ISSN: 2178-7727
DOI: 10.17648/acta.scientiae.6389

Criteria That Guide the Professor's Practice to Explain Mathematics at Basic Sciences Courses in Engineering Degrees in Peru: A Case Study

Walmer Garcés[a]
Vicenç Font[a]
Luisa Morales-Maure[b]

[a] Universitat de Barcelona, Facultat d'Educació, Departament de Didàctica de la Matemàtica, Barcelona, España
[b] Universidad de Panamá, Facultad de Ciencias Naturales , Exactas y Tecnología, Departamento de Matemática, Ciudad de Panamá, Panamá.

Received for publication 6 Jan. 2021. Accepted after review 24 Mar. 2021
Designated editor: Claudia Lisete Oliveira Groenwald

ABSTRACT

Background: Research on mathematics teaching in basic science classes within Peruvian engineering degrees was required prior to the identification of alternatives to improve the training of future engineers. **Objective**: To identify the criteria that guide the practice of professors in Peru when explaining mathematics in basic science classes within engineering degrees, with specific reference to derivatives. **Design**: Qualitative case-study research that seeks to understand current mathematics teaching through an analysis of and reflection on the participants' practices. **Setting and participants**: Professors who give classes within engineering faculties in Lima. One of these was selected as a case study. **Data collection and analysis**: The classes taught by these professors were filmed and the criteria they follow in the design and implementation of their classes were inferred by means of the didactic suitability criteria, which were also used to design a questionnaire to interview the teachers; triangulation was then performed between their words and their actions. **Results**: This professor was guided by ecological criteria (syllabus and profession) and mediational criteria (time available for classes), although he felt that his practice was based primarily on cognitive criteria (previous knowledge) and ecological criteria (future profession). **Conclusion**: The criteria that guided his practice help explain why basic science classes within engineering degrees are taught with a lecture-based and procedural approach, and why innovations are not included.

Keywords: Didactic suitability criteria; Derivatives; Mathematics teaching; Engineering; Reflection on practice.

Corresponding author: Walmer Garcés Córdova. Email: wgarceco7@alumnes.ub.edu

Critérios que orientam a prática de um professor quando explica matemática nas disciplinas de Ciências Básicas pertencentes aos cursos de engenharia no Peru: um estudo de caso.

RESUMO

Contexto: Com a finalidade de buscar alternativas para melhorar a formação de futuros engenheiros no Peru, são necessárias pesquisas sobre o ensino da matemática nas disciplinas de ciências básicas relacionadas aos cursos de engenharia. **Objetivo**: identificar quais são os critérios que norteiam a prática de um professor no Peru quando explica matemática nas disciplinas de ciências básicas dos cursos de engenharia e, especificamente, quando explica a derivada. **Metodologia**: Pesquisa qualitativa do tipo estudo de caso que busca compreender o ensino de matemática realizado, por meio da análise das práticas dos participantes e da reflexão sobre sua prática. **Ambiente e participantes**: Professores que ministram aulas nos cursos de engenharia na cidade de Lima, Peru. Um professor foi selecionado como estudo de caso. **Coleta e análise de dados**: A partir da filmagem e gravação das aulas desses professores, utilizando os critérios de adequação didática, identificaram-se os critérios que eles seguiam ao planejar e implementar suas aulas, critérios estes que também foram utilizados para elaborar um questionário para entrevistá-los. Após isso, realizou-se uma triangulação dos dados para contrastar o que o professor diz (no questionário) e faz (na implementação da aula). **Resultados**: Este professor orienta-se pelo critério ecológico (currículo e profissão) e pelo critério mediacional (tempo disponível para as aulas), embora, segundo ele, seja o critério cognitivo (conhecimentos prévios) e o critério ecológico (futura profissão) que orientam, acima de tudo, sua prática. **Conclusão**: Os critérios que norteiam sua prática explicam por que as aulas de ciências básicas estão sendo implementadas nos cursos de engenharia de forma expositiva e procedimental e por que as inovações não estão sendo incorporadas.

Palavras-chave: Critérios de adequação didática; Derivada; Ensino da matemática; Engenharia, Reflexão sobre a própria prática.

INTRODUCTION

Historically, mathematics teaching for engineers has been underpinned by the following dilemma: should classes be tailored to each branch of engineering, or should they address more general mathematics during the early stages of several different engineering degrees at the same time? The basic sciences have gradually been structured around a programme common to several branches of engineering, for a number of reasons. One argument, for example, is based on the assumption that context is not relevant when it comes to applying formal mathematical knowledge; in other words, it assumes that the answer to the question "Can people apply general knowledge to different contexts with relative ease?" is yes. The second argument seeks to avoid presentation of mechanistic or behavioural mathematics; in other words, based on the assumption that there are essentially three methods of teaching mathematics (formalist, mechanistic and realistic), it attaches considerable weight to formalist mathematics and, above all, avoids the second approach in the design of basic science programmes.

In the 20th century, the different branches of engineering were most commonly structured around an initial stage, called basic or general studies, which was designed to provide engineers with basic mathematical tools that they would then go on to apply in the other subjects in the degree and, later on, in their professional careers. Thus, the basic sciences were incorporated into engineering training such that, in theory at least, engineering students would acquire the strategies and knowledge they would need to tackle and resolve the challenges that would arise in their professional activity (Monforte, 2011).

After about a hundred years of this approach, however, doubts and dilemmas have arisen about whether this is the best option. Based on international trends in higher education engineering teaching, Capote et al. (2016) pointed out that, although engineers require in-depth knowledge of the basic sciences, today's society also demands engineering training that shapes professionals with the ability to respond to the requirements of contemporary development; this requires that teaching and learning processes and curricular models be organized in such a way that they are interactive, collaborative and student-centred and allow students to engage in lifelong learning.

Some of the key dilemmas concerning the role of the basic sciences, especially mathematics, in engineering training concern the following aspects (Font, 2019):

1) The question of whether basic general knowledge can be applied to different contexts with ease;

2) The high number of students who fail;

3) The teaching of content that is not subsequently used in practice;

4) The current approach to teaching, which focuses on skills development.

The dilemmas posed by basic science and general studies classes within engineering degrees have given rise to a research agenda on how to teach mathematics at this stage, and the possible alternatives. For this reason, a number of studies have explored the skills and knowledge of mathematics teachers at this stage, how mathematics is taught within the subjects included, and the possible alternatives. With respect to the approach to teaching differential and integral calculus as part of engineering degrees in Peru, the following statements reveal a somewhat vague consensus: a) students encounter many difficulties in the learning process; b) these difficulties are due to the fact that teachers clearly use an algorithmic, mechanistic and routine approach to teaching formulas and that the teaching method is highly rigorous and formalistic, among other factors; c) in both cases, the teaching method prevents students from acquiring an in-depth understanding of the basic notions of calculus and its applications, which means that these future engineers lack the skills to use mathematics to solve problems encountered in their professional lives.

However, there is insufficient research on mathematics teaching in basic science classes within Peruvian engineering degrees to support this consensus. Based on the literature review carried out, we concluded that research is needed to shed light on the state of mathematics teaching in basic science classes within engineering degrees. Such studies are required prior to the identification of alternatives for mathematics teaching at this stage.

