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An Analysis of Pre-service Teachers' Understanding of the Knowledge Needed for Teaching Mathematics: The case of Malawi

Uma Análise da Compreensão dos Futuros Professores sobre os Conhecimentos Necessários para o Ensino da Matemática: O caso do Malaui

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Abstract

In contexts of adversity, there is a need to educate pre-service teachers with specialized content knowledge so they can carry out the work of teaching effectively. This article draws upon a study with three pre-service teachers in Malawi that examines the understanding they develop of the knowledge needed to carry out mathematics teaching tasks, in particular, the knowledge needed sequence instructional tasks sequence and use mathematical representations in classrooms. The research was conducted in one teacher education college where the curriculum is under development and has an emergent demand for qualifying teachers in mathematics. The research methodology was based on a questionnaire, interviews, teaching observations, and group discussions. The data were thematically analyzed through two themes reflecting the knowledge pre-service teachers considered necessary for teaching mathematics. While the first theme reveals how pre-service teachers understand this knowledge as a reference for meeting curricular standards and students' needs, the second captures a form of understanding that attempts to go beyond conceptual knowledge. The findings can help better understand the pre-service teachers' learning and experience during teacher education and how theoretical constructs are conceived in challenging contexts of teacher education.

Keywords: Mathematics education. Malawian Teacher Education. Teacher Knowledge. Specialized content knowledge.

Resumo

Em contextos de adversidade, existe a necessidade de se formar professores em formação com conhecimentos de conteúdo especializado para que possam realizar o trabalho de ensino de forma eficaz. Este artigo baseia-se num estudo com três professores em formação no Malaui que examina a compreensão que estes professores desenvolvem sobre os conhecimentos necessários para realizar de tarefas de ensino da matemática, em particular, os conhecimentos necessários para sequenciar tarefas instrucionais e usar representações matemáticas em salas de aula. A pesquisa foi realizada num colégio de formação de professores onde o currículo está em desenvolvimento e há uma grande demanda de qualificados professores em matemática. A metodologia de pesquisa baseouse em entrevistas, observações pedagógicas e discussões em grupo. Os dados foram analisados tematicamente a partir de dois temas que refletem os tipos de conhecimentos que os professores consideram importantes para o ensino da matemática. Enquanto o primeiro tema revela como os professores entendem esses conhecimentos como referências para atender aos padrões curriculares e às necessidades dos alunos, o segundo tema capta uma forma de compreensão que busca ir além dos conhecimentos conceituais. Os resultados ajudam a entender melhor a aprendizagem e experiência dos professores durante programas de formação e como os conceitos teóricos podem ser mais bem compreendidos em contextos desafiadores de formação docente.

Palavras-chave: Educação Matemática. Formação de Professores Malauianos. Conhecimentos Docentes. Conhecimento especializado do conteúdo.

1. Introduction

Teacher education is critical for societies, but there is a shortage of effective teachers worldwide. Qualified teachers play a vital role in preparing the next generation to face societal and environmental problems and become proactive contributors to a more just, peaceful, tolerant, inclusive, secure, and sustainable world (UNESCO, 2014). An adequate supply of qualified teachers is essential to accomplishing all its SDG targets[1], showing that attention should be given to teachers' professional education, especially those working in areas with extreme adversities.

Sub-Sahara Africa has one of the highest rates of students teachers' ratio and underqualified and unqualified teachers. In Malawi—one of the poorest regions of the Sub-Saharan region, recent curricular reforms have included a high demand for pre-service teachers to learn and acquire specialized content knowledge to carry out tasks of teaching in crowded classrroms[2] with limit didactical resources (Malawian Institute of Education, 2017; Wamba & Mgomezulu, 2014).

In this scenario, the Malawian Initial Primary Teacher Education (IPTE) program appeared as a promising solution to improve the quality of education offered to primary pre-service teachers (Malawian Institute of Education, 2010). The pre-service teachers enrolled in the two-year IPTE program benefit from a combination of college-based education and teaching practice in primary schools. The IPTE program aims to ensure that, upon graduation, pre-service teachers are "academically well-grounded and professionally competence;" "flexible and capable of adapting to the changing needs and environment of the Malawian Society;" and "capable of creating and utilizing locally available resources suitable for the needs of their learners" (Malawian Institute of Education, 2017, p. 3).

The importance of combining theory and practice in the preparation of pre-service teachers is well recognized (Gravett & Ramsaroop, 2015; Korthagen, 2010), but little is known about the impacts of the IPTE program on the level of preparation of primary pre-

[1] UNESCO's (2016) agenda contains 17 Sustainable Development Goals (SDGs), number four of which is the global access to education, a target that includes ten sub-targets such as Target 4.1: Primary and Secondary Education, and Target 4.C: Quality of Teacher Education.

