MUDANZA SOCIOECOLÓGICA EN EDUCACIÓN MATEMÁTICA: REFLEXIONES ACERCA DE LA INNOVACIÓN CURRICULAR

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Resumen

Este artículo propone que la educación matemática ha alcanzado un momento de mudanza socioecológica. Se identifican tres factores condicionantes de esa mudanza. En primer lugar, los avances científicos recientes apuntan hacia la interconectividad de la vida y de las ecologías que viven dentro y fuera de los cuerpos humanos. En segundo lugar, los avances en las ciencias humanas indican la necesidad de que sean repensadas las relaciones humanas y no humanas. El tercer fator es que la precariedad ecológica del mundo sustenta la necesidad de que sean repensados los propósitos y objetivos de la educación matemática. En respuesta a esa mudanza socioecológica, son ofrecidas dos posibles innovaciones curriculares. En la primera de ellas el currículo comienza a partir de una apremiante problemática ecológica enfrentada por uma comunidad. En la segunda, em un contexto escolar padrón, se promueve una matemática comunitaria con énfasis em la construcción de preguntas por parte de los estudiantes. Las dos tienen en común la adaptación del currículo.

Palabras clave: Socioecología. Innovación curricular. Enseñanza de Matemática.

Uma Mudança Socioecológica em Educação Matemática: Reflexões Acerca da Inovação Curricular

Resumo

Este artigo propõe que a educação matemática tem alcançado um momento de mudança socioecológica. São identificadas três linhas para essa volta. Primeiramente, recentes avanços científicos apontam para a interconectividade da vida e das ecologias que vivem dentro e fora dos corpos humanos. Em segundo lugar, avanços nas ciências humanas indicam a necessidade de sejam repensadas as relações humanas e não humanas. E a terceira, a precariedade ecológica do mundo aponta para a necessidade de se repensarem os propósitos e objetivos da educação matemática. Em resposta a essa mudança socioecológica, são oferecidas duas possíveis inovações curriculares. Na primeira delas o currículo começa a partir de uma problemática ecológica premente enfrentada por uma comunidade. Na segunda, em um contexto escolar padrão, promove-se uma matemática comunitária com ênfase na construção de perguntas pelos estudantes. Em comum elas têm a adaptação do currículo.

Palavras chave: Socioecologia. Inovação curricular. Ensino de Matemática.

A SOCIO-ECOLOGICAL TURN IN MATHEMATICS EDUCATION: REFLECTING ON CURRICULUM INNOVATION

Abstract

This article proposes that mathematics education has reached a socio-ecological turn. I identify three strands to this turn. Firstly, current advances in the sciences point to the interconnectivity of life and the ecologies living within and without human bodies. Secondly, advances in the humanities point towards the need to re-think human and non-human relations. And thirdly, the ecological precarity of the world points to the need to re-think the purposes and aims of mathematics education. Two possible curriculum innovations, in response to a socio-ecological turn, are offered. In one case, the curriculum starts from a pressing ecological issue facing a community. In the second case, in a more standard schooling context, a communal mathematics is promoted, with an emphasis on students asking their own questions. A commonality is the dramatization of the curriculum.

Keywords: Socio-ecological. Curriculum innovation. Mathematics teaching.

Introduction

This aim of this article is to propose that mathematics education has reached a socio-ecological turn and to illustrate two examples of what this might mean in terms of curriculum innovation. The phrase *socio-ecological* was first used by Berkes and Folke (1998), in the context of management practices aiming for resilience. The phrase socio-ecological was used to point to the interdependence of social and ecological systems. Berkes and Folke proposed that links and relations between these systems are vital for analysis, in contrast to more typical approaches which take one or other side as fixed. They also noted that distinctions between what is taken to be "the social" and what is taken to be "natural" are not only artificial but also arbitrary. Any social system is constituted by natural agents, making it impossible to say where a natural, ecological system ends and a social one starts. Similarly, to take an example of an ecological system, we know that among trees there are vital social interconnections sustaining the natural system of a forest (Simard, 2021). The social is necessary to the ecological and vice versa. I will be using the term socio-ecological in this article, also wanting to hold in mind that the social and the ecological cannot be disentangled. I will at times use the individual words "social" and "ecological", for instance when they relate to the work of other authors, or to point to how aspects of the world have been kept separate in our thinking. But, to re-emphasise, my own preference is to move towards the use of the term socio-ecological both for largely human, social systems and for largely non-human, ecological systems.

