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POSSIBILITIES OF THE DGE USE IN MATH CLASS: BRAZILIAN AND GERMAN EXPERIENCES

POSSIBILIDADES DE UTILIZAÇÃO DE SGD NA AULA DE MATEMÁTICA: EXPERIÊNCIAS NO BRASIL E NA ALEMANHA

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Abstract

This article presents possibilities of dynamic geometry software (SGD) use in geometry teaching. It starts from a contextualization of the use of DGS in school and the reasons teachers use (or not) this technology in their practice. There is a discussion about the great potential of SGDs, especially the dynamics of dragging. The construction of geometric figures and the exploration of their invariants make it possible to study a range of concepts differently than with static resources such as paper and pencil. Based on the vast body of literature, they are synthesized the three main approaches that enhance the use of software, namely: exploration, proof and visualization. From the (non-comparative) example of two countries, Brazil and Germany, it is possible to note that SGD can be used in different ways, but the goal remains the same: to provide an environment for the student to construct knowledge of the field of geometry. When analyzing the various contexts, it is possibible to note that they complement each other and understand good experiences that can be adapted to the different realities of the classroom beyond these two countries. These types of activities may also be intertwined with practices developed in Brazil, with similar objectives, depending on the type of resource available by teachers and the type of proposal they consider most relevant to the class.
Keywords: Dynamic Geometry. Software. Mathematics Education.

Resumo

Neste artigo apresentamos possibilidades de utilização de softwares de geometria dinâmica (SGD) no ensino de Geometria. Partimos de uma contextualização do uso dos SGD na escola e as razões pelas quais os professores usaram (ou não) essa tecnologia em sua prática. Debatemos sobre o grande potencial dos SGD, especialmente pela dinâmica do "arrastar". A construção de figuras geométricas e a exploração de seus invariantes possibilitam o estudo de uma gama de conceitos de maneira diferente do que apenas com recursos estáticos, como papel e lápis. Com base no vasto corpo de literatura, sintetizamos as três principais abordagens que potencializam o uso de software, a saber: exploração, prova e visualização. A partir do exemplo (não comparativo) de dois países,

Brasil e Alemanha, é possível notar que os SGD podem ser usados de maneiras diferentes, mas o objetivo permanece o mesmo: proporcionar um ambiente para a construção do conhecimento do campo da Geometria por parte de o estudante. Ao analisar os vários contextos, devemos observar que eles se complementam e compreendem boas experiências que podem ser adaptadas às diferentes realidades da sala de aula, além desses dois países. A dinâmica desenvolvida na Alemanha e os diferentes tipos de atividades também podem estar entrelaçados com as práticas desenvolvidas no Brasil, com objetivos semelhantes, dependendo do tipo de recurso disponível pelos professores e do tipo de proposta que eles consideram mais relevante para a turma.
Palavras-chave: Geometria Dinâmica. Software. Educação Matemática

ORIGIN

This paper is a result of collaborative effort between the University of Education Freiburg and São Paulo State University. The project originated from an interest in the use of Dynamic Geometry Environment (DGE), analyzing its particularities and its use in Brazil and Germany. After a number of years, where each team study the characteristics of DGE and the possibilities of its educational application in middle and high school in his and her own country, we decided to summarize the possibilities of DGE use in mathematics classes, in order to discuss the potential of this resource and to showcase researches in Brazil and Germany related to this topic, as example (not in a comparative way, but complementarily). We begin with a short history of the field and then proceed to talk about the use of DGE in Germany and Brazilian schools.

THE TECHNICAL DEVELOPMENT OF DGE

In the 1980s, software for geometry education was, to some extent, already in use. Even though Papert (1980) considered the software Logo – which is now famously known as “Turtle Geometry” (see Abelson and Disessa (1981)) – to be an early general mathematical medium, it was not widely accepted in the curriculum of that time due to its “intrinsic” algorithmic geometry style. The problems were characterized by the fact that both Turtle Geometry and paper-and-pencil geometry having different meanings for the same terms.

A transition occurred in the software development in the years following Logo. Besides a universally used programming language, user-friendly applications were developed which were relatively unchangeable. These were gradually improved into more flexible and adaptable systems (see Biehler (1992)). Alongside these applications, graphical user-interfaces were developed, which, in hindsight, marked a major step in the development of today's dynamic geometry systems. Even though the software "Supposer" (YERUSHALMI; SCHWARTZ, 1985), for example, did not require the learning of a complex programming language, it still limited the user in its range of applications when constructing.

The content and software related limitations of Supposer were overcome in several ways by its successor Cabri-Géomètre (BAULAC; BELLMAIN; LABORDE, 1990). The navigation could be adjusted flexibly for content-related and didactic reasons. This way, tools could be freely removed or new macros could be added as required by the learning environment. The greatest achievement of this new geometry software is the hand-grip cursor, which enabled a continuous variation of constructions.

