AN INDUCTIVE REASONING MODEL IN LINEAR AND QUADRATIC SEQUENCES[1]

María C. Cañadas, Universidad de Zaragoza, Spain, mconsu@unizar.es
Encarnación Castro, Universidad de Granada, Spain, encastro@ugr.es
Enrique Castro, Universidad de Granada, Spain, ecastro@ugr.es

We present some aspects of a wider investigation (Cañadas, 2007), which is focussed on Secondary students' inductive reasoning when solving problems that involve linear and quadratic. We developed an inductive reasoning model constituted by seven states. In this paper, we show some results related to: (a) the frequencies of the states performed by students, (b) the relationships between the frequencies of states depending on the problems characteristics, and (c) the (in)dependence relationships among different states of the model of inductive reasoning.

It is usually to distinguish between two main kinds of reasoning: deductive and inductive. We will focus on the second one, although we are conscious that sometimes it is very difficult to separate both in practice. We refer to inductive reasoning as a process that begins with particular cases and produces a generalization from these cases, in the same sense that Pólya (1967) talked about induction. On the other hand, mathematical induction or complete induction is a formal method of proof based more on deductive than on inductive reasoning. Some processes of inductive reasoning conclude with mathematical induction but this does not always occur.

In this paper we describe some key aspects of a research study (Cañadas, 2007) related to inductive reasoning,

[1] This study has been developed within a Spanish national project of Research, Development and Innovation, identified by the code SEJ2006-09056, financed by the Spanish Ministry of Sciences and Technology and FEDER funds.

whose main objective was to *describe and characterize inductive reasoning used by Spanish students in years 9 and 10 when they work on problems that involved linear and quadratic sequences*. One theoretical contribution was a model constituted by seven states to analyze inductive reasoning described by Cañadas and Castro (2007).

We start with some general aspects of the theoretical and methodological framework of Cañadas (2007). Then we show some results of students' use of inductive reasoning related to: (a) a general description based on the frequencies of states performed by students, (b) the significant differences in the performance of such states depending on the characteristics of the problems, and (c) the (in)dependence analysis among the states considered in the model of inductive reasoning.

INDUCTIVE REASONING MODEL

One of our specific research objectives was to get a systematic way to describe the students' inductive reasoning in problem solving. We followed the idea of Pólya (1967) about induction process, considering four states in the first approximation of a model: (a) Observation of particular cases, (b) conjecture formulation based on previous particular cases, (c) generalization, and (d) conjecture verification with new particular cases. In the context of empirical induction from a finite number of discrete cases, Reid (2002) describes five more detailed states.

Finally, we determined seven states for the model of inductive reasoning from a finite number of discrete cases: (a) Work on particular cases, (b) organization of particular cases, (c) search and prediction of pattern, (d) conjecture formulation, (e) justification (conjecture validation based on particular cases), (f) generalization, and (g) demonstration (justification of the generalization).

These states can be thought of as levels from particular cases to the general case and its justification beyond the inductive reasoning process. Moreover, they have been successfully used for other kinds of conjecturing processes (Cañadas, Deulofeu, Figueiras, Reid, & Yevdokimov, 2007).

The states of the model are useful to analyze the students' performance in the inductive reasoning process, but not all of them necessarily occur, and they do not have to occur in the proposed order.

METHODOLOGY

Students

We selected 359 students intentionally in years 9 and 10 of four State Spanish Schools.

We got information about students' background related to inductive reasoning, problem solving and sequences, from four sources: (a) Spanish curriculum, (b) informal interviews to students' teachers, (c) mathematics textbooks used by students, and (d) students' notebooks.

Spanish curriculum includes reasoning as one of its main objectives. However, it contains just some actions related to inductive reasoning as: (a) to recognize numerical regularities, (b) to find strategies to support students' own argumentations, and (c) to formulate and to prove conjectures (Boletín Oficial del Estado, 2004).

Students had previously studied linear sequences but they had not worked on the quadratic ones; and they had worked on problems using inductive reasoning just occasionally, and usually in relation to sequences.

Questionnaire

We elaborated a written questionnaire with the purpose of analyzing inductive reasoning through the students' responses to the problems posed. This questionnaire had six problems that involved linear and quadratic sequences. We asked students to work individually in an hour of their usual mathematics classes.