While social perspectives (e.g., social constructivism or socio-cultural theory) within mathematics education are now commonplace (replacing more cognitivist approaches of the 1970s), the role of ecology is rarely discussed (Boylan & Coles, 2017). If the boundary between social systems and ecological systems is indeed largely arbitrary then the neglect of the ecological is potentially a significant drain on insight within mathematics education. Coupled with the potential for bringing the ecological in to view, there is also a pressing need to consider ecological issues, given that in many parts of the world the ecological precarity of the early twenty-first century is disruptive and destructive to life (e.g., droughts, fires, displacements). What would it mean for mathematics education if we viewed the division between the social and ecological as artificial, and did not take the ecological as a fixed background for social events?

In the next section, I set out some of the connecting influences which point to the significance of the socio-ecological at the current time (I am writing in 2021). Following this, I review past work linking mathematics education to ecological concerns. By way of summarizing this past work, I offer a map of possible futures, taken from Boylan and Coles (2017). I then exemplify two possibilities for curriculum innovation, drawing on current work taking place in Mexico and past work I have been involved with in England, and consider their similarities.

1 A socio-ecological turn

In this section, I trace three sources of thought, which all point to the necessity and significance of a socio-ecological turn in mathematics education. Firstly, current advances in the sciences point to the interconnectivity of life and the ecologies living within human bodies. Secondly, current advances in the humanities point towards the need to re-think human and non-human relations. And thirdly, the ecological precarity of the world points to the need to re-think the purposes and aims of mathematics education.

1.1 Science and life

Recent advances in science have pointed to a level of interconnectedness across human and non-human life that has not previously been appreciated. One avenue of insight has come from discoveries about micro-organisms living inside the human gut, our micro-biome or gut bacteria (e.g., Enders, 2017). It is estimated that, in terms of numbers of cells, there are more "non-human" cells living inside the boundary of the skin, than human cells. We are more non-human than human, on one way of reckoning. Another way of saying the same thing is that a human body supports an ecology of microbial life. One reason for the lack of study of the bacteria inside our gut is that many of them cannot live anywhere else and hence are not amenable to lab-based research. The bacteria are dependent on the conditions inside a human gut for life and they themselves are vital for human health. Gut bacteria exist in a relation of symbiosis to their hosts which has made them invisible to all but the most recent of scientific perspectives.

At the same time as new insights at a micro-level, scientists are becoming increasingly aware of the connectivity between human life and wider ecological systems; for instance, we are now aware that micro-plastics humans put into the environment end up in the human food chain and being ingested by humans across the world. Such interconnectivity was, again, all but invisible to the scientific perspectives of the past and assumptions, for example, of linear causation. Furthermore, the example of the interconnectivity of trees with each other and with fungal networks (Simard, 2021), mentioned above, points to the way in which humans have split the non-human world up into individual entities (such as trees) and analysed them as such, blinding ourselves, in the process, to the symbiotic and enmeshed relations which are actually sustaining them. Taken together, scientific advances across a range of fields suggest a view of humans as ecological systems, enmeshed in ecological systems, from scales spanning the micro to the global. Bateson (1972) coined the phrase "ecology of mind", my reading of which was to point to the way in which mental life extends beyond the skull and skin in vast circular re-entering circuits of influence. The sense of the symbiosis of life, emerging across disciplines, points the socio-ecologies of, not just mind, but action.

1.2 De-centring the human

A second strand of thought that points towards a socio-ecological turn is the call across a range of perspectives within the humanities for a *de-centring* of the human. One broad strand of such work goes under the umbrella term of post-humanism (e.g., Wolfe, 2010). There are some diverse sets of ideas all labelled post-human and there is not space to delineate them. One image that appeals to me, perhaps capturing a commonality across some versions of post-humanism, is the idea that the challenge of post-humanism is equivalent to the Copernican revolution. Whereas Copernicus displaced the false belief of the Earth being at the centre of the solar system, posthumanism proposes that human concerns need to be similarly displaced from being the sole and central focus of study and ethical consideration. Just as Copernicus helped us view the Earth in its place in the solar system, so post-humanism may help us view our role as humans in a more appropriate place (compared to assuming we are always front and centre). Moves to accord legal, personhood status to animals and to rivers are examples of what a post-human de-centring of the human might mean in practice.

Entertaining notions that animals may experience similar emotions to humans has long been tarnished, in the West, with the charge of anthropomorphism. And yet, as Safina (2015) argues, given many animals have similar brain and organ structures as humans and many have patterns of behaviour which are recognisable to us as humans (such as devotion to a partner, or grief) then the simplest explanation is that they experience emotions and thoughts which are recognisably human. At the least, this explanation should be considered the default position, on both scientific and ethical grounds.

Another disparaging term used for traditions which accord agency to the non-human world is "animism", the term carrying with it an implication, perhaps, of a category error being made (according animal status to what is not animal). However, with the recognition of the epistemological violence done by colonialism to indigenous ways of knowing, there is also recognition of the value and wisdom of some of these indigenous knowledge systems (Kimerer, 2013) which do not accord the human a primary position – a wisdom that feels sorely needed at this time.