All the other successors of Cabri-Géomètre took this manipulation style as a model. Currently, the best-known dynamic geometry systems worldwide, besides the perennially successful Cabri-Géomètre II (developed by Baulac, Bellemain and Laborde (1992)) are GeoGebra (developed by Borcherds and Hohenwarter (2001)), Cinderella (developed by Kortenkamp and Richter-Gebert (1998)) and the Geometers Sketchpad (developed by Jackiw (1991)). Even though most software programs vary in appearance, both conceptually and ergonomically, most current programs offer the same options:

- to make constructions in school geometry more flexible by means of a *"dragmode."* The features of the constructions (e.g. variables) stay the same through constant distortions of the individual constructions.
- to visualize the "*locus of points.*" When dragging the starting points, a line appears that shows other (dependent) points.
- to carry out modular constructions, in which the sequence of construction steps is summarized into a "*macro.*"

Software that possesses those special features is usually called Dynamic Geometry Software (DGS) or Dynamic Geometry Environment (DGE) in the Anglophone world. In contrast to the tutorial-oriented systems (e.g. GEOEXPERT (HOLLAND, 1993)), the so called tool software experiences a sustainable development. Contrary to its predecessors, it does not aim to illustrate or visualize special geometric facts, but to support a wide-range learning with the help of new media. By playing around with geometry, the students themselves are supposed to experience the links between the different constructions and develop an understanding of them with the help of the software. Currently, DGE is being used in most schools in 2D, even though the 3D equivalent is available on the software market.

But what do the everyday geometry lessons look like in schools? Are most teachers really using DGE? When looking at didactic literature for geometry, we see that many articles demonstrate the possibility of using DGS while teaching geometry. Equally, there is much literature in the area of empirical studies about the learning and teaching process combined with DGS (see Furinghetti and Paola (2003), Hadas, Hershkowitz and Schwarz (2000), Haug (2010, 2012), Hölzl (1994, 1999), Jones (2000), Laborde (1995, 2000), Marades and Gutierrez (2000), Mariotti (2000), Mariotti and Bussi (1998), Olivero (2001), Olivero, Paola and Robutti (2001)). Normal geometry lessons in schools look similar. Even though the use of computers in math lessons is becoming less of a rarity, most teachers still do not use DGE. One reason is many teachers are still not taught how to use DGE during their training. Another reason is the teacher would have to go to a special computer room with the whole class in order to use DGE in a math lesson. Nevertheless, using computers in math lessons is not nearly as controversial as it was in the 1980s. Using computers, especially DGE, to influence, complete and support math lessons is no longer so controversial. Even so, the question remains as to how this can be achieved. Due to an increasing differentiation within the individual classes, it is becoming more and more difficult when students have to learn the lesson content at the same time and speed. This is especially the case because the following can happen: one learner might prefer to solve the task with the computer, whereas another learner might want to do the same task without. In light of this, it will become clear whether or not the old idea of computer rooms will last or if, at some point, every student will get his/her own laptop or hand-held device (such as the TI-inspire by Texas Instruments, or a tablet or smartphone). Students can then decide when and for which tasks they want to use the computer. These devices already contain the main tools: “computer algebra systems,” “dynamic geometry systems” and “table calculations.”

Besides these unresolved technical and organizational questions, there are more questions from the didactic perspective. What do these new technological possibilities offer for a geometry lesson based on computers? Is there a “more” for didactic lesson options?

Or does the use of calculators not really have an advantage, apart from increased motivation (TAAKE, 1987), when it only causes additional complexity?

Looking at the relevant literature given, we can easily find a range of characteristics DGE offers in contrast to the static learning environment with paper and pencil. When constructing or drawing in a dynamic way, it opens up a new range of actions in various respects:

• Geometrical constructions and links are more easily discovered with the drag-mode. When dragging the starting points it shows a continuous series of images of the constructions, whose essential characteristics (e.g. invariables) stay the same (see Arzarello et al. (1999), Arzarello et al. (2002), Hölzl (1995), Sträßer (2001)).

• Relating to inductive methods, dynamic geometry systems support concepts of the visual-dynamic argument (see Beckmann and Elschenbroich (2001), Elschenbroich (2005)), as well as the experimental argument considering assumptions and reasoning strategies (see Ware (2004), Fulvia and Domingo (2003), Marrades and Gutierrez (2000)).

• Using the drag-mode and locus of points, new heuristic learning strategies can be applied very well to solve problems. This is shown through fitting exercises, for example, which can be solved by leaving out one condition and the additional variation of the drawing (see Hölzl (1994, 1999)).

• On the graphical and operational level, there's a tool that supports more complex constructions through modular construction for the first time. This way of working (forming modules through the creation of macros) in connection with the locus of points offers an expanded range of construction possibilities with a dynamic geometry system (see Weth (1992), Lutz and Weth (1994a), and Lutz and Weth (1994b)).