In this regard, our research addresses the case study of a teacher who gives differential calculus classes in a Peruvian engineering faculty. We filmed three classes on the subject of derivatives and their applications, we compiled information on his course materials and the curricular materials he was required to follow, and we conducted a semi-structured interview; these sources were triangulated with a view to answering the following question: What criteria guide the practice of this Peruvian professor and help him explain mathematics in basic science classes taught as part of engineering degrees, with specific reference to derivatives?

This introduction, which outlines the research question and its relevance, is followed by a review of the literature and the theoretical reference used, i.e. the didactic suitability criteria (DSC) proposed by the Onto-Semiotic Approach (OSA) to Mathematical Cognition and Instruction. The qualitative methodology used to conduct the case study is then presented, followed by triangulation of the sources. Next, the data are analysed and the results presented. The paper concludes with a discussion of the results and possible aspects to address in future research.

REVIEW OF THE LITERATURE

This section briefly outlines the literature review we carried out, which explored two aspects: 1) The teaching of differential calculus in engineering degrees, especially in Peru; and 2) Inference of the criteria that guide teachers' practice based on an analysis of their practices and their reflections on these.

With respect to the first aspect, several studies on mathematics teaching in engineering degrees have indicated that the high number of students who fail basic science subjects in these degrees is directly related, among other aspects, to the way in which teachers approach and teach mathematics, especially differential calculus, at the early stages of university. Based on a review of the literature on differential calculus, García (2013) stated that the learning and teaching of mathematics in general, and of calculus in particular, present one of the greatest challenges for university students, including engineering students.

According to the author, precise and effective solutions to this problem remain elusive, since the tendency to reduce algebra, which supports the calculus learning process, to arithmetic and algorithmic processes has further complicated the situation given that it leads to decontextualization of the discipline. This author also highlighted one of the dilemmas posed by calculus teaching within engineering degrees (previously mentioned by Artigue, 1995). He argued that, although it constitutes the foundation for future engineers' professional development, it has been taught through the use and abuse of algebra and mechanization, and an absence of modelling processes.

García (2013) claimed that the algebrization and arithmetization of calculus have lost sight of its origin and its role in engineering and resulted in learning without understanding by encouraging students to obtain mechanistic solutions and overlooking other aspects, such as cognitive, social, emotional and contextualization factors, in the teaching process.

A number of studies have been conducted on the skills and knowledge of mathematics teachers in these programmes (e.g. Arana, Ibarra & Font, 2020), the way mathematics is taught in the subjects within these programmes, and the possible alternatives (e.g. Camarena, 2013; Cooper, Levi Gamlieli, Koichu, Karsenty & Pinto, 2020; Juárez Ramírez, Chamoso Sánchez & González Astudillo, 2020; Rodríguez Gallegos, 2017). Much of this research has focused on the content of differential and integral calculus.

In the case of Peru, the literature review we carried out on research on mathematics teaching in engineering degrees revealed that such studies are in short supply. With respect to differential calculus, we came across proposals for innovative approaches to teaching mathematics to future engineers, but we found virtually no research on the subject. Villanueva (2019) carried out research on the teaching of derivatives in the first year of a telecommunications engineering degree at a Peruvian public university and found that one of the problems faced by students in their first year is that they fail to acquire meaningful learning of derivatives due to the traditional methods used, which are based on a highly procedural approach.

After expanding our search to the countries bordering Peru, we found that a study carried out by Vargas (2010) to assess the quality of mathematics teaching and learning processes in engineering classrooms at Chilean universities found that the lecture method predominated due to the high number of students; that tutorials were considered merely as a complement to teaching; and that the type of course, class sizes and the relationship between the teachers and students all influenced the methodology.

With respect to the second aspect, although mathematics teachers are not always able to clearly explain the reasons behind their teaching practice, various lines of research have inferred teachers' knowledge and skills based on an analysis of their teaching practices and their reflections on these, since this method can offer an insight into certain patterns or regularities that guide their teaching practices. Pepin, Gueudet and Trouche (2017) argued that teachers' implicit, unspoken considerations in the selection and implementation of task sequences can help shed light on the criteria that guide their practice and shape what the authors called "teacher design capacity", which, in turn, can grow through reflection-in-action (Schön, 1983).

Carlos Guzmán (2018) presented the results of three studies (one of which was carried out with teachers in engineering degrees) whose objective was to conduct in-depth interviews that were designed based on the theoretical framework used as a reference (i.e. the notions of effective teaching and good teaching practices). This was designed to identify the qualities and teaching methods of a group of teachers considered a priori to engage in good teaching practices and aimed to identify the criteria that guided their practice and, based on these, define suggestions for teacher training.

A number of studies have been carried out within the framework of the OSA and have revealed the following phenomenon with some regularity: the components of the didactic suitability criteria proposed by the OSA (see Section 3) function as regularities in teachers' discourse when they assess an episode or explain why a teaching proposal represents an improvement, or when they reflect on their practice, without ever having been taught how to use this tool to guide their thinking (Breda, 2020; Breda, Pino-Fan & Font, 2017; Seckel, Breda, Sánchez & Font, 2019).

In other words, their comments can be considered evidence of the implicit use of certain components of the didactic suitability criteria as a guide to inform their teaching practice.

In line with the study by Carlos Guzmán (2018), we developed and applied a semi-structured interview to identify the criteria that guide the practice of the case-study teacher. However, in accordance with the abovementioned phenomenon, the didactic suitability criteria tool was used as a theoretical reference to design the interview questionnaire.

THEORETICAL FRAMEWORK

The Onto-Semiotic Approach (OSA) to Mathematical Cognition and Instruction considers five types of analysis in instructional processes: 1) Identification of mathematical practices; 2) Development of the configurations of mathematical objects and processes; 3) Analysis of didactic trajectories and interactions; 4) Identification of the system of norms and meta-norms; and 5) Assessment of the didactic suitability of the instruction process (Font, Planas & Godino, 2010; Breda, Pino-Fan & Font, 2017).

The first type of analysis explores the mathematical practices carried out within a mathematical instruction process; the second focuses on the mathematical objects and processes involved in the execution of the practices, in addition to those that arise from said practices; the third type is based primarily on a description of interaction patterns, didactic configurations and their sequential expression in didactic trajectories (these configurations and trajectories are supported by a system of norms and meta-norms); the fourth type of analysis examines this system.

The first four types of analysis are tools for descriptive and explanatory didactics, while the fifth focuses on assessing didactic suitability. This latter is based on the four previous teaching analyses and offers a synthesis to identify potential improvements in the instructional process through new implementations.

In the OSA, the didactic suitability of a teaching and learning process is understood as the extent to which said process (or part of it) features certain characteristics that justify its categorization as suitable (optimal or appropriate) to achieve alignment between the personal meanings derived by the students (learning) and the intended or implemented institutional meanings (teaching), with due regard for the circumstances and the resources available (environment). It is a multidimensional construct that is broken down into criteria of partial suitability. The didactic suitability criteria can help, firstly, to guide mathematics teaching and learning processes and, secondly, to assess the implementation of these processes (Breda, Font & Pino-Fan, 2018).

