

— “Mathematics”? What do you mean? — Don’t play the fool;
everybody knows it.

— “Matemática”? O que quer dizer isso? — Não se faça de bobo;
todo mundo sabe.

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Abstract

We criticize the polysemy of the signifier “mathematics”. Its commonsense meaning should not be considered enough to ground mathematics education. We describe a form of speech emerging in Ancient Greece, originated from the social necessity to avoid intra-family clashes by means of precise agreements, written laws, and democratic dialog in a singular historical situation. This form of speech emerged together with coinage, was made numerically precise with the Pythagorean movement, and logically precise after the crisis unleashed by Russell’s paradox in the beginning of the last century. We show how this form of speech has developed in history together with communities that came to be known as “exact sciences”, among which is Mathematics, a distinguished community of speech. We end the paper suggesting a political agenda for mathematics education.

Keywords: Labor theory of value. Genesis of mathematics. Lacan. Sohn-Rethel. Ancient Greece.

Resumo

Criticamos a polissemia do significante “matemática”. Seu sentido do senso comum não deveria ser considerado suficiente para embasar a Educação Matemática. Descrevemos uma forma de discurso que emergiu na Grécia Antiga, originário da necessidade social de evitar conflitos intrafamiliares por meio de acordos precisos, leis escritas e diálogo democrático dentro de uma situação histórica singular. Essa forma de discurso emergiu junto com a cunhagem de moedas, tornou-se numericamente precisa com o movimento pitagórico e logicamente precisa com a crise desencadeada pelo paradoxo de Russell no começo do século passado. Mostramos como essa forma de discurso se desenvolveu na história junto com comunidades que vieram a se chamar “ciências exatas”, entre as quais a Matemática, uma comunidade de discurso proeminente. Terminamos o artigo sugerindo uma agenda política para a Educação Matemática.

Palavras-chave: Teoria do Valor Trabalho. Gênese da Matemática. Lacan. Sohn-Rethel. Grécia Antiga.

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1 Foreword

In order to develop our argument, we will take the point of view of one who does not know what people mean by “mathematics” and ask what “it” is that was born in Greece? The reader may also wonder whether a journal of mathematics education is the proper place for this discussion: would not a journal on ancient history, sociology, psychoanalysis, or philosophy of mathematics be more adequate? To state only the first arguments, we observe that the polysemy of “mathematics” does not concern mathematicians at all, but facilitates the production of a wealth of articles in mathematics education, which politically point in different directions under the common purpose of “improving teaching”. In the blur of polysemy, these directions risk contradicting one another. Besides, can we make justice if a polysemy sustains our summative assessment, which in turn affects students' lives?

We also recall that the so-called sociopolitical turn in mathematics education has opened the interdisciplinary dimension of classroom practice, as surveyed by Lerman (2000), Gutierrez (2013), Nortvedt and Buchholtz (2018) on assessment, Rubel (2017) on critique of meritocracy, and many others. The hope of this movement to solve the inequity problem by developing sufficient content ability in the students is what constitutes the present world of mathematics education. Mathematicians may think that they discover mathematics in Plato’s world of ideas. This is of no avail to us; there isn’t a socio-political movement in that world. Mathematics education, on the other hand, deals primarily with an interdisciplinary object that unites human beings under the constraint of the school credit system. In this article, we propose a new, but conceptually rigorous way to consider “mathematics” that leads to the possibility of basing summative assessment exclusively on rewarding effort. This transcends the humanistic hope of the social turn movement and focuses on the point where exclusion materializes itself. In the end, we propose an agenda for mathematics education; we are already implementing it in our daily practice, after two semesters of remotely teaching calculus under the constraints of COVID-19 (BALDINO; CABRAL, 2021).

