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Prospective high school mathematics teachers' assessment of the epistemic suitability of a proportionality textbook lesson

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ABSTRACT

Background: Every teacher should be able to use curriculum materials to guide instructional design and make reasoned pedagogical decisions about the limitations these resources may have. **Objectives**: In this paper, we describe and analyse a formative intervention with prospective high school mathematics teachers, aimed at developing their competence of didactic suitability analysis of mathematics textbook lessons. **Design:** The methodology followed is didactic engineering, furthermore, the content analysis methodology is applied to examine the response protocols of the participants. **Setting and Participants**: The experience was carried out within the framework of a University Master's Degree in Compulsory Secondary Education and High School; the sample was made of 30 students. **Data collection and analysis**: We proposed these prospective teachers systematically and critically analyse a lesson on proportionality. The written reports of the lesson suitability produced by 14 work-teams are compared with the a priori analysis carried out by the researchers. **Results**: The results suggest that the prospective teachers usually make more descriptive and less analytical analyses even while using a guide. The participants did not clearly identify the epistemic deficiencies of the lesson, thus revealing their limited didactic-mathematical knowledge on proportionality and their lack of critical evaluation of the textbook. However, based on the analysis previously conducted, prospective teachers managed to be quite accurate in preparing their proposals for the use of the textbook lesson. **Conclusions**: In this article we show the interest and usefulness of providing future teachers with a tool to systematically analyse a specific textbook lesson. However, in order for future teachers to acquire the necessary skills in

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critical analysis of the lesson, it is necessary to reinforce their didactic-mathematical knowledge related to proportionality.

Keywords: teacher education; didactical analysis; mathematics textbook; epistemic suitability; proportionality.

Avaliação dos futuros professores de matemática do ensino médio sobre a adequação epistêmica de uma lição de proporcionalidade do livro didático

RESUMO

Contexto: Todo professor deve ser capaz de usar materiais curriculares para orientar o desenho instrucional e tomar decisões pedagógicas fundamentadas sobre as limitações que estes recursos podem ter. **Objetivos**: Neste trabalho, descrevemos e analisamos uma intervenção formativa com futuros professores de matemática do ensino médio, visando desenvolver sua competência de análise didática da adequação das aulas de matemática. **Design**: A metodologia seguida é a engenharia didática, além disso, a metodologia de análise de conteúdo é aplicada para examinar os protocolos de resposta dos participantes. **Ambiente e participantes**: A experiência foi realizada no âmbito de um Mestrado Universitário em Ensino Secundário Obrigatório e Ensino Médio; a amostra foi composta por 30 alunos. **Coleta e análise de dados**: Propusemos a estes futuros professores uma análise sistemática e crítica de uma lição sobre proporcionalidade. Os relatórios escritos sobre a adequação da lição produzidos por 14 equipes de trabalho são comparados com a análise a priori realizada pelos pesquisadores. **Resultados**: Os resultados sugerem que os futuros professores geralmente fazem análises mais descritivas e menos analíticas, mesmo utilizando um guia. Os participantes não identificaram claramente as deficiências epistêmicas da lição, revelando assim seus limitados conhecimentos didático-matemáticos sobre proporcionalidade e sua falta de avaliação crítica do livro didático. Entretanto, com base na análise anteriormente realizada, os futuros professores conseguiram ser bastante precisos na preparação de suas propostas para o uso da lição do livro didático. **Conclusões**: Neste artigo mostramos o interesse e a utilidade de fornecer aos futuros professores uma ferramenta para analisar sistematicamente uma lição específica de um livro didático. Contudo, para que os futuros professores adquiram as competências necessárias na análise crítica da aula, é necessário reforçar os seus conhecimentos didático-matemáticos relacionados com a proporcionalidade.

Palavras-chave: formação de professores; análise didática; livro didático de matemática; adequação epistêmica; proporcionalidade.

INTRODUCTION

The textbook remains the teacher's preferred curriculum material. On the one hand, it constitutes a source of learning for teachers (Grossman

&Thompson, 2008; Nicol & Crespo, 2006). On the other hand, it largely determines what happens in the classroom, and acts as a mediator in the student's learning (González & Sierra, 2004; Thompson, 2014). The use of the textbook in the classroom is linked to their professional teaching work and involves a critical position on the part of the teachers (Braga & Belver, 2016; Monterrubio & Ortega, 2012).

A competent teacher should use the curricular materials as a guide for instructional design, in addition to being able to interpret these materials, establish criticisms and make adaptations that solve their limitations when considering the contextual needs (Choppin, 2011; Taylor, 2013; Thompson, 2014; Yang & Liu, 2019). However, studies have shown that these actions are often difficult for novice teachers (Beyer & Davis, 2012). In particular, when novice teachers use the curricular resources to plan instructional processes, generally omit weaknesses in the mathematical content of texts, do not make changes in the materials, or make untimely changes that alter their meaning (Grossman & Thompson, 2008; Nicol & Crespo, 2006). Furthermore, when analysing the textbooks, they base on their criteria and tend to have an intuitive and holistic approach to interact with the materials, instead of an analytical or critical approach (Lloyd & Behm, 2005; Schwarz et al., 2008).

Consequently, several authors recognize the need and importance of incorporating analysis tools in the design of training activities, which can guide teachers in identifying the curriculum materials strengths and weaknesses and in making adequate changes (Beyer & Davis, 2012; Braga & Belver, 2016; Schwarz et al., 2008; Shawer, 2017).

Despite that, in recent years, the teachers' analysis of textbooks has become an important object of study in Mathematics Education (Fan, Zhu & Miao, 2013), most of these research papers are mainly descriptive (Fan, 2013). Therefore, several authors suggest that teacher education programs should assume the responsibility of promoting the teacher's ability to analyse the problems included in the books, to identify the mathematical objects involved, and to recognize the possible comprehension difficulties. The aim is to ensure that teachers have criteria for making appropriate use of such material, since mathematics textbooks include shortcomings, and yet prospective teachers will face contexts where their use is required (Lloyd & Behm, 2005).

In this research work, we propose prospective teachers to analyse the relevance or adequacy of a mathematics textbook lesson on a specific topic, proportionality, and relying on this analysis, they make reasoned judgments and identify aspects in the management of the use of the lesson to increase the quality of the planned instruction process. The evaluation of the proportionality textbook lesson is carried out by using the didactic suitability components and criteria proposed by the Onto-semiotic Approach (OSA) of Mathematical Knowledge and Instruction (Godino et al., 2007).

In recent years, there is wide research in the field of teacher education, which use the suitability didactic tool to organize the teacher's reflection and to develop their competence to evaluate instructional processes and make improvement decisions (Breda et al., 2017; Burgos et al., 2020; Burgos et al., 2018; Giacomone et al., 2018; Morales-López & Font, 2019).