[2] In 2019, UNESCO reported an average teacher/pupil ratio is 1:77 in schools located in urban areas and about 1:150 for those in rural and remote zones in Malawi (see UNESCO, 2019).

service teachers in Malawi (Kasoka, Jakobsen, & Kazima, 2017). For example, a recent study conducted by Jakobsen, Kazima, and Kasoka (2018) involving all eight teacher training colleges (TTCs) in Malawi revealed that the pre-service teachers' level of mathematical knowledge for teaching was low, especially the knowledge needed to handle typical classroom situations. Thus, the authors called for more research on the IPTE curriculum and the pre-service teachers' learning process during teacher training (Jakobsen et al., 2018).

This article is a piece of larger research developed about Malawian pre-service teachers' learning of mathematical knowledge for teaching (Jacinto, 2020) and is guided by the question: *How do Malawian primary pre-service teachers develop their understanding of the knowledge needed to carry out the tasks of teaching mathematics?* In particular, the article focuses on two themes that entail knowledge components: Knowing how to sequence instructional tasks to help students solve mathematical problems and how to use locally available resources to teach mathematics representations. These themes were initially developed from Ball, Themes, and Phelps's (2008) concept of specialized content knowledge and were later revised and adapted according to the dataset.

2. The Teacher's Specialized Content Knowledge

Research on teachers' knowledge dates back to Shulman's investigation of pedagogical content knowledge (PCK), defined as a "knowledge base that any teacher needs for teaching a content" (Shulman, 1987, p. 4). Shulman (1986) considered it as "the most useful ways of representing and formulating the subject that make it comprehensible to others" that "includes an understanding of what makes the learning of specific topics easy or difficult" (p. 9). Since then, Ball et al. (2008) have worked to give a more comprehensive description of mathematical knowledge for teaching.

Mathematical knowledge for teaching combines subject matter knowledge (SMK) and PCK in mathematics. PCK is divided into knowledge of the content and curriculum (KCC), knowledge of the content and students (KCS), and knowledge of content and teaching (KCT), whereas SMK comprises common content knowledge (CCK), horizon content knowledge (HCK), and specialized content knowledge (SCK). Ball et al. (2008) described SCK as a unique component for teaching, whereas CCK can be used in areas other than teaching. According to Jakobsen, Thames, Ribeiro, and Delaney (2012), SCK is distinct from HCK, which refers to "An orientation to and familiarity with the discipline (or disciplines) that contribute to the teaching of the school subject at hand, providing teachers with a sense for how the content being taught is situated in and connected to the broader disciplinary territory" (p. 4642).

SCK is the type of knowledge and skill that makes "features of a particular content visible and learnable by students" (Ball et al., 2008, p. 400). Such knowledge goes beyond content knowledge itself, as it "addresses both mathematics substance and pedagogical appropriateness," aiming to unpack "a mathematical concept into its subcomponents to make it comprehensive for children" (Ding, 2016, p. 2), as shown in Figure 1.

Figure 1: Teaching tasks of mathematics

Presenting mathematical ideas
Responding to students' "why" questions
Finding an example to make a specific mathematical point
Recognizing what is involved in using a particular representation
Linking representations to underlying ideas and to other representations
Connecting a topic being taught to topics from prior or future years
Explaining mathematical goals and purposes to parents
Appraising and adapting the mathematical content of textbooks
Modifying tasks to be either easier or harder
Evaluating the plausibility of students' claims (often quickly)
Giving or evaluating mathematical explanations
Choosing and developing useable definitions
Using mathematical notation and language and critiquing its use
Asking productive mathematical questions
Selecting representations for particular purposes
Inspecting equivalencies

Fonte: Ball et al. (2008)

SCK has been widely acknowledged as a crucial component for pre-service teachers to develop during teacher education (Ding, 2016). Yet, how pre-service teachers think about SCK's items they are supposed to acquire in order to teach effectively remains poorly understood (Chapman, 2015; Ferguson & Brownlee, 2018; Mosvold & Fauskanger, 2013). Addressing this gap would help determine not only the relevance of such knowledge for pre-service teachers and if they fully understand the role of theories in teaching practice (Allen & Wright, 2014; Kwenda, Adendorff, & Mosito, 2017), but also whether or not they can understand the problems and situations that happen in classrooms, why it happens, what ought to happen, and how they can overcome those problems (Chapman, 2013).