As a guide for the reader, we anticipate that in paragraph 2 we criticize the polysemic character of the signifier “mathematics”. We argue that the question “what is mathematics?” is already loaded with meaning – otherwise it would not make sense, like in “what is XPTO?” We change this question and ask instead what is it that people dispute whether it was born in Greece, Mesopotamia, or Egypt? The paragraph ends with a further hint to guide the reader. In paragraph 3, we evoke elements of Ancient Geek history to show how the social fracture between rich and poor in that society became an intra-family clash requiring a new solution: coinage, democracy,

and a new form of speech that we call quilted speech. In Paragraph 4, we show how coinage led to the Pythagorean project to ground quilted speech on numbers. In paragraph 5, we briefly describe the historical trajectory of quilted speech and discern a community of speech that we call Mathematics, thereby concluding the exposition of its genetic process. Finally, in paragraph 6 we suggest a political agenda for mathematics education.

2 Introduction

The exact argument of this paper is that *the signifier “mathematics” is a polysemy whose meaning relies on common sense and that is insufficiently rigorous to ground our classroom research and summative assessment judgment*. Well-recognized exact sciences, such as Physics, Chemistry, and Biology are also subjected to judgments as to what they are. For instance, “*Physics [...] is the natural science that studies matter, its motion and behavior through space and time, and that studies the related entities of energy and force*” (PHYSICS, 2020), “[...] *Chemistry is the scientific discipline involved with elements and compounds composed of atoms, molecules and ions*” (CHEMISTRY, 2020) and “*Biology is the natural science that studies life and living organisms*” (BIOLOGY, 2020). However, under “mathematics” we find: “*Mathematics [...] includes the study of such topics as quantity (number theory), structure (algebra), space (geometry), and changes (mathematical analysis)*. *It has no generally accepted definition*” (our emphasis) (MATHEMATICS, 2020).

That is, Physics, Chemistry, and Biology “are”, but Mathematics only “includes”. Such distinction is inscribed into university programs that announce degrees in “sciences and mathematics”, thereby formalizing “mathematics” as distinct from the sciences.

Lacking a precise concept of “mathematics”, literature in mathematics education is constrained to multiple perspectives: mathematics can “be seen as” mental relations, association of ideas, forms of understanding, quasi-empirical stuff, calculations, rule following, etc. (RAVN; SKOWMOSE, 2019; GOLD, 2003). Bertrand Russell's motto lurks in the horizon: “mathematics” is the subject in which we never know what we are talking about, or whether what we are saying is true (RUSSELL, 2017).

Many texts indistinctly refer to “mathematics” in Mesopotamia (HØYRUP, 2007), India (MUMFORD, 2010), Greece, the Middle Ages, etc. Lately people are referring to “mathematics” in the brain (DE FREITAS; SINCLAIR, 2013), challenged by Pais (2019). “Too many people still think that *mathematics was born in Greece* and more or less slumbered until the Renaissance” (MUMFORD, 2010, p. xiii). “Mathematics” is also found in school, where

we, educators, are supposed to teach it. It is presented to teachers wrapped up in a mandatory ideology of importance, utility, and reputation that has been increasingly criticized (LUNDIN, 2012; DOWLING, 1997).

Efforts have been made at a more precise description of what people mean when they say “mathematics”. Dissatisfied when Courant and Robbins’ *What is mathematics?* (1996) failed to provide him an answer, Hersh (1999) published his own *What is Mathematics, Really?*. However, this book also follows the common trend: “mathematics must be understood as [...]” (HERSH, 1999, p. xi). Other people have been patient enough to collect answers to this question before stating what “mathematics should be” (GOLD, 2003).

In the cliché “mathematics can be seen as”, almost anything fits, provided it is interesting enough to momentarily captivate the reader. The result is the proliferation of research reports that tend to obfuscate the view of the *interdisciplinary object* that we are required to deal with in our classrooms as mathematics teachers. The polysemy of “mathematics” affects mathematics education practice and research, but it does not seem to hamper mathematicians. However, we are aware that a “critique of mathematicians’ notions of mathematics may endanger the very foundation and status of mathematics education as a research entity in its own right” (BATARCE, 2011, p. ii).