The choice of proportionality is due, on the one hand, to the importance that receives the study of ratio, proportions and proportionality in the Primary and Secondary Education curricula, given its transversal role concerning to other mathematical subjects and their relations with many curricula contents. On the other hand, few studies address the knowledge needed by teachers to teach proportionality in a relevant way (Weiland et al., 2020).

Various reasons, including the current approach to proportionality in textbooks, lead to classroom practice is biased towards rote learning of the cross-multiplication algorithm (Lamon, 2007; Riley, 2010). Proportionality is not usually adequately presented in the textbooks and point out to errors and weaknesses in the exposition of ratio and proportion (Ahl, 2016; Burgos et al., 2019; Shield & Dole, 2013). Many of these problems are related to key aspects of proportional reasoning, such as the use of representations to address multiplicative relationships within and between magnitude quantities, the distinction between additive and multiplicative situations, and differences between ratio and fraction (Lamon, 2007; Shield & Dole, 2013; Van Dooren et al., 2010).

In the following section, we present the elements of the OSA theoretical framework that underpins our research, as well as the specific research problem. We then describe the method used, the context, the participants, the data collection and the analysis tools that are part of a design-based research. This is followed by the results of the participants' analysis of the proportionality lesson and the reasoned judgements about the epistemic suitability of the lesson made by the participants. We finish the paper with some conclusions, limitations and possible lines of research.

THEORETICAL FRAMEWORK AND RESEARCH PROBLEM

The OSA framework assumed an anthropological conception of mathematics, in which the notion of meaning and its relation to mathematical practice plays a central role. Mathematical practice is "any action or manifestation (linguistic or otherwise) carried out by somebody to solve mathematical problems, to communicate the solution to other people, to validate and generalize that solution to other contexts and problems" (Godino & Batanero, 1998, p. 182). Mathematical object is any entity emerging from the subject' systems of practices to solve a class of problem situation. Hence, the meaning of a mathematical object is conceived in terms of the system of practices in which that object intervenes, playing a relevant role. A typology of primary mathematical objects: situation-problems, languages, concepts, procedures, propositions and arguments, is proposed extending the traditional distinction between conceptual and procedural entities (see Figure 1).

Figure 1

An onto-semiotics of mathematical knowledge (Godino et al., 2007, p.132)

When planning an instructional process for a mathematical object directed to students of a given educational level, the teacher must first delimit what that object represents for the mathematical and the didactic institutions. He should analyse the corresponding mathematical texts, the curricular orientations, and what the experts consider operative and discursive practices inherent to the object whose instruction is pursued. With all this information, the teacher should determine the system of practices that we designate as the *institutional reference meaning of the object*.

Basing on the reference meaning, the teacher selects, defines, and sequences the system of specific practices to be proposed to his/her students in a specific study process for this mathematical content. He takes into account the time available, the students' prior knowledge and means available for instruction. In this way, the teacher decides the intended *institutional meaning* for the mathematical object.

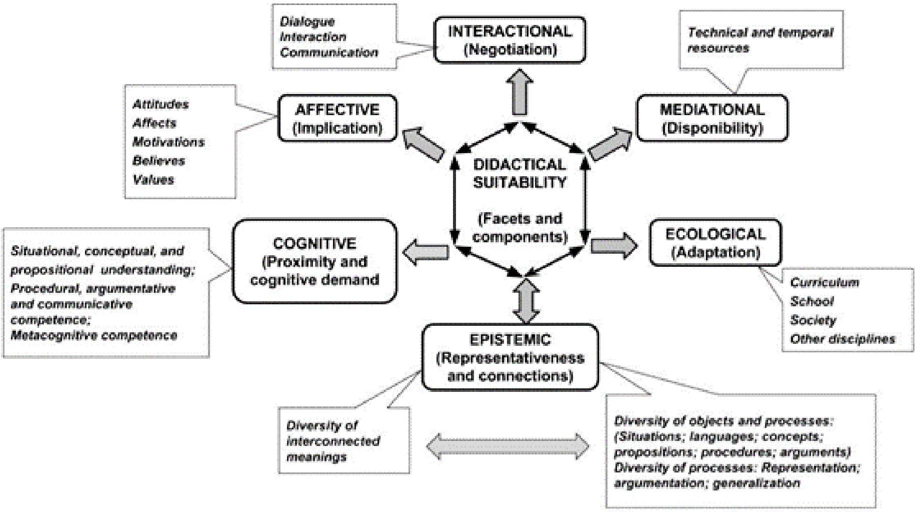
In the OSA framework, learning involves the appropriation of the institutional meanings by students, through their participation in the community of practices generated in the classroom. Thus, the students' system of practices in the resolution of mathematical tasks in which the object appears, determines the *personal meaning* achieved by the student.

In a mathematical study process, we often find some disagreement between the institutional reference meaning and the intended or implemented meaning, due to unfortunate didactic decisions. These mismatches, that we call *epistemic conflicts*, condition the study process and the student's learning.

Didactic suitability is defined in the OSA as the degree to which an instructional process (or a part of it) meets certain characteristics that allow it to be classified as optimal or adequate to achieve the adaptation between the personal meanings achieved by the students (learning) and the expected or implemented institutional meanings (teaching), while taking into account the circumstances and available resources (environment) (Breda et al., 2017; Godino et al., 2016). Didactic suitability involves the coherent and systemic articulation of six facets or dimensions (Godino et al., 2007): *epistemic, ecological, cognitive, affective, interactional*, and *mediational*. A system of components and empirical indicators are identified for each of these facets, which constitute a guide for systematic analysis and reflection, in providing criteria for the progressive improvement of the teaching and learning processes (see Figure 2).

Figure 2

Facets, components and basic didactical suitability criteria (Godino et al., 2016, p.3)



In this study, we focus attention on the epistemic facet. The degree of *epistemic suitability* of a mathematical instruction process is the extent to which the intended or implemented institutional meanings represent adequately a reference meaning. This reference meaning depends on the educational level and must be fixed by taking into account the different types of problems and contexts of use as well as the operative and discursive practices required of the teaching content. High epistemic suitability requires: a) including a representative and a well-articulated sample of problem situations and various representations, b) the fundamental definitions, procedures, and propositions for the topic are presented clearly and appropriately, c) the proposed tasks allow students different ways of approaching them and require to interpret, generalize, and justify their solutions.

Various authors suggest that reflecting on teaching practice is a key competence for professional development and improvement of teaching (Avalos, 2011; Gellert et al., 2013; Jacobs et al., 2010; Mason, 2016; Ramos et al., 2016). According to Breda et al. (2017), didactic suitability allows the teacher to reflect on their practice and serves as a guide to improve the teaching and learning process, while taking into account the context in which it develops.