Prior research on teachers' knowledge for teaching has involved epistemic beliefs about the certainty of teaching knowledge (Ferguson & Brownlee, 2018), perceptions of teaching knowledge during teacher education (Jacinto & Jakobsen, 2020), construction of ideas about knowledge and teaching (Kroll, 2004), knowledge and beliefs of students' prior knowledge and the potential to develop new knowledge while solving mathematical tasks (Lee, Coomes, & Yim, 2019), changes in beliefs about the teaching and learning (Tillema, 1998), and the development of mathematical knowledge for teaching teachers (Jankvist, Clark, & Mosvold, 2020). Such a body of works provides insights beyond what teachers can or cannot do in the classroom. Their findings help to better understand emergent issues involving the knowledge demands entailed in the work of teaching as well as on the nature of mathematical knowledge teachers and pre-service teachers hold; how and why they hold it; how does it develop; how and when do they use it in practice; how can it be supported, changed, or enhanced to carry out tasks of teaching effectively (Chapman, 2013).

Following this line of inquiry, the present study aims to contribute to this field by ascertaining how Malawian primary pre-service teachers develop their understanding of the knowledge necessary to carry out teaching tasks in mathematics, which is crucial for understanding the pre-service teaches' learning and experience in becoming mathematics teachers.

3. Methodology

This article's data comes from a Ph. D. project investigating the Malawian pre-service teachers' learning and experience during teacher education. In the larger project, twenty-three pre-service initially volunteered to a questionnaire survey about their teaching experience (TE), mathematics interests in high school (MIHS), and mathematics interest during college (MIC). The participants also gave individual interviews about their motives and objectives in becoming teachers and their views about the tasks and knowledge relevant to teach mathematics in primary schools in Malawi.

The three criteria (TE, MISH, and MIC) allowed us to arrange the participants into six classifications of profile (four pre-service teachers possessing TE, MIHS, and MIC; three having TE, but not MIHS and MIC; six with no TE, but with MIHS and MIC; four with no TE and MIHS, but with MIC; two with no TE, MIHS, and MIC; and four with TE and MIC, but no MIHS). This first sample of data and organization was conducted at the beginning of the teacher education program (first two terms of the TTC program) and was called Initial Moment (IM).

As the pre-service teachers were sent to teach in different schools across the country, one pre-service teacher per group was chosen to be followed and observed during their supervised teaching practice in primary schools, giving preference to those near the teacher college. The list with the six participants and their pseudonyms appears in Table 1.

Table 1: Selection of the Research Sample according to their profiles

Criteria of selection of the research sample	Pre-service teachers' anonymized names
TE/MIHS/MIC	Martin
TE/No MIHS/No MIC	Mario
No TE/ MIHS/MIC	Patrick
TE/No MIHS/MIC	Clara
No TE/No MIHS/MIC	Daniel
TE/ No MIHS/MIC	Denise

Fonte: Jacinto (2020)

The second moment of the study (SM) occurred during supervised teaching practices in schools (three and four terms of teaching program). It included observations of the pre-service teachers' mathematical lessons and post-lesson interviews. The lessons were video-recorded and the post-lesson interviews captured insights into the tasks the knowledge identified by the pre-service teachers as crucial for carrying out those tasks effectively.

The third moment (TM) was conducted at the end of the teacher education after pre-service teachers returned to the college (program's terms five and six) and consisted of a focus group discussion (Krueger, 1998) with the six pre-service teachers. Each participant gave a brief description of their learning and experience in the college, followed by discussions about their initial views during the beginning of the program and tasks of teaching that they covered while teaching in primary schools. This third moment aimed to determine the pre-service teachers' understanding of teaching knowledge and its development during the program. Table 2 synthesizes the three moments of the study.

Table 2: Moments of investigation and data collection instruments

	Initial Moment (IM)	**Second Moment (SM)**	**Third Moment (TM)**
Program's terms	Terms 1 and 2: Theoretical courses at the TTC	Terms 3 and 4: Teaching Practice in primary schools	Terms 5 and 6: Students come back to TTC
Data collection instruments	Questionnaire survey and individual interviews	Systematic Observations of mathematics lessons and individual interviews afterward	Focus Group Discussion
Numbers of participants	23	6	6

Fonte: Jacinto (2020)

