societies that, from the point of view, those praxeologies are considered similar” (WAZNIAK, 2005, p. 187). “The level *Societies* involves the belonging to some civilization, such as Brazilian society, German Society, etc. At the higher levels, we can still find the *Schools*, that includes all “the infrastructures provided by educational institutions to organize teaching systems and assist them in their functioning: teacher and student group organizations, course structures and modules, physical spaces and virtual, physical, virtual, times, compulsory final exams, [...] etc.” (CASABÓ, 2018, p. 4003). *Pedagogies* understood as everything used by the teacher and student for learning, which does not necessarily depend on a specific praxeology.

For example, many of the instructional formats that are generally proposed to improve university teaching practices (eg, 'interactive interactions', 'cooperative learning', 'discovery learning', 'participatory tutorials' etc.) are defined independently of the precept content to be taught and learned and can therefore be located at the pedagogical level. (Casabó, 2018, p. 4003).

Now, we move on to understand the lower levels of the scale. “The lower levels of the scale of didactic codeterminacy take these structures into account by distinguishing different “sizes” of the praxeological organisations: *disciplines, sectors, domains, themes and questions*” (Casabó, 2018, p. 4004). This paper, for example, deals with Mathematic subject, specifically the Probability sector, that includes a set of *Domains, Themes and Questions* (or subjects). Thus, by investigating the probability teaching proposal (s) in a collection of primary school textbooks, we take into account each of these levels of codetermination, that such knowledge is involved.

By presenting these first elements of ATD, we emphasize that in this paper we seek justifications for the importance of probability teaching in the primary school by looking at the higher levels of the didactic codetermination ladder. Next, we look specifically at the proposed praxeologies for the teaching of this theme in Brazilian textbooks. To perform this analysis at the lower levels (discipline, sector, domain, theme and questions), it is necessary to understand some more essential elements presented by ATD that enable the praxeological analysis of probability teaching. Such elements are presented below.

The ATD "places mathematical activity, and consequently the activity of the study of mathematics, within the broad set of human activities and social institutions" (Chevallard, 1998, p.1). Chevallard (1998) postulates, then, that human activities, carried out regularly, can be described by the means of a unique model, that is called praxeology. It consists of the following elements: type of tasks (T), technique ( $\tau$ ), technology ( $\theta$ ) and theory ( $\Theta$ ). Thus, according to Chevallard (1998), all human activity consists of fulfilling a task t (expressed through an action verb associated with an object) with a certain type T, which is performed by the means of a technique  $\tau$ . The justification is that this technique is valid and presented (explicitly or



not) by technology  $\theta$ , which is justified by a theory  $\Theta$ . The quartet  $[T, \tau, \theta, \Theta]$  is called praxeology or praxeological organization.

The pair  $[T, \tau]$  is called the practical-technical block, and is related to the "know-how"; and the pair  $[\theta, \Theta]$  is the technological-theoretical block, which refers to the "knowing". It is worth to emphasize, at that moment, that each praxeology lives in an institution and it can be understood by this term as been an organization that defines praxeologies. In this way, we consider a church, a school, a classroom, a school institutions. Thus, the textbook represents an institution, the publisher, when presenting praxeologies proposed by the group of people that have elaborated it. Analyzing what lives in an institution, we seek to analyze the mathematical organization (MO) related to the mathematical content, and the didactical organization (DO), regarding the didactic choices and how this mathematical organization is presented. Both MO and DO are analyzed through the quartet  $[T, \tau, \theta, \Theta]$ , but DO can also be analyzed through *didactic moments* or *moments of study*.

Given these elements of the ATD, we seek to model types of tasks and techniques as well as technological-theoretical block found for the probability of teaching in the textbook collection of Elementary school and also describe the educational choices made by the author in this process. The main results of production and data analysis are presented below.

### 3. Probability teaching and the Brazilian textbooks

Probability is part of Mathematics and Statistics subjects and it is essential to understand random events and phenomena that permeate our daily lives, Gal (2005). Probability teaching in the early years of elementary school becomes explicit, in official Brazilian curriculum documents, in the National Curriculum Parameters (1997), composing the content block entitled "Information Treatment" together with Statistics and Combinatorics. It is stated that "social demand is what makes this theme stand out as a block of content, although it could be incorporated into the previous ones. The purpose of the highlight is to highlight its importance, due to its current use in society". (Brasil, 1997, p. 40). Studies related to the notions of statistics, combinatorics and probability are integrated into this block. It is, therefore, a demand from the *society level* that gives this theme certain importance in the scenario of mathematics education in Brazil. This demand led to a change in the official curriculum documents in the *School* and *Pedagogy* levels, and that consequently changes at the lower levels.

