

CHANGES OF TEACHERS' DISCURSIVE PRACTICES LEARNING DYNAMIC MATHEMATICS IN A COLLABORATIVE, ONLINE ENVIRONMENT

MUDANÇAS DAS PRÁTICAS DISCURSIVAS DE PROFESSORES APRENDENDO MATEMÁTICAS DINÂMICAS EM UM AMBIENTE COLABORATIVO *ONLINE*

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Abstract

Using data from an inservice professional development course, we inquire into changes in secondary teachers' discursive practices in a latter part of the course compared to the beginning of the course. Participants interacted in a collaborative online environment, known as Virtual Math Teams with GeoGebra (VMTwG), focusing on discursive, mathematical, and collaborative practices. From a sociocultural perspective, we believe that teachers gradually develop their Technological Pedagogical Content Knowledge (TPACK) by interacting discursively in small teams. Using conventional content analysis of the teachers' review of their recorded discourse, we investigate changes in practices from the teachers' perspective. Our results show that teachers perceived that their discursive practices differed importantly from their practices at the beginning of the course.

Keywords: Classroom Discourse; Dynamic Geometry; Teacher Professional Development; Technology.

Resumo

Usando dados de uma disciplina de desenvolvimento profissional para professores de matemática em serviço, investigamos as mudanças nas práticas discursivas dos professores de Fundamental II e Ensino Médio em uma última parte da disciplina, em comparação com o início da disciplina. Os professores interagiram em um ambiente colaborativo online, conhecido como Virtual Math Teams com GeoGebra (VMTcG), com foco em práticas discursivas, matemáticas e colaborativas. De uma perspectiva sociocultural, acreditamos que os professores desenvolvem gradualmente o seu



conhecimento tecnológico, pedagógico e do conteúdo (TPACK), interagindo discursivamente em equipes pequenas. Usando a análise de conteúdo convencional da revisão dos professores sobre seu discurso gravado, investigamos as mudanças nas práticas da perspectiva dos professores. Nossos resultados mostram que os professores perceberam que suas práticas discursivas diferiam importantemente de suas práticas no início do curso.

Palavras-chave: Discurso em Sala de Aula; Geometria Dinâmica; Desenvolvimento Profissional de Professores; Tecnologia.

Resumen

Utilizando datos de una disciplina de desarrollo profesional para profesores de matemáticas en servicio, investigamos los cambios en las prácticas discursivas de los profesores de Fundamental II y Enseñanza Media en una última parte de la disciplina, en comparación con el inicio de la disciplina. Los profesores interactuaron en un entorno colaborativo en línea, conocido como Virtual Math Teams con GeoGebra (VMTcG), con foco en prácticas discursivas, matemáticas y colaborativas. De una perspectiva sociocultural, creemos que los profesores desarrollan gradualmente su conocimiento tecnológico, pedagógico y del contenido (TPACK), interactuando discursivamente en equipos pequeños. Usando el análisis de contenido convencional de la revisión de los profesores sobre su discurso grabado, investigamos los cambios en las prácticas de la perspectiva de los profesores. Nuestros resultados muestran que los profesores percibieron que sus prácticas discursivas diferían importantemente de sus prácticas al inicio del curso.

Palabras clave: Discurso en Sala; Geometría dinámica; Desarrollo Profesional de Profesores; Tecnología.

Introdução

The rapidity of social and technological changes necessitates that teachers continually learn new technologies, pedagogies, and content to broaden their perspectives on mathematics thinking and learning. The Conference Board of the Mathematical Sciences (2012) advocates that mathematics teachers "develop the habits of mind of a mathematical thinker and problem-solver, such as reasoning and explaining, modeling, seeing structure, and generalizing" (p. 19). The practices of a mathematical thinker and problem solver evolve from continual participation in the performance of mathematics through discourse. Sfard (2001, 2008) argues that learning mathematical concepts is discourse phenomena rather than the acquisition of mental objects. She suggests that "becoming a participant in mathematical discourse is tantamount to learning to think in a mathematical way" (SFARD, 2011, p. 4). Concerning new technologies, the Common Core State Standards for Mathematics (2011) recommends "students consider the available tools [such as] dynamic geometry software... to explore and deepen their understanding of concepts" (p. 7). To accomplish this, teachers need to engage students meaningfully in lessons that incorporate dynamic mathematics software. For professional development in mathematics, this and a focus on discourse raise questions about how to apprentice teachers to broader perspectives that use new technologies to serve the goal of engaging in mathematical discursive practices.



