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The Design of Statements and the (Re)Formulation and Resolution of Open Problems that Address Issues of Social Relevance with the Use of Digital Technologies in the Initial Formation of Mathematics Teachers

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ABSTRACT

This paper presents the results obtained with an investigation, in which a pair of Mathematics graduates, participants of an Extension Course, carried out the design of a statement of open problems. This investigation also addressed a theme of social relevance, in which technologies were used so that these problems were (re)formulated and solved, with the use of technological resources, by students of Basic Education. In order for this objective to be achieved, the training teachers carried out the design activities of problems with the use of digital technologies, planning of the pedagogical practice, in which these problems would be proposed, and of execution of this practice, which occurred through a pedagogical workshop. After completing these activities, the graduates had the opportunity to discuss and reflect on the experiences of designer and teacher, so that they contributed to producing knowledge about the design of open problem statements that address issues of social relevance, (re)formulation and resolution using digital technologies, and how to propose such problems.

Keywords: Design of statements, (re)formulation and problem-solving, digital technologies, initial teacher training in Mathematics.

O *Design* de Enunciados e a (Re)Formulação e Resolução de Problemas Abertos e que Abordam Temas de Relevância Social com o Uso de Tecnologias Digitais na Formação Inicial de Professores de Matemática

RESUMO

Neste trabalho apresentam-se os resultados obtidos com uma investigação, em que uma dupla de licenciandos em Matemática, participantes de um Curso de Extensão, realizaram o *design*

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de um enunciado de problemas abertos e que abordou um tema de relevância social, em que as tecnologias digitais foram utilizadas, para que tais problemas fossem (re)formulados e resolvidos, com o uso de recursos tecnológicos, por alunos da Educação Básica. Para que esse objetivo fosse atingido, os licenciandos realizaram as atividades de *design* de problemas com o uso de tecnologias digitais, de planejamento da prática pedagógica, em que esses problemas seriam propostos, e de execução dessa prática, que ocorreu por meio de uma oficina pedagógica. Após a realização dessas atividades, os licenciandos tiveram a oportunidade de discutir e refletir sobre as experiências de *designer* e professor, de modo que essas contribuíram para que produzissem conhecimentos, relativos à realização de *design* de enunciados de problemas abertos e que abordam temas de relevância social, para a (re)formulação e resolução, utilizando as tecnologias digitais, e acerca de como propor esses problemas.

Palavras-chave: *Design* de enunciados, (re)formulação e resolução de problemas, tecnologias digitais, formação inicial de professores de Matemática.

1 INTRODUCTION

The implementation of pedagogical practices, using methodological perspectives, that allows the production of mathematical knowledge associated to technological and related to social relevance, is a necessity in Basic Education since they can also contribute for the students to develop the ability to solve problems, in the school environment and beyond. For this reason, it is understood that, in the initial formation of Mathematics teachers, spaces should be offered so that the training teachers can study, discuss and reflect on such perspectives, such as the *design* of problems with the use of digital technologies, which can favor the production of knowledge and the improvement and/or development of teaching skills and abilities.

In this perspective, according to Figueiredo and Dalla Vecchia (2015), the statements of mathematical problems can be produced using technological resources and present the characteristics of an open-type problem, so that the students can make their choices and use those resources in the resolution process. Moreover, in the *design* stages of these problems, socially relevant themes can be valued in order to promote Critical Mathematics Education, and include (re) formulation and problem-solving activities, associated with the use of technological resources, among other characteristics and aspects that may be attributed (Figueiredo, 2017; Figueiredo & Groenwald, 2017, 2018).

In order to do so, it is accepted that the problems *designer* and teacher experiences, being provided and experienced by the training teachers in Mathematics, in their formative and educational process, are able to prepare them for the later realization of other *designs* of problems, which present characteristics and aspects, and pedagogical practices, in these problems are proposed to students of Basic Education (Figueiredo, 2017). In addition, such experiences can facilitate the development of the group, cooperative and/or collaborative decision-making, and the expression of creativity and innovation in these *designs* and the planning and implementation of these pedagogical practices.