Moreover, the primary purpose of probability is to make children understand that most everyday occurrences are random and that in many cases it is possible to identify probable outcomes of these events. It is also suggested that "intuitively manifesting notions of chance and uncertainty can be explored at school in situations in which the student performs experiments and observes events (in equiprobable spaces)" (BRASIL, 1997, p. 40). It is at this stage that children will develop the first probabilistic notions, through experiments and observations of events, and intuitively to observe and write all the possibilities of occurrence of any event (Brasil, 1997).

Santana (2011, p. 12) states that "understanding probability requires elaborate thinking of situation analysis, raising possibilities, and judging what is probable, unlikely, and impossible. In this sense, the teaching of probability can promote the

development of critical capacity and autonomy [...]”. Thus, we see the relevance of probability teaching in the early years of elementary school (which is equated to kindergarten), which led us to propose the research we are developing in the master's degree and which we bring in this text.

Also, we see that the textbook:

[...] has an important - or at least not negligible - place in different educational systems. Sometimes it is the main resource for teachers and students to teach and learn, as is the case in Brazil. But to exist, it is subject to certain conditions and restrictions. The textbook can be considered as a result of the interpretations of the authors of the programs and of what the other noospheric institutions say (Chevallard 1985, 1992, Chaachoua and Comiti, 2010). The sustainability conditions and constraints of a manual are not the same from country to country. For example, in France, regulation is done by users, ie teachers; In Brazil, this regulation is done through a national evaluation program. (KASPARY, 2020, p. 251).

From the ATD's point of view, we have that:

The publishing company and PNLD are considered two institutions in the noosphere,  $I_M$  and  $I_{PNLD}$ . These two noospheric institutions each express in their way the relations - which they consider appropriate - of a student or teacher to the objects of knowledge.  $I_M$  creates mathematical and didactic organizations in produced textbooks: where we find inert praxeologies; Similarly,  $I_{PNLD}$ , through the results of evaluations and recommendations, prescribes conditions and restrictions for objects that may or may not live in  $I_M$ . (Kaspary, 2020, p. 252).

Then we can see the relevance of these institutions in this research and it is from them that we identify the mathematical and didactic organizations proposed for the study of probability. It is noteworthy that the official Brazilian curriculum documents are produced by the noosphere<sup>3</sup> and prescribe conditions and restrictions on PNLD.

As for the textbook (LD), we agree with Frison et al (2009) in stating that this is one of the main tools to aid the teacher's work and this reflects on the teaching of the content being worked on. Also, “[...] it is perceived that it is one of the didactic materials and, as such, it becomes a facilitating resource for learning and an instrument to support pedagogical practice” (Frison et al, 2009, p. 4). Regarding the analysis of this feature, Bittar (2017, p.365 and 366) also states that:

[...] LD is the main material used by the teacher in the preparation of his classes, his study allows, among others, a certain approximation with what is taught by the teacher. Consequently, it is important to know the proposals of LD, especially to help in the elaboration of didactic interventions with students, because, regardless of the theoretical choice, it is necessary to take into consideration their teaching context.

Regarding the analysis of probability teaching in textbooks, we agree with Santana and Borba (2010, p. 4) in stating that:

It is relevant [...] to analyze how elementary school math textbooks approach the concept of probability, identifying which notions are worked by the authors and if they are proposed activities that have problem-solving as a central axis, in a broad view in general. around situations and notions worked [...].

Thus, we confirm the importance of the analysis of textbooks and the study of probability in these, to reflect on the choices of the authors and the role of this content

<sup>3</sup>According to Chevallard (1991), noosphere is that part of society that is responsible for the interrelationship between society and the spheres responsible for the production of knowledge; It includes representatives and drafters of teaching documents and guidelines, and the results are seen in official curriculum documents such as National Curriculum Parameters, Curriculum References of states, etc.

in the teaching of mathematics. Below, we briefly present the main theoretical elements that underlie our production and analysis of data.

#### 4. Production and analysis of data

In this work, we analyzed the proposal of the teaching of probability in the collection of textbooks from the Primary school, approved in the PNLD of 2016, that is named *Projeto Ápis*.