Purpose of the Study

In the past decade, there has been research on discourse in mathematics classroom (MARTIN, TOWERS & PIRIE, 2006; MICHAELS, O'CONNOR & RESNICK, 2007; POWELL, 2006; SFARD, 2008; STAHL, 2009). However, the research literature in mathematics education contains few investigations on teacher professional learning in the use of dynamic mathematics software. Among what exists, Powell and Grisi-Dicker (2012) report that current research literature focuses on analyses of secondary students' learning with dynamic mathematics environments through individual interaction with such software. Absent are analyses at the small-group level of the discursive interactions of learners—students or teachers—collaborating to solve problems in dynamic mathematical environments.

To contribute to literature on changes in teachers' discursive practices when learning with technology in a collaborative, online environment, this study investigates evidence that teachers provide of differences in the discursive practices resulting from their participation in an online course learning to use dynamic mathematics software and to focus on discursive interactions, collaborating in small teams to solve mathematics problems. The cyber-learning environment in which the course occurred is Virtual Math Teams with GeoGebra (VMTwG), a product of a collaborative research project among investigators at Rutgers University and Drexel University. The environment contains chat rooms with collaborative tools for mathematical explorations, including a multi-user, dynamic version of GeoGebra (see Figure 1).

Material: Add a tab 🛨					Current users:
Ta	ab 10.01	Tab 10.02	Tab 10.03	Summary Synchrono	
	File Edit View Options Tools Window Help				Chat: (0) Ceder 10/21/13 5:38:40 PM EDT: So I used the compass to make GJ the radius and then set I as the center and H as the center carrigsb44 10/21/13 5:38:40 PM
	Take turns dragging and DEF. Chat abor notice, what sense what you wonder at Construct a triangle that behaves the sa Chat about how you	g vertices of triangles ABC ut what dependencies you you make of them, and yout this figure. inscribed in a triangle me as this figure. u are constructing and why.	C F		EDT: Most impressive tab construction yet! Ceder 10/21/13 5:38:58 PM EDT: then the intersection points which makes them dependent then connect!!! Sunny blaze 10/21/13 5:39:18 PM
	□ Hint1 □ H	int 2	A	B	EDT: that was impressive indeed. good job christine ceder 10/21/13 5:39:26 PM EDT: Message:
0	<u>2</u> 2	Take Control	History: ceder had contr	ol 🔓 Move	

Figure 1. VMTwG environment



We analyze and report on changes in teachers' discursive practices from their perspectives. Our data include teachers' asynchronous reflections and synchronous chat. Our analysis was guided by this research question: What are teachers' perspectives of differences in their discursive, mathematical, and collaborative practices from earlier online sessions to a session in the latter half of the professional development course?

Conceptual Framework

The conceptual focus of our framework is discursive practices. Our perspective is that discursive practices involve uses of natural and symbolic language to achieve specific goals. Mathematics and collaboration are particular forms of discursive practices in that they occur and are shaped in natural and symbolic language. From a sociocultural perspective, we believe that discursive practices are learned socially, as part of interactions among people, and that knowledge is an evolving achievement of interpersonal meaning making. We view mathematics learning as a discursive, participatory process (SFARD, 2001, 2008) and believe that in learning environments norms can be established to engender productive, accountable mathematical discourse (MICHAELS, O'CONNOR, & RESNICK, 2007; RESNICK, MICHAELS, & O'CONNOR, 2010). Moreover, building on the Vygotskian notion of the zone of proximal development, a group of peers has the ability though its discourse in either presential or virtual collaborative learning environments to develop new knowledge that exceeds the capabilities of any one member of the group (MARTIN, TOWERS, & PIRIE, 2006; POWELL, 2006; STAHL, 2005, 2009).

Collaborative problem solving among learners working as small teams is an interactive, layered building of meaning. Through their discursive interaction, teams create objects and, in turn, these objects shape and advance the discourse. Further, the team's discursive interactions occasion their reflections on relations among objects and dynamics among relations, as well as reasoning and problem-solving heuristics. The interactive work leaves the team with tools for future collaboration (POWELL & LAI, 2009). The public, persistent nature of online mathematical collaboration allows colleagues to follow their own and other teams' mathematical accomplishments, observing and reflecting on their colleagues' developing knowledge, successful mathematical collaboration, and shifts in discourse (SILVERMAN, 2011).