The OSA includes six criteria of partial suitability: 1) Epistemic suitability, which assesses whether the mathematics taught is "good mathematics"; 2) Cognitive suitability, which assesses, prior to the instructional process, whether there is a reasonable gap between what is to be taught and what the students already know and, after the process, whether the learning outcomes are aligned with the teaching objective; 3) Interactional suitability, which assesses whether the interactions successfully address the students' queries and problems; 4) Mediational suitability, which assesses the appropriateness of the resources, in terms of materials and time, used in the instructional process; 5) Affective suitability, which assesses the students' involvement, in terms of interests and motivations, during the instructional process; and 6) Ecological suitability, which assesses the extent to which the instructional process is aligned with aspects such as the centre's educational project, curricular guidelines, and the conditions of the social and professional environment (Font, Planas & Godino, 2010).

For the implementation of the didactic suitability criteria, a set of observable indicators, components and descriptors serves as a guide for the analysis and assessment of instructional processes at any stage of education (Breda, Pino-Fan & Font, 2017). Table 1 below details the didactic suitability criteria and components for the abovementioned proposal (the indicators have not been included due to space limitations).

The notion of didactic suitability is a tool that is widely used, firstly, to analyse didactic sequences (and their redesigns) that have been designed and implemented by teachers with a view to improving mathematics teaching (Breda, 2020; Morales-López & Font, 2019; Sousa, Gusmão, Font & Lando, 2020), and, secondly, to organize the reflections of future or active teachers on their own practice in teacher training programmes (Esqué & Breda, 2021; Giacomone, Godino & Beltrán-Pellicer, 2018; Morales-Maure, Durán-González, Pérez-Maya & Bustamante, 2019; Seckel & Font, 2020), since it allows teachers to engage in systematic reflection on the complexity of the mathematical objects they teach and the factors involved in studying these. This tool has also been used for the analysis and assessment of textbook lessons

(Burgos, Castillo, Beltrán-Pellicer & Godino, 2020) and for the design and assessment of mathematical tasks (Gusmão & Font, 2020).

Table 1

Didactic suitability criteria and components. (Morales-López & Font, 2019, p. 5).

Criterion	Component
Epistemic	(ES1) Errors, (ES2) Ambiguities, (ES3) Diversity of processes, (ES4) Representativeness of the complexity of the notion to be taught.
Cognitive	(CS1) Previous knowledge, (CS2) Adaptation of the curriculum to individual differences, (CS3) Learning, (CS4) High cognitive demand.
Interactional	(IS1) Teacher-learner interaction, (IS2) Interaction between learners, (IS3) Autonomy, (IS4) Formative assessment.
Mediational	(MS1) Material resources, (MS2) Number of students, timetable and classroom conditions, (MS3) Time.
Affective	(AS1) Interests and needs, (AS2) Attitudes, (AS3) Emotions.
Ecological	(ECS1) Alignment with the curriculum, (ECS2) Intra/interdisciplinary connections, (ECS3) Social-professional usefulness, (ECS4) Teaching innovation.

We used the didactic suitability criteria as a theoretical tool to address the question raised in this research, i.e. what criteria guide this Peruvian professor's practice when he explains mathematics in basic science classes, with specific reference derivatives, taught as part of engineering degrees?

METHODOLOGY

The approach used in this research is qualitative and interpretative, since our purpose was not to explain, control or predict, nor to transform reality; rather we sought to understand the criteria that guide the teaching practice of the mathematics professor we chose as our case study during the basic stages of engineering degrees through an analysis of his teaching practices and his reflection on these.

Research stages

The method used to reconstruct the case-study professor's implicit, unspoken considerations in the selection, sequencing and execution of tasks consisted of the following phases:

Phase 1: Selection of participants

We contacted a group of 10 professors with extensive experience of teaching mathematics at the basic stage of engineering; eight graduated in mathematics and two graduated with a degree in education with a mathematics specialization. This group of professors give classes in differential and integral calculus in engineering faculties at various public and private universities in Lima, Peru. After we had presented the research objectives, we asked them to participate in the study and sought their consent to enter their classrooms and film their classes (between two and three, depending on the professor) on derivatives and their applications; they were also asked to provide us with course materials and to participate in a semi-structured interview.

Phase 2: Collation of curricular documents, materials prepared by the professor to be implemented in the classroom, etc.

All professors who participated in the research provided us with curricular materials such as the syllabus for the subject, timetables for the academic year, lesson plans, presentations used to teach derivatives, handouts on exercises, reference material on the subject for students, objective tests and graded practices, as well as material from workshops that involved teamwork in the classroom.

Phase 3: Filming of classes

The classes on derivatives and their applications given by these professors were filmed (between two and three classes for seven professors; for three professors, filming was not possible for various reasons). Classes had an average duration of 100 minutes.

Phase 4: Preparation of the tool for conducting the interview

Based on: 1) the didactic suitability criteria (the theoretical reference for this research); and 2) an initial observation of the classes that were filmed; a questionnaire was designed to serve as the basis for a semi-structured interview. This questionnaire was used to conduct a pilot interview with one of the 10 professors. This first questionnaire was revised based on the following:

a) comprehension problems and the redundancy of some questions; b) the opinion of an expert in the use of the OSA tools for researching the knowledge and skills of mathematics professors. This resulted in a second questionnaire of 46 questions, with three initial questions of a general nature and 43 based on the six didactic suitability criteria (epistemic, cognitive, interactional, mediational, affective and ecological), as well as their components and descriptors. This questionnaire was conducted with the 10 professors, with minor variations based on our observations of their classes (for example, a question was adapted depending on whether or not they had used a certain computer resource).

Phase 5: In-depth analysis (radiography) of the classes and identification of the criteria that guide the professor' practice

For each of the professors, an initial expert analysis of the filmed classes was carried out using the first four analysis types proposed in the didactic analysis model based on the OSA constructs, as in Breda, Hummes, da Silva and Sánchez (2021) and Pochulu and Font (2011), with a view to determining mathematical practices, objects and processes, teacher and student functions, didactic configurations, semiotic conflicts, patterns and norms.

The information obtained was used to infer the criteria followed by the filmed professors when designing and implementing their classes; the categories were based on the indicators and components of the didactic suitability criteria, although an expert assessment of the suitability didactic was not carried out (fifth analysis type in the didactic analysis model based on OSA constructs). This would serve as a reference for triangulating the data with the criteria the professor claimed to follow.

Phase 6: Selection of the case study

To prepare this document, one of the seven professors whose classes were filmed was selected as a case study. In these classes (three in total), the teacher explained the subject of derivatives and their applications. This consisted of explaining the notion of derivative as the slope of the tangent line and as the limit of the average rate of change, then applying the definition of derivative to certain functions to find the derivative of the sum and product of two functions; he then handed out the complete list of basic derivative rules, which were then mechanized through application in different exercises, and finally he presented the criteria of the first and second derivatives and their application to determine growth intervals, relative minimums and maximums, intervals of concavity (upwards and downwards) and points of inflection. The professor selected for this case study will be referred to as "Professor A" from now on.