Thus, it was decided to present, in this work, the cut of the results obtained with the *design* proposals, the elaboration of a lesson plan and the accomplishment of the planned pedagogical practice, which occurred by offering a workshop on Science Day,

in October 2018, at the Lutheran University of Brazil (ULBRA) in Canoas, Rio Grande do Sul. The activities were carried out by the working group denominated as graduates A and G and are part of qualitative research of the Graduate Program in Teaching Science and Mathematics (PPGECIM) of the Lutheran University of Brazil (ULBRA).

2 THEORETICAL framework

The *design* of problems with the use of digital technologies is a methodological perspective that, when used in the initial formation of Mathematics teachers, can help the training teachers to have the opportunity to exercise the roles of *designer* and teacher, discussing and reflecting and learning to elaborate their own statements and proposing them in pedagogical practices. Problem statements can be elaborated by teacher educators, so that they can be solved by the use of digital technologies by the training teachers, but when they have the opportunity to produce them, either individually or cooperatively or collaboratively, to propose them to students of Education Basic, they can learn to plan them, assigning characteristics and aspects, that allow them to develop and implement them.

In this process, the steps proposed by Figueiredo (2017), which were identified in line with those suggested by Filatro (2008) for the Design of Instructional Systems or *ISD*[1] (analysis of the need, design, development and implementation of the solution evaluation of this solution): *training of the workgroup*, the double or trio of graduates who will carry out the *design*; *analysis of the needs*, for the choice of the students that will solve the problem, the subject that will be approached, among others; *design/planning* of the utterance; *development* of each part of it; *implementation* of its first version; *evaluation* of this version; *discussion and reflection by the designers*, to make improvements or not in the statement; *and making modifications or re-design, to obtain the second version of the problem*, if so decided by the group and to obtain the final version. Still, in the course of these steps, the *storyboard* feature can be used, "[...] in the pre-production phase, [which] functions as a series of sketches (scenes) and notes that visually show how the sequence (sic) of actions must unfold" (Filatro, 2008, p.60).

In relation to the characteristics and aspects that can be attributed, the appeal to the visualization is indicated, through the images chosen and/or produced by the *designers* and that will be the own statement or will be part of it, and the production of phrases and questions, when they are read and interpreted, favour exploration, experimentation, simulation and research, among other aspects, in the search for information and data and even the choice of technological resources that contribute to the solution of problems (Figueiredo, 2017).

The statements can be planned to be closed, in which the resolution process is usually pre-determined and univocally determined, or open, that favours the exploration of contents, the valorisation of ideas and the resolver's choices (Allevato, 2008), but the

[1] *"Instructional System Design".*

choice of one of these options depends on the objectives to be achieved. However, it is understood that the open problems have potentialities, which need to be recognized by the training teachers and, when *designing* statements, using digital technologies, these become evident, since they will need to plan them to be predetermined (Figueiredo, 2017), as well as to admit multiple starting points and meet the needs of the diversity of pupils present in the same classroom (Van de Walle, 2009).

In the *design* of open problems with the use of digital technologies, subjects of social relevance can also be considered in order to contextualise them, and the mathematical and technological knowledge can be used or learned in the resolution process. Therefore, graduates must learn to choose subjects, recognising those that approach students' interests or experiences in daily life, which can value their prior knowledge and that are capable of correlating mathematical and technological knowledge in the process of solving the problems (Figueiredo, 2017). As examples, we mention the *Transversal Themes*, which are mentioned in the National Curricular Parameters (CPP), for the teaching of Mathematics in the 3rd and 4th cycles (5th to 8th grades) of Elementary Education (BRASIL, 1998): ethics, environment, sexual orientation, cultural plurality, and work and consumption. According to Olgin (2015), the Mathematics curriculum makes it possible to study topics such as Contemporary, Political-Social, Culture, Environment, Technological Knowledge, Health, Local themes and themes that involve the intra-mathematical knowledge.

In addition, teacher educators can guide them in approaching themes that can foster Critical Mathematics Education, so that utterances become a context or a setting for research, even if they present a semi-reality (Skovsmose, 2008). These scenarios are environments that contribute to the exploration, the formulation of questions, the search for explanations and "it is possible to make reflections with mathematics [...]" and this, therefore, can reinforce such reflections (Skovsmose, 2014, p.97, emphasis added).

