Validation of the Teaching Mathematical Modeling Self-Efficacy Scale (TMMSS)

Validação da Escala de Autoeficácia de Modelagem Matemática de Ensino

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Abstract

Mathematical modeling has many possible benefits for students when implemented in K–12 mathematics instruction. When teachers have positive self-efficacy for mathematical modeling it is more likely they will continue to implement it. In order to measure teacher's self-efficacy in the context of teaching mathematical modeling the development of an instrument specific to mathematical modeling is needed. The purpose of this study was to develop and validate the Teaching Mathematical Modeling Self-Efficacy Scale (TMMSS) as a tool for measuring perceived teacher preparedness for implementing mathematical modeling. The TMMSS was constructed through a thorough review of the literature regarding K–12 mathematical modeling and prior research on the development of teacher self-efficacy instruments. The participants for this study were from the USA. The TMMSS can be useful for evaluation of mathematical modeling professional development and school-based mathematical modeling research.

Keywords: K–12 Teachers; Mathematical Modeling; Self-Efficacy

Resumo

A modelagem matemática tem muitos benefícios possíveis para os alunos quando implementada no ensino de matemática do ensino fundamental e médio. Quando os professores têm autoeficácia positiva para modelagem matemática, é mais provável que continuem a implementá-la. A fim de medir a autoeficácia do professor no contexto do ensino de modelagem matemática, é necessário o desenvolvimento de um instrumento específico para modelagem matemática. O objetivo deste estudo foi desenvolver e validar a Escala de Autoeficácia de Modelagem Matemática de Ensino (TMMSS) como uma ferramenta para medir a percepção do preparo do professor para a implementação da modelagem matemática. O TMMSS foi construído por meio de uma revisão completa da literatura sobre modelagem matemática do ensino fundamental e médio e pesquisas anteriores sobre o desenvolvimento de instrumentos de autoeficácia do professor. Os participantes deste estudo eram dos estados unidos da américa.

O TMMSS pode ser útil para avaliação do desenvolvimento profissional de modelagem matemática e pesquisa de modelagem matemática baseada na escola.

Palavras-chave: Auto-eficácia; Modelagem Matemática; Professores do Ensino Fundamental e Médio.

1. Introduction

Internationally mathematical modeling is an important topic that teachers should have confidence in implementing (Stohlmann, Maiorca & Allen, 2017). Germany includes mathematical modeling as one of six compulsory competencies (Blum & Borromeo Ferri, 2009). Mathematical modeling has a long history of implementation in Brazil (Souza, de Almeida, & Klüber, 2018). In the United States, mathematical modeling is one of eight Standards for Mathematical Practice (Common Core State Standards for Mathematics [CCSSM], 2010). Australia has mathematical modeling included as part of the key ideas that students should know in the National Mathematics Curriculum (Australian Curriculum, Assessment and Reporting Authority, 2018).

There are many benefits of participating in mathematical modelling including developing mathematical understandings (Brown & Edwards, 2011; Lesh & Carmona, 2003), coming to appreciate mathematics more and see it as more applicable to real life (Kaiser & Schwarz, 2006; MaaB, 2010), and developing communication and life skills (English, 2006; Stohlmann, 2013). In order for these benefits to be realized, teachers need to feel confident in their abilities to implement mathematical modeling effectively.

Teacher self-efficacy measures a teacher's beliefs in his or her ability to achieve specific goals (Bandura, 1997), and this construct has been linked to important student and teacher outcomes. Multiple studies have demonstrated a significant relationship between teacher self-efficacy and student achievement (e.g., Ashton & Webb, 1986; Borko & Whitcomb, 2008). Self-efficacy has also been shown to influence teachers' instructional practices (e.g., Brown, 2005; Holzberger, Philipp, & Kunter, 2013; Tschannen-Moran & McMaster, 2009) and what they are willing to implement in their practice (e.g., Nie, Tan, Liau, Lau, & Chua, 2013; Stein & Wang, 1988).