Phase 7: Interview with the professor in which he described the criteria that, in his opinion, guide his teaching practice

The interview was carried out with Professor A and was filmed to determine the criteria that, in his opinion, guide his teaching practice. The interview lasted two hours and was divided into two clearly differentiated parts. In the first part, he was asked three general questions (about his training, the criteria that guide his teaching practice and a teaching model he identifies with). In the second part, he was asked more specific questions related to some of the components of the didactic suitability criteria.

Phase 8: Transcription of the interview with Professor A

In this phase, the interview was transcribed verbatim.

Phase 9: Inference of criteria from the interview

The content of the interview was analysed to infer the criteria that, according to the professor, guide his practice, in line with Breda (2020) and Seckel, Breda, Sánchez and Font (2019).

Phase 10: Triangulation of sources

In this last phase, the sources were triangulated (especially the results of phase 5 on the classes observed and the interview responses) to draw conclusions.

Data analysis

Phase 5 started with the first four analysis types proposed in the didactic analysis model based on the OSA constructs (identification of mathematical practices, identification of primary objects and processes, analysis of didactic interactions and conflicts, and, finally, analysis of the norms that regulate the teaching process). The first analysis type explores the mathematical practices carried out within a mathematical instructional process. This can be understood as the narrative a teacher uses to explain to another teacher what has happened from a mathematical viewpoint. The second analysis type focuses on the mathematical objects and processes involved in those practices, as well as those that arise from the processes. The third type of didactic analysis aims primarily to describe interaction patterns, didactic configurations and their sequential expression in didactic trajectories. The fourth level of analysis studies the system of norms that regulate the instructional process.

The tools of the first four levels of analysis proposed by the OSA are used to divide the transcript of a class session into a trajectory of didactic configurations and to study different aspects of each configuration. For example, the second didactic configuration (DC2) (see Figure 1) occurred just after the beginning of the first class and after DC1, when the professor recapped the meaning and calculation of the slope of a line, and ended when the professor began explaining the notion of derived function, which was institutionalized in DC3.

DC2 began when the professor drew the graph of a function and a secant line that passes through points $(a, f(a))$ and $(b, f(b))$, and the calculation of its slope, and ended when he began explaining the notion of derived function. It ran from line 1 to line 5 of the transcript (see Figure 1). We considered a line to be a complete paragraph that made sense as a whole.

The didactic suitability criteria tool was then applied to identify some of the criteria that guide the professor's practice. For example, the in-depth analysis (radiography) in Figure 1 allowed us to conclude that the teacher takes into account the existing knowledge needed to understand the notion of derivatives.

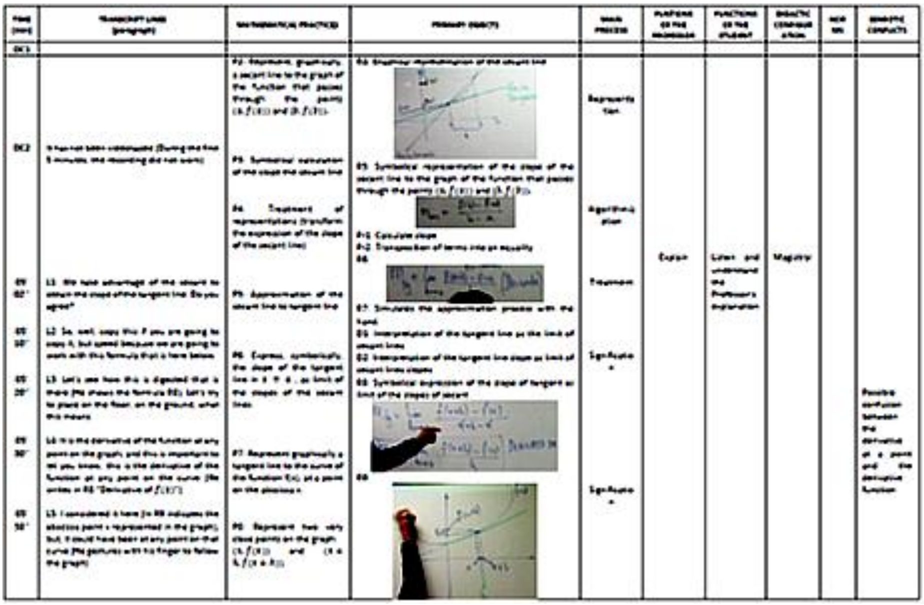
Phase 8 consisted of transcribing the interview with the professor, and phase 9 involved identifying the criteria that, in the professor's opinion, guide his teaching practice. For example, the following transcript allowed us to infer, among other things, that the professor considers it important to take the students' previous knowledge into account:

> I: Could you describe the main criteria you take into account when designing and implementing your classes in this engineering faculty?
>
> Professor A: (02'32") Regarding my criteria for designing class sessions, I really try to start by making sure the students feel comfortable with the topic. I mean, I try to make some kind of comment to establish a conversation with the students to avoid just diving straight in. My intention at the beginning is to create a fairly familiar environment for the students, so that the subject doesn't feel alien to them and they feel a connection with the teacher. Once I've managed to establish that rapport, I

ask the students what they know about the concepts we're going to study and perhaps also about why it's important to study this subject. Then we start the purely mathematical part, development of the concepts, explanation of theorems, some proofs and then the application.

Figure 1

Didactic configuration 2 of the radiography of Professor A's classes.



Finally, triangulation of sources was performed between the professor's statements in the interview and our class observations. Unlike other cases, there was some coherence in the example provided between the professor's words and his actions. This allowed us to infer that taking the students' previous knowledge into account is a valuable principle for the professor when it comes to designing and implementing his classes.

RESULTS AND ANALYSES

Tables 2a–f present a summary of the criteria that, in his own opinion, guide the professor's practice. They have been arranged primarily according to the didactic suitability criteria. These tables were created based on the transcript of the interview, as explained in the data analysis section.

Table 2a

Analysis of Professor A's responses in the interview – General questions

Components	Analysis of the Professor's Responses
Initial training	▪ Professor with undergraduate and postgraduate training in mathematics education, with teaching experience at secondary school, college and university levels, and undergoing regular on-the-job training.
Criteria for designing and implementing classes	▪ He takes existing knowledge into account; he creates a familiar atmosphere and transmits confidence to students; he follows a logical order in the sequence of content; he explains why this subject is taught rather than a different one.
Teaching model	▪ A dialogic approach that seeks to build knowledge through dialogue and to teach mathematics through real-world applications.