Due to this polysemy of mathematics, some people say that mathematics is everywhere, like the character Lucy (2014). While this is done with the good intention of empowering people by recognizing that they can also do mathematics, the result ends up being the opposite: the reinforcement of the importance of mathematics. That is, handcrafting, for instance, is important not in itself, but because it is also mathematics. The attempt to “democratize” mathematics ends up emphasizing mathematics as a social measure of importance.¹

Additionally, polysemy works hand in hand with summative assessment. Issues stemming from the recognition that “the educational system is, above all, a credit system” (VINNER, 1997, p. 68) have scarcely been addressed by authors who broached assessment and evaluation. “Mathematics” is called upon because “precision and reliability are essential assessment qualities for the credit system to accomplish its judicial function of bestowing/denying credit without raising protests from students, parents and courts”. (CABRAL; BALDINO, 2019). Polysemy avoids the dangers of stumbling upon the exclusion-in-action that is implied in the credit system.

We will base our argument on the point of view of one who does not know what people

¹ We thank a colleague who prefers to remain anonymous for the idea in this paragraph.

mean by “mathematics”. We stress that the reader should join us in this view, or our argument may become difficult to follow. Instead of asking what “mathematics” is or where “it” was born, we ask *what is this “thing” that people debate whether it was born in Greece or elsewhere?* An examination of the Archaic Period of Greek history will give us the answer.

To entice the reader's curiosity, we provide a partial sneak peek at our conclusion. At that ancient time and place, for the first and perhaps the only time in history, *the global social fracture (of rich vs. poor) inherent to all societies was reduced to an intra-family fracture, inherent to all families, namely, the challenge of the paternal authority*. We will take this as the starting point of a narrative of how such a *historical singularity* developed and unfolded into *communities of speech* that came to be called hard sciences, among which we place Mathematics, now with a capital M.

Physics, Chemistry, Biology, and Mathematics are all *communities of speech* that differ only in what their *implicit statutes* consider to be valid arguments. Such *reduction of epistemology to language* is the main outcome of the first part of the paper. Our theoretical framework is Hegel-Marx-Lacan (HML) theory as recently elicited by Žižek (2012).

3. Family fracture and the father's authority in question: quilted speeches

In order to supersede the polysemy of “mathematics”, we will develop a narrative on the history of Ancient Greece, looking for what people dispute whether it was born there. We will abide only by accepted facts. Since this is not an article on history or sociology, by “accepted facts” it suffices to consider those registered in schoolbooks and encyclopedias. However, it is necessary to remark that a narrative is not necessarily made by threading truths together; the narrative itself assigns a certain *degree of truth* to the elements that it assembles.

For what follows, we rely on Coulanges (2005), Monzani (2013), Thomson (1978), and on Wikipedia. The technology of iron started around 1200 BC. Within the next two hundred years, the Bronze Civilization collapsed; many sites were destroyed and abandoned, many palaces were consumed by fire. According to the testimony of tablets found in continental Greece, the language in that region, the so-called Linear-B², has been Greek since 1450 BC. No foreign invasions are likely to account either for the end of the Bronze Age or for the five-hundred-year period of scarcity of data that followed. A period of dark age of Greek culture ensued. The absence of data suggests a period of cultural recession matching the window of

² Linear B stemmed from Linear A, an older writing system used in Crete and not yet deciphered. <https://www.britannica.com/topic/Linear-A>

scarcity of archeological data: no pottery, no writing.

As an explanation for such a dark window of archeological data and for destruction having struck various places in very different manners, some scholars have suggested a wave of earthquakes around the Mediterranean. Based on our theoretical framework, we suggest an entirely different explanation. The technology of iron, being simpler than that of bronze, was developed in the outskirts of the centralized civilization and imposed a revolution in the means of production. It is easy to imagine that a unique contradiction between means and relations of production was confronted differently by politics in different places. Some of these places were destroyed and abandoned, others were reoccupied. It is remarkable that Athens, as well as the Aegean islands, the stage of our next scenes, were not destroyed as well.