1.3 Socio-ecological precarity

A third strand which points to a need for a socio-ecological turn in mathematics education is the incontrovertible fact of the ever-increasing precarity of the planet. I do not use the word "crisis" here, to avoid any implication that effects of, e.g., climate change are new, or that they are potentially fixable. In marginalised communities across the world, the effects of climate change are all too present, in the form of reduced crop yields, failed harvests, land dispossession, water scarcity. While in affluent parts of the world it may be possible to experience the impacts of climate change as relatively recent, this has not been the experience of the majority.

Growing awareness of socio-ecological precarity provokes questions about what role mathematics has, and could have, in understanding what is taking place, in communicating about what is taking place and in influencing what is taking place, questions raised as part of a critical mathematics education (Skovsmose, 1994). There has been work done on these questions, which is the focus of Section 3. What I want to draw out here, in terms of a socio-ecological turn in mathematics education, is the almost certainty that the world in 20 or 30 years' time will not look like it does today. No matter what mitigation strategies the global community is able to agree, past emissions have locked in a rise in global temperatures which will have an ever more destabilising effect on life. We know change is coming, but we have little idea what it will look like. In such a context, how might mathematics education help prepare students for an uncertain future?

1.4 Towards a socio-ecological turn

The three strands of contemporary thought, sketched briefly above, point towards the importance of the ecological coming into dialogue with the social and political in our thinking and action. The phrase socio-ecological points to a desire to think about human and non-human together and a key belief driving this paper is that such a perspective is necessary for a mathematics education that is relevant to the future. In the next section I consider how the socio-ecological relates to mathematics education by reviewing past work within mathematics education that has taken on the challenge of bringing the ecological into focus alongside the social.

2 Reviewing mathematics education and the living world

While a body of research and scholarship within mathematics education has developed, focused on socio-political issues, ecological issues have been largely neglected, but with some notable exceptions (Boylan & Coles, 2017). In a review of past work (Boylan & Coles, 2017), Mark Boylan and I identified four themes. The first was work relating to critical mathematics education (Skovsmose, 1994), the second was work linking ecology to mathematical modelling, the third was the idea of a sustainable mathematics education (Renert, 2011) and a fourth strand specifically looked at the question of climate change within mathematics education (Barwell, 2013). A fifth strand could be added, which is work done within ethnomathematics. Ethnomathematics, even recently, has tended not to address explicitly questions of ecology, yet the respect given to indigenous ways of knowing within an ethnomathematical perspective is in keeping with section 2.2 and, hence, is connected to the argument for the need for a socio-ecological turn and a de-centring of human perspectives.

A common reference across many authors working in the strands above, is the work of Freire (1970) and his notion of a pedagogy of the oppressed and proposals for a problem-posing pedagogy, i.e., a conception of education where students are able to work on questions of relevance to their lives and their communities, developing skills and confidence to become active citizens. The link between a socio-ecological turn and education as a site of political activism is important. Freire wrote about finding the "meaningful thematics", or "generative themes" (p. 96) of a community, when planning the content of a non-oppressive education. Freire, pre-figured some of the developments sketched in this paper, by proposing that generative themes "can only be apprehended in the human-world relationship" (p. 106). I take human-world relationship to be socio-ecological.

In Boylan and Coles (2017), we mapped a possible future for the development of a socio-ecological mathematics education. The rows of the map indicated a deepening engagement in ecological issues. My sense is that, as a field, we have barely moved from the first row in the 4 years between its publication and this current writing. The map makes no claim to clairvoyance, but was offered, and is repeated here, as a provocation to scholars to place their own work within the grid and to consider what alternatives they would see as a mapping of possible futures.

Table 1. "Mapping a future"

Development of curriculum, pedagogy and practice	**Teacher education and professional development**	**Creation and fostering of networks**	**Research including theoretical work and empirical study**
Small-scale, classroom-based projects and trials take place, using resources that open up mathematics to global issues, in the context of a subject-oriented curriculum.	Small-scale programmes are developed, on isolated sites, linking learning to teach mathematics with a questioning of the role of mathematics in the world.	A mathematics education conference hosts a symposium on mathematics education and the living world. Mathematics educators connect with others engaging in environmental education. There is a global sharing, online, of resources and experiences.	A research agenda is established to explore ways in which mathematics classrooms can become sites for exploring uncertainty, risk and global, ecological issues.
Some schools experiment with a curriculum in which mathematical development is balanced with interdisciplinary work. A range of models are tried and tested globally and the results are shared.	Teacher training courses internationally shift to include how the study of mathematics can relate to wider ecological issues and the living world.	Scholars from across the globe document, trial and share experiences of linking mathematics education to the living world, both within their own communities and internationally.	The first large scale research project is funded, to study the ways in which mathematics education can broaden to encompass ecological awareness.
One country changes its national curriculum to put ecological awareness and stewardship of the planet at the core of all teaching and learning.	Professional development, that supports inter-disciplinary ways of working in schools, becomes a norm, on multiple sites.	Research in this field, internationally, matures providing a range of forums for the on-going linking of teaching and learning mathematics to inter- and trans-disciplinary ways of working and the living world.	Schools become sites where new research takes place on trans-disciplinary questions and issues. Students and teachers and researchers collect evidence in their own communities about change, documenting oral histories and sharing findings.