Table 2b

Analysis of Professor A's responses in the interview – Epistemic

Components	Analysis of the Professor's Responses
Errors	He accepts that he makes mathematical errors in his classes on derivatives (although he confuses mathematical error with didactic error).
Ambiguities	He considers intuitiveness to be necessary in mathematics because it serves as the basis for developing the subject. He admits that, by resorting to intuition, he sacrifices formality and rigour and that this can lead to ambiguity.
Diversity of processes	He approaches the subject of derivatives and their applications in an instrumental way since he believes that engineers should have, above all, procedural knowledge. He places emphasis on the resolution of exercises and

Components	Analysis of the Professor's Responses
	the use and algebraic manipulation of the table of basic derivative rules. He only teaches proofs once in a while, since he is restricted by time and the problems associated with using the limit definition of the derivative. He rarely focuses on problem solving and does not work on modelling due to time restraints and a lack of knowledge. He focuses, above all, on algorithmization.
Representative sample of the multiple meanings of the mathematical object to be taught	He focuses on the resolution of exercises in an intra-mathematical context by applying derivation rules. The few contextualized problems he poses relate to physics and economics and are not cognitively demanding for students. He uses few extra-mathematical contexts in his classes. In accordance with the syllabus, he gives the limit definition and geometric definition of the derivative. He addresses the interpretation as instantaneous velocity at the end, in applications, but only very occasionally. He considers that different modes of expression and representation of the derivative should be used. In his classes, he addresses conversion of the graphic register to the symbolic register, in addition to different symbolic expressions. He feels that it is important to teach different forms of representing the derivative because of the students' different learning rates, among other reasons. He considers that the content of derivative classes and the way he teaches it are conditioned by the syllabus. In his view, it should be modified to accommodate a variety of application problems in different contexts.

Table 2c

Analysis of Professor A's responses in the interview – Cognitive

Components	Analysis of the Professor's Responses
Previous knowledge	▪ He takes students' existing knowledge into account when planning and implementing his derivatives classes. However, if the students lack prior knowledge, he does not modify his plan to explain the topic to them

Components	Analysis of the Professor's Responses
Adaptation of the curriculum to individual differences	due to a lack of time. In such cases, he accepts that these students are lagging behind. ▪ He feels that he manages to ensure that most (if not all) students bridge the gap between what they already know and the new concepts of derivatives he intends to teach.
Learning	▪ He strives to embrace the diversity of students in the classroom by proposing reinforcement activities to do outside of class hours. For enhanced learning, he proposes both algebraic and geometric scenarios to facilitate visualization. He knows the extent of the students' understanding based on their comments, questions and observations. ▪ The summative assessment tools he uses (tests common to all groups) prevent him from being certain about whether the students have learned or not. When he realizes that students are not learning properly, he seeks to create an affective environment to facilitate communication with them.
High cognitive demand	▪ He points out that the tasks he proposes to his students have been prepared in advance by the subject coordinators; these are derivative tasks that prioritize algebraic practice and manipulation with derivation rules and, therefore, in his opinion, do not activate the students' key cognitive processes.

Table 2d

Analysis of Professor A's responses in the interview – Interactional

Components	Analysis of the Professor's Responses
Teacher-learner interaction	▪ He strives to teach the content on derivatives in class so that it follows a logical order. Through interaction, he can spot when students have queries, even by their facial expressions; in the event that he observes queries, he tries to clarify them. ▪ He uses argumentative resources such as questions, follow-up questions, metaphors, stories and anecdotes to engage, include and involve most students in his derivative classes.

Components	Analysis of the Professor's Responses
Interaction between learners	▪ He states that, in his classes on derivatives and their applications, he seeks to ensure that there is dialogue and communication between the students
Autonomy	▪ Through teamwork, he creates moments to foster student autonomy and moments for exploring, formulating and validating their conjectures on derivatives.
Formative assessment	▪ He observes whether students understand derivatives based on the interaction generated in the classroom when he presents examples, counterexamples and conjectures, and also when the students struggle to back up their statements.

Table 2e

Analysis of Professor A's responses in the interview – Mediational

Components	Analysis of the Professor's Responses
Material resources	▪ He uses free-to-access mathematical software such as Desmos and GeoGebra in his classes on derivatives.
Number of students, timetable and classroom conditions	▪ He feels that the number of students in the classroom is a decisive factor in good teaching, which he cannot do because there are more than 40 students. ▪ The classroom elements and student distribution within it are neither adequate nor motivational for derivative teaching. He suggests that it is necessary to consider more innovative spaces.
Time (for group teaching, tutoring, learning)	▪ In the face-to-face phase, he focuses on the core, most important part of derivatives, and leaves complementary tasks and activities for the virtual phase. ▪ Through his many years of experience teaching in engineering degrees, he is clear that a lot of time has to be spent on the algebraic manipulation of the derivation rules, on the procedural part. He also feels that he spends time on derivative content that implies more difficult conceptual problems for students.

Table 2f

Analysis of Professor A's responses in the interview – Affective

Components	Analysis of the Professor's Responses
Interests and needs	▪ The subject coordinator provides him with a list of exercises, and he tries to include those he thinks will be of interest to the students. He also tries to include exercises to encourage reflection and critical thinking. ▪ In the optimization part, he says he proposes situations that allow the students to assess how the derivative works in the workplace. ▪ To involve students in the mathematics activities he proposes, he assigns grades for tasks completed outside of class hours (extrinsic motivation).
Attitudes	▪ He states that the mathematics tasks he proposes to his students have worked well for him, as he has noticed how they encourage responsibility and engagement in most students. ▪ He creates moments for participation, reasoning and critical thinking, although he says that not all students are involved in these.
Emotions	▪ He seeks to enhance student self-esteem by creating an environment of familiarity and confidence to help them lose their fear, rejection and phobia of mathematics.

Table 2f

Analysis of Professor A's responses in the interview – Ecological

Components	Analysis of the Professor's Responses
Alignment with the curriculum	▪ He believes that there is a relationship between the content, assessment and implementation of derivatives, and the curricular guidelines for engineering degrees, but he also believes that the assessment method should be modified.

Components	Analysis of the Professor's Responses
Intra/interdisciplinary connections	▪ He states that the derivative content he teaches is widely used in other subjects taught as part of engineering degrees and that, in addition, it will serve them in their professional careers.
Social-professional usefulness	▪ He strives to ensure that the future engineers are competent and that they can solve problems using derivatives, and he considers that this skill will serve them when they are in the labour market.
Teaching innovation	▪ He has been introducing technological resources such as GeoGebra and tools to gamify the assessment of derivatives such as Quizizz and Kahoot, but he feels that he has not been able to change the assessment method since it is established by the subject coordinator and is the same for all teachers.

Tables 3a–g present the results of the triangulation of the teacher's answers in the interview and the authors' conclusions based on observations of his classes.

Table 3a

Triangulation between Professor A's answers in the interview and our observations of his classes – General questions.

Components	Class observations
Criteria for designing and implementing classes	▪ It was evident that, to some extent, he does follow the criteria he says he follows (he starts the class by connecting with the students' existing knowledge; he creates a familiar atmosphere and transmits confidence to students; he follows a logical order in the sequence of content; and he explains why this subject is taught rather than a different one).
Teaching model	▪ Although he involves the students to some extent, his classes are lecture-based rather than dialogic, since transmission of knowledge was observed above all. Similarly, according to our observations, he does not attempt to teach mathematics through real-world

Components	Class observations
	applications (i.e. the model he claims to follow is not consistent with our observations).

Table 3b

Triangulation between Professor A's answers in the interview and our observations of his classes – Epistemic.