Concerning teacher professional development with technology, the research literature describes a long learning curve for dynamic mathematics environments (LABORDE, 2007). Mishra and Koehler (2006) argue that using technology in teaching requires a specific type of knowledge that teachers need to acquire, what they term, Technological Pedagogical Content Knowledge (TPACK). This knowledge is the integration of content, pedagogical, and technological knowledge that teachers use to teach effectively using technology. Acquiring TPACK helps teachers create



suitable representations of concepts with technology and be aware of and address epistemological difficulties that students face.

Methods

This study is based on four components of a larger project, Computer-Supported Math Discourse among Teachers and Students, a collaboration among researchers at Rutgers University and Drexel University. The first component is a discourse-based model for professional learning consistent with the TPACK model. The second, third, and fourth components are analyses of teachers' perspectives of changes in their discursive practices from earlier in the course compared to the seventh week of the course, as recorded in documents described below.

In the online professional learning course, 32 inservice secondary mathematics teachers engaged in small teams of four within an online environment, Virtual Math Teams with GeoGebra (VMTwG), in which each "chat room" has a chat panel, a whiteboard tab for summaries, and a GeoGebra tab, where team members can define dynamic objects and drag their base elements around the screen.

The course is 11 weeks long and consisted of six modules. Except for the first and last, each module lasts two weeks and engages teachers in a cycle of activities, consisting of two pairs of asynchronous and synchronous interactions. In the first cycle, team members do an individual GeoGebra activity in their own tab in VMTwG and post their noticings and wonderings to an asynchronous discussion forum. The team then meets synchronously, and members chat about their noticings and wonderings. In this synchronous session, the team also collaborates to solve an openended mathematics problem in GeoGebra, guided by prompts to discuss the mathematical ideas in which they engage, for team members to take turns to accomplish the activity, and to explain reasons for their GeoGebra actions. Each team member is accountable to the whole team, ensuring that every member is capable of accomplishing each task. Team members reflect on their experience, how their experience relates to assigned readings such as the Common Core State Standard Mathematical Practices (COMMON CORE STATE STANDARDS INITIATIVE, 2011) and accountable talk (MICHAELS, O'CONNER & RESNICK, 2007), how they will structure a similar activity for their students, and comment on each other's reflections. Finally, team members reflect on the logs of their prior week's synchronous discursive interactions, captured in VMTwG, to identify successful discourse moves and discourse moves that may have hindered progress, posting their reflections in the asynchronous discussion forum. In a final synchronous session, the team discusses the interesting reflections of their discursive practices. Each course module repeats this cycle.

These cycles of problem solving followed by analysis, discussion, and reflection in small teams were designed to move teachers towards the goal of facilitating the transition from doing



mathematics and supporting each other's mathematical development to synthesis and reflection on the significant mathematical ideas that transcend particular solutions or solution methods (SILVERMAN, 2011). These cycles integrate TPACK and involve teachers in authentic problem solving with technology (LEE & HOLLEBRANDS, 2008; MIRSHRA & KOEHLER, 2006).

To investigate teachers' perspectives of changes in their discursive practices in the seventh week of the course compared to the beginning of the course, we analyzed the second, third, and forth components of our data. The second component was individual teachers' asynchronous reflections on their team's prior week's chat logs and the third component was segments of their prior week's chat logs the teachers chose as evidence to support their statements. The fourth component was the team's synchronous discussion of each other's reflections and evidence. For each of the nine teams consisting of two to four teachers, a document was created containing each member's reflections and evidence and the chat log from the team's synchronous discussion of these reflections. For each team, this combined the second, third and fourth data components.

To analyze these data, we employed conventional content analysis (BERG & LUNE, 2012; HSIEH & SHANNON, 2005) since we sought to understand how teachers describe and provide evidence for changes in their discursive practices when solving problems with technology in a collaborative, online environment within small teams. We avoided preconceived codes, categories, and themes and instead allowed them to emerge from the data. After we constructed a guiding research question, we read to immerse ourselves in the data for one team, highlighted key words, wrote analytic memos; based on the key words, we created codes and defined them, using the teachers' words, and identified exemplars of each code. We then arranged related codes into categories and similar categories into themes and created a tree diagram to organize the themes, categories, and codes into a hierarchical structure. Two coders analyzed one team's data until intercoder reliability was over 70 percent, considered reliable (DE WEVER, SCHELLENS, VALCKE, & VAN KEER, 2006). Then the codes and categories were uploaded from an Excel file to the software Dedoose and the coders divided the remaining eight teams for analysis within Dedoose, identifying exemplars for codes that were different from those previously coded or identifying new emergent codes. We compared our work and modified the tree diagram within Dedoose to reflect the new codes. In the next section, we present the results of our analysis based on the themes and categories of our tree diagram and follow with a discussion of our findings.