Another aspect that can be valued in the realization of this design is the reformulation of problems that, according to Silver (1994) and Brown and Walter (2009), consists in the elaboration of questions and in the presentation of one or more problems that reconstruct the proposed task / problem, the information and conditions presented therein. In this reconstruction, students can be guided to make decisions, to take actions and to use resources in the explorations and the elaboration and the use of strategies (Brown & Walter, 2009).

Such an aspect or focus may occur before, during, or after a problem is solved, but when reformulation occurs in the resolution process, there is a plan for how to get a new version of the problem, in which students embody, recreate, and determine the goals that will be achieved (Silver, 1994). In addition, in the course of the course, the interpretation of what was proposed focuses on decision making and, consequently, on the elaboration of strategies, employment and / or teaching and learning of mathematical and technological knowledge, and on the subject of social relevance addressed and skills and abilities (Figueiredo & Groenwald, 2017, 2018).

According to Ayllón, Gómez and Ballesta-Claver (2016), the reformulation of a problem or the formulation of other problems from the proposed task/problem favour the development of the capacities of analysis, creativity, innovation, reasoning, abstraction, reflection and the establishment of relations between different mathematical concepts or numerical structures. However, it is necessary for the teacher to incorporate it in the pedagogical practices and provide environments that encourage and stimulate the students.

Still, according to Nunes (2016), in reformulating and solving problems, the teacher should encourage research, communication and reflection, as well as guide learning. Also, students need to be encouraged to develop their own ideas and points of view, to face error and overcome it and to gain the confidence to express creativity.

The steps proposed by Figueiredo (2017), for the *design* of problems with the use of digital technologies, whose purpose is to value characteristics and aspects, such as those mentioned above, can also be complemented by others, of the *pedagogical practice*, that the problem will be proposed to the students of Basic Education, the accomplishment of the *pedagogical practice* and the *discussion and reflection by the resolvers and the designers* of the results obtained. The occurrence of these stages allows the training teachers in Mathematics pedagogical planning, using the problems they have elaborated, and the accomplishment of the practice, guiding the students in the process of solving these problems with the use of digital technologies, as well as the verification of the results of the discussion and reflection on the part of the students in the course and/or after the solution and, mainly, can contribute to them to discuss and reflect on the experiences of *designer* and teacher acquired, "[...] in order that the potentialities and/or the limitations of this perspective in Mathematics Education can be identified "(Groenwald & Figueiredo, 2017, p.3).

With regard to the reflection on the experiences of *design* and teacher, these, when guided by the teacher teachers, can cause, as Hartman (2015, p.13) advocates, "[...] a process of introspection. Through critical analysis and evaluation of past, present and/or future thoughts, postures, and actions, the teacher strives to get new ideas and improve performance in the future." For the author, reflective thinking presents as components: *observation*, seeing and hearing, which benefit the evaluation of various situations; the *memory*, when using the memory to recover the observations occurred at other times; and the *combination and consideration* of the observations that have occurred that may help in understanding factors that affect or favour the teaching and learning process. Also, in relation to the teaching of Mathematics, the teacher can even execute it in phases: *pre-active*, which involves planning the pedagogical practice; interactive, which requires the monitoring or verification and self-control of this practice as it occurs; and post-active, in which it is evaluated and rethinking about how new practices will be performed, to be effective.

Thus, it is believed that the theoretical-practical study, the discussion and reflection on the *design* of problems with the use of digital technologies, the characteristics and aspects that can be attributed to it and about the accomplishment of pedagogical practices, in which these problems will be solved by students of Basic Education, they are means for the graduates in Mathematics to build, as affirmed Imbernón (2011), the specialized basic knowledge. According to the author, the use of methodologies and the

experiences provided in the training of future teachers should value forms of cooperation and teamwork, foster reflective processes on Education and social reality and develop skills that lead them to make decisions, and have investigative, interactive, and dialectical attitudes that help them shape their own pedagogical options.

Therefore, it is reiterated that teacher educators need to guide them in the course of these practices in order for Mathematics graduates to recognise the potentialities and/or limitations of this perspective so that they produce knowledge that makes them capable of accomplishing others problem *designs* and pedagogical practices. On the other hand, graduates can enhance and/or develop skills and abilities, if they are encouraged to express creativity and innovation, make decisions and pedagogically plan, choose and use digital technologies, discuss and reflect on *design* and pedagogical practices, among others (Figueiredo, 2017; Groenwald & Figueiredo, 2017).