Investigating teacher's self-efficacy about teaching mathematical modeling is important because there are still a large number of teachers who do not implement mathematical modeling (Ng, 2013), which could because of low-self efficacy for teaching mathematical modeling. Therefore, a self-efficacy instrument specific to mathematical modeling can be a useful tool to measure perceived teacher preparedness for mathematical modeling. There are currently several self-efficacy instruments, but to effectively measure self-efficacy the instrument should be domain and context specific (Bandura, 1997). In order to more adequately evaluate teachers' beliefs in their abilities to implement mathematical modeling, an instrument for measuring teacher self-efficacy for mathematical modeling needs to be developed. The purpose of this paper is to describe the development and initial validation of the Teaching Mathematical Modeling Self-Efficacy Scale (TMMSS) for K–12 teachers. We define mathematical modeling self-efficacy as a teacher's personal belief in his or her ability to positively affect students' learning of mathematics through mathematical modeling.

2. Self-Efficacy

The concept of self-efficacy was introduced in Bandura's (1977) theory of social learning, and has been an important topic in education ever since. He defined it as one's personal belief about one's capability to take an action toward an attainment (Bandura, 1977). Teacher self-efficacy has drawn interest from researchers because of findings that indicate its direct relationship with teachers' classroom practices, which in turn have a direct influence on students' understanding (Holzberger et al., 2013; Muijis & Reynolds, 2002). For example, Holzberger et al. (2013) found that high-efficacy teachers demonstrated higher instructional quality whether instruction was rated by teachers themselves or by their students.

Self-efficacy can affect teachers' personal goals, amount of effort, and persistence in teaching students. Teachers' practices are affected by their self-efficacy and the outcome of teaching then shapes the foundation of new sources of self-efficacy (Tschannen-Moran et al., 1998). Teacher self-efficacy can vary by subject, teaching practice, and environment. Thus, teacher self-efficacy varies by context and must be defined appropriately (Yoon, Evans, & Strobel, 2014). Since mathematical modeling is one part of effective mathematics teaching, it is important to determine self-efficacy specifically for mathematical modeling.

Multiple teacher self-efficacy instruments have been developed, validated, and utilized for various purposes in education. Tschannen-Moran et al. (1998) note that general self-efficacy instruments are not as useful tools for assessing aspects of context specific self-efficacy. This provides support for the development of more context specific tools. Riggs and Enochs (1990) developed their Science Teaching Efficacy Belief Instrument (STEBI) to measure self-efficacy specifically for teaching science at the elementary grade level. The STEBI was designed to measure two constructs, outcome expectancy and self-efficacy, on the basis of Bandura's (1997) theoretical claim that behaviors are affected by both personal expectancy about the outcome and personal beliefs about teaching. Since the development of the STEBI, several variants have been developed and tested, each calibrated for a specific content area and different target population.

Wilhelm and Berebitsky (2019) developed and validated the Mathematics Teacher Sense of Efficacy Scale (MTSES) based on adapting the more general teacher self-efficacy instrument, the Teachers' Sense of Efficacy Scale (TSES) (Tschannen-Moran & Hoy, 2001). They noted that the items in the TSES are aligned with the theoretical framework of social cognitive theory and the work of Bandura (Bandura, 1997). The TSES had validity and reliability that had been demonstrated but Wilhelm and Berebitsky wanted to adapt the survey for mathematics self-efficacy specifically while keeping many of the items similar. The researchers inserted the word "mathematics" into the items and also made some small phrasing changes. The researchers argued that the demonstrated validity of the TSES provided support for the validity of the MTSES given their similar items and the alignment of the scale with social cognitive theory.

Given the increased implementation of engineering education in K–12 education, Yoon et al. (2014) developed the Teaching Engineering Self-Efficacy Scale (TESS) for K–12 teachers based on previous instruments. The researchers considered the following instruments in the development of the TESS: Teacher Efficacy Scale (Gibson & Dembo, 1984), STEBI (Riggs & Enochs, 1990), Teacher Self-Efficacy Scale (Bandura, 2006), TSES (Tschannen-Moran & Hoy, 2001), and Teaching Technology Self-Efficacy (Teo, 2009). The reviewed instruments provided the researchers ideas on items and also possible factor structures for the TESS. The validation of the TESSS was demonstrated through exploratory and confirmatory factor

analyses using structural equation modeling and high Cronbach's alpha scores (Yoon et al., 2014).

In summary, the context-specific instruments mentioned above were developed on the basis of previous instruments. The types of constructs, the total number of items, and the phrasing of the statements in each item were tailored to fit the content and population targeted in each instrument. These modifications were necessary in order to address self-efficacy appropriately in the particular teaching context.