Results

Guided by our research question—what are teachers' perspectives of differences in their discursive, mathematical, and collaborative practices from earlier VMTwG sessions to a session in the latter half of the professional development course?—the data from the nine teams showed a wide



range of differences from teachers' perspectives. Analysis of these data indicated shifts in their practices in each of the three main areas—discursive, mathematical, and collaborative—from earlier VMTwG sessions to a session in the latter half of the professional development course. Here, we discuss the major changes in each area.

Discursive practices

By the seventh week into the course, the 32 teachers cited many changes in their discursive practices. In general, they believed that their mathematical discourse had become more productive as one teacher wrote: "By reviewing our old logs and discussing what we did well and how we could improve we have gotten better at employing productive math talk." The teachers reported important improvements in their accountable talk, referring to specific accountable talk questions as evidence of change, such as these two examples: "I also notice implementation of prior article's hints about accountable talk, where we say 'I haven't heard from you, what do you think' or 'Does anyone have anything else to add?" and "We are interested in knowing that we all are involved. IF someone is silent too long, we try to find out why." They made certain everyone understood before moving on, offered or asked for control of GeoGebra: "in this part of the chat other teammate took control to really comprehend the movement of the figure." The teachers used these examples as evidence of their improved accountable to each other.

Accountable talk was one communicative practice that shifted, and the teachers reported substantial shifts in other communication practices. Most noticed changes in the quality of their questions. One teacher said: "We also continue to get better at asking each other questions." They noticed that they questioned each other more in latter session. In addition, teachers noted changes in their listening, responding, and explaining to each other. A teacher said "I chose the following segment because it shows how we have grown to explain things better before we do them, ask more questions and answer them better." They arrived at the idea that it is important for the team member who is constructing an object in the GeoGebra tab or window (called the controller) to explain what he or she is doing. In the early sessions, the controller would work in the GeoGebra tab and the others usually would not be able to follow exactly what was being constructed and why. A teacher provided evidence of this that by saying:

In this part of our VMT session, Dave is the controller and is in the process of drawing a square. He does a great job of "talking" about his construction and why he is doing certain things in the GG [GeoGebra tab]. He is also very responsive to our questions and he clarifies when Dan and I don't understand or ask for more information. This is a tremendous difference from our first few VMT sessions where we struggled to communicate and couldn't really see what the controller of the GG was doing during constructions. It shows not only Dave's understanding of the underlying mathematics used in the construction, but also the collaboration involved in the activity.



This shows how teachers recognized improvement in their communicative practices and the important of explaining and justifying their GeoGebra constructions.

Mathematical practices

To shed further light on teachers' descriptions of changes in their discursive practices, we report separately the teachers' described changes in their mathematical and collaborative practices. The teachers identified differences in their mathematical practices on several levels. Teachers identified sequences from their team's chat log from the sixth week that demonstrated Common Core State Standards Mathematical Practices, such as "This was tied into the 'original conjecture' from another section showing problem perseverance (MP1)", where MP1 refers to Mathematical Practice 1 (Make sense of problems and persevere in solving them). The teachers discovered new mathematical relationships as a result of collaboratively manipulating GeoGebra, such as the following:

This is the first session in which we realized (as a team) that we can create a square using a line segment and the rotation tool. Also, in line 220 Morgan states "When we change the scale factor both dimensions are multiplied by that specific number. Because of this the area changes by the square of the scale factor". ... I realized that I had never recognized that relationship before. Morgan had also never realized it. I think this is a great example of mathematical collaboration because we really learned something from each other.

Another mathematical practice that teachers recognized in their discursive interactions was their use of different representations in GeoGebra. A teacher stated "We also used different representations to explore the idea, using both words and diagrams to clarify the thinking that was being developed." Their interactions in VMTwG over the seven weeks of the course yielded changes in their mathematical practices.

Collaborative practices

The teachers cited many differences in their collaborative practices. One teacher described how they used to approach the problems at hand:

When we first began our synchronous collaborative group, there would be a long pause while the GeoGebra controller created the object. Once the object was created the next person would do their part. There was little to no explanation about how the shape was created, so unless you were the geocontroller, you didn't know how to create the object. Conversely, in this segment, Marie asked, "how did you go about constructing the square?"