Boylan & Coles, 2017, p.13

Reflecting on what is written in Table 1, a change I might make, were I to be involved in writing a new map now, would be to place greater emphasis on activism or action for change, within each of the columns.

Before moving to consider some practical examples of what a socio-ecological perspective in mathematics education might mean for curriculum innovation, I will recap the argument so far. Firstly, I hope to have shown that there is a need for breaking down the barriers which have separated the social and the ecological in much of the theorising to date within mathematics education. Secondly, I have pointed briefly to some current and recent strands of research within mathematics education which are working towards such an aim. And, thirdly, I believe there is an urgency for moves towards curriculum innovation in mathematics education that pays attention to the precarious present time of the planet. All four strands of the map are relevant, and perhaps even necessary, for curriculum innovation. In the next section, I offer two examples (from the first row of the map) of school-based innovations "in the context of a subject-oriented curriculum". The purpose of these examples is to draw out one practical implication of a socio-ecological turn.

3 Curriculum innovation towards the socio-ecological

Given the arguments above, what are some possibilities for the mathematics curriculum? What would an innovation look like, which respected or took account of the socio-ecological? Without feeling able to answer these questions with any generality, in this section I compare and contrast two examples of curriculum innovation, one from Mexico and one from the UK. These were chosen partly for their contrasts and partly for being two innovations with which I have some personal connection. After setting out briefly the context of the work, in each case, I consider which mathematics is planned for students working on the curriculum; I use the categories of that mathematics in order to analyse the detail of the planning for the innovation (there is not the space to consider the actual work of students).

3.1 Example of a community-based project: mathematics education with the living world

The work in Mexico I report on here is a two-year (2019-2021) project, funded by United Kingdom Research and Innovation (UKRI). The Principal Investigator is Armando Solares, and I am a Co-Investigator and the only team member based in the UK (i.e., the work takes place in

Mexico). A project aim is to bring together interdisciplinary teams of scientists, university educators, teachers, non-governmental organisations, community leaders and policy makers – in order to design curriculum innovations which bring the concerns of the community into the life of its schools. The particular work I report below is taking place in a community which is in the catchment area of what has become a highly polluted river (The River Atoyac). As a direct result of (illegal) pollution in the Atoyac, there are higher than usual levels of cancer, miscarriage and childhood leukemia in the surrounding areas. Before the project began, there were few opportunities for primary schools near the river to bring questions of its pollution into the curriculum. The project is providing support for curriculum change called for within these communities.

3.1.1 Which mathematics?

The view of mathematics informing the project is closely linked to the perspective of critical mathematics education (Skovsmose, 1994) and this is the perspective used to frame an analysis of the work being planned. Researchers brought an awareness that mathematics is far from neutral in socio-ecological questions. Mathematics has a role in the modelling, description, analysis, communication, even creation of global challenges – and this role is linked to human decisions, by actors with their own agenda. The curriculum innovation in Mexico was co-planned by three groups, whose work is described below. The tasks they developed, although taking place in schools where the curriculum is subject-oriented, start off from an inter-disciplinary problem (the pollution of the river) that is keenly felt in the community. Mathematics emerges out of a wider socio-ecological problem facing a community, rather than being present as a set of objectives driving planning.

Critical mathematics education has considered three forms of knowing and these forms of knowing provide a framework with which to reflect on the tasks for students in the Mexico project:

> (1) Mathematical knowledge, which refers to the competencies we normally describe as mathematical skills. (These include competencies in reproducing mathematical thoughts, theorems and proofs, as well as in performing algorithms for calculations. The advanced competence of inventing and discovering new mathematics is also included.)
> (2) Technological knowledge, which refers to the ability to apply mathematics and formal methods in pursuing technological aims. (We

> concentrate our discussions on the applications of formal methods because they characterise highly technological societies.)
> (3) Reflective knowledge, which has to do with the evaluation and general discussion of what is identified as a technological aim, and the social and ethical consequences of pursuing that aim with selected tools. (SKOVSMOSE, 1994, pp.100-101)

In the next section, I describe how the planning for the curriculum innovation and relate this description to the forms of knowledge (above) which are being pushed or provoked. The planning described below took place early in the academic year 2020-21, in one region of Mexico, near the Atoyac River.