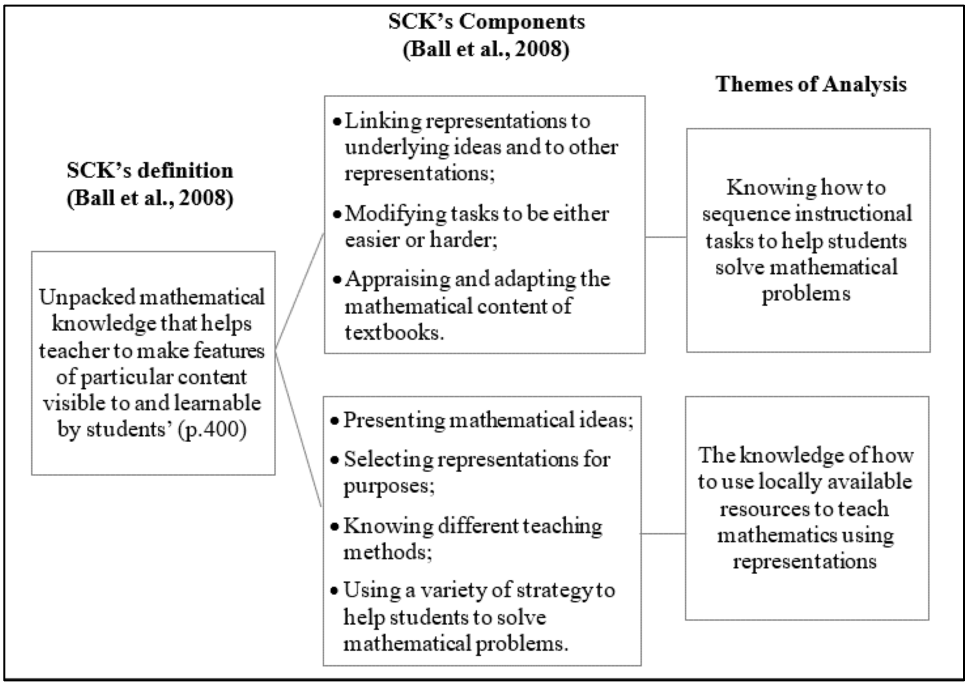
4. Data Analysis

The larger study's findings revealed that although possessing different profiles, five pre-service teachers presented a similar understanding of the tasks and knowledge needed to teach mathematics in primary schools while one (Mario) tended to have a broader understanding of these instances (see Jacinto, 2020). Rather than discuss the similar understanding across the five pre-service teachers, I decided, instead, to represent these cases through two cases (Stake, 2006), written in the form of narratives (Clandinin, 1992). The two pre-service teachers (Denise and Martin) were selected because the understanding they underwent was reflective of the other pre-service teachers' understanding.

While the larger research focused on data from around the six domains developed by Ball et al. (2008), this article focused on two tasks (designing the sequencing of the instructional tasks and use of locally available resources in classrooms) that are subjects in the mathematics syllabus of the program (Malawian Institute of Education, 2017) and are crucial for pre-service teachers to learn during teacher education in Malawi (Jacinto & Jakobsen, 2020).

The data were thematically analyzed (Braun & Clarke, 2012) upon two themes that were initially crafted from the list of SCK's components the in general described the forms of sequencing instructional tasks and use representations in mathematics teaching (Chinnapan, White, & Trenholm, 2018, p. 147). These components were refined and adapted according to the pre-service teachers' understanding (see Figure 2). These final versions of the themes pertained to the pre-service teachers' understanding of (1) Knowing how to sequence instructional tasks to help students solve mathematical problems and (2) knowledge of how to use locally available resources to teach representations.

Figure 2: Themes of analysis crafted by the SCK's components and refined according to the pre-service teachers' understanding



Fonte: Ball et al. (2008)

5. Findings and Discussion

5.1 Knowing how to sequence instructional task to help students solve mathematical problems

The ability to sequence instructional tasks to facilitate learning toward the lesson objective is a crucial characteristic of effective teaching (Hebert, Landin, & Solmon, 2000). Thus, this first theme is related to the knowledge needed for an effective sequence in mathematical instruction tasks. For Jenkins and Veal (2002), "effective task progression, or extension, requires the teacher to see the content through the student's eyes" (cited in Jenkins & Haefner, 2011, p. 48) and adjust the content to students' level of understanding.

In this theme, Denise and Martin acknowledged that tasks should be presented to the students in an easy-to-difficult/simple-to-complex sequence. Both pre-service teachers argued that these models are shaped by teaching practice and are referential forms to meet curricular demands and help students learn mathematics. Both Denise and Martin's understanding of the knowledge needed to carry out instructional tasks sequence are similar, both characterized from different angles.

The case of Denise. In the IM individual interview, Denise expressed a broader view of the relevance of the knowledge needed to carry out task progression: "The teacher needs to know how to teach from simple to complex, so everyone [students] can learn and use it [the content]." However, after Denise's teaching experience, she acknowledged that "teaching mathematics from simple to complex does not mean dividing the lessons into sections and that's it. It means to think about it in a way learners can understand the content gradually . . . so they can solve problems step-by-step" (Denise, SM post-lesson interview). During the TM focus group discussion, she explained that organizing tasks for students in mathematical lessons should focus on "helping learners to see the steps as a whole. . . . They [students] need to understand that, if they cannot solve a problem at once, they should divide the problem into parts, so that they can solve it one by one. Later, they put everything together as a whole. . . . When you give, for example, a problem like two plus three multiplied by four minus three, it can be very confusing for the learners. They can first solve two plus three, and then four minus three. Later, they can solve the multiplication and so on."