Components	Class observations
Errors	▪ He makes some errors in his explanations, but he realizes and corrects these.
Ambiguities	▪ He engages in dynamic discourse (metaphors, gestures, etc.) and also relates the notion of slope to the students' experiences.
Diversity of processes	▪ He prioritizes mechanization and the algebraic resolution of exercises and gives two initial demonstrations of basic derivation rules. ▪ He engages in little reasoning, gives intra-mathematical examples, a lot of algorithmization but does not cover modelling.
Representative sample of the multiple meanings of the mathematical object to be taught	▪ In each of his classes, it was noted that he prioritizes the resolution of intra-mathematical exercises. ▪ It was observed that he uses the different meanings of derivative (limit of the average rate, slope of the tangent line and instantaneous velocity). ▪ He uses various modes of expression, conversion and treatment. ▪ He uses different ways to represent the derivative, but not the table form. ▪ It is evident that he implements his classes in accordance with the syllabus.

Table 3c

Triangulation between Professor A's answers in the interview and our observations of his classes – Cognitive.

Components	Class observations
Previous knowledge	▪ He does not carry out an initial assessment of the students' existing knowledge; he assumes that they already know it. However, he starts the topic with a recap (the slope).
Adaptation of the curriculum to individual differences	▪ He proposes achievable content within the students' zone of proximal development. ▪ He uses geometric graphics to accompany the algebraic resolution of derivatives exercises to enhance student visualization. He proposes tasks to be completed outside of class hours. When he thinks a student hasn't understood some aspect, he tries to explain it again, but in a different way.
Learning High cognitive demand	▪ ▪ It was evident that the assessments that carry most weight are summative tests common to all groups. It was noted that if the teacher detects that a student is struggling to learn something, he approaches the student and speaks to or helps him/her at any time in an attempt to create a familiar environment. ▪ He follows a guide of tasks common to all groups, which mainly contains exercises for mechanization and application of basic derivative rules. These tasks promote mechanization and are not cognitively demanding.

Table 3d

Triangulation between Professor A's answers in the interview and our observations of his classes – Interactional.

Components	Class observations
Teacher-learner interaction	▪ From the classes that were observed, it was clear that he makes an effort to ensure that the students understand his explanations, which are organized in a logical way. In addition, it was clear that he helps the students with any queries they have.

Components	Class observations
	▪ He uses a range of argumentative resources, but uses a lecture-based approach in his explanations.
Interaction between learners	▪ The class is fundamentally lecture-oriented; only occasionally does he encourage dialogue between the students.
Autonomy	▪ He creates work teams to resolve some tasks in class (called workshops), which count towards the students' grades. Given that these are tasks that count towards assessments, the groups work autonomously. Students work autonomously in these workshops and in assignments completed outside of class hours. However, given that the classes are basically lectures, most of the time the teacher does not encourage student autonomy.
Formative assessment	▪ It was evident in the classes that the teacher interacts with the students when he feels that they have not understood him; he uses questions and follow-up questions.

Table 3e

Triangulation between Professor A's answers in the interview and our observations of his classes – Mediational.

Components	Class observations
Material resources	▪ The use of software was not observed in the classes that were filmed (although we were able to infer that he uses it in other classes).
Number of students, timetable and classroom conditions	▪ The number of students in the classes is very high. ▪ The classroom conditions are reasonable, although the whiteboard is small and reflects light. The quality of the multimedia projector is also low. However, there is enough space for students to work in collaborative groups in the workshops.

Components	Class observations
Time (for group teaching, tutoring, learning)	▪ He works on the core content in his classes. The content students work on in tasks completed outside of class hours has already been covered in class. ▪ The teacher spends a lot of time on solving procedural exercises and focusing on aspects of derivatives that pose the biggest problems for students.

Table 3f

Triangulation between Professor A's answers in the interview and our observations of his classes – Affective.

Components	Class observations
Interests and needs	▪ The list of tasks he follows consists mainly of procedural exercises. We noted that just one contextualized task based on derivatives was included in this list. ▪ Although the teacher claimed that, in the optimization part, he presents situations that allow students to see how derivatives function in the workplace, there was no evidence of this in the classes we filmed. ▪ He applies extrinsic motivation when he includes tasks for completion outside of class hours in the assessment of the subject.
Attitudes	▪ The students complete the tasks proposed by the teacher to carry out outside of class hours. ▪ Within lecture-based classes, the teacher makes an effort to encourage student participation (he asks them to approach the whiteboard to solve tasks, asks them questions and follow-up questions, etc.).
Emotions	▪ Indeed, he was friendly, attentive and kind to students in all classes we filmed.

Table 3g

Triangulation between Professor A's answers in the interview and our observations of his classes – Ecological.

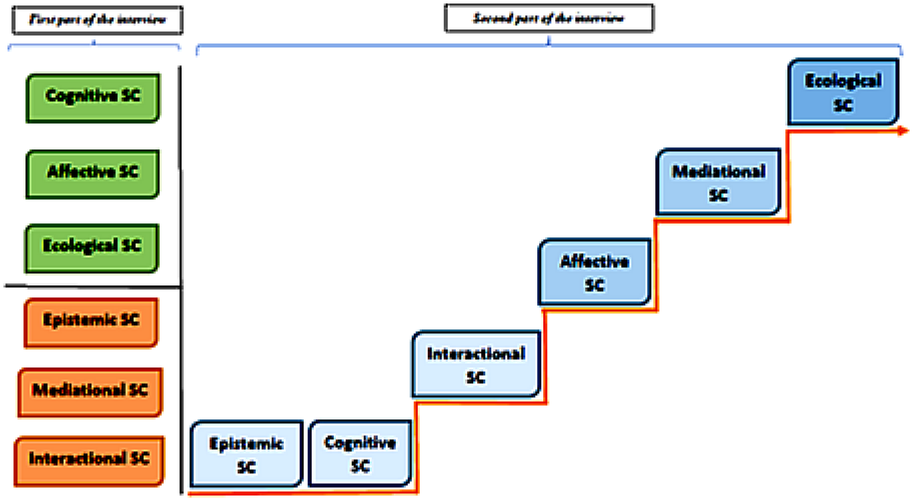
Components	Class observations
Alignment with the curriculum	▪ The content he implements on derivatives and his assessments are consistent with the curricular guidelines for the degrees.
Intra/interdisciplinary connections	▪ He makes statements about the future applicability of content relating to derivatives, but these are general comments about applicability to other subjects in the degree.
Social-professional usefulness	▪ There was no evidence that he focuses on developing his students' mathematical competence in such a way that they can apply derivatives to solve a variety of problems.
Teaching innovation	▪ There was no evidence that he uses the technological resources he claimed to use.

CONCLUSIONS

Our results allowed us to answer the question we sought to address in this article, i.e. what criteria guide the practice of this Peruvian professor and help him explain mathematics in basic science classes taught as part of engineering degrees, with specific reference to derivatives? In addition, the triangulation we performed between his own words and his actions, as observed in the classroom, allowed us to conclude that these criteria do indeed play a major role in his practice (Figure 2).