In this research work, the results of the design, implementation, and evaluation of a formative intervention with prospective high school teachers to promote the competence of the didactic suitability analysis of the proportionality textbooks lessons are described and analysed. Since a textbook lesson can be viewed as an (intended or planned) "instructional process", we can apply didactic suitability, its components, and indicators, to systematically analyse the textbook lessons. We are interested in providing a methodology for critically analysing textbook lessons and reflecting on their use. The central research question in this paper is:

Does this methodology help prospective teachers to develop a critical and constructive analysis of mathematics textbook lessons?

METHODOLOGY

Taking into account the research problem, the methodology followed is the *didactic engineering*, understood in the generalized sense proposed by Godino et al. (2013). This interpretation distinguishes four research phases: a preliminary study in its different dimensions (epistemic-ecological, cognitive-affective and instructive), design of the experiment (task selection, sequencing and a priori analysis of them), implementation (observation of interactions between people and evaluation of the learning achieved), and retrospective analysis (derived from the contrast between what is foreseen in the design and what is observed in the implementation).

Furthermore, the content analysis methodology is applied (Cohen et al., 2011) to examine the response protocols of the prospective teachers who participated in the training experience.[1]

Context

The experience was carried out within the framework of a University Master's Degree in Compulsory Secondary Education and High School[2]

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[1] The Informed Consent Form (TCLE) was not signed, because the identity of the participants is not revealed, in any case, we exempt Acta Scientiae from the consequences derived from it, including comprehensive assistance and eventual compensation for any damage resulting from any of the research participants, according to Resolution No. 510, of April 7, 2016, of the National Health Council of Brazil.

[2] This constitutes the initial training that all university graduates must complete in order to be able to work as secondary school teachers in Spain.

(speciality in Mathematics), in December 2019. The sample was made of 30 students in the Learning and Teaching Mathematics in Secondary School course, where the analysis of the textbook, as a resource in the mathematics classroom, and its connection with the curriculum organizers, is considered.

The formative intervention concerning with the didactic suitability theory and its application to the mathematics textbook lesson, lasted two and a half working hours in the classroom. The participants were also given two additional weeks, to prepare their written reports questions, in which they could pose questions through the Moodle platform. Most of the questions posed by the participants were related to their lack of didactic-mathematical knowledge about proportionality. Prior to this session, the participants had received training (two sessions of two and a half hours) on the above exposed idea of meanings (as a system of practices) and about the different types of mathematical objects considered in the OSA framework. It was also explained to the prospective teachers that if they did not consider it appropriate to apply an indicator in a unit of analysis, because they felt it was not adequate or had already been applied, they should indicate this. We compiled the analysis of the proportionality textbook lesson reports produced by 14 student's teams (referred as T1, T2, ..., T14). Each team was made of 2 or 3 students who did this voluntary work to increase their course score.

Preliminary study and design of the training intervention

The facets, components, and indicators of Didactic suitability should be enriched and particularized according to the specific topic to be taught (Breda et al., 2017). For this reason, before starting this research work, the authors prepared a Mathematics Textbooks Lesson Analysis Guide adapted to the theme of proportionality (TLAG-Proportionality). TLAG-Proportionality guides the reflection on the strengths and weaknesses of the lesson and supports the teacher to decide on appropriate changes that optimize a text-based instructional process. Although this guide includes indicators for the six suitability components considered by the OSA, in this article we focus our attention on the epistemic dimension. Given that the indicators of the TLAG-Proportionality should reflect the scientific community consensus on best educational standards, their concretion involved the review and synthesis of relevant research on didactic-mathematical knowledge on proportionality and proportional reasoning (Cramer & Post, 1993; Fernández & Llinares, 2011; Freudenthal, 1983; Lamon, 2007; Ruiz & Valdemoros, 2004; Shield & Dole, 2013; Weiland et al., 2020).

Data collection instrument

After presenting to and discussing with the students the notion of didactic suitability and the TLAG-Proportionality guide, we proposed them to work in teams of 2 or 3 students, to respond to the following tasks:

(1) In each analysis unit (ratio and proportion, direct proportionality, inverse proportionality) in which the proportionality lesson, taken from Arias and Maza (2015) textbook, has been split and for each facet of didactic suitability:

a) Identify the appropriate components and subcomponents, following the order in which they are presented in the TLAG-Proportionality.
b) To what extent is each indicator of didactic suitability met in the analysis unit? Assign a score 0, 1, 2 in the evaluation column to express the compliance degree of each indicator, according to the following criteria: 0: The indicator is not met; 1: It is partially fulfilled; 2: It is fully fulfilled. In providing your scores, please consider if there is any disparity (epistemic or cognitive conflict) between the meaning planned by the authors, and the reference meaning, in the corresponding epistemic component.
c) Make a reasoned judgment about the lesson didactic suitability in each facet. Take into account the information obtained in the previous task and the didactic suitability criteria (TLAG-Proportionality).

(2) How do you think the textbook lesson should be used to increase the suitability of the study process? Suggest feasible changes in the study process to solve the epistemic, cognitive, and instructional conflicts that have been previously identified.

A priori analysis of the lesson epistemic suitability

Below we present the analysis of the proportionality lesson epistemic suitability that will serve as a reference to interpret the students' responses to the assessment of the lesson suitability. This a priori analysis was carried out independently by each author and finally confronted to decide on a common assessment.

The textbook lesson was divided into three analysis units: U1 corresponds to the ratio and proportion; U2 refers to direct proportionality, and

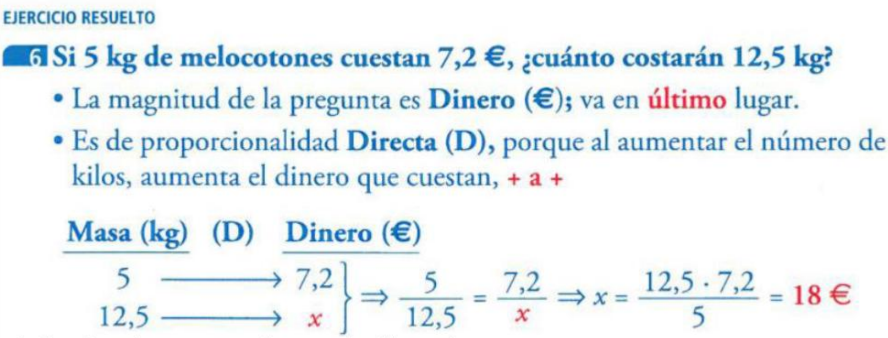
U3 concerns with inverse proportionality. We use the initials I (indicator) accompanied by their corresponding numbering according to tables 1 and 2 (see next section) to assess the different subcomponents.

Problem- situations.