In the teacher's manual (TM) of the volume for the first year, it is stated that “we introduce current themes, such as statistics and possibilities, combinatorial reasoning” (DANTE, 2014, v.1, p. 287). In the third year's volume, it is further stated that “ideas about statistics and probability, through the exploitation of tables, graphs and 'chance', are worked informally throughout the Collection, due to their great importance in modern society” (DANTE, 2014, v.3, p. 386). Since there are no specific chapters or times for the probability study, we did a page-by-page search of the proposed activities, concepts, procedures, and speeches for this study. For this search, we considered that the probability study involves “a diversity of other contents, such as the concepts of fraction, ratio, proportion, percentage, notions of chance, random, possibilities, randomness, among others. These articulated concepts are necessary to provide broad learning of probability” (SANTANA, 2011, p. 19).

We then seek to find the occurrence of activities that bring in their utterance, teacher guidance or topic in some section with the words “possibilities”, “chance”, “random” or even “probability” and situations that exploit randomness. We check whether such concepts or study focuses are related to probability. Those, we believe to be related to the study of probability and we model through the praxeological model. The analysis of volumes from the first to the third year is presented together, as they are part of the mathematical literacy cycle, in the 4.1 section. This same search procedure was performed in the last two volumes, referring to the fourth and fifth years and the analysis is presented in 4.2 section. We point out that the PNLD/2016 Guide points out, in the review of this collection, that “only in book 5 the contents of the [Information Processing] field are studied at a specific time dedicated to it, although throughout the collection they are found as part of activities of the other fields” (BRAZIL, 2015, p. 229).

We also emphasize that we found activities throughout the collection with the occurrence of the word “possibility”, but not all situations were proposed with the focus on the study of probability. We cite as an example the activity found in the first volume of this collection, which works the composition of quantities:

**Figure 2:** Activity that does not focus on probability study

**2** VAMOS CONSTRUIR O “MURINHO DO 5”?

ELE SERVE PARA ESCREVER TODAS AS POSSIBILIDADES DE OBTER 5 COM UMA OU DUAS BARRINHAS.

→	0	MAIS	5	OU	$0 + 5 = 5$
→	1	MAIS	4	OU	$1 + 4 = 5$
→	2	MAIS	3	OU	$2 + 3 = 5$
→	3	MAIS	2	OU	$3 + 2 = 5$
→	4	MAIS	1	OU	$4 + 1 = 5$
→	5	MAIS	0	OU	$5 + 0 = 5$

MURINHO DO 5

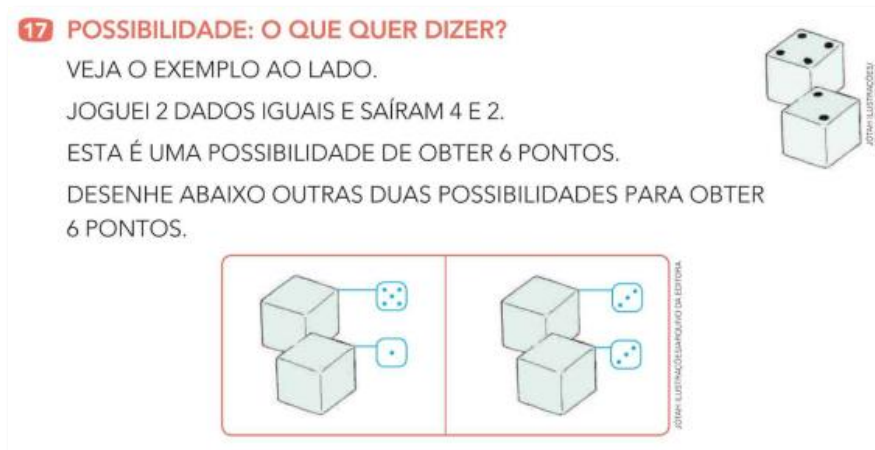
In this activity, the student is asked to write “all possibilities of getting 5”. In the TM it is pointed out that in this first stage of mathematical literacy, the composition of numbers is important for the study of the addition of natural numbers. In activities of this type, the goal is to work on the composition of numbers. Such situations were not considered in our search as activities that aim or prepare for the study of probability. That is, we not considered as a probability task because it does not involve randomness. The following describes what was identified in volumes 1 through 3.

#### 4.1 Literacy stage: Tasks and Techniques Preparing for the Probability Study

In the literacy cycle volumes, when looking for tasks that used “possibilities”, “random”, “chance”, “uncertainty” or even “probability” in their utterance, teacher guidance, or topics in a study section, we just found activities with the word “possibilities” that prepare for the probability study, for a total of one activity in the second volume and four activities in the third volume. Following the tasks and techniques were identified and modeled.