Considering these viewpoints, what does the use of DGE offer in geometry lessons? Can process competencies such as geometrical argument, discussion and reflection be aroused and supported with this kind of software? After all, it is the character of geometry that offers a great chance to visually experience abstract mathematical links concretely with examples. In this respect, the hope of dynamic geometry systems lies in the implementation of the following:

• Promotion of heuristic problem-solving strategies by discovering invariants, making assumptions or by drawing help lines.

• Promotion of the ability to reflect on mathematical content with the help of interactive learning environments that lay the foundation for discussing, arguing and communicating.

• Promotion of an argumentative working through of functional dependencies.

• Promotion of exploration of a divergent way of working and monitoring.

• Promotion of experimental work through generating assumptions.

• Promotion of searching for special cases by making constructions dynamic.

If one wants to summarize all the points mentioned above that militate for the use of DGE in maths lessons, then the analysis can be interpreted to the effect that the use of computers can clearly enhance maths lessons.

However, the use of DGE can also have a very ambivalent character. On one hand, DGE enhances the heuristic potential of the mathematical radius of operations. On the other hand, the interactivity of the drag-mode poses the danger of distracting learners from their goals. Hölzl refers to the experiences of Hoyle and Sutherland (1989), who described this behavior of missing the goal as well as the playing around with the drag-mode as "degoaling." Because of the unavoidably large complexities of teaching, there is a need for professionally and didactically reflected concepts for the use of DGE. When learners do not understand the difference between a starting point, a crossing point and a glider within

DGE, they will only be able to develop analytical solving strategies to a certain extent. This type of "computational transposition" shows how hard it is for learners to understand the connection of their own geometrical imagination, the exploratory experiences and the already learned theorems on one hand, and the representation of geometric objects in DGE on the other hand (see Balacheff (1993), Balacheff and Sutherland (1994)). Laborde (1999), one of the developers of the Capri-Geometry, also spoke about this connection, claiming that one of the most important decisions in the development of the software was the distance between the Euclidean Geometry and the Cabri-Géomètre. This was the case because everyone already knew at this time that the success of such software depends on, among other things, how well the learners can analyze the constructions and geometric figures within the Cabri-Géomètre and link this with their pre-knowledge (Euclidean Geometry). An *increase* in complexity, through an underlying foundational understanding of geometry of DGE, necessitates an *increase* in pre-knowledge and DGE experience. This special concept of geometry puts itself between the subjective imaginations of a learner and the knowledge that has to be constructed when working on a mathematical task.

Some Characteristics of DGE and the Beginning of Its Use in School

With the creation of the first Dynamic Geometry Software in the early 1990s, began the first experiments in the use of the software in the mathematics classroom. This use began to approach the contents of plane geometry, but work was developed with contents that are not “geometric field,” such as, for example, the study of some types of functions, such as linear function (SILVA, 1997).

When teachers encountered the software, they began to realize that with this tool it was possible to explore another type of activity, different from those usually developed in class. The ability to construct a square and move it and the possibility of viewing it in different positions without having to build back up each one, as with paper and pencil, opened up the possibility of performing mathematical explorations that would require much time with this media. Therefore, new issues could be investigated by students. Also, geometric construction activities gained prominence in geometry classes because the construction time was much lower, increasing students' familiarity with this type of activity.

Reviewing the literature produced during the last two decades, we sought to identify what led teachers to start / continue their practices with the software. Justifying the integration of this media in mathematics classes, we synthesize the arguments into three main approaches to the use of software: exploration, proof and visualization.

One possibility is to harness the potential of software to make explorations. With it, students can formulate their own conjectures and try to see if they are valid. That is, the student him/herself will conduct the verification and validation of the conjecture that is made. This is possible due to features such as drag-mode, which allows the simulation of different cases of figures, as if the student were checking for “all” possible cases of the same family configuration.

The exploration with software, in general, can be done in one of two ways: the student performs some construct and then explores its properties, or the student receives a construction previously performed, called a “black box,” and is invited to explore some properties. Marrades and Gutiérrez (2000) point out that the two major contributions to DGE are, firstly, providing an environment where students can experiment freely, checking their intuitions and assumptions, and, second, providing non-traditional ways of teaching and learning of mathematical concepts and methods. A study by Olivero et al. (1998) concluded that the activities of free exploration of geometric invariants of a construction open space for the selection and formulation of conjectures could lead to the proof of experimental results (depending on the level of students' education).

This leads us to deal with another approach, which is to use software features to address activities that involve reasoning and proof. With DGS came a strong re-emphasis on the deductive aspect in the classroom. Many researchers have followed this direction, pointing out the potential of DGE for this type of work in the classroom (HANNA (2000); MARIOTTI (2000); SINCLAIR; ROBUTTI (2013); VILLIERS (1998)). In this perspective, the need to *justify* can be related to the need to *explain why* a certain property applies – that is, why the drag-mode passed the test in the case of DGS (MARIOTTI (2000); VILLIERS (1998)). This kind of explanation reinforces collective activity, where you can discuss the different explanations and make comparisons. As Mariotti (2000, p.32) adds, when construction is crafted using a DGS, the justification of a correct solution "requires an explanation of why some buildings work and others do not".