The problems of the questionnaire were selected attending to our research objective and using the characteristics arisen through the subject matter analysis (Gómez, 2007) of natural number sequences:

1. The order of the sequence. We selected linear or quadratic sequences for problems, attending to our research objective.
2. The representation system in the statements. To analyze inductive reasoning, we considered statements with particular cases expressed verbally, numerically or

graphically, the three possible representation systems for particular cases in natural number sequences.

3. The task proposed. We identified four different tasks related to inductive reasoning and sequences: Continuation, extrapolation, generalization, and particularization. We selected continuation and extrapolation for the first task of each problem because generalization and particularization were part of our analysis through continuation and extrapolation.

The problems had a second task consisting on justifying their responses, which allowed us to complete the description of the inductive reasoning model[2].

DATA ANALYSIS

In this paper, we focus on the quantitative data analysis developed in Cañadas (2007). First, we identified the states performed for each student in his/her responses to each problem of the questionnaire. This information allowed us to:

1. Get frequencies of the states performed by students in the six problems.
2. Analyze the relationship among states performed by students, representation used in the problems statements, and the order of the sequence involved in problems. We used a logarithmic-linear analysis of three factors: States*Representation*Order. This analysis revealed the effects of each factor in the performance of the states, considering the possible interaction of a pair of factors, and the interaction among the three factors.
3. Analyze the (in)dependence in the performance of different states on each problem. We identified the dependence or independence relationship among different states in relation to previous states considered in the model.

RESULTS

Frequencies of States

The frequencies of states identified in different problems show a general tendency, as we observe in Figure 1.

[2] The questionnaire is collected in Cañadas (2007), Append B.

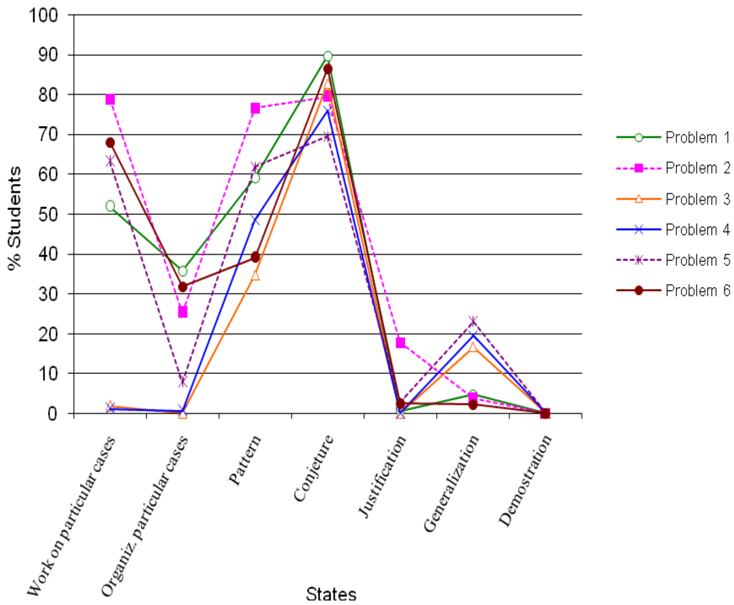

Figure 1. *Percentage of frequencies of states*

The states more frequently used by students were: (a) Work on particular cases, (b) search and prediction of pattern, and (c) conjecture formulation.

Just a few students used particular cases to justify their conjectures and no students demonstrated them.

Logarithmic-linear Analysis States*Representation *Order

Given that the residual values are null, the adequate logarithmic-linear model is the saturated one, which includes the three factors. To study the partial associations, we considered the Chi-square test. We present the results of this analysis in Table 1.

Table 1. *Results of the partial associations*

Effect	Freedom grades	Partial Chi-square	Prob.
Order*Repres	2	175,130	0,0000
Order*States	6	69,441	0,0000
Repres*States	12	255,956	0,0000
Order	1	7,469	0,0063
Repres	2	100,442	0,0000
States	6	4222,179	0,0000

The values of “Prob.” are lower than 0,05, so all the partial effects are significant. Representation*States and Order*States associations are the most significant ones involving two variables (see Partial Chi-square in Table 1). In this paper, we analyze these effects basing on the λ parameter and the z values.