In the second volume we identify the following activity:

**Figure 3:** Probability study first activity



Source: Ápis Collection – Mathematics Literacy, v. 2, p. 24

We consider this task to be the first of the probability study because it involves randomness and possibility. In this situation, we understand that "getting the sum 6" or "six points" is the specific event that belongs to the randomized experiment "rolling two six-sided dice". By presenting both dice as being equal, it was inferred that the roll order of the dice is not being taken into consideration, i.e., obtaining “5 and 1” is the same as “1 and 5”. As “4 and 2” have already been presented as results, two more possibilities remain, which are “5 and 1” and “3 and 3”. We model as subtype T<sub>1.1</sub>:



- $T_{1.1}$ : Write/draw some possibilities of occurrence of a certain event.

We consider this to be a subtype because it is not required that all the possibilities of the event be written, but that "two other" be written. Thus, we understand that  $T_{1.1}$  is a subtype of  $T_1$  tasks, which we model as:

- $T_1$ : Write all the possibilities of occurrence of a specific event.

We then sought to indicate techniques for solving this first task that involves the probability study. In the teacher's book, shortly after the statement of the activity, two possible results are presented for this sum in the ostensive figure. Also, in the TM it is said that:

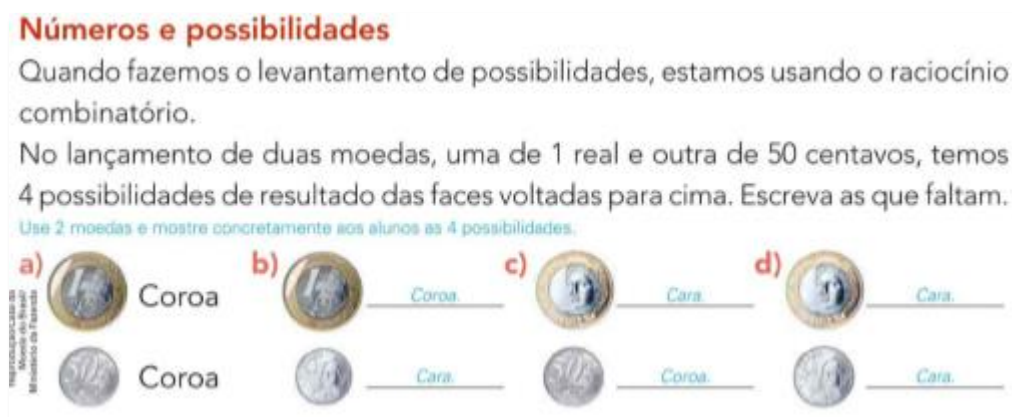
On page 24, activity 17 explores possibilities. For example, in tossing a coin there are two possibilities: heads off or heads out. It is interesting to provide a concrete exploration of these materials. To do this, gather the class into groups and give each group one dice. Students should explore possibilities such as those presented in the activity. Discuss with them the impossibility of another solution to get 6 points. (Dante, 2014, v.2, p. 382).

Thus, it is pointed out the importance of providing the concrete exploration of these materials, in this case, the dice. For this, it is suggested that students be arranged in small groups and given two dice to each group so they can explore these possibilities by playing. Thus, we understand as a first technique for solving tasks of this subtype as  $\tau_1$ :

$\tau_1$ : Manipulate ostensible - dices, coins or notes -, looking for possibilities of the event occurrence.

In the third year textbook, the first task identified is in the first chapter of the volume, which focuses on the study of numbers.

**Figure 4:** Possibility Activity on Two Currency Posting



Source: Ápis Collection – Mathematics Literacy, v. 3, p.17

In this situation, it is proposed that the child complete the missing possibilities in the two coin toss. For this, the child is already presented with the total possibilities (which in this case are four) and the ostensible figurative of the coins also reinforce this result. Representing the first currency being one real and the second being fifty

cents indicates that this order is being taken into account. We point out this task as belonging to subtype  $T_{1.2}$ :

- $T_{1.2}$ : *Name the possibilities for a given event.*

We consider it to be a subtype for requesting completion of the possibilities of the specific event. Thus,  $T_{1.2}$  is contained in  $T_1$ . As shown in the teacher's book, the technique for solving this task is what we call  $\tau_2$ :

$\tau_2$ : *Write the name corresponding to the face represented in the figure plate.*

In addition to this technique, another technique was identified in the TM, when it is proposed that: “If convenient, give students the toy coins to manipulate them and perform the activity concretely” (Dante, 2015, v. 3, p.389). In this technique occurs the manipulation of the ostensive coins to obtain the missing possibilities, that is, the  $\tau_1$  (*Manipulate ostensible - dices, coins or notes -, looking for possibilities of the event occurrence*), previously modeled.