Teachers observed other changes in the seventh week of the course as compared to earlier chats using evidence from the earlier chats. These changes include growing as a team, everyone contributing to the team's work, members asking if anyone has anything to add, trying different approaches, discussing and agreeing on the team's problem-solving strategy before beginning GeoGebra manipulations. One teacher said: "I find that we as a team have become more productive



in our weekly tasks because of this communication and collaboration." They encouraged a frustrated teammate, as another teacher said: "Through suggestions and encouragement Pasquale did not shut down and instead went on...It was through our collaboration and working together that we got Pasquale to that point and in turned pushed the activity forward." These are examples of affective changes that teachers reported.

Teachers were asked to identify an instance where everyone in their team contributed to moving the problem-solving process forward, referring to an assigned reading about problem-solving stages. Many teachers identify those stages in their team's work. The following is representative of what teachers noted:

This is an example where we all worked together to solve a problem. We were talking about why the ratio of the area to the length of the square is always equal to the length of the side of the square. We went through Stahl's problem-solving steps, but not necessarily in the right order all of the time. We kind of went forward and then backed up and clarified and then tried again. At one point, Jade kind of jumped to seeking the equation before we had Identified the pattern. I think this is due to the familiarity we as math teachers have with this concept. It was good but difficult to go back and think more deeply about the underlying why behind the memorized formula.

This teacher identified Stahl's problem-solving steps (STAHL, 2011) in the team's work as flexible guidelines to approach problem solving. Another teacher responded that the team's process informed it pedagogically:

I think this was a big step, because instead of one person constructing and explaining while the other questioned the STEPS, we instead discussed the mathematics behind the constructions and why it worked and how we could better discuss with our students.

Teachers noted that discussing mathematical reasoning while problem solving as important and how to extended this discussion to their classrooms.

In the synchronous session, as teachers discussed each other's perspectives on their changes, they discussed how, at that point in the course, they were applying what they have learned. They became more attentive to each other's chat messages and used the persistent feature of these messages to scroll back and read questions that were posted that they might have missed while working in the GeoGebra tab.

I think we've all become more aware of the chat, and we look back frequently while working to keep the conversation going while we're working, and I think we are reading up more than the last line typed and catching more of the questions, even when other questions have been asked before a response is given.

They saw benefits to the persistent feature of the chat messages. They also reported shifts in their use of mathematical vocabulary and encouraged similar improvements with their students: "I find my self really enforcing proper mathematical vocabulary more than ever! And I think it's really helping the kids understand topics more thoroughly". Teachers also reported that they more closely followed instructions in the VMTwG activities, responding to the prompts with improved noticings



and deeper understanding. They reported similar changes in their teaching practice, allowing their students more time to reflect, as demonstrated in these two excerpts: "I also find myself thinking extremely more with my students. I pause a little more often and I'm trying to get the kids to do the same" and "i try to give my students more wait time than i had previously instead of just explained the solution." Teachers reported that they have already begun to apply what they learned in their classrooms.

The following example reminds us that the teachers used the chat logs as evidence to verify their perceptions. This teacher recognizes that the evidence in the chat did not substantiate his perception of how he works with teammates:

As an aside, I cringed as I read through our dialogue. I find myself to be too impetuous, too controlling. It can be a challenge for me to work in groups...It didn't feel that way during the session, but looking at the dialogue is another matter.

This teacher was surprised about his own work; he recognized that his perception was inaccurate. The chat logs were the main source of support for teachers' perspectives on their change.

These examples show that teachers recognized important differences in their practices from early VMTwG sessions to a session in the latter part of the professional development course including improvements in their discursive, mathematical, and collaborative, practices.

Discussion

Our research intends to contribute to literature on changes in teachers' discursive practices from their perspective. In our work, teachers participated in a discourse-centered course where they learned cutting-edge technology while engaged in collaboratively solving challenging mathematics problems. Our analysis indicates that teachers recognized changes in their mathematical, collaborative, and discursive practices. As these changes occurred in natural and symbolic language, the shifts they described are improvements in their discourse.