3. Self-Efficacy for Teaching Mathematics

Self-efficacy for teacher mathematics is a teacher's personal belief in his or her ability to positively affect students' learning of mathematics. Helping teachers enhance their beliefs about their ability to teach mathematics is an important topic (Huinker & Madison, 1997) as mathematics teacher beliefs and practice are related (Beswick, 2012). Mathematics teachers who have higher levels of self-efficacy are more likely to overcome challenges in teaching by persisting in their efforts to improve (Huinker & Madison, 1997). Teachers with high teaching efficacy are more likely to try new teaching strategies, particularly techniques that may be difficult to implement and involve more student-centered learning (Swars, Daane, & Giesen, 2010).

Research has investigated what affects mathematics teachers' self-efficacy. Swars, Daane, & Giesen (2010) found a moderate negative relationship between mathematics anxiety and mathematics teacher efficacy of elementary teachers. In general, the teachers with the lowest degree of mathematics anxiety had the highest level of mathematics teacher efficacy. It has also been found that elementary teachers' efficacy for teaching science and mathematics is lower compared with other subject areas (Bass, 2010). Bandura (1996) asserted that efficacy beliefs are primarily shaped by an individual's previous performance and experiences. Support for this has been found in that elementary teachers with low self-efficacy for teaching mathematics have reported past negative experiences with mathematics (Swars, 2005). At the secondary level, mathematics teachers' ratings of their classroom management and students' feeling that the mathematics required cognitive activation predicted teachers' subsequent self-efficacy (Holzberger, Philipp, & Kunter, 2013). Various ideas have been found to increase mathematics teachers' self-efficacy including positive experiences participating in mathematics, observing and discussing effective mathematics teaching, and positive teaching experiences (Huinker & Madison, 1997). The research demonstrates the importance of self-efficacy for teaching mathematics well in order for mathematics teachers to continue to improve and focus on effective teaching practices for mathematics.

4. Mathematical Modeling

The CCSSM (2010) state that, "Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace" (p.7). Specifically the CCSSM (2010) states modeling as "the process of choosing and using appropriate mathematics and statistics to analyze empirical situations, to understand them better, and to improve decisions" (p.72). Also highlighted for modeling is the importance of making assumptions and approximations, revisions, identifying important information, interpreting mathematical results in the context of the situation, and reflection. Our definition is aligned with this description of mathematical modeling in the CCSSM. "Mathematical modeling is an iterative process that involves open-ended, real world, practical problems that

students make sense of with mathematics using assumptions, approximations, and multiple representations" (Stohlmann & Albarracín, 2016, p.2).

Prior work with teachers in developing their understanding of mathematical modeling and how to implement it has identified several important topics. The Learning and Education in and through Modelling and Applications (LEMA) was a project that involved mathematics educators and researchers from six countries working together on a set of professional development materials for primary and secondary school teachers. The LEMA professional development materials focused on understanding mathematical modeling and why it is important, mathematical modeling tasks, implementing mathematical modeling, and assessment of students' work on mathematical modeling (Garcia & Ruiz-Higueras, 2011). In regards to tasks, teachers learned about selecting or creating appropriate tasks for their students; as well how to anticipate multiple student solutions on the tasks including difficulties and mistakes. Teachers learned about how to structure mathematical modeling, how to support students while working, how to give helpful feedback, how to monitor students while working, and how to facilitate a discussion of students' ideas. Teachers also learned how to assess students' work and to incorporate mathematical modeling as a formative or summative assessment (Garcia & Ruiz-Higueras, 2011). Blum (2015) describes additional items for implementing mathematical modeling well including an effective student-oriented classroom management approach and not to be overly directive on students' solution development. Others echo these ideas including facilitating discussion, providing strategic guidance, asking good questions that build on students' ideas, and planning for the diversity of approaches that students might take with modeling tasks (Burkhardt & Pollack, 2006; Doerr, 2007).

Mathematical modeling has moved, and will continue to move, to the forefront of K–12 mathematics education (Sole, 2013). Mathematical modeling can help students to analyze, reason, and communicate ideas effectively. Research has shown that mathematical modeling increases students' interest in mathematics, students' mathematical understanding, problem solving, technological literacy, communication and teamwork skills, and allows students to apply knowledge from other content areas (Biembengut & Hein, 2007).