3.1.2 Curriculum tasks

The overall framing for the curriculum innovation was co-created at a meeting of teacher educators and teachers and this was the idea of developing a Memorial Museum to the River Atoyac. It is important to note that the Spanish word "memoria" could also be translated "memory", so an alternative title would be: Memory Museum of the River Atoyac. The decision was made (at a network meeting) to design a year-long project, which would run alongside the children's usual (subject-based) curriculum, and to create an exhibition or temporary Museum, which could then become something which is preserved and made suitable to be a travelling exhibition around the region. The Memorial Museum was split into three galleries. Each gallery had a team to develop it, comprising university educators, teachers, and members of a community-based non-governmental organization (NGO). The teams developed the plans below, for each gallery, near the start of the academic year.

Gallery 1: Once upon a time a river

Objective: to reconstruct memory and identity of my community in relation to the environment.
Triggering questions: What was the river like? What did the community members do around the river? What kinds of animals and plants inhabited its environment?
Links with Mexico curriculum content:

- Interviews of grandparents and elders from the communities.
- Historical accounts (using maps, drawings, photographs, videos).
- Radio scripts, journalistic articles, reports.
- Geographic and environmental space of local communities.

- Information analysis (interpretation and elaboration of graphs and tables of quantities).

Gallery 2: The Atoyac river contamination

Objective: Analyze the river pollution process, its characteristics and the health risks

Triggering questions: What is pollution? What types of pollution exist? Who pollutes? How do they pollute?

Links with Mexico curriculum content:

- Describe how humans transform nature by obtaining resources to nourish and protect ourselves (third grade).
- Explain the relationship between water, air and soil contamination due to the generation and improper handling of waste (third grade).
- Analyze the deterioration of ecosystems from the use of resources and technical advances in different stages of human development: hunter-gatherer, agricultural and industrial (fourth grade).

Gallery 3: Children's right to a healthy and toxic-free environment

Objective: to recognize that our universal rights are part of each one of us, as people.

Triggering questions: Do you know your universal rights? Have you heard about the right to a healthy environment? How to exercise a universal right?

Links with Mexico curriculum content:

- Assembly and reflection on the rights of children.
- Wall newspaper, journalistic notes, interviews.
- Geographic and environmental space of local communities.
- Information analysis (keywords, flowchart representation).

Reflections on the Memorial Museum

In the context of this article, one obvious reflection is that the presence of mathematics is not central to the design of the curriculum innovation and this was, of course, deliberate. However, there are elements of mathematics in each gallery. In terms of mathematical knowledge (as defined by Skovsmose, 1994), there are mathematical skills which will be developed or practiced (interpretation of graphs and tables in gallery 1; analysis of data in gallery 2 and 3). The second of Skovsmose's categories is technological knowledge, which is linked to technological "aims". I would interpret one of the technological aims, relevant to the work planned within the Memorial Museum, as being how industrial by-products and waste are disposed. Hence, in gallery 2, when the students consider who pollutes and how, and combine this with their analysis of the pollution levels themselves, they will be applying methods of analysis, interpretation and calculation, in order to make sense of the technological aim of waste disposal. However, it is clear from the Museum design that the aim is to go beyond an understanding of the technological aim of waste disposal alone, and to consider the environmental, social and ethical implications of how this aim is pursued in their community. Such a broad, overall aim is a clear example of a plan to develop reflective knowledge. In other words, as students learn about the history of the river, how European and national companies have used it to deposit waste, at no cost, over decades, they are learning about the impacts of technological aims bound up with the globalized economic system, which meant that international companies found it beneficial to locate factories near the Atoyac. Indeed, it is not too far of a stretch to say that one of the reasons Mexico presented as an attractive site for European companies is that they were able to dump waste in the river, reducing their running costs compared to having to dispose of waste in a responsible, costly and legal manner.

The social and ecological come together in this curriculum innovation through the focus on the river itself. The river is a living entity and one way community activists frame their work, which aims to reverse the current situation of the Atoyac, is to draw attention to the rights of the river. It is actually not possible to draw a boundary between the ecological and the social either in the causes or the effects of pollution. There are social and ecological conditions that lead to companies making decisions to dispose of waste in the way they do and the ecological and social impacts of the accumulation of toxins are similarly impossible to tease apart. A childhood death from leukemia can be traced through the ecology of water in the region and through the socio-economic conditions of the region, and both intertwine with each other.