3 METHODOLOGICAL PROCEDURES

The results presented in the course of this work come from an investigation, conducted under the qualitative approach, in which the objective was to investigate, through *design* activities and reformulation and problem solving, what is the knowledge produced by future teachers in the which refers to the mathematical, methodological and technological aspects related to the approach of socially relevant topics that can promote Critical Mathematics Education. In order to perform it, the *case study* method was used, because an experiment was planned to collect the data, which was the Extension Course *Design of mathematical problems with the use of digital technologies, under the focus of the reformulation of problems in Mathematics Education*, held in 2018, in the blended mode and promoted by PPGECIM/ULBRA.

The course took place in 13 meetings, which totalled 60 hours and were distributed in: five face-to-face meetings, of 25 hours, in which the activities of designing and reformulating and solving problems with the use of digital technologies; 8 4-hour meetings, totalling 32 hours for non-face-to-face meetings, extra-class, involving the execution of other activities, in groups or individually, in the *Virtual Moodle Learning Environment* (ULBRA, 2018) (reading texts on the theme research, participation in forums, filling in questionnaires and records of problem solving and planning), as well as in the Primary Education schools, where future teachers proposed the problems they produced to students of Basic Education. In the course and after the activities, the future teachers discussed and reflected on the implications of the evidenced methodological perspective.

The participants were ten students of the ULBRA Mathematics Degree Course, who were studying between the 5th and eighth semesters, and the researchers exercised the roles of trainers. Among the instruments used to collect the data, the following stand out: the participant observations of the researchers and the licensees, who recorded them in *word* documents; the records of the activities carried out by the training teachers, made in the *Moodle* Virtual Learning Environment; and the semi-structured interviews that occurred after the pedagogical practices, in which the problems produced by the groups of graduates were proposed for students of Basic Education.

For the organization and analysis of data, we considered the analytical phases and their interactions, which are mentioned by Yin (2016): *compilation*, in which the data collection and organization occurs; *decomposition*, which involves fragmentation or separation of data into smaller groups; *recomposition*, whose fragments or elements are reorganized, in groups and sequences differentiated from the original organization; *interpretation*, which would be to use the recomposed data to produce narratives, tables and graphs (if necessary) and to determine the initial interpretations; and *conclusion*, which requires the use of the interpretations of the fourth phase and the definition of conclusions on the research carried out. In addition, the elaborated theoretical framework was used and, according to the intended objective, it was possible to construct the categories of analysis: Process of reformulation and resolution with the use of digital technologies, carried out by the future teachers, as well as the discussion and reflection about such process; Characteristics and aspects identified by the future teachers, regarding the *design* and the reformulation and resolution of problems with the use of Digital Technologies; and Mathematical, methodological, and technological knowledge about the approach of socially relevant topics that were produced and the professional skills and abilities demonstrated and/or developed by the future teachers.

The cut of the results obtained is related to the design of the problem statement, the lesson plan, the pedagogical workshop held on Science Day[2] and that these problems were reformulated and solved by students of the Final Years of Elementary Education and the discussions and reflections carried out by the graduates A and G after the design and pedagogical practices. These activities were proposed between the 5th and 13th Extension Course meetings in the first half of 2018 but culminated on that date.

4 THE PROBLEM PRODUCED BY THE PRE-SERVICE TEACHERS AND THE RESULTS OBTAINED BY THE PEDAGOGICAL OFFICE

With regard to the proposed activities of the fifth to eighth meetings of the extension course, these were carried out by the training teachers in order to carry out the *design* of open problems, addressing a theme of social relevance and using digital technologies, whose problems Critical Mathematics Education through the reformulation and resolution of these problems, with the use of digital technologies, as well as for the training teachers to discuss and reflect on the activity performed. In these meetings, the stages of *design* took place: *formation of the working group; needs analysis; project/planning; development; Implementation; evaluation of the first version of the problem; discussion and reflection*

[2] Occurred on October 17, 2018, this Day was part of the National Science and Technology Week, whose motto was "Science for the reduction of inequalities", and counted on the participation of teachers and students of the Final Years of Elementary and Secondary Education , from schools in RS. There were pedagogical workshops in Mathematics, Chemistry, Physics and Biological Sciences, which were promoted by PPGECIM-ULBRA, in Canoas-RS-BR, and had the support of the National Council for Scientific and Technological Development (CNPQ).

by designers; and making modifications or re-design, to obtain the second version of the problem (Figueiredo, 2017).