Figure 2

Scheme of the suitability criteria that guide the practice of Professor A



The top part of the left-hand column of the diagram in Figure 2 shows the didactic suitability criteria mentioned in Professor A's answer to the question about the criteria that guide his teaching practice (cognitive, affective and ecological), while the bottom part reveals the didactic suitability criteria that were not mentioned in his answers (epistemic, mediational and interactional) (first part of the interview).

The column on the right contains the suitability criteria mentioned in his answers to more specific questions related to some of the components of the didactic suitability criteria (second part of the interview). As expected, given the questions asked, the six criteria appear in this column. Professor A did not attribute equal importance to all of these when he reflected on and explained his practice. A step diagram has been used to represent the different degrees of importance of the criteria in guiding the teacher's practice; the most important criteria appear on the upper steps and the least important on the lower steps. In addition to indicating the level of importance attached to the criteria, the way they have been arranged on the steps is also intended to represent how some suitability criteria were subordinated to others. The order in which the criteria appear on the steps is the result of the triangulation performed between the professor's answers and our conclusions from observing his classes.

In the initial interview, Professor A explained that his practice is guided by cognitive, affective and ecological criteria. However, when more specific questions were asked about cognitive criteria in the second part of the interview, we were able to conclude that: 1) Although he takes previous knowledge of a topic into account, he does not spend time teaching the topic if students are not up to speed; 2) Similarly, he does not address diversity in the classroom; 3) He does not propose scenarios involving a high cognitive demand, etc., which he justifies by having to follow the syllabus and the lack of time. These findings, together with the triangulation of our observations of his classes, show that cognitive criteria are subordinated to mediational and ecological criteria in Professor A's classes.

Moreover, epistemic criteria, which this professor initially did not mention in his answers to specific questions, gained importance as he considered different meanings of derivatives, the use of different representations, etc.; at the same time, however, he said that he does not ask students to carry out relevant mathematical processes such as problem solving and modelling, since he presents the subject in a very procedural way, which he blamed on a lack of time and compliance with the syllabus. In other words, although the second part of the interview indicated that epistemic criteria play a certain role in guiding his practice, these are subordinated to mediational criteria (especially lack of time) and ecological criteria (compliance with the syllabus); even the fact that he explains different interpretations of the derivative is justified by its inclusion in the syllabus.

Although interactional criteria are not mentioned in professor A's initial responses, our observations of his classes and the answers he gave when asked specifically about these aspects allowed us to conclude that they play a role in the implementation of his classes. For example, we observed that he used different argumentative resources (questions, follow-up questions, metaphors, stories and anecdotes) to engage, include and involve most students in his classes and encourage them to participate. In addition, he created moments to promote student autonomy through teamwork. However, his interaction was also limited by the classroom conditions, the number of students and the need to follow the syllabus within a set timeframe. In other words, these criteria are also subordinated to mediational and ecological criteria.

The Professor attached some importance to affective criteria in his initial responses, and his answers to specific questions in the second part of the interview also indicated that he takes these into account; this was corroborated by our observations of his classes. For example, he tries to build students' self-esteem by creating an environment of familiarity and confidence to help them lose their fear, rejection and phobia of mathematics, and he proposes tasks he thinks may be of interest to them because they relate to their future profession. Nevertheless, this is carried out within a lecture-based model and without actually focusing on the modelling of tasks related to the profession. We were able to conclude that these criteria are also subordinated to mediational and ecological criteria.

Although media' criterion did not feature in professor A's initial responses, our observations of his classes and the answers he gave when asked specifically about this aspect reveal that it plays a major role in guiding the implementation of his classes. For example, he uses software such as Demos and GeoGebra in his classes, although this was not observed in the classes we filmed. He does not cover modelling due to a lack of time, etc. Moreover, he considers the classroom conditions and the number of students to be detrimental to good teaching of the subject.

By contrast, ecological criteria did appear in his initial responses and the triangulation performed showed that they play a major role in guiding his practice, which he justified primarily by the need to comply with the syllabus, although in his reflections he also explained it in terms of his students' future careers. The triangulation performed between the professor's answers in the interview and our observations of his classes reveal that these two criteria (ecological and mediational) carry the greatest weight in guiding his practice and relegate the other criteria (epistemic, cognitive, interactional and affective) to lower positions, although interactional and affective criteria play a more significant role than cognitive and epistemic criteria.

With respect to ecological criteria, it should be noted that the teacher uses the "Social-professional usefulness" component to justify his highly mechanical, somewhat irrelevant presentation, but not his failure to include modelling processes. In other words, the effect of including these criteria is to reduce epistemic suitability (i.e. diversity of processes), when the effect could be to increase this if his classes included modelling. This finding is consistent with that of García (2013); on the one hand, he considers that the students are future engineers who will need mathematics but, on the other hand, he implements his teaching through the use and abuse of algebra and mechanization and a disregard for modelling processes.

The diagram showing the importance the professor attributes to the different didactic suitability criteria in guiding his practice could provide a plausible explanation as to why basic science classes taught as part of engineering degrees are implemented in line with the findings of Villanueva (2019) and Vargas (2010) (i.e. lecture-based with an emphasis on procedural aspects) and also why innovations such as those proposed by Camarena (2013) and Rodríguez Gallegos (2017) are not included.

Finally, the analysis method used for Professor A is also being carried out with the other participants, which will allow us to perform a cross-sectional analysis of the data in the future. In particular, it will allow us to study how the level of importance attached to the different suitability criteria in guiding teaching practice varies from professor to professor.

ACKNOWLEDGEMENTS

This work was carried out within the framework of the research project on teacher training: PGC2018-098603-B-I00 (MCIU/AEI/FEDER, UE).

AUTHORS' CONTRIBUTIONS STATEMENTS

W. G. C. carried out the field work and participated in all other phases involved in preparation of the article. V. F. M. participated in all phases except the field work. L. M. M. participated in the analysis of the results.

DATA AVAILABILITY STATEMENT

Anyone who makes a reasonable request to the first author of the article will be provided with the data that support the results of the study.

REFERENCES

Arana-Pedraza, R. A., Ibarra, S., & Font, V. (2020). Conocimientos y Competencias Didáctico-Matemáticas del profesor de matemáticas en ingeniería: un primer acercamiento. In Y. Morales-López & A. Ruiz, *Educación Matemática en las Américas 2019* (p. 928-935). Comité Interamericano de Educación Matemática.

Artigue, M. (1995). La enseñanza de los principios del cálculo: problemas epistemológicos, cognitivos y didácticos. En M. Artigue, R. Douady,

L. Moreno y P. Gómez (Eds.). *Ingeniería didáctica en la educación Matemática*. Iberoamérica.

Breda, A. (2020). Características del análisis didáctico realizado por profesores para justificar la mejora en la enseñanza de las matemáticas. *Bolema, 34*(66), 69-88. https://doi.org/10.1590/1980-4415v34n66a04.

Breda, A., Font, V & Pino-Fan, L. R. (2018). Criterios valorativos y normativos en la Didáctica de las Matemáticas: el caso del constructo idoneidad didáctica, *Bolema, 32*(60), 255-278. http://dx.doi.org/10.1590/1980-4415v32n60a13.