3.2 Example of a school-based, symbolically structured environment

The second example, in this section, points to possibilities within a more narrowly defined mathematics curriculum, which is the case in many parts of the world. There may be contexts where teachers cannot engage with communities, or where national curricula are highly constraining. In these contexts, what can be done to bring the socio-ecological into the mathematics classroom? What relevant skills might students gain, through the study of mathematics, which might help them bring the social and ecological together? The project described below was a curriculum innovation which took place in the city of Bristol, within a single state school (Hayling School, a pseudonym), serving some of the more deprived catchment areas in the region. The work of this school was part of my doctoral study (Coles, 2013).

The aim behind the curriculum innovation in Hayling School, was to "Think Mathematically" and this was made explicit to staff and students. Students were offered the purpose of the year of "becoming mathematicians" where this phrase was interpreted as pointing to skills such as noticing patterns, making predictions and considering proof. The words "conjecture", "counter-example" and "theorem" were introduced to students in the first year of the school (students aged 11-12) and used repeatedly by staff, until they became words that were used by students and ideas that guided their actions (Coles, 2013). There was evidence of students seeing their role in mathematics classrooms as coming up with conjectures, testing them, sharing them and then trying to think why they worked. Of course, for such mathematical activity to take place, there must be tasks that make such work possible. The framework below, of a symbolically structured environment (SSE), is one I developed with Nathalie Sinclair – it captures features of the tasks used (the notion of a SSE was not developed with Hayling School in mind, but rather as a way of thinking about the work of educators such as Caleb Gattegno, Seymour Papert and Robert Davis). I will explain briefly what we mean by a SSE and then offer one example from the mathematics department teaching plans of Hayling School.

3.2.1 Which mathematics?

The phrase SSE comes from the work of the anthropologist Catherine Bell (1991). Bell developed the notion of ritualization, which Sinclair and I (Coles & Sinclair, 2019a) have applied to mathematics teaching. A "ritual" in a mathematics classroom often has negative connotations of an unthinking sequence of actions, in contrast to more exploratory of meaningful activity (Sfard, 2008). In contrast, Bell's notion of "ritualization" is about privileging some activities over others and distinguishing between the "sacred" and "profane"

(1991, p.74) but far from being set in opposition to thought, Bell saw ritualisation as a particular form of thinking. For Bell, it is non-discursive thinking:

> Ritualisation is embedded within the dynamics of the body defined within a symbolically structured environment. An important corollary to this is the fact that ritualisation is a particularly 'mute' form of activity. It is designed to do what it does without bringing what it is doing across the threshold of discourse or systematic thinking. (BELL, 1991, p. 93)

With Bell, Sinclair and I resist making the jump from acknowledging something does not cross "the threshold of discourse or systematic thinking" to assuming it is therefore meaningless or unthinking (Coles & Sinclair, 2019). We have argued that ritualisation is a helpful concept for thinking about teaching and learning mathematics in contexts where students become engaged in the study and experience some sense of being able to make decisions and choices, and ask their own question. We proposed the notion of symbolically structured environments (SSEs) in the mathematics classroom as ones where:

(a) symbols are offered to stand for actions or distinctions;

(b) symbol use is governed by mathematical rules or constraints embedded in the structuring of the environment;

(c) symbols can be linked immediately to their inverse;

(d) complexity can be constrained, while still engaging with a mathematically integral, whole environment;

(e) novel symbolic moves can be made.

Rather than exemplify each line now, I illustrate them with one of the tasks used in the curriculum at Hayling School.

3.2.2 Curriculum tasks

The text below is taken from the guidance for teachers at Hayling School, which was created by the teachers in the school. The task was for use with 12-13 year old students. The mathematical context is matrix transformations. Matrices do not appear on the curriculum in the UK until at least age 17, and then only for students specializing in Mathematics. Therefore, the purpose of the task was not so that students learnt matrices but rather that they learnt about a wide range of geometrical transformations, had practice plotting co-ordinates, dealing with multiplication by negative numbers and zero, and had opportunities for noticing patterns and asking their own questions. The task was designed originally by Laurinda Brown (see Brown, 1991) and at Hayling School we continued to use the same starting shape and matrix that Brown had originally chosen.

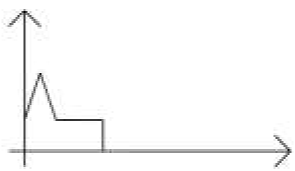
Figure 1. a first lesson

A first lesson:

Draw a co-ordinate grid in your books from 0 to 12 on x and y ...

Draw one on the board as you say this, or have one already drawn.

... and draw this shape at exactly the same points.



What is it? A church, yes!