In this activity (Figure 4), although the TM suggested the use and manipulation of the ostensible *coins* to obtain the missing possibilities, we believe that we present the “ready” possibilities, and only ask the child to write the names of the coin faces, does not allow it to come to a solution on its own. Silva (2016, p.6) concluded in his research that “the use of games made it possible for intuitive [randomness] notions to emerge naturally, but that there is a need for instruction, which may also include this resource, to construct coherent probabilistic knowledge”. The book is inconsistent when presents a statement that is important to explore and manipulate concrete materials to obtain the possibilities, but the possibilities of the event are already given.

The next two identified tasks work with monetary values and possibilities. The context brought up is related to the idea of random events, because it is not known the amount that will come out in the withdrawal of two notes (since the possibilities are two hundred, one hundred and ten, or twenty reais), nor in the withdrawal of three notes (whose possibilities are three hundred, two hundred and ten or one hundred and twenty real). Thus, the child will have to write all these possibilities of the two-note withdrawal and three-note withdrawal experiments, without looking, randomly. By requesting all possibilities of these situations, we understand that in these tasks the child should write all the elements of the sample space of such an experiment. Bryant and Nunes (2012, p.5) state that:

We can only calculate the probabilities of particular events if we know what all the possibilities are. The complete set of possibilities in a probability problem is called its ‘sample space’. Working out the sample space is the essential first step in solving any probability problem (Keren, 1984; Chernoff, 2009), and in many, it is the most important, since the solution is often quite obvious to someone who knows all the possibilities.

We then consider these two tasks belonging to a new type that we model as  $T_2$ :

- $T_2$ : *Write all elements of the sample space of a randomized experiment.*

Since context is associated with amounts, in the teacher's book it is suggested that “ask students to use the cut-out money from *My Block* to develop the following activities. For example, in activities 4-7, ask them to reproduce the amounts concretely

”(DANTE, 2015, v.3, p. 41). “My block” is an attachment at the end of the book that contains representations of money bills from the Brazilian monetary system. This indicates that the technique  $\tau_1$ (*Manipulate ostensible - dices, coins or notes -, looking for possibilities of the event occurrence*) allows the manipulation of ostensive notes for the composition of monetary values with the amounts of one hundred and ten real. With this technique, the child can write all the possibilities of sums in the withdrawal of two notes and three notes of the amount of three notes of one hundred real and two notes of ten reais.

Next, the last task identified in this volume also deals with the context of withdrawing amounts without looking. This time the statement goes as follows: “In a little bag there are four coins like these beside it. Taking two of them without looking, how much money can we get? Use *My Block* coins to represent the possibilities and write down all the amounts we can get below ” (DANTE, 2015, v.3, p. 187). In this task it is also asked to obtain the possibilities of amounts in the random situation “withdraw two coins without looking”. This is a task that also belongs to  $T_2$  (*Write all elements of the sample space of a randomized experiment*) and for its resolution, it is proposed to manipulate the ostensible coins available at the end of the book. This points again to the mobilization of the technique  $\tau_1$ (*Manipulate ostensible - dices, coins or notes -, looking for possibilities of the event occurrence*) in the resolution of this task.

Thus, we identified in all three volumes related to the literacy stage the occurrence of one type of tasks, two subtypes of tasks and two solving techniques that prepare children for the study of probability, involving the idea of related “possibilities”.

Below, we present the data produced and analyzed in the last two volumes of the initial years.

#### 4.2 Fourth and fifth years: possibility activities and introduction to the probability concept

In the volumes for the fourth and fifth years of this collection, we performed the same process of searching for activities that used “possibilities,” “random,” “chance,” or “probability” in their utterance, teacher guidance, or topics in a section study. In this search, we identified two activities of “possibilities” that prepare for the study of probability, one activity in each work. These are tasks that contribute to the study of probability and, in particular, we believe they are tasks that contribute to the understanding of how to determine the sample space in situations involving randomness.


In the volume for the fourth year, in the last study unit, we found a proposed activity with the following statement:

**Figure 5:** Possibility Activity on Four Currency Posting

**1 Possibilidades**

Quando jogamos uma moeda, há duas possibilidades para a face voltada para cima: pode sair cara e pode sair coroa.

Imagine agora se jogarmos 4 moedas iguais. Uma das possibilidades é saírem 3 caras e 1 coroa:



Em seu caderno, escreva todas as possibilidades e indique quantas são.

São 5 possibilidades: 3 caras e 1 coroa; 4 caras; 2 caras e 2 coroas; 1 cara e 3 coroas; 4 coroas.