However, it is easier to explain the collapse of the Bronze Age than the revival of culture starting on the 8th century BC. Coulanges (2005) tells us how important and mandatory were both tradition and rituals in Archaic Greece (880-480 BC). There was only one class, comprised of Greek citizens. Slaves had no rights; there were no words for them to plea for better conditions. They were considered a useful and necessary function of the Greek families. The eldest of the brothers inherited the patriarch's power and responsibilities. He was to watch over the family's sacred fire, preside over the rituals of sowing and harvesting, pay tribute to the dead (usually buried beside the house), decide about marriages, etc. Within three centuries, this social structure generated enough power to rebuff two Persian invasions and build the Parthenon, inaugurated in 432 BC.

The commonsense explanation refers to an “increase in commerce” as the cause of the revival. However, commerce depends on material causes. Polanyi (2021) explains the political and economic development of the West in terms of a resistance of local commerce against the rise of the present self-regulated market. Kennedy (1989) explains the outcome of wars during the last five hundred years in terms of the economic power of nations. Marx emphasizes the self-movement of capital (an *automatices Subjekt*). The transformation of society's ideological superstructure, including mathematics, during the two hundred years from Pythagoras to Euclid and the two thousand years from Euclid to us, deserves an explanation in terms of economy, but this is beyond our scope here.

This much is what history tells us; the following is our narrative. After the destruction due to the introduction of iron, three artifacts were decisive in reorganizing production: the iron plough, the spoked wheel and iron clamps. These artifacts meant better crops, easier transportation by land and lighter vessels. However, the indiscriminate use of the iron plough rapidly exhausted the soil. Some Greek noble families went bankrupt and were threatened to be

enslaved by other families of equal nobility. It is not hard to imagine that, while the patriarch had to remain at home fulfilling his religious functions and watching over the sacred family fire, his younger brothers and descendants were free to go into commercial activities, and *they returned home rich*. The following *logical scene*³ condenses the historical development of that society. *During a heated discussion to decide the family's next enterprises, the patriarch felt his authority contested and physically threatened the younger brother. The young calmly replied: "strike, but listen"*.

No weapon could solve this dispute; *the social fracture became an intra-family conflict*; dialogue was necessary; agreements should be clearly stated and obeyed; signifiers ought to have precise meanings. Soon, a new form of discourse became necessary that would aim at the impossible mission of controlling the slide of the signified under the signifier, not for leisure or pleasure, but as a necessity of survival of the whole society. We call these *quilted speeches*. The quilting point (*point de capiton*) is the stitch that makes a sack into a cushion. Lacan borrows this concept from the mattress industry to refer to the point of speech where the signified becomes attached to the signifier and determines the meaning of what has been said so far. Accordingly, a quilted speech is one where many such points of control of meaning are available; in other words, at many points the fundamental operation of understanding may be launched: what exactly are you saying? "This is the basic paradox of the *point de capiton*. [...] Itself it is nothing but a 'pure difference'; its role is purely structural, its nature is purely performative — its signification coincides with its own act of enunciation" (ŽIŽEK, 1999, p. 99).

In summary, due to very particular historical and geographical circumstances, the intra-family conflict was mitigated by the strategy of quilting speeches. As a result, from the time of the first Olympiad in 776 BC, when the cultural revival began, to the reforms of Dracon (622 BC), Solon (594 BC), and Cleisthenes (508 BC), democracy in Athens was consolidated and the first known written legislation in history was enforced as a general agreement by the whole of society. At least in Athens, the Archaic Period can be seen as the historical development of a single principle ruled by the need of dialogue and condensed in the formula "*strike, but listen*".