Given the importance of mathematical modeling there is a need for the development of an instrument to measure teachers' self-efficacy in teaching K–12 mathematical modeling. To the best of our knowledge there is one current instrument for self-efficacy related to mathematical modeling, the self-efficacy scale for mathematical modeling competencies (Koyuncu, Guzeller, & Akyuz, 2017). However, this scale measures self-efficacy in regards to teachers' beliefs in their ability to successfully participate in mathematical modeling themselves. There is also a Mathematical Modeling Attitude Scale (Asempapa, 2018) that measures teachers' feelings toward mathematical modeling in regards to constructivism, understanding, relevance and real life, and motivation and interest. This instrument does not look at a teacher's personal belief to positively affect student learning through mathematical modeling though.

The importance of a self-efficacy instrument specific for teaching mathematical modeling is based on three main points. First, it has been found that self-efficacy instruments are context and domain specific (Bandura, 1997; Pajares, 1997; Wang & Pape, 2007). Second, prior research has found that teachers' self-efficacy differs by subject and environment (Bass, 2010; Midgley, Anderman, & Hicks, 1995). Teacher's self-efficacy can also be affected by implementation of tasks and the perceived outcomes of these tasks (Tschannen-Moran et al., 1998). This is important to investigate for mathematical modeling implementation. Third,

within mathematics education, mathematical modeling is a separate research line (Burkhardt & Pollack, 2006; Doerr & Lesh, 2011). The modeling process is specific to mathematical modeling and not other aspects of mathematics instruction (Blum & LeiB, 2007; Lesh & Doerr, 2003). Similarly, while engineering is usually integrated in science education, engineering has the engineering design process and a separate self-efficacy instrument has been developed (Yoon et al., 2014). Mathematical modeling has received increased focus in the last decade but there are current teachers who have misconceptions about what mathematical modeling is and how to implement it (Gould, 2013). This necessitates the importance of professional development for mathematical modeling and ways to assess outcomes from these professional development experiences (Stohlmann, 2019). Self-efficacy is an important construct that provides valuable information for understanding the development of mathematics teaching expertise (Holzberger et al., 2013).

There is a need then for an instrument focused on self-efficacy for teaching mathematical modeling. This instrument can be beneficial for researchers and practitioners who are involved in teacher education or professional development programs to measure the effects of mathematical modeling programs on teachers' self-efficacy for teaching mathematical modeling.

5. Methods

5.1 Instrument Development

We undertook several steps to develop an instrument to measure teachers' self-efficacy for teaching mathematical modeling by following recommendations set for developing self-efficacy instruments (Bandura, 2006; Tschannen-Moran et al., 1998). First, we reviewed studies that reported processes for developing new teacher self-efficacy instruments (Koyuncu et al., 2017; Wilhelm & Berebitsky 2019; Yoon et al., 2014). As mentioned previously, these studies made use of existing instruments in their development process. A 6-point Likert-type scale (strongly disagree, moderately disagree, disagree slightly more than agree, agree slightly more than disagree, moderately agree, and strongly agree) was chosen to be used. This decision was made following the recommendation of Boone, Townsend, and Staver (2010). Through reliability and Rasch analyses, they showed that the six-point response option, provided better measurement properties than four or five point response scales.

Second, we drew on reviews of research that the first author had done on K–12 mathematical modeling education (Stohlmann & Albarracín, 2016; Stohlmann et al., 2016) as background knowledge; as well as prior research on developing teachers understanding and ability to implement mathematical modeling (Blum, 2015; Garcia & Ruiz-Higueras, 2011). We found that the factors from the Teaching Engineering Self-Efficacy Scale (Yoon et al., 2014) connected well with the prior work in mathematical modeling. The initial five factors from this instrument were used with wording changes from engineering to mathematical modeling. Because "there is no all-purpose measure of perceived self-efficacy" (Bandura, 2006, p. 307), various aspects of teaching mathematical modeling are included.

Mathematical modeling content knowledge self-efficacy- teachers' personal belief in their knowledge of mathematical modeling that will be useful in a teaching context.

Mathematical modeling planning self-efficacy- teachers' personal belief in their ability to identify and select appropriate mathematical modeling tasks to use.

Instructional self-efficacy- teachers' personal belief in their ability to teach mathematical modeling to facilitate student learning.

Management self-efficacy- teachers' personal belief in their ability to cope with a wide range of student behaviors during mathematical modeling activities.

Outcome expectancy- teachers' personal belief in the effect of teaching on students' learning of mathematics through mathematical modeling.