In this situation, first, the possibilities are pointed out in the launch of only one coin: heads or tails. Already in the launch of four coins, it is pointed out that one of the possibilities is to come out three heads and tails. The first task is for the child to write down all the possibilities of the “four-coin toss” experiment and the other to indicate the total number of possibilities. We observe that in this situation when treating the coins as being equal, the order of the throws is not being taken into account, that is, coming out “head, head, tail, head” is the same as coming out “head, head, head, tail” or “head, tail, head, head”, etc. Thus, the total possibilities of this experiment are five, as shown in the teacher's textbook (Figure 5).

In this activity, we understand that two distinct tasks are proposed. First, by requesting all possibilities of the “four equal coin toss” experiment, the task belongs to type  $T_2$  (*Write all elements of the sample space of a randomized experiment.*). In this task, and as we have seen in tasks in the volume for the third year, the figural ostensive was used to represent one of the possibilities. From this we can infer that to solve such a task the child could draw the other possibilities. In the TM we do not find comments on the possible ways of solving this activity, but as this volume also provides concrete representations of coins and banknotes of the Brazilian monetary system, we believe that the technique  $\tau_1$  (*Manipulate ostensible - dices, coins or notes -, looking for possibilities of the event occurrence*) the resolution of this type of tasks can be mobilized.

Then, by requesting to indicate the total amount of possibilities, we understand this part as a new task, which we model as subtype  $T_{2.1}$ :

- $T_{2.1}$ : *Indicate / write the amount of sample space elements of a randomized experiment.*

To indicate how many total possibilities there are in the experiment of the situation in Figure 5, the student only needs to count how many elements are from the elements he had already listed in the previous task (type  $T_2$ ). For this, the resolution technique of this subtype task  $T_{2.1}$  was modeled as  $\tau_3$ :

$\tau_3$ : *Count the elements of the sample space.*

This was the only activity in the fourth volume with tasks that seemed to contribute to the probability study.

We now turn to describe what was identified in the last volume of the early years' collection, which refers to the fifth year. In this volume, we identify activity of “possibilities” that works, again, the idea of a random situation and asks “all possibilities of a specific event”. This activity is found in unit 3 whose title is *Addition and Subtraction with Natural Numbers* that take up some ideas of addition and subtraction, explores addition properties, rounding and approximations involving addition and subtraction, as well as working with the algorithm and the addition and subtraction and problem-solving.

**Figure 6:** Sum Eight Possibilities Activity in Two-Sided Dice Rolling

**4 Possibilidades**

Desafie os alunos perguntando: Quantas são as possibilidades de sair soma maior do que 12? E menor do que 2? Eles devem responder que não existem essas possibilidades no lançamento de 2 dados, ou seja, o número de possibilidades é zero.

Bia jogou 2 dados de cores diferentes e obteve soma 8, com a face 4 no primeiro dado e a face 4 no segundo.

Indique todas as possibilidades de obter soma 8.

Depois escreva quantas são as possibilidades.

*Possibilidade além de  $4 + 4$ :  $2 + 6$ ,  $3 + 5$ ,  $5 + 3$ ,  $6 + 2$ ; ao todo, 5 possibilidades.*

Estúdio Félix Reimers/Arquivo da Editora



In this activity, the randomized experiment is to "throw two six-sided dice" and the specific event is "sum eight". All possibilities are requested to be indicated and then the total possibilities identified. We consider, then, that this first part of the task is of type  $T_1$ : *Write all the possibilities of occurrence of a specific event*. We also observed that in this situation the dices are presented in different colors (red and blue), which shows that the order of the releases will be taken into account, i.e., "5 and 3" is different from "3 and 5". In the TM it is pointed out that this activity "relates the operation of addition with possibilities (combinatorial reasoning)" (DANTE, 2014, v.5, p.363). We found that at this stage of schooling the decomposition algorithm is worked on quantity decompositions as in this situation and, being in a chapter on addition, we understand as a new resolution technique that we modeled as  $\tau_4$ :

$\tau_4$ : *Write all the possibilities of decomposition of the requested sum.*

The second part, by requesting that the totality be written as well, we see that there is a new task that we believe belongs to the subtype of task  $T_{1.3}$ :

- $T_{1.3}$ : *Indicate / write the total number of possibilities of occurrence of a given event.*

To solve this subtype task  $T_{1.3}$ , the child only needs to count the elements already listed when writing "all the possibilities of decomposition of sum eight". Thus, the technique  $\tau_{4.1}$  was defined:

$\tau_{4.1}$ : *Count the elements (possibilities) of a specific event.*

Still in this situation, it is proposed to the teacher to "Challenge students by asking: How many possibilities are there for a sum greater than 12? And less than 2? They must answer that there are no such possibilities when rolling 2 dice, i.e., the number of possibilities is zero" (Dante, 2014, v.5, p.61). Such questions refer to the notion of an impossible event, albeit informally.