According to the observations made, in the *design* of the statement, licensees A and G discussed and reflected on the subject and decided to address "the planning of an international trip or the purchase of a zero kilometer", considering that the main character, named Elisa, could choose one of these options and would have the value of 20 thousand reais to make the payment, which had earned as graduation gift graduation. When proposing the choice of one of the options, the graduates wanted that the students of an eighth or of the ninth year of Elementary School or even of the second year of High School (the requests would be different and according to the year and level of teaching), make decisions as if they were experiencing the same situation as the character. In this bias, they chose a cross-cutting theme related to consumption (BRASIL, 1998), which presents a context and could provide a scenario for research, aiming at Critical Mathematics Education (Skovsmose, 2008, 2014) and even the Financial education.

In relation to the digital technologies used, the graduates A and G chose to use the resources offered by the *site Powtoon*[3], to produce the statement in the form of an animated video. As for the mathematical knowledge that could be worked on, they identified that the topic addressed favoured the employment or learning of new knowledge on Financial Mathematics (Monetary Values, Percentage, Interest, among others).

After revising spelling and improving the aesthetic aspects of the first version, the graduates obtained the final version of the statement, which was entitled "Elisa graduated from college." The main *slides* of the same can be visualised in Figure 1, but the result in its entirety is available at https://www.youtube.com/watch?v=Rhe_vgvLm-E.

[3] https://www.powtoon.com/home/?

Figure 1. Slides of the Problem "Elisa graduated from college".

The statement produced has two options, but students, as a group or individually, should choose only one, so that they can make their own choices and carry out investigations of information and monetary values on the *Internet* regarding the expenses they determine. The questions presented can lead to the search for answers and guide them in the use of digital technologies, in the process of reformulation and resolution of the problem, as pointed out by Silver (1994) and Brown and Walter (2009).

To guide the students, the graduates A and G decided to prepare two spreadsheets, one for each option, to guide decisions and actions and favour discussions, reflections, investigations and written records. Figure 2 shows the spreadsheets produced in Microsoft Office Excel, where the first, called "My Own Car", refers to the planning of the purchase of a zero kilometre automobile, how it would be paid and the expenses with the Tax (IPVA), private insurance and/or Personal Injury caused by Land Vehicle (DPVAT) and fuel (monthly), and the second, entitled "Elisa's trip", deals with planning of an international trip, in which the expenses with the passport, visa, transport, lodging and meals must be foreseen in the suggested period.

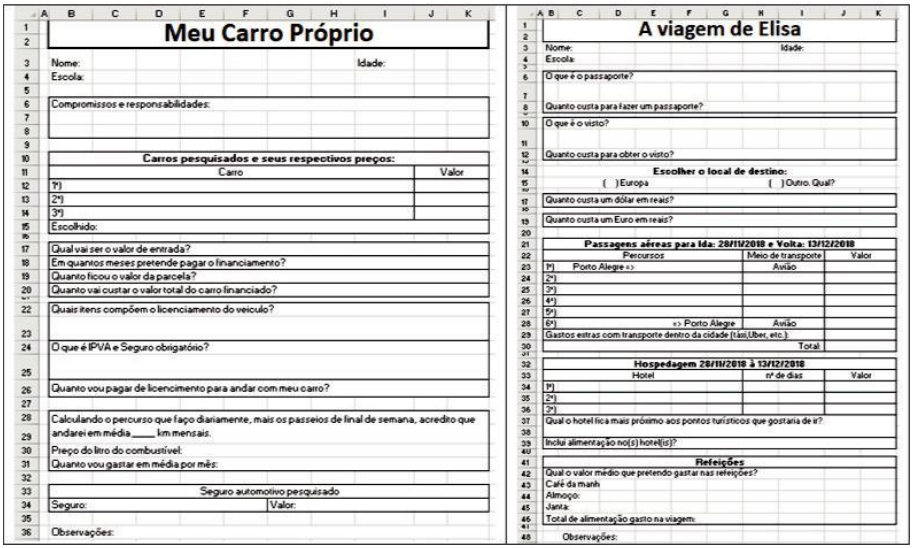

Figure 2. Automobile and travel buying spreadsheets.