Third, we modified existing items from self-efficacy instruments, in particular the TESS, to situate them in the context of teaching mathematical modeling. In the TMMSS all items are positively worded (e.g., "I can" instead of "I can't"). This was done because prior research found that negatively worded items can cause confusion for respondents (Netemeyer, Bearden, & Sharma, 2003) and distort reliability and validity evidence (Herche & Engellend, 1996). In total, there were 36 items grouped under the five factors.

Fourth, cognitive interviews (Desimone & Le Floch, 2004) were done with teachers in order to reword any items to improve clarity. The interviews were also used to identify items that were considered redundant. The interviews led to twenty-six items total being chosen for the five factors to be used for the TMMSS in this study.

To summarize, the evidence for validity was based on three points from the Standards for Educational and Psychological Testing (American Educational Research Association, American Psychological Association, & National Council on Measurement in Education, 2014). First, evidence of validity of test content can be found in the organized way in which the TMMSS was created and revised. The TMMSS was created to align with Bandura's social cognitive theory (Bandura, 1977) by using relevant literature as a basis and also using previously published instruments as a guide (McGee & Wang, 2014). Researchers have noted that self-efficacy instruments based on a prior valid instrument and aligned with relevant research, provide support for validity (McGee & Wang, 2014; Wenner, 2001; Wilhelm & Berebitsky, 2019). The TMMSS was also developed with the background knowledge of a thorough review of the literature on mathematical modeling. Also, the first author is an expert in mathematical modeling and judged the content validity of the instrument based on his own work and the prior research work of experts in mathematical modeling. Second, validity evidence based on response processes was investigated by using cognitive interviews with teachers (Desimone & Le Floch, 2004). Third, validity evidence based on internal structure was done through the use of factor analysis to determine the underlying factor structure of the TMMSS.

5.2 Sample and Procedure

The sample used to validate the TMMSS was K–12 teachers who teach mathematics. A snow-ball technique was used to maximize the spread of recruitment. Recruitment e-mail messages were sent to teachers that the first author had worked with in professional development settings. These teachers were also asked to forward the recruitment message to appropriate colleagues. Because of this, we were unable to determine the number of teachers the recruitment reached or a response rate.

Teachers completed the TMMSS through Qualtrics, a web-based survey program, and also provided demographic information through Qualtrics. Sixty-three participants completed the TMMSS, with a total of 59 participants completing all of the questions. The sample size of 59 was deemed appropriate because this was a new measurement scale and there is little

discussion in the existing literature for how to determine appropriate samples for pilot studies in scale development (Converse & Presser, 1986; Johanson & Brooks, 2010; Winter, Dodou, & Wieringa, 2009). A recent study focused on initial validation of a survey scale had a similar sample size (Asempapa, 2018).

The sample of teachers included 43 females and 16 males. The sample of teachers being seventy-three percent female is close to the average for the USA where approximately seventy-six percent of K–12th teachers are female (National Center for Education Statistics, 2021). Sixty-eight percent of the teachers were white, eleven percent were Hispanic, ten percent were Asian, six percent were black, and six percent were multi-racial. Forty-two percent of the teachers had 1 to 2 years of teaching experience and 41% of the teachers had 6 or more years of teaching experience. Sixty-one percent of the teachers were at the K–5th grade level, 17% at the middle school level, and 22% at the high school level. Eighty percent of the teachers taught in urban schools. Sixty-seven percent of the teachers had some professional development experience in regards to mathematical modeling. These teachers had experience with mathematical modeling in a formal professional development setting or in a university course.

5.3 Data Analysis

Exploratory factor analysis (EFA) and Cronbach's alpha reliability analysis were used to demonstrate the usefulness and reliability of the scale items. An exploratory factor analysis (EFA) was conducted to investigate the underlying factor structures of the instrument and to identify irrelevant items that did not fit into any factors (Yoon et al., 2014). Cronbach's coefficient alpha was used in this study to measure the internal consistency reliability of the scale; one of the common forms of reliability mostly used in social science research (Cronbach, 1951).