An implication of our study concerns the amount of time teachers require to learn computersupported collaborative learning (CSCL) and dynamics mathematics software. Some researchers have found a long learning curve for teachers learning dynamic mathematics (LABORDE, 2007). We found that the CSCL environment that we used in the course (VMTwG) mitigated and shortened the learning curve. Moreover, the course supported teachers' enhanced integration of technological, pedagogical, and content knowledge. An advantage that teachers noted was the persistent feature of their discursive interactions in the chat messages, which contrasts with the ephemeral nature of classroom discourse.

The first cohort of teachers will participate in a second course in which they will apply what they have learned by using VMTwG with their students. We will analyze their implementation and



their students' chat logs to understand how the teachers structure and guide their students' collaboration and discourse.

Endnote

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References

BERG, B. L.; LUNE, H. *Qualitative research methods for the social sciences* (8th ed.). Boston: Pearson, 2012.

COMMON CORE STATE STANDARDS INITIATIVE. (2011). *Common core state standards for mathematics*. Retrieved, 2011. From http://www.corestandards.org/assets/CCSSI_Math Standards.pdf>

CONFERENCE BOARD OF THE MATHEMATICAL SCIENCES. *The mathematical education of teachers II*. Washington, DC: American Mathematical Society, 2012.

De Wever, B., Schellens, T., Valcke, M., & Van Keer, H. Content analysis schemes to analyze transcripts of online asynchronous discussion groups: A review. *Computers & Education*, 46(1), p. 6-28, 2006.

HSIEH, H.; SHANNON, S. F. Three approaches to qualitative content analysis. *Qualitative Health Research*, 15(9), p. 1277-1288, 2005.

LABORDE, C. The Role and Uses of Technologies in Mathematics Classrooms: Between Challenge and Modus Vivendi. *Canadian Journal of Science, Mathematics & Technology Education*, 7(1), p. 68-92, 2007.

LEE, H.; HOLLEBRANDS, K. Preparing to teach mathematics with technology: An integrated approach to developing technological pedagogical content knowledge. *Contemporary Issues in Technology and Teacher Education*, 8(4), p. 326-341, 2008.

MARTIN, L.; TOWERS, J.; PIRIE, S. Collective mathematical understanding as improvisation. *Mathematical Thinking and Learning*, 8(2), p. 149-183, 2006.

Michaels, S.; O'Connor, C.; Resnick, L. Deliberative discourse idealized and realized: Accountable talk in the classroom and in civic life. *Studies in Philosophy and Education*, 27(4), p. 283-297, 2007.

MISHRA, P.; KOEHLER, M. J. Technological Pedagogical Content Knowledge: A Framework for Teacher Knowledge. *Teachers College Record*, 108(6), p. 1017–1054, 2006

POWELL, A. B. Socially emergent cognition: Particular outcome of student-to-student discursive interaction during mathematical problem solving. *Horizontes*, 24(1), p. 33-42, 2006.



POWELL, A. B.; GRISI-DICKER, L. *Toward collaborative, discourse-focused learning with dynamic geometry environments*. Paper presented at the 12th International Congress on Mathematical Education, Seoul, Korea, 2012.

POWELL, A. B.; LAI, F. Inscriptions, Mathematical Ideas and Reasoning in VMT. In G. Stahl (Ed.), *Studying Virtual Math Teams* (p. 237-259) New York: Spring, 2009.

RESNICK, L. B.; MICHAELS, S.; O'CONNOR, M. C. How (well structured) talk builds the mind. In D. D. Preiss & R. J. Sternberg (Eds.), *Innovations in educational psychology: Perspectives on learning, teaching and human development* (p. 163-194). New York: Springer, 2010.

SFARD, A. There is more to discourse than meets the ears: Looking at thinking as communicating to learn more about mathematical learning. *Educational Studies in Mathematics*, 46(1-3), p. 13-57, 2001.

SFARD, A. Thinking as communicating: Human development, the growth of discourses, and mathematizing. Cambridge: Cambridge, 2008.

SILVERMAN, J. Supporting the Development of Mathematical Knowledge for Teaching through Online Asynchronous Collaboration. *The Journal of Computers in Mathematics and Science Teaching*, 30, p. 61-78, 2011.

STAHL, G. Group cognition in computer-assisted collaborative learning. *Journal of Computer Assisted Learning*, 21, p. 79-90, 2005.

STAHL, G. (Ed.). Studying Virtual Math Teams. New York: Springer, 2009.

STAHL, G. *How a virtual math team structured its problem solving*. Paper presented at the international conference on Computer-Supported Collaborative Learning (CSCL 2011). Hong Kong, China. Proceedings p. 256-263, 2011.

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