Moreover, the fact that students are convinced, by the test of drag-mode, that a property is true does not stop them from proving it. As Villiers (2001, p.32) points out, "the proof is not a necessary requirement for conviction, on the contrary, the conviction is most often a prerequisite for seeking a proof". This is borne out historically, where on many occasions we see that it is the belief that motivates the proof.

For the conviction, the drag-mode is the key point. Marrades and Gutiérrez (2000, p.63) confirm that,

> Dragging was sufficient to convince most of pupils of the correctness of conjectures, so questions like "why is the construction valid?" or "why is the conjecture true?" were important to induce students to elaborate justifications beyond the simple checking of some examples on the screen by dragging.

Moreover, Hadas et al. (2001), agreeing with Goldenberg, Cuoco, and Mark (1998), argue the drag-mode can convince, but stressed that "in DGE is possible to create opportunities for the creation of uncertainty, leading students to seek explanations" (p.129).

Thus, activities may be prepared to "create situations of conflict followed by surprise or uncertainty that lead students to seek explanations" (p.130).

Thus, in order that the use of DGE not do become an obstacle to the proof, we must not overemphasize the conviction, which is often acquired with the handling of the software. We must excite students to explain the reason for the veracity of their conjectures (VILLIERS, 1998).

Another possibility that can be explored with DGE is its visualization feature. According to official Brazilian documents in the area of Mathematics Education, mathematical thought is initially developed by visualization (BRASIL, 1998). For Fonseca et al. (2001, p.75), who wrote about geometry in primary school, visualization

> embraces the formation or conception of a visual, mental image (of something that is not in front of the eyes at the moment). That happens because, in fact, it is in the exercise of studying geometric forms that constitute the space, and in the description and comparison of their differences, that kids will construct a mental image, which will enable them to think about the object in its absence.

Visualization is part of "mathematical doing." Further studies stress the role of visual thinking and visual representations as mediating artefacts for the teaching and learning of mathematics and the relevance of spatial thinking in early mathematical development (GÓES; DAVID, 2010). When studying mathematics, the visualization is associated with the ability of interpreting and understanding pictorial information. For that, two processes can occur: to interpret a visual image or create a visual image from a non-figural information. Visualization is also considered as a "process of forming images (mentally or with paper and pencil or other technology), used in order to get a better mathematical understanding and stimulate the process of mathematical discovery" (BORBA; VILLARREAL, 2005, p.80).

The act of visualizing can consist of a mental construction of objects, or of the processes associated with them, perceived by the individual as external. Alternatively it can also consist of a construction, in an external medium such as paper, black board or computer screen, of objects or events that the individual identifies with objects or processes of his mind. However, a distinction is made between what is external (paper, computer etc.) and what is internal (mental). The individual (and not another person who sets it) perceives those objects as internal or external (BORBA; VILLARREAL, 2005).

In Mathematics Education, visualization is part of the students' processes of teaching and learning of mathematical production, which explains its relevance in this area:

> • "Visualization constitutes an alternative way of access to mathematical knowledge;
> • The comprehension of mathematical concepts requires multiple representations, and visual representation may transform understanding in itself; • Visualization is part of the mathematical activity and a way to solve problems;
> • Technology with powerful visual interfaces is present in schools, and its use for teaching and learning mathematics requires comprehension of visual processes;
> • If the contents of the mathematics itself may change due to computers, as proposed by some mathematicians, it is clear at this point that mathematics in schools will undergo at least some kind of change." (BORBA; VILLARREAL, 2005, p. 96)

It is noteworthy that, in Mathematics Education, visualization has educational value and is related to the understanding of students, which can translate into internal or external representations, with or without the use of media. With advances in technology, however, it has been closely associated with media, especially the computer. Therein, for Borba and Villarreal (2005), there is no dichotomy between internal and external representation. "We think-with" mental and external images and with different media, in various types of collective thinking. The two are so closely related that the dichotomy does not make sense. When we do not have access to external representations, identifiable to the eye, we appeal to internal representations, built over mathematical experiences. Thus, visualization is considered a resource for understanding mathematics, not just associated with graphical representations, but also with numeric and symbolic representations. Moustaki and Kynigos (2011, p.257) also note that "spatial and visualization abilities have long been considered as being connected to mathematical learning and aspects of it, such as geometrical thinking and problem solving". However, there are studies that point to the difficulty in visualizing and forming a mental representation, such as Ho and Har's (2011), which focus on the study of volume.