6. Results

EFA was used to determine the interrelationships among items and examine the internal structure of the items on the TMMSS. Since the data are ordered categorical variables, polychoric correlations coefficients among the 26 items were calculated. The correlation matrix indicated that the coefficients were all positively correlated; this correlation result meant that putative factors identified through an EFA are not independent. Since no correlations exceeded .85, multicollinearity was not observed, and hence no two items measure the same aspect of a construct, and each item contributes to a unique aspect of a factor. The two criteria used to extract the number of factors underlying the data included the point of inflection of the curve in the scree plot (Cattell, 1966) and the number of eigenvalues greater than one (Kaiser, 1960). Following Kaiser's (1960) criteria, we retained factors with eigenvalues greater than one. This factor extraction yielded three factors considered for inclusion in a factor structure for the TMMSS instrument. The Kaiser-Meyer-Olkin (KMO) measure of sampling adequacy of 0.87 was acceptable and supported factor analysis (Kaiser, 1974). In addition, Bartlett's test was statistically significant ($p < 0.05$). Thus, applying common factor analysis was appropriate to this data set.

Once a factor structure for the TMSSS was identified, the factor loadings of the items were checked for each factor to decide which items constitute which factors. On the basis of Stevens' (2002) guideline about the relationship between the sample size and cutoff factor loading, items with a factor loading greater than 0.40 were considered significant for the designated factor. This cutoff functioned to suppress as irrelevant any items that did not fit well

into the designated factor. If an item loaded onto more than one factor, then the item was excluded. Factor analysis revealed three independent factor structures with eigenvalues 8.4, 1.7, and 1.4, respectively for the factors.

The analysis resulted in 15 items, out of the original 26, that fit into one of three factors (Table 1). All 15 items had significant factor loadings onto one of three factors. In other words, each item had a unique contribution to one of the factors. The three-factor structure accounted for about 75.16% of total variance in the data set. The first 8 items, clustered on Factor 1, were related to mathematical modeling pedagogical content knowledge self-efficacy, which is a new factor. The items in this factor were items from the original factors of content knowledge self-efficacy, planning self-efficacy, and instructional self-efficacy. We described Factor 1 as pedagogical content knowledge self-efficacy as the items relate to mathematical modeling content knowledge instruction related to students, teaching, and curriculum. The four items on factor 2 were associated with management self-efficacy, and the three items on factor 3 were associated with outcome expectancy. Following the factor analysis, additional reliability analysis was conducted for each of the three factors using Cronbach's alpha coefficients. The eight items under factor 1 had a reliability of 0.93, the four items under factor 2 had a reliability of 0.96, and the three items under factor 3 had a reliability of 0.81. The reliability analysis indicated that the factor's Cronbach's alpha ranged between 0.81 and 0.96 with an overall scale coefficient of 0.94. All items were worthy of retention because removal of any item would not increase Cronbach's alpha for any factor.

Table 1

Factor structure and loadings of the 15 items on the TMMSS

Item	Factor loading
Factor 1: Mathematical modeling pedagogical content knowledge self-efficacy (Cronbach's alpha: 0.93) (56.03% of variance explained)	
1. I can explain what mathematical modeling entails	0.838
2. I can recognize and appreciate how mathematical modeling is connected to other subjects	0.819
3. I can identify and select appropriate tasks for mathematical modeling for my students	0.776
4. I can discuss how assumptions and approximations affect the outcome of a mathematical modeling solution.	0.758
5. I can facilitate my students' solution development with the mathematical modeling process.	0.718
6. I can gauge student comprehension of the mathematics in mathematical modeling I have taught.	0.694
7. I can assess my students' mathematical modeling solutions.	0.692
8. I can encourage my students to have productive struggle when engaged in mathematical modeling.	0.692
Factor 2: Mathematical modeling management knowledge self-efficacy (Cronbach's alpha: 0.96) (11.05% of variance explained)	
	0.891

9. I can calm a student who is disruptive or noisy during mathematical modeling.

10. I can get through to students with behavior problems while teaching mathematical modeling. 0.886

11. I can keep a few problem students from ruining an entire mathematical modeling activity. 0.873

12. I can control disruptive behavior in my classroom during mathematical modeling activities. 0.859

Factor 3: Mathematical modeling outcome expectancy
(Cronbach's alpha: 0.81)
(8.08% of variance explained)

13. When my students do better than usual in mathematical modeling, it is often because I exerted a little extra effort. 0.816

14. If I increase my effort in teaching mathematical modeling, I see significant change in students' mathematics achievement. 0.771

15. I am generally responsible for my students' achievements in mathematics. 0.759

7. Discussion

The purpose of this study was to develop and validate the Teaching Mathematical Modeling Self-Efficacy Scale (TMMSS) in order to provide an instrument to measure K–12 teachers' self-efficacy for teaching mathematical modeling. Teacher self-efficacy varies by context and must be defined appropriately (Yoon, Evans, & Strobel, 2014). Since mathematical modeling is one part of effective mathematics teaching, it is important to determine self-efficacy specifically for mathematical modeling.