4. Coinage and Pythagoras' project

It is generally held that coinage was invented to facilitate commerce. Of course, but why

³ This is actually a historical phrase; it was uttered by Themistocles of Athens to Eurybiades of Sparta in a debate during Xerxes's invasion of Greece in 480 BC (HANSON, 2013, p. 12).

in Greece and why around the 6th century BC, and not elsewhere or sooner, since commerce had been going on for millennia? Coinage has also been evoked as a factor for the emergence of “mathematics”, but this doesn't follow automatically. We will allow our theoretical framework to speak for itself on this topic.

Commodities tend to be exchanged according to their *value*, that is, according to the quantity of average social work necessary for their production and “crystallized” in their material bodies. In ancient times, coins, especially gold coins, were no exception. In principle, their value was the value of their bodily metal, a property that Babylonian clay tablets used as “money” could never have had. However, paying with a single coin is rather the exception. *Coins must be counted*. This was knowledge held by slaves who took care of the incipient banking accountability. The essential point here is that while counting coins, people had to overlook that they were counting *lumps of crystallized human work*. This sort of abstraction was occurring every day in the sphere of production as necessary for the act of buying and selling; Sohn-Rethel calls it *real abstraction*. “I set out to argue that the abstractness operating in exchange and reflected in value does nevertheless find an identical expression, namely the abstract intellect, or the so-called 'pure understanding' — the cognitive source of scientific knowledge” (SOHN-RETHEL, 1978, p. 34).

In money-based societies, an act of buying is the representation of a subject through a signifier (a bill or a coin) to another signifier (the society which recognizes the value of money). Lacan defines the signifier as that which represents the subject to another signifier. We consider this act of signification to be a speech. In buying, if one pays with a bill, the seller will only realize the received value when he spends this bill in a second buying act. Before this moment, the possibility of the bill being false remains open. This second buying moment retroactively assigns a meaning to the first act, namely, the meaning of an actual transference of value. The essential point here is that quilting comes is always offset in time; there is always a delay between the moment of payment and the seller collecting the transferred value when he spends the bill in another purchase. However, suppose that instead of paying with a bill, the buyer uses a gold piece. Since the coin has an intrinsic value, the transfer becomes immediate; nothing can be intercalated between paying and receiving. The statement coincides with its utterance; quilting is absolute. In this first society based on a monetary economy, we have the emergence of *numerically quilted speeches* present in the sphere of production as a real abstraction.

The dissemination of instant numerical quilting opened the possibility for an intellectual project aiming at *absolute quilting* at the level of language. A speech would consist of a sequence of quilting points so that nothing else could be intercalated between them, just like in

counting coins. Speeches would mean what they say, and the meaning of statements would coincide with the meaning of their enunciation. “Philosophy, in its historical function, is this extraction, I would almost say betrayal, of the slave's knowledge to get its transmutation into a master's knowledge” (LACAN, 1991, p. 22). Such an “extraction” was for the first time in history attempted by Pythagoras through what we call Pythagoras' project (PP). The Pythagoreans “saw the true grounds of the things in this world in their *countableness*” (KLEIN, 1968, p. 64).

Pythagoras is known as an almost legendary historical personality because he took the risk to take his motto “the being of everything is number” as an enunciation into the *agora*. He collided with Polycrates, the dictator of Samos, his home city, and moved to Italy where his sect was persecuted. It is important for what follows to realize what kind of *jouissance*⁴ supports PP as an in-action project of *political epistemology*; we call it *intellectual jouissance*.