Excerpt I complements these preliminary findings and shows how Denise's understanding of the knowledge needed to sequence instructional tasks has evolved (highlighted in bold) by the end of the program. Numbers on the left column of the except helps to identify the traces of this development.

Excerpt I. Denise's statements during the TM focus group discussion.

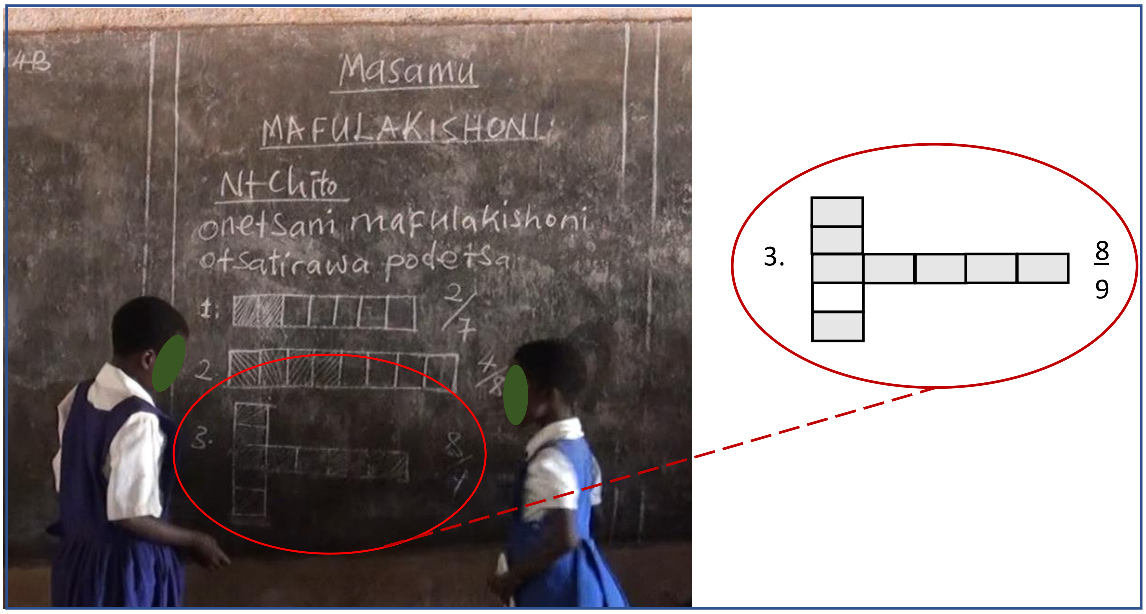
	Interlocutor	Speech
1	Researcher:	Do you think your idea about teaching from simple-to-complex has changed since we first met?
2	Denise:	Yes, it has changed a lot! In the beginning, I tried to do what my secondary teacher did, you know . . . but now, I think it is not so easy. It is more than using simple words. You should think about how the learner will receive the message. Is there too much information here? How can we make it easy for them to understand it? Should they learn this first? How about or this? A good teacher must know it because it is what the curriculum tells us to do.
3	Researcher:	Ok, you gave the example of two plus three multiplied by four minus three. This is supposed to be taught in a higher class, right? Why did you give this example when I asked you about teaching from simple-to-complex?
4	Denise:	Yes, that example was from a much more advanced level. We used to teach it in Standard 5 [5th grade] and 6 [6th grade]. . . . I used it because teaching from simple-to-complex is a very effective method. But of course it can be done in many ways, you know . . . . For me, the teacher has to think about what is best for the students. It is not only about dividing the lessons into segments. They should know why you are doing it. The learners should also keep in mind what is the main goal and why you are dividing it, why you are doing it step-by-step.

Denise's responses in Excerpt 1 illustrate how her views changed as she progressed through her teacher education. She provided insights into a situation from her teaching practice, where she had to teach multiplication of numbers by three. Then, she argued that teachers should think about how learners acquire the content (Line 2 in Excerpt I) and acknowledged that teaching from simple to complex is not a simple task

(Line 2 in Excerpt I). An effective teacher should be capable of making content gradually accessible to the students, allowing them to solve mathematical problems, she explains (Line 2 in Excerpt I). This view aligns with a direct teaching approach that includes instructional design and alignments with students' learning needs. In this context, a teacher can break down the content into meaningful and appropriate segments to assist students' understanding of the mathematical content (Line 4 in Excerpt I). The knowledge needed to carry out this task seems to include elements of an explicit form of mathematics instruction that is intentionally and logically organized (Archer & Hughes, 2011). Such knowledge manifests as systematic expertise with the emphasis on proceeding in small steps and checking for students' understanding, with the goal of promoting their problem-solving skills and reasoning (Line 4 in Excerpt I).