The TMMSS has three factors: mathematical modeling pedagogical content knowledge self-efficacy, mathematical modeling management knowledge self-efficacy, and mathematical modeling outcome expectancy. Prior research noted that teachers need to understand mathematical modeling and why it is important, how to select appropriate mathematical modeling tasks, how to implement mathematical modeling, how to assess students' work on mathematical modeling, and use a student-oriented classroom management approach (Blum, 2015; Garcia & Ruiz-Higueras, 2011). The final version of the TMMSS has items connected with all of these ideas. Ideas connected to planning, facilitating, assessing, and understanding mathematical modeling are in the mathematical modeling pedagogical content knowledge self-efficacy factor. The factor of mathematical modeling management knowledge self-efficacy involves classroom management items. The third factor, outcome expectancy, is connected to teachers believing in the value of mathematical modeling and its positive benefits. Table 2 contains the definition for each construct and the overall definition for the teaching mathematical modeling self-efficacy (TMSS) construct. Appendix A has the final version of the TMMSS, with 15 items listed in order of the constructs to aid a logical flow of thought as participants respond to the survey. Directions for scoring the TMMSS follow in Appendix B.

Table 2

Constructs of the Teaching Mathematical Modeling Self-Efficacy Scale (TMMSS)

Construct	Abbreviation	Definition

Mathematical modeling pedagogical content knowledge self-efficacy	PC	Teachers' personal belief in their ability to teach mathematical modeling to facilitate student learning, based on knowledge of mathematical modeling that will be useful in a teaching context.
Mathematical modeling management knowledge self-efficacy	MS	Teachers' personal belief in their ability to cope with a wide range of student behaviors during mathematical modeling activities.
Mathematical modeling outcome expectancy	OE	Teachers' personal belief in the affect of teaching on students' learning of mathematics through mathematical modeling.
Teaching mathematical modeling self-efficacy	TMMS	Teachers' personal belief in their ability to positively affect students' learning of mathematical modeling that reflects a multifaceted nature of self-efficacy of teaching mathematical modeling.

The TMMSS was validated with a population of in-service teachers who mainly taught in urban schools in the United States. Future work could investigate the validity of the instrument with other populations as well as utilizing confirmatory factor analysis. Validity evidence based on relations to other variables could also be investigated. This involves investigating relations to other variables established in previous studies. For example, it has been found that in comparisons between veteran and novice teachers that veteran teachers who have more teaching experience tend to have higher self-efficacy (Tschannen-Moran & Woolfolk Hoy, 2007). This finding could be investigated to see if it holds true for the TMMSS as well.

People who see themselves as efficacious set challenges for themselves and are more likely to persist in their efforts to be successful (Huinker & Madison, 1997). This is important for mathematical modeling as this type of teaching is more demanding for teachers than traditional instruction (Doerr & Lesh, 2011). Professional development for mathematical modeling is needed to provide teachers with needed feedback and knowledge in order to develop understanding for implementing mathematical modeling well. This can increase teachers' self-efficacy beliefs that will make it more likely teachers will continue to implement mathematical modeling. Mathematical modeling has a focus on student thinking and this has been noted as an effective mechanism for teachers to embrace more student-centered pedagogy (Sowder, 2007).

Teacher self-efficacy can vary by subject, teaching practice, and environment. Thus, teacher self-efficacy varies by context and must be defined appropriately (Bandura, 1997). Given the many benefits of mathematical modeling there was a need for the development of an instrument to measure teachers' self-efficacy in teaching K–12 mathematical modeling. The use of the TMMSS, as a teacher self-efficacy instrument specific for the mathematical modeling teaching context, is expected to contribute to the literature on K–12 mathematical modeling education. First, the TMMSS can be used with in-service teachers in university courses or in professional development programs. The instrument allows researchers to examine teachers' beliefs upon entering such programs, and then to assess how the programs have changed them. Second, after determining the current status of teachers' self-efficacy, the measure will be beneficial in helping trainers determine the best approaches to increase the self-efficacy of teachers according to which construct area they are weakest in. For example, teachers with low efficacy in outcome expectancy may need different approaches in training from teachers with

low efficacy in classroom management. Third, researchers using the TMMSS can explore the relationship between mathematical modeling teachers' self-efficacy and students' achievement and mathematical modeling competencies. In conclusion, we expect that the TMMSS will be a useful instrument for research on K–12 mathematical modeling education.