- I1: The formulation of problems is neither addressed, nor there are introductory problems that serve as motivation and contextualize the topics covered throughout the lesson.
- I2: No comparison situations to be solved are included in U1, U2, U3. In U1 some exercises include ambiguous instructions.
- I3: There are no situations that help students to distinguish between additive and multiplicative comparisons.
- I4: Ratio is not accurately established as a multiplicative relationship. This relationship is not made explicit in some situations in U1, U2 and U3.
- I5: The situations proposed in U2 and U3 mainly involve external rations. The use of internal ratios is just implicit in the solution of some situations. For example, in the worked exercise 6 in U2 when establishing the ratios $\frac{5}{12.5}$ and $\frac{7.1}{x}$ (see Figure 3).

Figure 3

Worked exercise 6 in U2



EJERCICIO RESUELTO

6 Si 5 kg de melocotones cuestan 7,2 €, ¿cuánto costarán 12,5 kg?

- La magnitud de la pregunta es **Dinero (€)**; va en **último** lugar.
- Es de proporcionalidad **Directa (D)**, porque al aumentar el número de kilos, aumenta el dinero que cuestan, **+ a +**

Masa (kg)	(D)	Dinero (€)
5	⟶	7,2
12,5	⟶	x

$$\Rightarrow \frac{5}{12,5} = \frac{7,2}{x} \Rightarrow x = \frac{12,5 \cdot 7,2}{5} = 18\ €$$

If 5 kg of peaches cost €7.2, how much will 12.5 kg cost?

- The magnitude of the question is Money (€); which is located in the last place.
- It a direct proportionality (D), because as the number of kilos increases, the money they cost increases, + to +

- I6: No situation promotes the transition from a qualitative to a quantitative approach.
- I7: Only a mental calculation exercise related to ratio is proposed and just in U2.
- I8: Problem-posing tasks are not considered in U1 and U2. Only one situation is included in U3 in which the student must write two inversely proportional quantities.

Language.

- I9: In U1 only symbolic (numerical and algebraic) representations are used; U2 and U3 additionally include tabular representations. Throughout the lesson, the sign = is written between numbers and quantity measurements; measurement quantities are confused with the numerical value of the measurement (see Figure 4).

Figure 1

Exercise 4 worked at U2

4 Se venden cajas de bombones del mismo peso según la tabla:

N.º de cajas	1	2	3	4	5	10	15	20
Coste (€)	6	12	18	24	30	60	90	120

Halla la constante de proporcionalidad.

Las dos magnitudes son directamente proporcionales porque al aumentar el número de cajas en el doble, triple, etc., el coste de las cajas aumenta en el doble, el triple, etc.

La constante de proporcionalidad directa es:

$$\frac{6}{1} = \frac{12}{2} = \frac{18}{3} = \ldots = 6 \text{ €/caja}$$

Boxes of chocolates of the same weight are sold according to the table. Find the proportionality constant.

Both magnitudes are directly proportional because as the number of boxes increases by double, triple, etc., the cost of the boxes increases by double, triple, etc. The constant of direct proportionality is: $\frac{6}{1} = \frac{12}{2} = \frac{18}{3} = \cdots = 6€ / box$

Furthermore, the translations between representations used are not explained. For example, the general character of the sequence of values in the proportionality table is merely indicated by an ellipsis. Likewise, the passage of the sagittal representation (arrows) of the correspondence between the magnitudes to the proportional equation expression may be conflicting.

- I10: In general, the author introduces the definitions of concepts by using literal symbols to generalize the quantities involved; the domain of these symbols is not specified.
- I11: The lesson is limited to the management and interpretation of symbolic and tabular representations.
- I12: Multiplicative relationships are not distinguished by means of any representation.

Concepts.

- I13: In U1 ratio is defined as a division; in the exposition, the quantities of magnitudes are reduced to numbers, regardless of the measurement units (see Figure 5).

Figure 5

Concept of ratio in U1

1.1 Razón de dos cantidades

Una **razón** es la división entre dos cantidades comparables.

Se representa $\frac{a}{b}$ y se lee «*a* es a *b*». Al número *a* se le llama **antecedente,** y al número *b*, **consecuente.**

Ratio of two quantities

A ratio is the division of two comparable quantities. It is represented as $\frac{a}{b}$ and read «*a* is to *b*». The number *a* is called antecedent, and the number *b*, consequent.

Furthermore, the last presented as the quotient of the antecedent and (its) consequent (see Figure 6).

Figure 6

Concept of constant of proportionality in U2

La **constante de proporcionalidad** es el cociente entre el antecedente y su consecuente.

The constant of proportionality is the quotient between the antecedent and its consequent.

In U2 and U3, neither the concepts of direct nor inverse proportionality constant are clearly defined. The definition of directly proportional magnitudes is not related to the ratio and proportion concepts introduced before.

- I14: There are no situations where students have to enunciate their own definitions of concepts.
- I15: In U1 the ratio as a multiplicative relation between two quantities of magnitudes is not stated. In U2 and U3 the functional relationships $y = kx$, $y = k/x$ are not established, neither formally nor intuitively (see Figure 7).

Figure 7

Defining directly proportional quantities in U2

2.1 Magnitudes directamente proporcionales

Dos magnitudes son **directamente proporcionales** cuando:

a) Al **aumentar** una cantidad de una de ellas el doble, el triple, etc., el valor correspondiente de la otra queda **aumentado** de igual forma.

b) Al **disminuir** una cantidad de una de ellas la mitad, un tercio, etc., el valor correspondiente de la otra queda **disminuido** de la misma forma.

La **constante de proporcionalidad directa** se calcula dividiendo una cantidad cualquiera de la 2.ª magnitud entre la correspondiente de la 1.ª

Two magnitudes are directly proportional when:

a) by increasing one of them by double, triple, etc., the corresponding value of the other is similarly increased..

b) by decreasing the quantity of one of them by half, a third, etc., the corresponding value of the other decreases in the same way.

The direct proportionality constant is calculated by dividing a quantity of the 2nd magnitude by the corresponding quantity of the 1st magnitude.

Propositions.

- I16: The fundamental property "in a proportion, the product of the means is equal to the product of the extremes" lacks a clear explanatory discourse in U1; in general, there are no propositions that make the multiplicative relationship between quantities of magnitudes explicit. In U2 and U3 the propositions do not reflect the multiplicative relationship between quantities of magnitudes.
- I17: The justification assumed in U2 to consider when two magnitudes are directly proportional is incorrect since it is limited to verifying that: "double, triple, etc. corresponds to double, triple, ..." or that the

  Figure 2. Defining directly proportional quantities in

  condition "+ to + or – to – "is true (see Figures 3 and 8). The same thing happens for inversely proportional magnitudes in U3. In both units, the regularity condition is assumed. This is also evident in the statements of the proposed situations, which miss the reflection on the linear or non-linear nature of the problematic situations. Since this reflection is forgotten, it is possible that a conflict of "linearity illusion" -that is, the tendency to use linear models in situations where its application is not relevant (Van Dooren et al., 2003)- may be generated.