The case of Martin. Another pre-service teacher who demonstrated a new form of understanding of the knowledge needed to sequence instructional tasks was Martin. In his first interview as a part of the IM, he asserted that "as the teachers' guide already tells how the content should be taught, there is no need for the teacher to do much about it. . . . It is very important to follow the guide." However, Martin's view changed as he gained practical experience teaching mathematics. In his SM's post-lesson interview, Martin acknowledged, "I saw no connection between the examples in the teachers' guide, so I had to include my own examples" (Figure 3).

Figure 3: An example of task added by Martin that was not in the school curriculum



Fonte: Jacinto (2020)

Martin's attempt to add to teachers' guide with his own examples marked a considerable shift in attitude. While this is a positive outcome, Zuya, Kwalat, and Attah (2016) cautioned that such practice might negatively influence student learning if the teacher has a low sense of mathematics teaching self-efficacy. This negative side of instructional tasks emerged in the focus group discussion during TM, when Martin observed, "The teachers' guide is very useful, but of course it cannot predict everything in the classroom. . . . Teachers need to ask themselves: 'Will this [example] help the learners to understand the problem?' Can they [learners] see the connections?' If not, the teacher should decide what can be included to make it easier for the learners." Excerpt II complements this evidence.

Excerpt II. Martin's comments during the TM focus group discussion.

	Interlocutor	Speech
1	Martin:	When I was planning the lesson, I noticed that some examples were too far from the others, so learners were having problem in understanding it [the content]. They could not see the connections between one example and others. . . . So, I started to think why they have no problem understanding this example, but they have a problem understanding this one. It is practically the same thing, but this one is slightly different. Then, I tried to help them with my own examples.
2	Researcher:	How do you create these examples?
3	Martin:	Well . . . I try to look at the content as if I am learning it for the first time, right?
4	Researcher:	Interesting!
5	Martin:	For example, if I am teaching in Standard 4, I think: 'Ok, I am a Standard 4 learner, and I already know this and this. So, what do I need to do to solve this problem?' . . . And if I see that the teachers' guide is not enough, I try to find other ways to help them to learn it.
6	Researcher:	But how do you know what they need to learn?
7	Martin:	You know . . . asking them what they already know about the content, what they find difficult. And later, somehow, you will discover what is important for them to get good grades and succeed in the final exams!

This excerpt from the TM focus group discussion illustrates that the way Martin perceives the knowledge needed to carry out task sequence evolved from IM and SM to TM. Initially, Martin believed that the teachers' guide should be strictly followed when conducting lessons in primary school classrooms. However, during teaching practice, Martin realized that the knowledge of the curriculum was insufficient to carry out teaching tasks, confirming the limitations of isolating knowledge components in teaching (Hill, Ball, & Schilling, 2008).

To carry out instructional task progression, Martin argued that the teacher needs to be able to identify and predict any problem or difficulty students might encounter during the lesson, for instance, inability to link examples provided in the teachers' guide (Line 1 in Excerpt II) and how students might recognize the similarities and differences among examples within the lessons (Lines 1 and 5, Excerpt II). Moreover, Martin opined that teachers needed to make use of the knowledge of content and students (KCS) to decide how to organize and present the tasks in the classroom (Line 3, 5, and 7, Excerpt II). Those views revealed that Martin's understanding of the knowledge needed to carry out instructional task progression at the TM was aligned with a particular SCK characteristic: "an unpacked mathematical knowledge that involves making features of particular content visible to and learnable by students" (Ball et al., 2008, p. 400).

5.2 Knowledge of how to use locally, available resources to teach mathematics using representations

Ball et al. (2008) discussed in detail the knowledge needed to represent mathematical ideas in an age-appropriate manner, a special knowledge component for teaching mathematics. Hudson and Miller (2006) explained that representations could be categorized as concrete, pictorial, or abstract. For instance, if a teacher uses a plastic cube to represent problems on a basic operation s(he) is utilizing concrete representation of the concept of operation. A picture of the plastic cube could be pictorial, while numbers or symbols could be abstract representations. Each of these categories possesses unique characteristics that contribute to the teaching and learning of mathematics.

The discussions that follow provide insights into how Mario—a pre-service teacher with experience in teaching but with no interest in mathematics—developed his understanding of the knowledge needed to teach mathematics during teacher education. He did not only acknowledge the importance of knowing about how to represent mathematics ideas but also demonstrated a unique grasp of the value of understanding, creating, and using unique forms of representation when teaching mathematics.