At the Forum "Reflecting on the *design* of problems with the use of digital technologies", graduates A and G declared that, by exercising the role of *designers*, they could learn to elaborate problems with the use of digital technologies, which stimulate students to reformulate them and solve them and to understand everyday issues, using Mathematics and digital technologies. Graduating G also stated, "[...] as a problem *designer*, I have been able to hone my skills to this perspective, preparing myself to use in the classroom in the future." According to the above, it can be inferred that such

experience allowed them to learn to elaborate statements that instigate the study of themes and that approach or are part of the students' possible experiences (Figueiredo, 2017), as well as provide environments that encourage and stimulate the reformulation and solving mathematical problems (Ayllón, Gómez & Ballesta-Claver, 2016) using digital technologies.

With regard to the activities proposed from the ninth to the twelfth meetings of the course, we highlight the ones that had the purpose of allowing the training teachers to plan and carry out a pedagogical practice and to discuss and reflect on it. In these meetings, the other stages of *design* took place (Figueiredo, 2017): *planning of pedagogical practice; realisation of pedagogical practice; and discussion and reflection by resolvers and designers.*

In order to elaborate the lesson plan, the graduates A and G were instructed to make use of the statement and the worksheets and to write down the objectives, contents, methodology, the expected time and resources that would be necessary to carry out the pedagogical practice. The elaborated plan is shown in Figure 3.

CLASS PLAN

Objectives: To value students' knowledge so that they learn new knowledge about how to plan the purchase of a desired product. To elaborate strategies of resolution for the problem, that contribute to the definition of the best form of payment among the possibilities that are offered. Communicate strategies used to solve problems. Discuss and reflect in the course and after the process of reformulation and resolution.
Contents: Payment planning, when choosing to purchase and pay for an international trip or a zero-kilometre car. Monetary values. Percentage. Compound interest. Tables.
Methodology: View the video "Elisa graduated from college", to answer the question: travel or own a car? From this response, students will be instructed to conduct research on the Internet, to verify the expenses they would have with the payment, in documents, among others, as well as to fill in the tables that appear in the worksheet of their choice, with the information and the values that represent the expenses and how they will be paid. They will have autonomy in the decisions but should observe that each decision can impact the others.
Time: 3 hours.
Resources: Computer, notebook, pencil, picture, Word and Internet.

Figure 3. Classroom plan developed by students A and G.

According to the lesson plan, the objectives determined by the A and G graduates reaffirm their pretensions, as they were established in the *design* of the statement, since they wanted the students to use their previous knowledge, to use strategies and to discuss and reflect in the reformulation and solving the problem, on the process carried out and the theme evidenced. In the contents, they mentioned the knowledge that would be used or taught and learned by the students and considered that the payment planning of the chosen purchase was also a content to be worked on. In the methodology, they highlighted how digital technologies would be used and their main actions as teachers and what were expected of the students. In addition, they established a timetable that enabled the resolution and cited the resources that would be used.

To implement the plan, graduates A and G were invited to give a pedagogical workshop on Science Day on October 17, 2018. The workshop lasted 3 hours and took place in a classroom of PPGECIM/ULBRA-Canoas-RS-BR, in which 22 students

participated, who were attending the eighth and ninth grade of Elementary School and came from state schools and deprived of the metropolitan region of Porto Alegre and the Rio Pardo-RS-BR Valley. The graduates organised the classroom environment, providing the necessary resources, and mediated the process of reformulation and problem solving with the use of digital technologies.

Initially, they were distributed in nine groups, composed of two or three students, and each group received a *smartphone*, sheets of paper and with the elaborated worksheets, pen, pencil and eraser. The pre-service teachers presented the statement and requested that they make the records on the sheet of the spreadsheet that was referring to the choice of the group, considering the amount of R$ 20,000.00 that the character would have available and all possible expenses that would have occurred, according to the decisions taken.

As for the records of the observations made during the workshop, it was possible to recognise that the students made the decisions in a group and, when necessary, requested the help of the graduates. Among them, mention is made of the registration of training teacher G, who stated that the reformulation occurred during the resolution process (SILVER, 1994) and presented itself when the students, in a group and in a collaborative way, opted for the option that came to the meeting their own wishes, as well as planning to pay for an international trip or a zero kilometre car without excessive spending.