In this volume, in addition to the activity of possibilities, we also find two sections that belong to the same unit (Unit 6) in which an introduction to the concept of probability is presented. We analyze the didactic organization and the mathematical organization proposed in these sections. For this analysis, we considered the definitions, explanations, techniques and didactic choices to present the theme and the proposed activities for the probability study.

Unit six, entitled *Fraction and Percentage*, is divided into fourteen sections and, among the objectives pointed out for this unit, one has *to explore the idea of probability and solve problems involving fraction, percentage and probability*. We have identified such exploration and activities in the sections entitled *Fraction and probability* and *Fraction and percentage*. The TM states that:

The idea of chance is part of everyday life. People want to know what chance they have of getting it right or not. They try to make predictions about whether this or that is more likely to occur. This subject is part of the "mathematical literacy" needed in practical life. In this volume we work on the idea of chance, arriving at the measure of chance or probability, initially expressed by a fraction. (Dante, 2014, v.5, p. 373 and 374).

We have identified new types of tasks and techniques, presented in the table below:

**Table 1:** New task types and techniques in the fifth year

Task Types	
T <sub>3</sub> : Determine the probability of occurrence of a specific event.	T <sub>3.1</sub> : Identify higher or lower chance/probability of occurrence of a given event.
T <sub>4</sub> : Create a situation that involves notions of probability.	
T <sub>5</sub> : Interpret / discuss terms, concepts or situations related to the study of probability.	
Techniques	
τ <sub>5</sub> : Associate specific events with: <ul style="list-style-type: none"> <li>• higher chance of occurrence, when it has the most elements;</li> <li>• lowest chance of occurrence when it has the least amount of elements.</li> </ul>	
τ <sub>6</sub> : Write the fraction (or ratio) whose numerator represents the number of favourable cases and the denominator represents the total number of possibilities.	
τ <sub>7</sub> : Calculate the percentage from a fraction representing the probability of an event.	

Source: Made by the author

We also note that the justification for using the overt fraction (or ratio) appears in the definition of probability and the example of a fraction of a set of elements. This points to the beginning of constitution of the technological-theoretical environment that validates the techniques that mobilize fraction and percentage in the representation of the measure of chance, the probability.

The following is a summary of the data produced in the collection of textbooks intended for the early years of the elementary school regarding the probability teaching proposal at this stage of basic schooling.

### 4.3 Summary of observed results

In the following table, we present the number of task types, task subtypes, and techniques that mobilized throughout the Ápis collection:

**Table 2: Quantitative task types, task subtypes, and techniques throughout the Ápis Collection**

Grade	Types and subtypes of tasks proposed	Techniques expected to be mobilized	Total
2nd year	T <sub>1.1</sub>	τ <sub>1</sub>	1
3rd year	T <sub>1.2</sub>	τ <sub>1</sub> or τ <sub>2</sub>	1
	T <sub>2</sub>	τ <sub>1</sub>	3
4th year	T <sub>2</sub>	τ <sub>1</sub>	1
	T <sub>2.1</sub>	τ <sub>3</sub>	1
5th year	T <sub>1</sub>	τ <sub>4</sub>	1
	T <sub>1.3</sub>	τ <sub>4.1</sub>	1
	T <sub>3.1</sub>	τ <sub>5</sub>	1
			2
	T <sub>3</sub>	τ <sub>6</sub>	8
		τ <sub>6</sub> and τ <sub>7</sub>	1
	T <sub>4</sub>		1
T <sub>5</sub>		1	
<b>Total of tasks</b>			<b>23</b>

Source: Made by the author

In the first column, the volumes are indicated, in the second the types of tasks and the third column the techniques that can be mobilized for the resolutions. In the last column, we indicate the number of times that the type of task, solvable by the given technique was proposed in each volume. The gray parts indicate the techniques used to solve tasks of type  $T_{3.1}$  (*Identify higher or lower chance/probability of occurrence of a given event*),  $T_4$  (*Create a situation that involves notions of probability*) e  $T_5$  (*Interpret/discuss terms, concepts or situations related to the study of probability*), which could not be described as involving students strategies.

We note that in the volume for the fifth year there is a significant growth of tasks and techniques proposed for the study of probability because this is where the first concepts related to this theme are introduced.