The case of Mario. In his IM's interview, Mario expressed an intuitive interpretation of the knowledge needed for mathematical representation: "It is very important for teachers to know how to introduce a concept. They must think of many ways of doing it so learners can learn the content. For example, you can use locally available resources to represent numbers, fractions, and operations. . . . In Malawi, it is a very effective way for larger classrooms as we have here. . . . When you show the learners how many forms a concept can take, they will see and use it to solve real-life problems." At this initial stage, Mario seemed to appreciate the value of the mathematical representation to foster students' learning of mathematics. When asked about the relevance of such knowledge to the teacher, Martin stated: "It will make teachers better prepared to teach the topics. But you know . . . if they know different strategies, they will be able to assist as many learners as they can." This is in line with the arguments put forth by Greeno and Hall (1997).

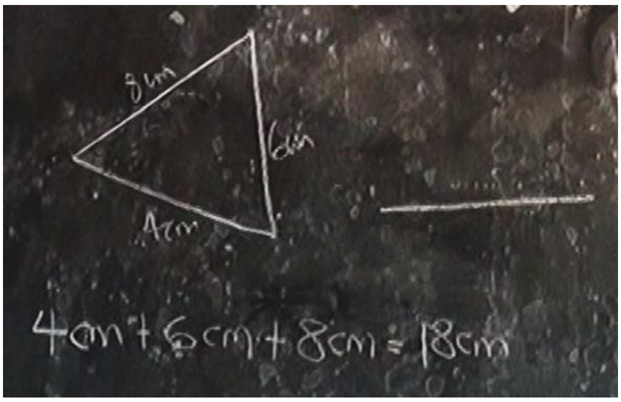
During teaching practice, Mario's understanding of the knowledge needed for representing mathematical ideas expanded to include a new aspect—the connections between mathematical representations. An example from Mario's teaching illustrates how Mario interprets a situation from which he introduces the concept of perimeter to Standard 5 students by using unique forms of representation. He first shared a story with his students about John, who was asked to calculate the total length of a bent wire. As he was telling the story, Mario used a cardboard television he had made for this purpose, intending to capture students' attention (see Figure 4).

Figure 4: Mario using a cardboard television to introduce the concept of perimeter

Fonte: Jacinto (2020)

As the story progressed, Mario actively engaged students in a discussion about John's problem. To help them find the answer, Mario drew a triangle on the blackboard with sides measuring 4, 6, and 8 cm, respectively. Below the drawing, he used these values to show how the perimeter is calculated (see Figure 4). Once the students agreed that the answer was 18 cm, Mario, using the cardboard TV, told the students that John brought the bent wire to his home, put it on the table and straightened it, so that he could measure its length. The story ended with John coming back to school and telling the teacher his findings. At the end of the lesson, Mario emphasized that the strategy for calculating the bent wire's length could be applied to other forms of objects. Then, he presented his students with a drawing of a rectangle measuring 15 × 6 cm, prompting them to calculate its perimeter. A female student went to the board, checked the rectangle dimensions, and answered 42.

Figure 5: Representations of the concept of perimeter produced by Mario



Fonte: Jacinto (2020)

During the post-lesson interview, when Mario was asked about his intentions in using multiple representations to introduce the concept of perimeter, he responded: “Many learners in Malawi don't know how to solve problems. I tried to make it [perimeter concept] meaningful, so they can think of the content in many ways and use it to solve problems” (Mario, SM’s post-lesson interview).

In Mario’s initial understanding, creating opportunities for students to interpret the concept of perimeter through multiple representations seemed to play a crucial part in the teaching knowledge. This is a unique component of mathematics teaching since concepts and procedures are enhanced in teaching to help students consider not only concrete but also pictorial and abstract representations. For Mario, “As the result is the same, the teacher can help learners see the content in different forms. In my case, I introduced perimeter in the form of a bent wire, a triangle, an expression, a straight line, and a rectangle” (Mario, SM’s post-lesson interview).

The experience of teaching perimeter in a Standard 5 classroom contributed to Mario’s understanding of what teaching mandates in terms of knowledge. Working with multiple mathematical representations gave him useful insights into students’ reasoning when solving a problem, in line with arguments put forth by Greeno and Hall (1997). Mario’s understanding of the knowledge needed to use representation in mathematics teaching evolved beyond what he was taught at the TTC and what the teachers’ guide suggests for mathematical instruction. Locally, available resources were used by Mario as a complementary tool to introduce mathematical ideas. Seeing their practical value motivated Mario to design concrete activities to develop a more robust understanding of mathematics. In this sense, Mario’s knowledge of multiple representations seems to have equipped him with useful mental models of his students’ appreciation of the relationship among concrete, pictorial, and abstract mathematical representations (Adu-Gyamfi & Bossé, 2014).

In the last phase of the study (TM), Mario demonstrated new insights into the knowledge needed to carry out multiple representations in the classroom. The following excerpt shows how Mario recognized such knowledge as a useful source for promoting generalizations of mathematical concepts. The excerpt was extracted from one of Mario’s responses to the TM’s focus group discussion.