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Appendix A

Teaching Mathematical Modeling Self-Efficacy Scale (TMMSS)

Directions: This survey contains statements about teaching mathematical modeling self-efficacy. Here, teaching mathematical modeling self-efficacy is defined as a teacher's personal belief in his or her ability to positively affect student learning of mathematical modeling. A brief definition of mathematical modeling is real world mathematics problems that have more than one possible answer that students solve with choices or assumptions. Please indicate the degree to which you agree or disagree with each statement by circling the appropriate number to the right of each statement.	**1** = Strongly Disagree **2** = Moderately Disagree **3** = Disagree slightly more than agree **4** = Agree slightly more than disagree **5** = Moderately agree **6 =** Strongly agree

1. I can explain what mathematical modeling entails.   1 2 3 4 5 6
2. I can recognize and appreciate how mathematical modeling is connected to other subjects.   1 2 3 4 5 6
3. I can identify and select appropriate tasks for mathematical modeling for my students.   1 2 3 4 5 6
4. I can discuss how assumptions and approximations affect the outcome of a mathematical modeling solution.   1 2 3 4 5 6
5. I can facilitate my students' solution development with the mathematical modeling process.   1 2 3 4 5 6
6. I can gauge student comprehension of the mathematics in mathematical modeling I have taught.   1 2 3 4 5 6
7. I can assess my students' mathematical modeling solutions.   1 2 3 4 5 6
8. I can encourage my students to have productive struggle when engaged in mathematical modeling.   1 2 3 4 5 6
9. I can calm a student who is disruptive or noisy during mathematical modeling.   1 2 3 4 5 6
10. I can get through to students with behavior problems while teaching mathematical modeling.   1 2 3 4 5 6
11. I can keep a few problem students from ruining an entire mathematical modeling activity.   1 2 3 4 5 6
12. I can control disruptive behavior in my classroom during mathematical modeling activities.   1 2 3 4 5 6

13. When my students do better than usual in mathematical modeling, it is often because I exerted a little extra effort. 1 2 3 4 5 6

14. If I increase my effort in teaching mathematical modeling, I see significant change in students' mathematics achievement. 1 2 3 4 5 6
 1 2 3 4 5 6

15. I am generally responsible for my students' achievements in mathematics.

Appendix B

Directions for Scoring the Teaching Mathematical Modeling Self-Efficacy Scale (TMMSS)

There are two ways of scoring the TMMSS: scoring a raw mean score of each construct and the overall raw score of self-efficacy in teaching mathematical modeling.

Method 1 assesses a teacher's self-efficacy in one of the three constructs that the TMMSS is designed to measure. This is done by first computing the unweighted means of a teacher's score on the items that load on each construct (subscale factor). Table B matches each item to a construct.

Table B

Constructs and corresponding items that constitute Teaching Mathematical Modeling Self-Efficacy (TMMSS).

Construct (Subscale factor)	Abbreviation	No. of items	Items
Mathematical modeling pedagogical content knowledge self-efficacy	PC	8	1–8
Mathematical modeling management self-efficacy	MS	4	9–12
Mathematical modeling outcome expectancy	OE	3	13–15

The mean PC score = (sum of scores from items 1–8)/8.

The mean MS score = (sum of scores from items 9–12)/4.

The mean OE score = (sum of scores from items 13–15)/3.

The TMMSS uses a 6-point Likert-type scale, (1 = strongly disagree, 2 = moderately disagree, 3 = disagree slightly more than agree, 4 = agree slightly more than disagree, 5 = moderately agree, and 6 = strongly agree), so the score for each item ranges from 1 to 6. Thus, the mean score for each construct has the same range.

Method 2 computes the overall self-efficacy in teaching mathematical modeling (TMMS) of a teacher. This is done by first completing Method 1 for each construct and then summing each of the scores. Thus, the maximum TMMS score is 18 and the minimum is 3.

The overall TMMS score is the sum of all the three mean construct scores.