Students conducted searches on the *Internet* in order to obtain the information they deemed necessary and the actual amounts of the expenses they determined for the character. All groups recorded the information and values in the spreadsheets and performed the calculations, which involved the Four Operations with the Decimal Rational Numbers and Percentage.

Among the decisions taken by the three groups that opted for the purchase of a car, we highlight the choice of those who had the lowest value and the use of *online* simulators, to calculate the value of the financing instalments, IPVA value and insurance. By using these simulators, they were able to verify the value of the interest that is paid and, therefore, the doubles, reworked the price surveys, to choose the cheapest car and so that the value that surpassed the R$ 20,000.00 (which Elisa possessed) did not have to be paid in several instalments. Also, the trio of students checked the number of kilometres that the chosen car travelled with 1 litre and, from there, calculated how many litres were needed to cover the number of miles they determined for the month.

The majority of the groups, six in all (three double and three trios), wanted to plan an international trip, destined to Europe or North America, and carried out searches in *websites*, of companies of air transport, of lodging and, agencies, which offer complete travel plans, among others. Because the spreadsheet presented expenses separately, the groups had to make decisions, record the value of each expense, and perform the necessary calculations.

In addition, in the records, it was identified that a trio considered it unnecessary to determine extra expenses, as for example with the trips of the taxi, and a pair wrote that they would have expenses with the particular classes, of English Language, since they

would want to communicate. Only one couple determined the expenses they would have with transportation and the hotel in more than one city they would visit in the chosen country.

In this way, it is understood that the groups were creative, presenting their own versions for each of the choices that were the guiding problem (Ayllón, Gómez & Ballesta-Claver, 2016, Nunes, 2016). Also, the discussions and reflections were essential for its reformulation and resolution.

In Figure 4, students' actions can be observed as they record, search, and calculate, and using the *smartphone* calculator.

Figure 4. Images of workshop accomplishment.

In the semi-structured interview, which occurred after the pedagogical workshop, the graduates A and G were asked by the researcher about the reformulation and resolution of problems, using digital technologies, and on the subject addressed. The excerpt can be checked below.

Researcher: *[...] How did students use digital technologies to reshape and solve problems [...], each option that was presented in the statement?*

G: *[...] Everyone needs Internet research to be solved.*

A: *[...] Regarding the car, they were discovering things they did not know about the IPVA, the value of the dollar and the euro [...]. In the financing part, they could see the value of the instalments, the interest. When they thought that they were going to pay a lot of interest, they gave a greater value of entrance; they parcelled in fewer times.* (Refers to the use of online simulators, which were used to calculate the financing of the cars chosen).

G: *[...] in the trip, the girls chose a package, and it was necessary to investigate every detail. However, the students reformulated because the words "trip" or "automobile" already directed the choices, the researches [...]. So much, that they spoke a price that would be spent in Reais, I asked to verify how much it would be in the currency of the country that they would go and did so [...].*

Researcher: *So, did the subject matter help them acquire new knowledge [...]?*

A: *[...] The question of real life, we took the purchase of a car or a trip, many ride with their parents, but do not know how much they spend fuel, how much they paid for it and IPVA, how much they financed and gave entry [...]. We took a real situation, but thinking about the future [...].*

G: *[...] So much so that we asked who had travelled by plane and few had [...].*

A: *[...] We thought about making a problem with other topics, first was planning a party of 15 years, after a trip to the World Cup in Russia, but this was closer to everyone's interest.*

According to the answers of the graduates A and G, they recognized that the option chosen by each group guided their searches on the Internet, the search for information, values in reais and dollars and euros to convert them into reais, and the use of online simulators, i.e., the reflections and decisions made by the students, workshop participants, guided them in the explorations, the use of resources and the strategies employed (Brown & Walter, 2009). Also, the graduates understood that the social relevance issue addressed favoured reflection on expenses and how to reduce them, preparing them to live similar situations that may meet their interests in the future.

In addition, the researcher asked about the teaching experience acquired and whether or not they had produced knowledge produced as mathematics teachers. In the excerpt, below, it is verified that the graduates A and G pointed out that the experience of proposing the statement they produced allowed them to understand that the teacher should follow the process of reformulation and resolution, directed students to use digital technologies , make the records and the calculations, and reflect on their decisions, just as it is necessary to consider the resources that the school has, the interests and the previous knowledge of the students, to then carry out the design of the statements and propose them, with the intention that propitiate these activities.