In this sense, visualization is considered a resource for understanding mathematics and the computer can be used to test assumptions, to calculate and to decide issues that have visual information as a starting point. According to Lourenço (2002), for example, the validity of a proposition is given great credence when it is supported by visual factors.

However, it is not possible to draw conclusions with the visual alone. Thus, the search for justification should not only prioritize the concept, but also the explanation, as we said before (HANNA, 2000; MARIOTTI, 2000; MARRADES; GUTIÉRREZ, 2000; VILLIERS, 1998).

To Marrades and Gutiérrez (2000), one of the advantages of DGE, confirming the ideas of Laborde (1998), is the ability to build complex figures and see them in different positions without having to construct them again, following changes in real time by dragging. Thus, this possibility makes the drag-mode potentially different from the traditional use of the paper and pencil.

HOW THE DYNAMIC GEOMETRY ENVIRONMENT IS USED IN GERMANY

In the German-speaking world, the abbreviation "DGS" or "DGE" is translated with the title "dynamic learning environment" alongside its literal translation "dynamic geometry system." A learning environment is a learning situation with different contextual factors "which can be planned and arranged to a different extent" (REINMANN-ROTHER; MANDL, 2001, p.603). This consciously arranged learning situation by the teacher involves teaching methods, teaching techniques, learning material and media to create a high-quality learning environment, which includes the relevant cultural context as well. Reinmann-Rother and Mandl (2001) make the lesson the center of interest in their writings.

They assume that such situations in lessons in general can be seen as learning environments, "in which learning processes are initiated, encouraged and facilitated by professional teachers with pedagogic intent and organized methods within a certain institutional frame" (REINMANN-ROTHER; MANDL, 2001, p.603). The planning and arrangement of a lesson, and the learning environment as well, is therefore dependent on the prevailing pedagogic and professional goal.

In the area of mathematical didactics, the term "learning environment" was influenced by, among others, Wittmann (1995), who spoke about "substantial learning environments" in the sense of innovative school books. But this viewpoint has been continuously broadened in the past years. Today, mathematical didactics subsume all conditions for teaching and learning processes under the term "learning environment," which offers the possibility from a constructional perspective for the learners to create their own learning paths by themselves. In order for the process to be successful, teachers have the task of setting up the appropriate learning environment, which depends on the individual goals and contents of a lesson. According to Leuders and Ulm (2007), these environments comprise three components:

• Tasks and their supporting media (e.g. worksheets, the objects under examination such as various fields, etc.).

• Forms of learning organization (e.g. methodical sequence, social format, potential room change, etc.).

• Support offered by the teachers as well as all available media and tools (e.g. books, flash cards, computer, calculator, projector, internet, OHP, etc.).

When discussing computer-supported learning environments as special forms of learning environments in math lessons, the focus in respect of this model is on the use of “new media.” In this sense, it is especially the conditions and effects of such use of new media that play a huge role. Alongside the question of an effective knowledge acquisition when using multimedia, the view of the learners’ handling with media and materials is of particular interest. If the computer is being used for exploratory learning as a dynamic tool, for example, special demands are placed on the learners. They then have to use the components autonomously if they are not guided closely through the multimedia learning offers. Barzel, Hußmann and Leuders (2005) speak about “digital learning environments” in this context and relate this term, above all, to the part of a learning environment that is prepared with media. This means, in essence, “the available presentation of problems and information, which is being used and broadened by the learners according to their interests, abilities and skills with results and experiences” (BARZEL et al., 2005, p.30). According to Barzel et al. (2005), digital learning environments can be differentiated by, among other factors, their degree of openness:

• With open tasks (see *computer tools to work on open-ended problems*)

• With partially guided tasks (*exploration of Black Boxes*)

• With clearly delimited, pre-made exercise sheets and learning units (*working with dynamic worksheets*)

This categorization is obviously only valid for the use of DGS in a digital learning environment. Dynamic geometry systems within such a learning environment can principally be used in the sense of a dynamic tool by constructing the figure under examination with the help of the software. DGS can also be presented in a relatively closed format in a digital learning environment, in which limited fields of application of the components are made available to the learner. In order to better describe these options in the German-speaking world with the help of the literature, options are shown exemplarily in the next three sections.

Use of a DGE as a dynamic tool

When using a DGE as a dynamic tool, the learners have to have command of both the navigation and the tool-components of a DGE. If learners want to examine the specialities of the "Pythagorean theorem" for example, this means that learners have to carry out the whole construction (or partial construction) of a right-angled triangle with its squares on the legs and the square on the hypotenuse themselves. In doing so, they will be able to obtain deepened insights into constructive and functional dependencies. The didactic function of such an arrangement of the learning environment is that the learners are confronted with the main aspects of the problem in the construction phase.