Breda, A., Pino-Fan, L., & Font, V. (2017). Meta didactic-mathematical knowledge of teachers: criteria for the reflection and assessment on teaching practice. *Eurasia Journal of Mathematics, Science & Technology Education, 13*(6), 1893-1918. https://doi.org/10.12973/eurasia.2017.01207a.

Breda, A., Hummes, V., da Silva, R. S., & Sánchez, A. (2021). El papel de la fase de observación de la implementación en la metodología estudio de clases. *Bolema, 35*(69), 263-288. https://dx.doi.org/10.1590/1980-4415v35n69a13.

Burgos, M., Castillo, M. J., Beltrán-Pellicer, P., & Godino, J. D. (2020). Análisis didáctico de una lección sobre proporcionalidad en un libro de texto de primaria con herramientas del enfoque ontosemiótico, *Bolema, 34*(66), 40-68. https://doi.org/10.1590/1980-4415v34n66a03.

Camarena, P. (2013). A treinta años de la teoría educativa "Matemática en el Contexto de las Ciencias". *Innovación Educativa, 9*(46), 17-44.

Cooper, J., Levi Gamlieli, H., Koichu, B., Karsenty, R., & Pinto, A. (2020). Instructional innovation in mathematics courses for engineering programs – a case study. En Inprasitha, M., Changsri, N. & Boonsena, N. (Eds). (2020). *Interim Proceedings of the 44th Conference of the International Group for the Psychology of Mathematics Education*. PME.

Capote, G., Rizo, N., & Bravo, G. (2016). La formación de ingenieros en la actualidad. Una explicación necesaria. *Revista Universidad y Sociedad, 8*(1), 21-28.

Carlos Guzmán, J. (2018). Buenas prácticas de enseñanza de los profesores de educación Superior. *REICE. Revista iberoamericana sobre calidad, eficacia y cambio en educación, 16*(2), 133-149. https://doi.org/10.15366/reice2018.16.2.008.

Esqué, D. & Breda, A. (2021). Valoración y rediseño de una unidad sobre proporcionalidad utilizando la herramienta Idoneidad Didáctica. *Uniciencia, 35*(1), 38-54. https://doi.org/10.15359/ru.35-1.3.

Font, V. (2019). Dilemas sobre las matemáticas entendidas como una materia general (ciencia básica) para diferentes carreras (profesiones). En Arango, A. J., Sánchez, O., Vargas, H., Ariza, M. X., Díaz, S. & Canoles, J. C. (Eds.), *Memorias III Congreso ciencias básicas en un mundo globalizado. Investigación experimental y simulación matemática*. Ediciones Usta-Universidad Santo Tomás.

Font, V., Planas, N., & Godino, J. D. (2010). Modelo para el análisis didáctico en educación matemática. *Infancia y Aprendizaje, 33*(1), 89-105.

García, J. (2013). La problemática de la enseñanza y el aprendizaje del cálculo para ingeniería. *Revista Educación, 37*(1), 29-42.

Giacomone, B., Godino, J. D., & Beltrán-Pellicer, P. (2018). Developing the prospective mathematics teachers' didactical suitability analysis competence. *Educação e Pesquisa, 44*, e172011. https://doi.org/10.1590/s1678-4634201844172011.

Godino, J., Batanero, C., & Font, V. (2007). The onto-semiotic approach to research in mathematics education. *ZDM. The International Journal on Mathematics Education, 39*(1), 127-135.

Gusmão, T. C. R. S. & Font, V. (2020). Ciclo de estudo e desenho de tarefas. *Educação Matemática Pesquisa, 22*(3), 666-697.

Juárez Ramírez, J. A., Chamoso Sánchez, J. M., & González Astudillo, M. T. (2020). Interacción en foros virtuales al integrar modelización matemática para formar ingenieros. *Enseñanza de las Ciencias, 38*(3), 161-178. https://doi.org/10.5565/rev/ensciencias.3041

Monforte, G. L. (2011). Debates sobre el papel de las matemáticas en la formación de los ingenieros civiles decimonónicos. In *Técnica e ingeniería en España: El Ochocientos: de los lenguajes al patrimonio*. (pp. 255-298). Prensas de la Universidad de Zaragoza.

Morales-López, Y. & Font, V. (2019). Valoración realizada por una profesora de la idoneidad de su clase de matemáticas. *Educaçao e Pesquisa, 45*, e189468. https://doi.org/10.1590/s1678-4634201945189468.

Morales-Maure, L., Durán-González, R., Pérez-Maya, C., & Bustamante, M. (2019). Hallazgos en la formación de profesores para la enseñanza de la matemática desde la idoneidad didáctica. Experiencia en cinco regiones educativas de Panamá. *Inclusiones, 6*(2), 142-162.

Pepin, B., Gueudet, G., & Trouche, L. (2017). Refining teacher design capacity: Mathematics teachers' interactions with digital curriculum resources. *ZDM, 49*(5), 799-812. https://doi.org/10.1007/s11858-017-0870-8.

Pochulu, M. & Font, V. (2011). Análisis del funcionamiento de una clase de matemáticas no significativa. *Revista Latinoamericana de Investigación en Matemática Educativa, 14*(3), 361-394.

Pochulu, M., Font, V., & Rodríguez, M. (2016). Desarrollo de la competencia en análisis didáctico de formadores de futuros profesores de matemática a través del diseño de tareas. *Revista Latinoamericana de Investigación en Matemática Educativa-RELIME*.

Rodríguez Gallegos, Ruth. (2017). Repensando la enseñanza de las matemáticas para futuros ingenieros: actualidades y desafíos. *IE Revista de investigación educativa de la REDIECH, 8*(15), 69-85. https://doi.org/10.33010/ie_rie_rediech.v8i15.55.

Schön, D. (1983). *The reflective practitioner*. Basic Books.

Seckel, M. J., Breda, A., Sánchez, A., & Font, V. (2019). Criterios asumidos por profesores cuando argumentan sobre la creatividad matemática. *Educação e Pesquisa, 45,* e211926. http://dx.doi.org/10.1590/S1678-4634201945211926.

Seckel, M. & Font, V. (2020). Competencia reflexiva en formadores del profesorado en matemáticas. *Magis, 12*(25), 127-144. https://doi.org/10.11144/Javeriana.m12-25.crfp.

Sousa, J.R., Silva Gusmão, T.C.R., Font, V., & Lando, J.C. (2020). Task (Re)Design to Enhance the Didactic-Mathematical Knowledge of Teachers. *Acta Scientiae, 22*(4), 98-120. https://doi.org/10.17648/acta.scientiae.5711

Vargas Muñoz, D. (2010). *Evaluación de la calidad de los procesos de enseñanza-aprendizaje en las aulas de ingeniería de las universidades derivadas chilenas*. (Tesis de doctorado) - Universidad de Sevilla, Sevilla. https://idus.us.es/handle/11441/24043

Villanueva, I. Z. (2019). *Sistema de tareas docente en el aprendizaje de la derivada en los estudiantes del primer año de la escuela profesional de ingeniería en telecomunicaciones, UNSA-2018.* (Tesis de doctorado) - Universidad Nacional de San Agustín, Arequipa. http://repositorio.unsa.edu.pe/handle/UNSA/10328