Representation*States Association

The use of verbal representation is associated to a significant low frequency in the work on particular cases and in the conjecture justification. On the other hand, this representation is associated to a high significant frequency in the pattern recognition and in the generalization states.

The numeric representation is associated to a low frequency in pattern recognition, in the conjectures formulation, and in the generalization. However, this representation system is associated to frequencies higher that the mean in work on particular cases, and in conjecture justification.

Conjecture formulation is the only state associated to graphic representation in a significant way. As the value of λ shows, it is higher than the mean.

We present the aforementioned results in Table 2.

Table 2. *Significant associations Representation*States*

	States						
Repres	Work Partic. Cases	Organiz. Partic. Cases	Pattern	Conject.	Justif.	Gen.	Dem.
Verbal	-		+	+	-	+	
Numeric	+		-	-	-	-	
Graphic				+			

“-” indicates negative significant association
“+”indicates possitive significant association

Order*States Association

The values of z reveal that the generalization is the only state that presents significant differences associated to the order of sequences in the posed problems. Observing the the λ value, we conclude that the number of students that generalize in problems that involve linear sequences is higher than the mean; and the number of students who generalize in problems that involve quadratic sequences is lower than the mean.

(In)Dependence Analysis

We analyze the (in)dependence among states through the Chi-square test of statistic independence with a level of significance of 95%. This analysis considered each state related to the previous states considered in the inductive reasoning model, and it was carried out on each problem independently. We do not consider demonstration because there were no students who performed this state.

We concluded that there was evidence of dependence between two states when there are more than three problems that reveal this characteristic. In other case, we consider that there was no evidence of dependence.

Through the described dependence analysis, four states were dependent of the previous ones: (a) organization of particular cases and work on particular cases, (b) pattern and organization of particular cases, (c) pattern and work on particular cases, and (d) generalization and pattern.

CONCLUSIONS

Despite students used to work on and to organize particular cases to get general terms of sequences in classes, they tend to not organize particular terms of sequences involved and even in two problems, students do not work with particular cases frequently.

Related to kind of problems, one conclusion is that students perform states more frequently in problems with particular cases expressed numerically. One reason for that can be the treatment of sequences in current Spanish Secondary classes.

Frequencies of pattern identification and generalization are higher in problems with graphical statements. In these problems, they reach a generalization, in many cases, without performing previous states. Most of these cases respond to inadequate patterns. Moreover, there are no significant evidences in problems that involve linear sequences and problems that involve quadratic sequences. Just in the generalization state, we identify significant evidence in the sense that generalization frequency in problems with linear sequence is higher than in problems with quadratic sequences. However, they are not able to generalize in problems that involve quadratic sequences. This can be consequence of the students' background because they use to work on generalization activities related to linear sequences.

In many cases, states are statistically independent from previous states. This confirms that the considered model for inductive reasoning is not linear.

REFERENCES

Boletín Oficial del Estado (2004). *Real Decreto 116/2004 de 23 de enero, por el que se desarrolla la ordenación y se establece el currículo de la Educación Secundaria Obligatoria* (BOE, nº 35, pp. 5712-5791). Madrid: Ministerio de Educación y Ciencia.

Cañadas, M. C. (2007). *Descripción y caracterización del razonamiento inductivo utilizado por estudiantes de educación secundaria al resolver tareas relacionadas con sucesiones lineales y cuadráticas*. Granada: Universidad de Granada. (Available at http://cumbia.ath.cx/mcc.htm)

Cañadas, M. C., & Castro, E. (2007). A proposal of categorisation for analysing inductive reasoning. *PNA, 1*(2), 67-78. (Available at ww.pna.es)

Cañadas, M. C., Deulofeu, J., Figueiras, L., Reid, D., & Yevdokimov, A. (2007). The conjecturing process: Perspectives in theory and implications in practice. *Journal of Teaching and Learning, 5*(1), 55-72.

Gómez, P. (2007). *Desarrollo del conocimiento didáctico en un plan de formación inicial de profesores de matemáticas de secundaria*. Granada: Universidad de Granada.

Pólya, G. (1967). *Le Découverte des Mathématiques*. París: DUNOD.

Reid, D. (2002). Conjectures and refutations in grade 5 mathematics. *Journal for Research in Mathematics Education, 33*(1), 5-29.