After they finish constructive tasks, the learners can carry out explorations with the help of a simulation. If learners develop assumptions in such a learning phase about how changes of the construction leads to new links, they can carry out these changes with the help of the tool-components. After such a (re-)construction of the learning environment, they can simulate the construction for the purpose of testing the assumed links with the drag-mode. Learners who internalize such heuristic work techniques in this way use a DGS to solve problem situations on changing levels as a simulation or a dynamic tool. The change of individual levels is a factor that especially opens up a more differentiated and individual solution-method for demanding, open-ended problems to the learner (see Haug, 2012).

From this perspective it becomes clear why learners, and therefore the mathematical research, is faced with particular challenges. If simulations can be constructed, and when necessary de-constructed, by the learner, the complexity of the potential way to learn increases. To master it successfully, the learners have to be helped in particular areas so that they can reach the content-related goal (see Haug (2012)).

Another difference arises in the quality of the learning environment that uses such open tools. The types of learning with these tools is seen as self-regulated, problem-solving and exploratory learning of concepts and terms (including their application). Besides content-related concepts, problem-solving, arguing and modelling competencies are being supported and – this is the central point – required in a certain sense. Therefore, the question of the requirements of learners cannot be reduced to using the tools, but comprises a complex, conceptually appropriate application in exploratory, problem-solving situations (see Hölzl (1994, 1999)).

Use of DGE as Black Boxes

Knipping and Reid (2005) follow a different, rather phenomenological starting equation for the construction of computer-based learning environments. They give the learners pre-made Black Boxes to work on: computer-based interactive learning environments whose behavior is not at first apparent. Additionally, they pose open questions that help the learners to discover mathematical structures by themselves. The results are discussed in partner or group work and documented in learning protocols. The goal of such a simulation is the discovery, for example, of mathematical connections of geometrical illustrations or constructions. Instead of having the learners create their own constructions, they are prompted to explore and describe the pattern of dependent construction elements by changing dependent elements (e.g. certain points in a triangle) with the help of "Black Boxes." When doing so, they learn to characterize illustrations and constructions by means of their special features (see Haug (2010)).

This type of exploratory work puts the learners in a position to discover mathematical patterns and structures independently. To successfully put this into practice, Knipping and Reid (2005) use a mixture of instruction and construction. On the one hand, they lead their learners to clear courses of action (observing phase) with the help of pre-structured questions; on the other, they give them enough space to independently explore new things (creating connections phase). In the last phase (analyzing), the learners try to ascertain the mathematical structures and patterns behind the individual tasks, which are not recognizable purely by working with a "Black Box." Therefore, "Black Boxes" offer the option of carrying out a change of perspective within a computer-supported learning environment and, seeing as geometry essentially consists of constructions, it strengthens the ability to look at the features of geometrical objects.

With the help of computers, a range of actions is created to examine mathematical phenomena independently. This type of phenomenological approach (exploring and describing) is especially motivating for learners and focuses first in its proceeding on the whole phenomenon of a mathematical content or connection. The subsequent mathematical exploration and scrutinizing of the gained knowledge and experience is sometimes easier for learners, since they already have a pre-knowledge through the completed exploration phase, which they have gained primarily through active operation.

Use of a DGE as a dynamic worksheet.

Elschenbroich (1999), like Miller and Ulm (2006), created very closed learning environments, in which the learners are given little space to explore independently, by using limited and pre-made electronic worksheets. Working with such dynamic worksheets aims to make mathematical connections visible by using a simple application of the individual tool-components and to make mathematical laws discoverable. The free use of tools is very limited for learners, often in order not to expect too much of them. These learning environments also aim to prevent an overexertion of the teacher when dealing with learners' divergent results. The learners only need a small amount of pre-knowledge about the relevant tool-components. However, they are less involved in the construction due to the strong guidance; the meaningful function of a problem-oriented exploratory procedure, which includes learners when generating the questions, is not really considered. Also, it often happens that learners think they have understood everything, as the finished editing of the content seems to make sense on the surface. The full extent of the underlying mathematical connections, structures and features possibly remain hidden as they are never asked to produce their own constructions (see Baptist (2004)). Teachers who use such dynamic worksheets as a learning environment in math lessons therefore focus on the use of a DGS as a tool to help visualize mathematical contents. The independent discovery and exploration of mathematical connections, or the exploratory working in relation to problem solving, will be excluded with such an application. When using these content-related reduced learning circumstances, the focus is rather to help the learners repeat or deepen what they have already learned.

The aforementioned description of different concepts for the use of a DGE within a learning environment shows the current development in the field in the German-speaking world. In German didactic literature, these three approaches are therefore a foundation for the creation of a suitable DGE-based learning environment. It depends on the teacher how and with what intention a DGE is used within his or her lesson.

HOW DYNAMIC GEOMETRY ENVIRONMENT IS USED IN BRAZIL

Discussion about the inclusion of Information and Communication Technology (ICT) in teaching practice and issues surrounding it have been considered by various studies for some time in the field of Mathematics Education in Brazil. We are going to highlight some of them, in a long difference of time, such as Penteado (2000), who investigated the implications of its inclusion in the practice of teaching and Javaroni and Zampieri (2015, 2016) that developed a large analyzing of use of the technology at school, in a collaborative research, which has the effect of encouraging the relationship between graduate students, teacher training and public schools teachers.