The technique  $\tau_1$  (*Manipulate ostensible - dices, coins or notes -, looking for possibilities of the event occurrence*) was present in the second, third and fourth volumes, and could be mobilized in six tasks. We believe that tasks such as  $T_1$  (*Write all the possibilities of occurrence of a specific event*),  $T_2$  (*Write all elements of the sample space of a randomized experiment*), and subtypes  $T_{1.1}$  (*Write draw some possibilities of occurrence of a certain event*),  $T_{1.2}$  (*Name the possibilities for a given event*),  $T_{1.3}$  (*Indicate/write the total number of possibilities of occurrence of a given event*) e  $T_{2.1}$  (*Indicate/write the amount of sample space elements of a randomized experiment*) prepare for the calculation of the possibilities of specific events in experiments of random nature and aim to work the idea of sample space and events in experiments.

Silva (2016, p.15) states that:

[...] knowing and understanding the sample space is an essential part of finding the solution of probability situations, since the calculation is based on the analysis of the problem sample space. Thus, a good way to solve many probability problems is to know the sample space that can be seen as listing all possibilities of occurrence of all possible events, such as the head and tail faces in the tossing a coin.

The task type  $T_3$  (*Determine the probability of occurrence of a specific event*) and the task subtype  $T_{3.1}$  (*Identify higher or lower chance/probability of occurrence of a given event*) already work on the notion of chance and its measure, the probability, since at this moment such a concept is institutionalized.  $T_3$  was the type of task most proposed in this volume and the technique  $\tau_6$  (*Write the fraction (or ratio) whose numerator represents the number of favorable cases and the denominator represents the total number of possibilities*), as the way to solve, which points to The moment of exploration of this kind of tasks and work of this technique also indicates the valorization of the classical view of probability. The other types of tasks,  $T_4$  (*Create a situation that involves notions of probability*) e  $T_5$  (*Interpret / discuss terms, concepts or situations related to the study of probability*), which also deal with ideas related to probability, did not present describable techniques. since different strategies can be carried out for their resolutions. We observed that only one task type  $T_4$  and one type  $T_5$ , appeared, indicating little attention throughout the study proposed in this volume.

We then see that the representations using the ostensible fraction (or ratio) of the desired probabilities were the most valued. And, by proposing tasks that mobilize ostensive manipulation, it is believed that there is an intention to value the ostensive manipulation work for the initial probability study. This is because in the identified

tasks the handling of coins, dices and banknotes were proposed in activities of possibilities determination and sample space of events.

As for the moment of constitution of the technological-theoretical environment, we observe that the justifications for the techniques of solving the types of tasks modeled, is superficially seen when the use of the ostensive fraction (or ratio) and percentage as representations of probabilities of the events are validated. We believe that at this early stage of basic education, the techniques presented for the study of probability are self-technological and therefore there is no need for justification moments. For example, in tasks of type  $T_2$  (*Write all elements of the sample space of a randomized experiment*) the manipulation of overt dice, coins and notes already show the validity of the technique  $\tau_1$  (*Manipulate ostensible - dices, coins or notes -, looking for possibilities of the event occurrence*).

In general, taking into account the didactic moments identified and the model of didactic organizations proposed by Gascón (2003), we believe that in this collection *an empiricist didactic organization* prevailed for valuing the exploration of task types, elaboration and work of techniques for the study of probability.

## 5. Final considerations

The concepts of probability are relevant in all stages of basic education because they are part of our daily lives, due to the random events to which we are subjected. Thus, in the early years of elementary school, children can already begin to be familiar with notions of probability by experimenting and observing events with the same chance of occurring, as suggested by Nacional Curriculum Parameters (BRASIL, 1997). In this article, we present the results of the analysis and production of data related to the probability teaching proposal in a collection of elementary school textbooks, approved in PNLD/2016, which are part of an ongoing master's research.

In the collection of textbooks from the early years, we sought to identify the proposed praxeologies for the study of probability. As we have seen, in this collection there were no specific chapters to study this theme. With the readings taken to understand probability, we realize that some words and concepts are strongly related to this study, which is the case of the ideas of possibilities, chance, chance, uncertainty, among others, in contexts involving randomness. When looking for the occurrence of these words throughout the collection, in the volumes referring to the second, third, fourth and fifth years, we identified activities of "possibilities" that prepared for the study of probability in random situations.

The analysis of this collection does not allow us to infer the teaching of probability in the early years of elementary school since we study a specific case: a collection of textbooks. We can say that in the case of this collection the teaching of probability in the early years was started intuitively with proposals for activities of possibilities and experiments with dices and representations of monetary values. The formalization of the concept of probability was presented in the last volume of the collection, with activities associated with the idea of fraction, ratio, percentage, as well as activities to discuss the use of the term probability in daily life, as well as the probability that involves personal beliefs and opinions.

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