- I18: The identification and enunciation of properties by the students are not encouraged.

Procedures.

- I19: some procedures are not explicitly shown in the text; Divisions, multiplications, and finding of unknown factors are carried out without explanation. The rule of three method (see Figure 8) and the reduction to unity are characterized by the absence of justifications for their relevance and for the steps to be followed.
  The expression "+ to +, − to −" to determine if the proportionality is direct, can be potentially conflicting since it eliminates the reflection on the multiplicative relationship between the magnitudes. The formulation of the proportional equation is neither explained (nor why the unknown factor should appear in the last position).

Figure 8

Introducing the Direct Rule of Three Method

2.4 Método de regla de tres directa

Para resolver los problemas de regla de tres directa se sigue el procedimiento:

a) Se identifican las magnitudes que intervienen y sus unidades.

b) Se colocan las magnitudes y los datos poniendo en **último** lugar la incógnita.

c) Se determina si la proporcionalidad es **directa.** Es **directa** cuando va de **+ a +** o de **− a −**

d) Se forma la proporción y se calcula el cuarto proporcional.

Magnitud *A* (Unidad)	(D)	Magnitud *B* (Unidad)
Cantidad conocida: *a*	⟶	Cantidad conocida: *c*
Cantidad conocida: *b*	⟶	Cantidad desconocida: *x*

$$\Rightarrow \frac{a}{b} = \frac{c}{x} \Rightarrow x = \frac{b \cdot c}{a}$$

To solve the problems of direct rule of three, the procedure is:
a) to identify the magnitudes involved and their units
b) to place the magnitudes and the data, putting the unknown factor last,
c) to determine if the proportionality is direct. It is direct when it goes from + to + or – to -,
d) to form the proportion and to calculate the fourth proportional.

- I20: Although the proportional equation is introduced after reduction to unit, not enough space is devoted to this or others more intuitive methods before introducing algebraic procedures.
- I21: No situations where students have to generate or negotiate procedures characteristic of proportional situations are proposed.

Arguments.

- I22: The propositions and procedures are presented in U1 without explanations or argumentative discourse by the authors. In U2 and U3 they are justified using specific examples, with little argumentation, particularly in the rule of three (Figure 8).
- I23: The justification of the statements and propositions is not favoured, the explanations of the definitions are usually given as an example, in an exercise after the statements.

Relationships.

- I24: There is a poor connection to fractions and rational numbers; the relationship between the concepts of ratio, fraction, division, and quotient is not clear.
- I25: The relationship between arithmetic and magnitude contents is not made explicit; the transition from measures of quantities (ratios) to numerical values of measures (fractions) is overlooked.
- I26: There is no treatment of the four approaches to proportionality (intuitive, geometric, arithmetic, and algebraic-functional); the focus presented in the lesson (and only partially) is arithmetic.

Processes.

- I27: We only found two tasks that ask the student to interpret the answer (formulate propositions and develop arguments) in U1. In the rest of the lesson, the student is not asked to justify his/her answers to the exercises or to make mathematical conjectures.
- I28: There are no situations where the student needs to use the linear function model.
- I29: The study process goes from general (definitions and statements) to particular; students are not requested to generalize.

- I30: Not all the approaches of proportionality are progressively promoted until reaching the algebraic-functional level (linear function).

For I31, all the previously analysed conflicts in the respective components and subcomponents must be considered. The scores given by the research team, based on these evaluations, are presented in tables 1 and 2, where they are also compared with those assigned by the prospective teachers.

RESULTS

In this section, we present the results of the assessment analysis given by the prospective teacher to the proportionality lesson, using the TLAG-Proportionality tool. First, we compare these assessments with the a priori analysis performed by the authors. Second, we present the results of the reasoned judgments about the epistemic suitability of the lesson prepared by the participants.

Lesson analysis using the TLAG-Proportionality

In tables 1 and 2 we present the scores given by the teams to each indicator in the different components of the epistemic suitability. The score given by the researchers is highlighted in bold in those indicators that the research team considered applicable in a specific unit of analysis. The meaning of the ordinal scores given by the teams indicates the presence or absence of each indicator in the lesson (0 never; 1 sometimes, partially; 2 always, fully). In some indicators, the sum of the frequencies is lower than 14 (total number of teams) because some teams have not assessed that indicator.

We will pay special attention to those suitability indicators that most teams valued differently from the research group. This suppose that we need to reflect on the causes of the discrepancies in their formulation and application to the analysis of the lesson. A limitation that we can specify is that, in general, the teams when evaluating each indicator, do not provide arguments or observations of their rating, although such justification was requested. Having such argumentations would have allowed us to more accurately detect the causes of disagreements.

Table 1

Teams' scores frequencies to the different indicators in each unit

Meaning component and indicators	Lesson analysis units								
	U1			U2			U3		
	Scores								
	0	1	2	0	1	2	0	1	2
Problem Situation									
I1. Inclusion of problems	0	**13**	1	0	**7**	7	0	**9**	5
I2. Varied, representative tasks	1	**9**	4	1	**5**	8	1	**5**	8
I3. Multiplicative/additive tasks	**12**	1	0	**8**	6	0	**5**	9	0
I4. Explicit multiplicative relationship	0	**8**	6	1	**6**	7	0	**6**	8
I5. Internal/external ratio	2	2	**10**	0	**3**	11	2	3	8
I6. Transition qualitative /quantitative	**10**	4	0	**7**	5	2	**10**	3	1
I7. Mental calculus	1	2	**11**	5	**6**	3	**9**	5	0
I8. Encouragement to pose problems	**10**	1	0	**9**	3	2	5	**6**	3
Language									
I9. Different representations	**5**	8	1	2	**10**	2	4	**9**	1
I10. Appropriate level	2	**4**	8	2	**1**	11	3	**1**	10
I11. Constructing/ interpreting language	**5**	7	2	3	**10**	0	4	**9**	1
I12. Within/Between differentiation	**6**	3	2	**4**	5	5	**3**	7	4
Concepts									
I13. Fundamental concepts	**3**	7	4	5	**4**	5	4	**2**	6

Meaning component and indicators	**Lesson analysis units**								
	U1			**U2**			**U3**		
	Scores								
	0	1	2	0	1	2	0	1	2
I14. Recognize, apply concepts	4	**7**	3	4	**6**	4	1	**8**	5
I15. Multiplicative relationship in comparisons	**2**	8	3	1	**6**	6	3	**4**	6
Propositions									
I16. Fundamental propositions	**4**	6	4	4	**6**	3	4	**6**	4
I17. Sufficient/necessary propositions	3	5	5	1	**4**	8	0	**9**	5
I18. Generate, apply properties	0	1	5	1	**7**	6	1	5	2
Procedures									
I19. Fundamental procedures	1	**5**	8	**6**	2	6	**5**	3	6
I20. First arithmetic, later algebraic	3	**4**	3	5	**2**	7	4	**2**	7
I21. Generate or negotiate procedures	**10**	3	1	**10**	2	2	**9**	3	2
Arguments									
I22. Adequate argumentation	**3**	9	2	**6**	6	2	**4**	7	3
I23. Justification is promoted	**7**	5	2	**8**	3	3	**8**	1	5

A total of 31 indicators have been assessed for each analysis unit in the epistemic facet. Teams' evaluation disagreed with the a priori research team assessment in 15 indicators in unit 1, 18 in unit 2 and 17 in unit 3.