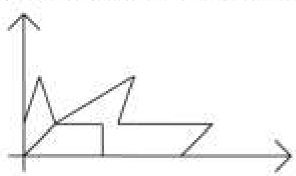
We are going to do a process involving these numbers that will change the shape. $\begin{pmatrix}2 & 0\\ 1 & 1\end{pmatrix}$

We will do it point by point. But first, so we know what we are talking about we need to label each point. Call this point A, what are its co-ordinates? Okay, in this project we write co-ordinates vertically. We are going to work together until everyone can do this process. Copy this into your books as we do it.

A *A'*

$$(1,5)\begin{pmatrix}2 & 0\\ 1 & 1\end{pmatrix} = (1x2 + 5x1, \quad 1x0 + 5x1) = (7, \quad 5)$$

Repeat these instructions for every point. Invite volunteers to complete.
Label and draw each of the new points:



What has happened to the shape?

You may want to introduce the language of stretches or shears and write up things like 'What happens to the area?' if students mention that kind of thing.

Try out your own numbers instead of $\begin{pmatrix}2 & 0\\ 1 & 1\end{pmatrix}$ *and see what happens to your own shape.*

You may want to discuss sensible shapes to try.

Challenge: *In 5 lessons time I will come in and write a set of four numbers (a matrix) on the board and your task will be to tell me what effect it will have just by looking at it.*

Reflections on a symbolically structured environment (SSE)

Each component of a SSE is present in the way this task was used at Hayling School. The students are introduced to the concept of a 2x2 matrix, which is a symbol standing for the action (condition a) of a geometrical transformation on a shape (in the first example, the shape being a 'church'). There are precise mathematical rules about how to operate with matrices (condition b) and these are taught to students explicitly at the start of the task. Having seen the transformation implied by a matrix, an obvious question to ask (and this either comes from the students or is prompted by the teacher) is, what matrix would transform the shape back to where it started (condition c)? The mathematical possibilities of matrix transformation are initially limited to 2x2 matricies. And within that constraint, it is possible to further constrain the situation, for example by insisting there are two zeros initially (constraint d). And, finally, students are encouraged to think of their own matrix to try and their own shape to try it on (constraint e).

In other words, the task above has the potential to fulfill all the aspects of a SSE. A SSE makes no mention of ecology or even the world outside the classroom. As such, it is potentially a framework for innovation which is possible to implement within a subject-oriented curriculum. And yet, if students take on the challenge of asking their own questions, working within a SSE can allow for a problem-posing pedagogy (Freire, 1970). And, where the task is designed to include elements well beyond the curriculum year of students, it is possible to square the circle of both allowing genuine student choice and covering a standard curriculum.

Drawing on a recent theorisation, this is a task that allows for *differentiation from an advanced standpoint* (Coles & Brown, 2021), which means choosing a topic well beyond expectations of what students need to be able to master, in order to use that topic as a context for practicing the skills needed on the curriculum, but in a manner where there can be genuine student choice – because whatever students end up doing (so long as it is within the SSE), they will be practicing the skills they need to know in the curriculum for their year. So, in the matrix example, it does not actually matter what matrix students choose, because whatever matrix they take, they will be plotting co-ordinates, multiplying numbers, comparing transformations, and more. My experience is that students can become highly engaged, working within a SSE, as they become interested in asking their own questions. It is possible for students to collaborate, across widely different levels of prior-attainment, particularly if there is a mechanism for the common collection of responses. At Hayling School, one mechanism used frequently was for students to pin up work, or results, on a board available for the whole class to see. In the matrices task, images of transformations, and the matrices which created them, can be gathered, allowing the whole class to engage in questions about similarities and differences. The common boards point are elements contributing to a communal mathematics, students working together as a class, rather than pursuing tasks as individuals.

The ecological aspects of a SSE are perhaps not so clear as in the Mexico example. However, if the ecological is read in its broadest terms, to include everything non-human, then one aspect of the melding of the social and ecological in the matrices task, comes from the mathematics itself. The process of matrix multiplication and its application to the co-ordinates of shapes in the Cartesian plane, mean that there are constraints built into the environment. If students choose matrices of a particular form (ones that are all equivalent to enlargements, say) then there are inevitable patterns which will be presented to them. Resources for students, to support mathematical thinking, come from both human communications and the structure of the mathematics being explored, with each one necessary to the other, in terms of occasioning something with which students might engage.

5 Discussion

The two examples sketched above could hardly be more different. In the first, mathematics plays a subservient role, and a socio-ecological issue drives the work and the engagement of both staff and students. In the second, the mathematics is what provides the structuring of the learning environment. Students are invited to engage in activities which will mean they, figuratively, hit up against the mathematics. The work they do will mean they get feedback about the patterns, regularities and boundaries of the mathematical space in which they are operating.