Brazil has a tradition of government programs for the training in and implementation of ICT in schools, such as Proinfo[1], which have been implemented in schools to achieve greater dissemination of knowledge of ICT. With regard to the subjects of the curriculum, the potential of the computer placing the teacher faced with the challenge of reorganizing the emphasis given to certain contents and to seek explanations for the "why" of privileging certain topics over others. This aspect of math course is a complicating factor for the use of computers in education in Brazil. But we know that computers make possible the investigation of mathematical concepts in differentiated ways.

The exploration of investigative activities in the classroom has been highly recommended by the literature in Brazil (e.g. FIORENTINI; LORENZATO, 2006; ROSA, 2008; COSTA, 2008; DOMINGUES et al., 2013). In this sense, the DGE stands out, since it can be a strong ally in investigations in geometry. Since its first release many works in this area of research have emerged, linking teaching geometry with the use of technology.

[1] The National Educational Technology (ProInfo) is an educational program created by government to promote the educational use of ICT in public high schools.

These studies have presented evidence that the teaching of geometric concepts together with the construction and exploration of dynamic geometry environments conduces to students' understanding about these mathematical concepts (ALMOULOUD; HANURA, 2000; BALDIN, 2006; BELFORT, 2000; BELFORT; GANI, 2000; BRUM, 2001; COSTA, 2008; FERREIRA; SOARES; LIMA, 2009; GRAVINA, 1996; HARUNA 2000; ROLKOUSKI, 2002; SANGIACOMO, 1996, 1998; SILVA, 1997; SILVA; CAMPOS, 1998).

Furthermore, by using it, it is possible to investigate, discover and rediscover, formulate conjectures, analyze results, make simulations, and especially issues related to structure its application (BRIDGE, 1996; MINGA, 1996; RESENDE; STOLFI, 1994; SCOTT, 1992; VALENTE, 1996; VELOSO, 1994). Balacheff and Sutherland (1995) further add that the Cabri-Géomètre, an example of such software, is an environment where the geometrical knowledge can emerge from the development of activities, and can be used from the earliest grades of elementary school, an insight that complements that of Sant (1995).

In Brazil, the term "dynamic geometry" has often been associated with Dynamic Geometry Software (e.g. AMORIM, 2005; BRAVIANO; RODRIGUES, 2002; ISOTANI; BRANDÃO, 2003; ZULATTO, 2002) and its main reference for many years was the CabriGéomètre. This software has become widespread in schools across the country, especially in São Paulo State, with the government having purchased the software for all state public schools. There have been many continuing education courses (FERREIRA; SMITH; LEE, 2008; LAKE, 2012; MULLER; LIEBAN, 2012) and events (e.g. Conference CabriGéomètre, 1999) to demonstrate the capabilities and experiences of the medium.

In recent years, GeoGebra has taken a prominent position, with specific events having been held (e.g. Latin American Conference GeoGebra, 2011) and its widespread installation in schools. The fact that it is free has no doubt contributed to this prominence. In São Paulo State, computers in public schools already have this software installed (CHINELLATO, 2013). However, given Brazil's size, it is difficult to generalize how the practice with this technology has developed. For this paper, we relied on the experiences of teachers disclosed at conferences and similar events and in research published in dissertations and theses, journal articles, and books.

We begin by dividing the pedagogical actions in two main areas: computer use by students and computer use by teachers.

Computer use by students, especially in the computer lab

When students use DGE, many practices are carried out by teachers. Schools often have many students per class (approximately 35-40), while the labs are usually equipped with a few machines (5-10). Thus, there are occasions on which the teacher works with the whole class collectively, and some in which he/she would rather divide the class into two groups, and, while one of them carries out activities in the traditional classroom, the other goes to the lab.

What can be noticed are two main perspectives:

- Introduction of content
- Checking content that has been previously studied

These two approaches are explored in the literature (ZULATTO, 2002). Some teachers prefer to allow students to investigate a concept, leading to the exploration of "what happens if ...?" In such cases, the concepts studied are constructed from investigation with the software and later formalized by the teacher in the traditional classroom.

From this perspective, the activities have more open questions and students have no initial idea of what direction the activity will go. They know that it is a guide that guides their exploration, and that something new must be observed. They are looking for something without knowing in advance what it is, some concept, some property to be construct. This type of activity requires attention in their preparation, to prevent students from feeling lost in this process.

Research and experience highlight that this possibility is not limited to geometry. This was shown by Stivan (2013), who conducted a study with GeoGebra, to allow students to see (graphically and numerically) what happens to a function by varying its parameters.