Let us briefly consider the possible causes of some of these discrepancies: the difference in I2, could be explained by the number of tasks proposed, which was favourably scored by the students, without considering their representativeness; another factor is that the students have difficulties in identifying other possible situations that can be covered in the topic of proportionality. The origin of the differences in the evaluation of indicators I3, I4, I5, I12, and I17, may come from the participants' low didactic-mathematical knowledge about proportionality, which prevents them from interpreting or distinguishing additive and multiplicative situations, internal to external ratios, within and between multiplicative relationships between magnitudes, as well as, the necessary and sufficient conditions for a situation to be proportional (Fernández et al., 2012; Izsák & Jacobson 2017; Nagar et al., 2016). In particular, the students may consider the multiplicative relationship is present, when interpreting the relationship "when an amount of one variable increases twice, triple, etc., the corresponding value of the other variable is increased in the same way (+ to +)" as direct proportionality (see Figure 7).

We were surprised by the evaluation given to I10, where the majority of students always rated the language level as adequate, in omitting possible conflicts caused by the use of literal symbols. The assessment given to I20 shows the need for prospective teachers to recognize the relevance of starting with an intuitive approach, of pre-proportional nature (Fernández & Llinares, 2011; Ruiz & Valdemoros, 2004) before presenting the rule of three; a key aspect to consider in the development of the pupils' proportional reasoning.

Regarding table 2, the discrepancy in the evaluation of I24 may be due to the fact that participants did not appreciate the importance of clearly distinguishing concepts such as ratio, fraction, division, and decimal. In the case of I25, prospective teachers considered that the simple presence in the textbook of several units dealing with magnitudes establishes the necessary relationships with the arithmetic and magnitude content, for assuring the student understanding of the assumptions justifying the transition from measures of quantities (ratios) to numerical values of measures (fractions). Another possible cause that would justify the discrepancy in I26 is a lack of knowledge of the different approaches to proportionality.

Table 2

Teams' scores frequencies given to the other components

	Lesson analysis units								
	U1			**U2**			**U3**		
	Scores								
Components and indicators	**0**	**1**	**2**	**0**	**1**	**2**	**0**	**1**	**2**
Relations									
I24. Fractions related with rational	2	**4**	7	1	3	10	1	3	10
I25. Link to numbers and magnitudes	**1**	7	6	**0**	3	9	**0**	4	10
I26. Different approaches related	**10**	3	0	**4**	9	1	**8**	6	0
Processes									
I27. Argue and formulate conjectures	11	**3**	0	**10**	4	**0**	8	6	**0**
I28. Linear function model	12	2	0	11	3	0	12	2	0
I29. Describe and make generalization	**13**	1	0	13	**1**	0	11	**3**	0
I30. Foster higher algebraization levels	6	6	1	**8**	4	1	7	3	3
Epistemic conflicts									
I31. No errors, contradictions, ….	**4**	6	3	**3**	9	2	**3**	7	4

In tables 1 and 2, we observe 14 indicators to which the majority of the teams have assigned a maximum score (2). We remark that the research team only considered two of these indicators (I5, I7) with score 2, and only specifically for U1. However, more than half the teams considered that a diverse sample of tasks is used to contextualize and apply proportionality or that the multiplicative relationship in proportional situations appears explicitly in different types of problems (U2, U3). Most teams assessed the language level as adequate for the students in all the units, Moreover, 8 teams agreed that sufficient and necessary propositions are established to characterize a situation as proportional (U2).

Half of the teams thought that sufficient experience in intuitive arithmetic procedures has been acquired before introducing algebraic treatment (direct and inverse rule of three). For example, T12 team points that "as for the procedures, except for the first unit that cannot be evaluated, they are correctly introduced after having gained experience in other intuitive and arithmetic procedures".

The majority of the participants considered that the appropriate relationships with fractions and rational numbers are established, as well as that the relationship with the magnitude block is made completely explicit (U2, U3).

The indicators that were scored by most of the teams with 0, correspond to the process subcomponents: communication, argumentation, modeling, and generalization; this occurs in at least three analysis units. Most of the team also valued with 0 the indicators of languages, arguments, and relationships subcomponents, concerning with: the presence of representations to distinguish multiplicative relationships within and between magnitudes, the justification of statements and propositions, and the identification and articulation of different approaches to proportionality, respectively. For the situations' subcomponent, students also scored 0 the indicators related to the inclusion of situations that promote: the distinction of additive from multiplicative comparisons, the transition from a qualitative to a quantitative approach, mental calculation in some units and problem-posing tasks.

Finally, most teams rated with zero, the indicator related to students' generation of characteristic procedures in situations of proportionality. Since all the indicators scored 0 were also considered with the same score by the research group, we consider that most of these evaluations carried out are appropriate.

Assessment of the lesson epistemic suitability

Taking into account the information obtained through the application of the TLAG-Proportionality, described in Section 4.1, the teams had to make a reasoned judgment on the didactic suitability of the lesson in each facet, specifically in the epistemic facet, that is the focus of this article. Content analysis of the teams' responses served to classify the positive and negative characteristics that the participants expressed in their reports.

Most of these characteristics correspond to components or subcomponents of the epistemic facet, and therefore, the students' responses have been organized in this way. Likewise, some characteristics did not correspond to the facet and were classified in the category of other opinions in tables 3 and 4.

In table 3 we summarize the positive features identified by the teams when judging the epistemic suitability. Nine out of the 14 teams suggested at least some positive characteristics of the lesson, although they did not usually justify their appreciation.

Table 3

Positive characteristics highlighted by the teams in the different subcomponents

Subcomponents	Description	Frequency
Situations-problems	Situations are included: of context in U1, to introduce the topic in U2, for students' generation of problems in U2 and U3, to develop the contents in U3.	2
	A large number of exercises are offered; mental calculation is promoted.	3
	Internal and external ratios are used in U1, U2 and U3.	1
Languages	Adequate language level; various types of representations.	3
	Appropriate representations to distinguish multiplicative relationships within and between quantities.	1
	Students are encouraged to handle, construct, and interpret different expressions and representations in U1 and U2.	1
Relations	Explicit relationship between the arithmetic and magnitude blocks.	2
Other aspects	It is the facet best valued concerning others.	5
	Lesson is adapted to the curriculum.	2
	Proper presentation of content.	2

In the "other aspects" category, when the participants describe the epistemic facet as "the best" about the other facets, they refer to averages, sum of points and, to tables of quantitative summaries that they have designed based on to their previous analysis and that the participants attached as part of their reports.