Researcher: *[...] Has the experience of proposing the problem they produced contributed to their training as teachers of Mathematics or not [...]?*

A: *[...] Contributed [...]. We can produce the problem by using a different feature, the Powtoon [...]. In practice, I saw the results [...], which the teacher needs to guide and request to research information, record the values and do the calculations, reflecting on the expenses. [...].*

G: *[...] Yes. [...] If it is with our class, we will see the resources that the school has, the interests of the students and according to what they know [...]. Those who solved the problem liked it, we provided the resources, but if it was in a school, it might not have [...].*

In view of the above, it is recognized that the pedagogical practice enabled the graduates A and G to verify the results of the reformulation and resolution of the problem they produced, even if this practice was carried out through a workshop, where the expected duration did not allow the discussions and further reflection and after that process, involving the participation of all. Its implementation facilitated the identification that, because it was open and dealing with a topic of interest to the students, the proposed problem favoured the choice of one of the options and the other decisions are taken, which occurred in a conscious way, from discussions and reflections that resulted in obtaining a solution.

Thus, it is stated that the steps taken in the *design* of the problem, the preparation of the lesson plan and the implementation of the planned pedagogical practice, through the creation of a workshop, contributed to enable the A and G graduates to build specialized basic knowledge (Imbernón, 2011), regarding the *design* of problems with the use of digital technologies, of the open type and that address issues of social relevance, for Critical Mathematical Education, that can propitiate the reformulation and resolution of these problems with the use of technological resources. The experiences they acquired as a *designer* and teacher were driven by teamwork and demonstrated investigative, interactive, and dialectical attitudes.

The discussions and reflective thoughts of the graduates were permeated by the memories, combinations and considerations of the observations they made (Hartman, 2015), during and after the execution of the activities proposed in the Course. These observations contributed to the understanding of the potentialities and even the possible limitations of this perspective, since the teaching and learning process of Mathematics had the phases: *pre-active*, that involved the *design* of the statement to use it in the elaboration of the class plan; *interactive*, in which this plan was executed, through the pedagogical workshop; and *post-active*, which required the analysis and evaluation of the *design*, the characteristics and aspects attributed to it, the objectives sought and achieved with the workshop, as well as rethinking and pointing out how other practices can be performed.

5 FINAL CONSIDERATIONS

The *design* of problems with the use of digital technologies is a methodological perspective that allows the assignment of characteristics and aspects to the statements, such as the approach of social relevance, aiming at Critical Mathematics Education, visualization through the images used and/or produced by *designers* and teachers, the presentation of information, incomplete data and sentences and questions that make them open or predetermined and the reformulation and resolution of problems with the use of technological resources, among others. However, for this perspective to be used in Basic Education and to contribute to the teaching and learning process of Mathematics, it is understood that, in the initial formation of teachers, spaces should be offered for theoretical-practical study, discussion and reflection on this perspective and for its use in pedagogical practices.

The problem "Elisa graduated in college" and the lesson plan present indications that the graduates A and G planned pedagogically since its reformulation and resolution with the use of digital technologies provided the obtaining of creative solutions for the same one that presents the results of their critical reflections and decisions. The objectives delineated in the stages of the *design* of the problem were presented in the elaborated lesson plan and were reached, according to the results obtained with the accomplishment of the pedagogical workshop.

In this way, the steps proposed by Figueiredo (2017), for the *design* of problems with the use of digital technologies, the elaboration of the lesson plan and the accomplishment of pedagogical practices, to propose the reformulation and resolution of these problems with the use of digital technologies, enable the discussions and oriented reflections, so that they focus on the formative and educational process of the future teachers of Mathematics (Hartman, 2015). Experiences as a *designer* and teacher are also a means for the expression of creativity and innovation, which are skills that contribute to the improvement and / or development of teaching skills and abilities, such as decision making and problem-solving planning and of pedagogical practice, of using digital technologies to meet the desired goals, of discussion and reflection on the *design* and pedagogical practices, among others (Figueiredo, 2017, Groenwald & Figueiredo, 2017).

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