Excerpt III. Mario’s views during the TM focus group.

	Interlocutor	Speech
1	Researcher:	Mario, what are the benefits for a primary school teacher in knowing different forms of representing a mathematical idea?
2	Mario:	Teachers can explore the connections between these different forms and their use in the classroom. They can think in new ways of teaching the content . . . For example, when I was teaching the concept of perimeter in Standard 5, I tried to create a situation that they [learners] could study the concept from different perspectives. So, if a learner does not understand one way, he or she might understand the other. […] The main character, John, was asked to measure a bent wire’s total length.

		Later there was a triangle . . . then, he made a straight line of that triangle and then he calculated the sizes . . . many ways of thinking about the perimeter.
3	Researcher:	So, you moved it from simple to complex, right?
4	Mario:	Yes, I gave it from concrete to abstract!
5	Researcher:	Why? You could just follow the teachers' guide, right?
6	Mario:	Yes, I could just follow the teachers' guide, but I wanted them to understand what perimeter means and why it is important. So I used the teachers' guide and other ideas to introduce the concept of perimeter. I created it and I think it was different**!**
7	Researcher:	But what about the 'proof'? You also included it in the story, and later, you showed them a different object, a rectangle.
8	Mario:	Yes, I included proof in the story because I wanted them to make sure they have the right answer so they can avoid mistakes. And the rectangle, I included it so they can generalize it to other forms, not only a triangle! In Malawi, it is very common for children to be asked about distances between places or areas of land. It somehow includes the idea of the perimeter, so it is very important for a teacher not to give the definition right away. Otherwise, he or she will confuse them.

Excerpt III indicates that, in Mario's view, the connections between different forms of mathematical representations help teachers think about their lessons from different perspectives (Line 2 in Excerpt III). By exploring similarities and differences among representations, teachers can introduce non-standard situations to prompt students to think about the connections between different forms of the same concept (Line 2 in Excerpt III). While teachers benefit from gaining knowledge about principles and relations between various forms of the subject matter, students benefit from learning different approaches that might help them better understand mathematics. For Mario, such knowledge about connections among representations is essential for effective teaching that gives originality to lessons, a component that goes beyond just knowing the content and curriculum (Line 6 in Excerpt III).

Mario's understanding of the knowledge about mathematical representation also seems to be driven by the specific purposes of promoting generalizations (Line 2, 6, and 8, Excerpt III). One of these aspects is the meaning production of representations (Line 6 in Excerpt III). Such knowledge builds bridges from teachers' personal representations to a more conventional one (Line 2 and 8, Excerpt III), contributing to improving teaching (Tillema, 1998). These approaches can foster generalizations of mathematical concepts, which is crucial when children learn abstract concepts (Vygotsky, 2000).

6. Conclusions and Implications

As SCK has been widely recognized as essential for teacher education, there is a great need to comprehend how pre-service teachers acquire and employ SCK in mathematics teaching. This article contributes to this body of literature that examines the views and understanding of primary pre-service teachers in Malawi about the knowledge needed to carry out mathematics teaching tasks. Two themes were generated from the SCK items and the data: the knowledge of instructional task sequence and how to use locally available to teach mathematics using representations. The findings revealed that pre-service teachers developed a nuanced and well-articulated understanding of what is needed to carry out instructional task sequence and mathematical representations in Malawi. The results also show that pre-service teachers gradually develop their understanding as they progressed through the teacher education program. This development process seemed to be associated with the need to attend to curricular demands and make sense of teaching in contexts of adversity.

The ways pre-service teachers developed their understanding of mathematical knowledge for teaching defined their maturation and professional development during teacher education. Although all three pre-service teachers demonstrated different forms of understanding, the first two cases shared a similar feature indicating a tendency to see teaching knowledge as a support and reference to meet curricular requirements and satisfy students' needs. The third case, however, associated the idea of teacher knowledge as a component that helps teachers have a broader sense of teaching purposes. These three cases revealed which aspects of pre-service teachers' education can contribute to a cognitive achievement of theoretical constructs and work of teaching. Moreover, shedding light on how pre-service teachers understand the teaching tasks and the knowledge needed to handle these tasks contributes to a better understanding of the link between theory and teacher education.

From this present work, teacher educators can use the data and insights to introduce discussions about the changing nature of the knowledge demanded for teaching, aiming to equip pre-service teachers with proper skills to ensure that they continue learning and adapting to the rapidly changing world. Thus, more longitudinal studies are needed to integrate theory and practice and whether theoretical constructs help pre-service teachers learn how to carry out the work of teaching more effectively. Knowing what teaching is and how it works might change how teaching and teacher education should be approached in challenging contexts.

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