Some teams referred to lesson attributes that actually correspond to other facets. For example, T11 specifies that an "adequate presentation of the contents is carried out through a short initial reflection, and progressively promotes the concepts towards greater complexity", which should be taken into account when qualifying the instructional facet. At the same time, the adaptation of the lesson to the curriculum corresponds to the ecological dimension. This misapplication suggests that some participants present difficulties in distinguishing the characteristics of each didactic suitability facet, which is not strange, given their limited training in didactic-mathematical aspects.

In table 4 we summarize the negative points that the teams highlight when judging the epistemic suitability of the lesson. They describe missing aspects, inconvenience, or negative qualification of the lesson that have been organized by components and subcomponents.

Table 4

Negative characteristics highlighted by the teams

Subcomponents	Description	Frequency
Situations-problems	Few problems are used to introduce concepts	2
	There is no diversity of exercises, they are repetitive	5
	The student is not encouraged to pose problems	4
Language	Absence of situations to distinguish multiplicative from additive comparisons	2
	Inadequate language for the educational level, insufficient types of expression and representations	4
	Students are not encouraged to handle, construct, or interpret representations	1
Concepts	The concepts are not clearly introduced	7
Procedures	The procedures are not clearly explained	1

Subcomponents	Description	Frequency
	There are no situations to generate procedures characteristic of proportionality	1
Arguments	The propositions and procedures are not adequately explained or discussed	1
	The justification of statements and propositions is not promoted.	3
Relations	The four approaches to proportionality are not dealt with in an organized way; unrelated content	3
Processes	Lack of situations that encourage communication processes, argumentation, etc.	8
Modeling	Modeling is not promoted	2
Other’s aspects	The epistemic facet does not reach 25% of the desired score	1
	Inappropriate presentation of content	2
	Students' creativity is not encouraged; cooperative work is not proposed in the exercises	2

In all, 12 teams provided attributes that disqualify the lesson in some respect, although in general terms, they do not support with arguments their evaluations.

Some characteristics mentioned by the prospective teachers in their reports correspond partially or incompletely to the indicators. For example, in the first characteristic of the problem-situations subcomponent, T1 and T13 indicate that “there are few problems in introducing concepts”, pointing out a specific lack of indicator I1, which provides an argument to consider that the first indicator should not be rated as optimal.

The second characteristic included in table 4 corresponds partially to indicator I2. In this sense, T4 suggests:

> These [problems] have to a large extent a similar structure, therefore, their solution is facilitated in a mechanized way,

> without favoring understanding. Only in a few exercises, the students are asked to interpret the statement or to indicate proportional magnitudes.

This sentence indicates the absence of a diverse and representative sample of tasks that allow the contents of ratio and proportionality (I2) to be contextualized and applied, as well as the lack of situations in which the student must argue and formulate conjectures about the proportional relationships (I27). It should be noted that, although in their explanation, T4 indicates the low diversity of situations, they gave the corresponding indicator the highest rating in all units. We raise the possibility that this indicator is not clear enough for the participants.

Besides, T3, T8, and T12 teams refer to the fact that the student is not encouraged to pose problems related to ratio, proportionality. In this sense, T8 suggests that "the students' creativity is not encouraged since they are not asked to create problems to better assimilate knowledge or suggested to do cooperative work on the exercises".

Regarding language, T2, T3, and T13 indicate that the types of expressions are insufficient and that graphic representation is not covered.

Concerning with relations, T3 and T12 remark that the four types of approaches are not developed in an organized way. In particular, T3 indicates that "the lesson only proposes the arithmetic approach, leaving aside the geometric and algebraic perspectives", referring to the lack of a first approximation based on the similarity of figures (geometric meaning) and the absence of the linear function model (algebraic-functional meaning).

When considering the communication and argumentation processes, the teams indicate that there are no situations asking the student to present their conclusions or reasoning (T8), justify their procedures (T1), interpret their results or statements (T1, T14, T8), make conjectures (T3), investigate and justify conjectures (T5). Moreover, T8 adds that, in U3, "the students are not encouraged to actively participate in the development of content because they are not asked to interpret the data", indicating why they did not consider this indicator. Similarly, T11 specifies that the student is not encouraged to argue their responses.

Finally, we identified some negative aspects not specifically concerned to the epistemic facet but related to other facets such as the affective or interactional components, which have been described as “other aspects” in table 4. For example, in the category “inadequate presentation of the contents”, we found appraisals such as "incongruence in the hierarchy when presenting concepts, exposing in the same way concepts with different relevance "(T13) or "the lesson does not put more emphasis on the most important concepts or procedures or those that may be more difficult" (T3). These are aspects related to the mediational dimension (propose sufficient time for the contents that present greater difficulty of understanding).

Analysis of epistemic conflicts

A fundamental aspect when evaluating the epistemic suitability is the identification of epistemic conflicts. Detecting and considering them supposes that the prospective teacher must recognize the source of potential learning difficulties in the students and considers possible actions to solve them.

Although, in general, the participants did not reflect in their reports a large part of the epistemic conflicts identified a priori by the research team, the analysis of their responses allowed us to identify the discrepancies or mismatches detected by the prospective teachers.

Regarding language T1, T8, T13, and T12 suggest that it is not adequate, although only T8 and T13 explicitly describe the problems they detect in their reports. In this regard, T8 specifies that in U1 "the transition from every day to mathematical language does not take place because of the way the problems are presented."

The conflicts related to concepts are pointed out by 8 teams in their assessment. However, only two of them (T2 and T4) justify their identification. Thus, T2 indicates that the definitions "can be confusing since they are not clear and precise (for example, the definition of two directly proportional magnitudes or the explanation of the rule of three)". On the other hand, T4 highlights that “the definition of inversely proportional magnitudes refers to the double-half, triple-third etc. duality, without hinting that the increasing and decreasing may also consider non-natural values”. This example points out a specific drawback of the definitions of inversely proportional magnitudes. The remaining teams only indicate that there are drawbacks to the concept’s subcomponent. We observe that they use qualifiers for concepts such as: "they are not clear" (T1, T3), "they are not precise" (T3), "they are inadequate" (T5), "they are technical and extensive" (T11), "are wrong" (T12), "are superfluous or redundant" (T13).