In the first case, the link to the socio-ecological is obvious. However, what is significant is the careful design to mean that students will be engaging in reflective knowing. It would be possible to design tasks or contexts which are linked to socio-ecological issues, without such reflective knowing being part of what is envisaged. It is the identification of aims and the reflection on the implications of those aims that raise the engagement to that of reflective knowing. In the second example, it is the maintenance of a SSE which will mean students are able to experience their own ideas about mathematics, are able to plan their own work and engage in reflection on the results of their choices.

There are some commonalities I would like to point towards, despite the differences. The first is a common feature of reflection. In the Mexican example, the tangibility of the polluted river provides ample opportunities for reflection, but such activity still needs to be planned, supported and, where appropriate, documented. In the UK example, the aims around mathematical thinking and developing habits of conjecturing and testing again provide opportunities for reflection on the outcomes of predictions, for example, and an altering of ideas as a result. And this leads me to a second commonality. In neither case, from the plans, is there any inevitability of either reflective knowing, or of the development of a SSE. It would be entirely possible to present the Memorial Museum to students in a manner that was controlled and where their attention was on the actions of fitting into a pre-determined plan, rather than on reflection about what they were finding out. Similarly, in the matrix task, it would be possible to constrain the task for students so that they all work on the same matrices and the space for making and testing predictions is constrained into what is already planned and known by the teacher. In both cases, the plans are to prompt communal action.

The implication of the observation that neither plan guarantees the hoped-for aims is, firstly, that the role of the teacher is fundamental. And, secondly, that in both examples of curriculum innovation there is a need to *engage* students in the plan. There is a need for a dramatization of the curriculum (Stengers, 2011), in the sense of making the tasks being offered dramatic enough to draw in students' attention and energy. As in a theatre, there is always a need for some kind of suspension of dis-belief in a schooling context. In the Memorial Museum, there is a need to suspend, for instance, any belief that the river pollution cannot be changed. In the matrices SSE, there is a need to be prepared to learn how to do matrix multiplication with little context or reason for doing so, outside the task itself. However, the patterns and connections and possibilities within the task can soon take over and become absorbing. Both tasks offer ample occasion for such a dramatizing, but nothing is guaranteed by plans alone.

5.1 Preparing for a future that is not what it what it used to be

"The future is not what it used to be" is the title of a book (Friedrichs, 2017), which argues that we cannot rely on the comfort of believing the future will follow the pattern of the past. Of course, that is only a comfort if your past has been comfortable and, for many communities around the world, a future that is different to the present may be something to be welcomed. Nonetheless, the phrase captures, for me, some wisdom that disruptions may be here to stay. This article has argued, in light of the inevitability and yet the uncertainty of change, that there is an obligation to consider what role the study of mathematics in school or university can play, in preparing students for a precarious future.

I have argued for a reconceptualization of the connection between what are often taken to be the separate spheres of the social and the ecological. A socio-ecological turn is taking place across the sciences, as we recognize, in one sphere after the other, the interconnected, symbiotic meshwork of living and non-living. This article has been an attempt to think through what such a recognition could mean for mathematics education. The map of a future, in Figure 1, offers one set of possibilities for a socio-ecological mathematics education.

I suggest that one element of re-considering possibilities for teaching and learning mathematics in school is to bring the agency of the non-human aspects of the socio-ecological (whether this be a river, or a mathematical structure) into play. I offered two examples of plans for curriculum innovations where the non-human was central to the classroom. In one example, a polluted river is the theme of a memorial museum, which allows reflection on past, present and future actions. In the second example, a mathematical structure provides constraints and feedback to students, which allows for the noticing of pattern and the raising of questions, and with reflection on actions leading to possibilities for new actions. In both cases there is a sense of a dramatizing of the curriculum (Stengers, 2011).

A socio-ecological turn implies viewing, as socio-ecological, what might more typically be taken to be either social, or ecological. So, for example, rather than considering communications in a lesson as a purely social system, a socio-ecological view might ask questions about the socio-ecological challenges facing the school's communities, and how these impact, or could impact, what tales place in the classroom; or, might ask about how the constraints of the conceptual structures in the intended curriculum might come to play an active role in students' work and discussion.

In this article I have focused on implications of a socio-ecological turn for curriculum innovation, via offering two examples. In terms of a socio-ecological turn more broadly, there is, of course, further work to be done, for example, to elaborate methodological implications for the doing of research. Also, the epistemological and ontological implications need much more detailed elaboration. In this article, I have merely pointed to the symbiotic, enmeshed, interconnecting view of knowing and being that emerges from taking seriously the role of the ecological in the social and the social in the ecological. In future work, I hope to elaborate more fully these philosophical implications as well as continuing to support work in classrooms.

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