Another approach is to start with the concepts in the classroom, as the teacher usually does, and thereafter taking students to the computer lab to carry out explorations that lead them to view these that have been concepts previously studied. The properties, already known, are then experienced a dynamic way with the software, from buildings that, with the drag-mode, allow the visualization of a huge number of cases, without having to build them one by one.

In this case students know with property they are working with, and the software is designed to provide another approach to the same property – a way to view, from another perspective, a concept previously studied in a static way. Here, crucial to the planning of the activity is the need to think of an activity that is not identical to those developed in the classroom, but to bring a different approach to the subject, emphasizing aspects that gain a new dimension when performed on a DGE.

We emphasize that neither of the two perspectives should be considered better or worse. Methodologies as expectations differ, but they can both be powerful at different moments of math study. "In line", a portal for teachers, reported that "ICT has a big impact, at times serving to confirm results or to reinforce concepts, and in other situations being used by the students to build their own knowledge autonomously" (cited by Eduteka, 2007[2]).

Computer use by teachers using multimedia projector

As mentioned above, many schools have only a small number of computers in the lab. Despite government incentives (for example, the purchase of equipment) often what is perceived are situations that hinder its use, such as lack of maintenance of the machines.

In addition, other factors lead teachers to choose not to take their students to the computer lab. Many of them do not feel secure enough, because they understand that this practice can lead students to ask many questions that they do not know how to answer (PENTEADO, 2000). In addition, much time needs to be spent on the preparation of any of the didactic proposals mentioned above, which, due to the high teaching load of teachers, also tends to be a drawback in this type of practice.

Nevertheless, believing that DGE can contribute to the learning of their students, many teachers have chosen to use the computer as a resource in their own traditional classroom. To do so, they make use of the projector so that students can see the constructions held by the teachers.

[2] http://www.eduteka.org/EntrevistaWilliam.php

These types of constructions vary. They can happen “live,” with students following the construction step by step and their possible holdings, with probing questions from teachers. It may also happen that the teachers submit applications previously built (by themselves or by others), so that the teacher has the goal of showing students a property, for example, with support from the dynamism of the software. Oliveira (2010) is an example. She explored the properties of a unit circle with her students through the colours used in construction. Students were “spectators,” and did not handled the software, but able to follow a course different from the usual viewing of the properties in a different context of the paper and pencil and chalk-board.

As we mentioned, Brazil is a large country with different political realities to Germany. There are some schools with many computers and others with almost nothing, and there are also places where the teachers training to use technology in school are encouraged more than others, although it should be noted that there are national projects for the implementation of technologies in the classroom. We have tried to do a comprehensive portrayal of current research and practice, but certainly many others avenues are being developed, often without disclosure. As with anywhere in the world, we know that these practices depend on many factors, such as the lived experience of teachers, resources available, support for school, student participation and others. In this paper we hope to bring a sketch of Brazil to contribute to this broad subject.

FINAL REMARKS

In this paper we have discussed about possibilities of the use of DGE in math class to teach geometry. We presented the contextualization of the use of this technologic resource in the school, and reasons why teachers have (or have not) used this technology in their practice.

We discussed the major potential of DGE, especially enabled by the dynamics of the DGE by the "drag-mode". The construction of geometric figures and the exploration of their invariants enable the study of a range of concepts in a different way than with only static resources, such as paper and pencil. In the same way, to view the “locus of points” and to carry out modular constructions using a “macro” are possibilities that were born out of the dynamics of this kind of environment.

We raised issues that led to reflection on what use we expect of DGE beyond motivation: what are the various potentials of DGE in comparison to the traditional pencil and paper? Based on the vast body of literature, we synthesized the three main approaches that potentiate the use of software, namely: exploration, proof and visualization. Only this classification makes room for a new article, to examine different types of exploration, among the ones mentioned here, that could be made by teachers and students in geometry classes (such as black boxes, for example). Likewise, different kinds of visualization with DGE could be studied, such as simulation, animation, etc., which bring different viewpoints and concepts for geometry.

Intersecting with the perspectives that aim to provide a DGE, these points could be discussed considering the promotion of heuristic problem-solving strategies, such as the ability to reflect mathematical content with the help of interactive learning environments, argumentative getting through functional dependencies and exploration of a divergent way of working and monitoring, and so on.

From the example of two countries, Brazil and Germany, it is possible to note that DGE can be used in different ways, yet the goal remains the same: to provide an environment for the construction of knowledge of the field of geometry on the part of the student. In analyzing the various contexts, we should note they complement each other and comprise good experiences that could be adapted to different realities of the classroom, in addition to these two countries. The dynamics developed in Germany and the different types of activities could also be intertwined with the practices developed in Brazil, with similar goals, depending on the type of resource teachers have available and the type of proposal they think is more relevant to their class.

We hope this paper may encourage further research to examine aspects which were not considered here, but would complement the global scenario for the potential development of geometric concepts in the classroom.

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