Finally, only two groups suggest deficiencies in the procedures: T3 indicated that “they are confusing” and T14 mentioned “the procedure about how to algebraically describe direct and inverse proportionality, as well as the calculation of their corresponding constants, are really confusing", although they did not justify what aspects generate confusion. Finally, T14 adds:

> All the statements, except one, are of the type" + to + "following the author's nomenclature", which leads the reader to believe that direct proportionality only applies in increasing relationships. In inverse proportionality, the questions are asked only for quantities less than those given, from which "could incorrectly deduce that for inverse proportionality the magnitudes decrease." A greater variety of problems should be incorporated in this regard to avoid erroneous deductions.

Proposals to manage the textbook lesson

The analysis and evaluation of the didactic suitability of the lesson -as a planned instructional process- should lead to a proposal for managing this resource to improve the teaching and learning process. Hence, in the second question proposed to the teams, they had to propose changes to increase the epistemic suitability of the lesson, which implies, in particular, thinking about how to overcome the epistemic conflicts.

Half the teams consider that the textbook lesson can only constitute a guide on the content to be covered, which should be complemented by other resources and good teaching work. In their proposal to use the material to increase the didactic suitability, they mention some components and subcomponents of epistemic suitability. In table 5 we include the characteristics identified in the team's reflections.

Table 5

Proposals to improve the lesson epistemic suitability

Subcomponents	Description of proposals	Frequency
Situations-problems	Vary the typology of situations	11
	Propose activities for the student pose problems	4
	Propose activities that allow different solution methods	2
Languages	Adapt the language to the student's level	2
	Employ various forms of representation	4
Concepts	Include clearer definitions	4

Subcomponents	Description of proposals	Frequency
	Consider prior concepts before approaching the topic	3
	Include situations that allow understanding, applying, and differentiating concepts	3
Relations	Begin the study of proportionality with intuitive experiences, linked to other areas of knowledge	2
Processes	Modeling using the linear function	1
	Introduce situations that motivate the student to justify their responses and procedures.	6

We briefly present some prototypical responses from prospective teachers. For example, T3 indicates: "Regarding epistemic suitability, it is advisable to propose students' activities in which they have to formulate their own proportionality problems." Meanwhile, T6 requires:

> You could use some of the problems raised in the textbook. But, including graphing, comparison, qualitative, and quantitative approach exercises. Besides, there should be problems to distinguish between additive and multiplicative comparisons.

About language, prospective teachers indicate the relevance of including graphic representations. For example, T10 suggests that "the worked examples to understand the techniques or contents must use graphics, images, tables, figures, etc. ... to make them easier to understand".

Several teams consider it pertinent to remember previous knowledge ("before introducing new content, remember the previous knowledge that will be used, in this case, the concept of fraction, rational number and decimal metric system", T3), and also to relate the contents of the topic to each other and other areas of knowledge:

> A more contextualized approach based on the reality of the students, related to other areas of knowledge and articulating the differences, similarities and various forms of representations of the different concepts, could be the correct way to improve the theme (T11).

For prospective teacher, it is important to dedicate space to the collective discussion and the justification of the procedures used:

> Dedicate sections of the activities to the students discussing the results they have obtained with their classmates. Thus, they will have other ways to carry out the exercises, as well as the possibility of knowing their possible errors (T4).

CONCLUSIONS

In this paper, we have described the design, implementation and results of a formative intervention aimed to promote in prospective mathematics high school teachers the competence of didactic suitability analysis using as a resource a proportionality textbook lesson.

According to Fan (2013) research in textbook analysis is usually categorized into: questions about textbooks themselves, questions about how different factors affect the development or production of textbooks, questions about how other factors are affected by textbooks. In this study, we obtain information on how the way a mathematical content is approached in a textbook can influence an instructional process based on the use of this resource, whether teachers are aware of these influences and able to propose appropriate changes according to the outcome of the analysis. Thus, our study represents an advance in this field of research as it is not merely descriptive.

It was expected that the TLAG-Proportionality tool and the assignment of the rating (0, 1, 2) in each of the indicators and units, would help the participants to state and organize their discourse in relation to the assessment of the suitability of the textbook lesson analysed. In this sense, we observed some teams that, by providing a reasoned judgment, agreed with their own quantitative evaluation, and therefore, for these prospective teachers the instrument served as a guide to synthesize the lesson weaknesses and strengths. However, we also noticed that the judgment of other teams did not agree with their previous evaluations, for example, when indicating as a negative characteristic an indicator that they had scored 2. Consistent with previous research (Lloyd & Behm, 2005; Nicol & Crespo, 2006; Schwarz et al., 2008; Yang & Liu, 2019), the results suggest that the participants usually make more descriptive and less analytical analyses even with the support of a guide. The prospective teachers omit the lesson weaknesses of mathematical content, they identify few epistemic conflicts, and when they do it, they do not specify them. However, when describing negative attributes, some participants specify information regarding the type of situation, representations, or approaches not addressed in the lesson).

Furthermore, we observed that prospective teachers had difficulty in differentiating components and understanding of some criteria for the didactic suitability analysis of the lesson. This fact is reflected not only in the evaluation and comparison with the a priori analysis but also because some participants explicitly describe this difficulty, pointing out the lack of familiarity.

We agree with Breda et al. (2017) the need of theoretical tools for teachers to focus on the most relevant aspects of instructional processes. We consider that TLAG-Proportionality constitutes one of them and that its optimization (taking into account the results obtained in this research) may have positive implications and better effects in future interventions.

Reflection, adaptation, and decision-making processes generate learning and reflective competence in teachers (Nicol & Crespo, 2006; Remillard, 2000). As Yang and Liu (2019) point out, it is important to analyse which training strategies improve the criticism of mathematics teachers of textbooks, and how this judgment influence the adaptation of these resources. In the intervention carried out, the prospective teacher managed to be quite precise when preparing their proposals to use the textbook.

The potential of this research work resides in equipping prospective teachers with a tool to systematically analyse a specific textbook lesson, by applying indicators resulting from a consensus in the research community in mathematics education. The idea is allowing prospective teachers to reflect and acquire the necessary knowledge and skills, not only in the analysis of textbook lessons but also in their critical management. It would be desirable for future interventions, to have spaces to share with the participants our results as learning opportunities and showing them the relevance of reinforcing didactic-mathematical knowledge related to proportionality.

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AUTHORS' CONTRIBUTIONS STATEMENTS

M.J.C: Conceptualization, Validation, Formal analysis, Data Curation, Writing – Original Draft, Visualization. M.B: Conceptualization, Validation, Investigation, Writing – Review & Editing, Supervision. J.G: Conceptualization, Validation, Writing - Review & Editing, Supervision.

DATA AVAILABILITY STATEMENT

The data supporting the results of this study will be made available by the corresponding author, [M.J.C], upon reasonable request.

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