Lacan (2007, p.62) stresses that “no truth can be localized except in the field in which it is stated as best it can. [...] I mean that the true can only be found outside all propositions”. In other words, truth lies in the enunciation, not in the statement; truth is what emerges from the *agora*. The meaning of what is said is a function of the listener, not of the speaker. Additionally, in a subtitle of this Seminar XVII, we read: “truth, sister of *jouissance*” (ibid., p. 54). That is, *jouissance* is also found in enunciation, “because we are beings born of surplus *jouissance*, as a result of the use of language” (ibid., p. 66). The dog eats first what it likes most; humans can balance their tastes in eating. Following Lacan, we argue that the emergence of humans and language was a single event on Earth; there has never been a human without language, nor a turtle without a shell.

However, speaking in public carries a risk. “The Greek thinker claimed objective validity for his statements; but they were his own personal contribution to the subject, and he was prepared to defend them as such” (SOHN-RETHEL, 1978, p. 99). Such presentation occurred in the *agora*, formerly a place to announce social agreements, later a central marketplace of the Greek polis. Presenting and defending one's opinion in the *agora* was taken as a serious risk, the risk of “dying of shame”, as Lacan says. There were winners and losers. “The relation of both self-consciousnesses is in this way so constituted that they prove themselves and to each other through a life-and-death struggle” (HEGEL, 2012, p. 51). Lacan, as a reader of Hegel, discerns *jouissance* in this theoretical combat: “*Jouissance* is very precisely correlative to the first form of emergence of what I call the mark, the unary trace

⁴ *Jouissance* is a central French term of Lacan's psychoanalysis. It is difficult to translate into English. It means *enjoyment* but has a sexual connotation. The French original is increasingly used in mathematics education.

which is mark for death, if you want to give it its meaning. Note that nothing acquires meaning unless death enters the stage” (LACAN, 1991, p. 206). “The Greek thinker who advanced an opinion stood behind the opinion himself” (SOHN-RETHEL, 1978, p. 99). As a reward for this life and death struggle, inserting and subjecting his *jouissance* to the place of the Other, the “specific quality of scientific thinking begins to be recognized” (ibid., p. 99). *En passant*, we remark that this paragraph applies, in particular, to our efforts to publish this paper.

The philosophers' and scientists' intellectual *jouissance* resides in their constant attempt to “save truth”, as Lacan says (2007, p. 63), that is, to avoid the act of enunciation to contradict its statement. Insofar as they can rely on some real abstraction to grant them audience, philosophers (and today scientists) try to construct a metalanguage that allows them to withdraw themselves from their discourse. Of course, this is an impossible operation, since “from every academic statement by any philosophy whatsoever [...] the *I-cracy* emerges irreducibly” (ibid., p. 63). Nevertheless, when well disguised, this sort of quilted speech may have definite political effects, such as the so-called “absolutism of mathematics” (ERNEST, 2016). Such an autonomous ego who thinks himself the author of a *subject-less absolutely quilted speech* “is an effect of the exchange mechanism through which man loses control over the social process” (SOHN-RETHEL, 1978, p. 93). Ancient Greeks were unaware that, from being a mere facilitator of commerce, coinage would become the mandatory way of thinking for their entire society.

Of course, the agency of the signifier in the world did not start only after Lacan defined it; it was in operation long before Pythagoras. He may not have realized it, but signifiers are also things in the world; they can be counted. Furthermore, no existence can be proved without the agency of the signifier, that is, without language. If everything is numbers, signifiers are numbers too. Each proposition would be distinguished from another according to its number and propositions of equal meaning would be joined together in the same *arithmos* (KLEIN, 1968). This is the paroxysm of PP. Such argument opens a new dimension of PP, namely, the possibility of *semantically quilted speeches*.

5. History of quilted speeches à vol d’oiseau

In this paragraph, we cannot but sketch the main episodes of the recent history of mathematics that flare the development of what we will call *identity-quilted speech*. Readers unfamiliar with this history may wish to look into the concise Rooney (2012).

The historical fact is that, almost 2500 years after PP, Gödel assigned meanings to a range of arbitrary numbers in the proof of his theorems (NAGEL, 2001). Actually, semantic quilting had started with Cauchy's "*on dit que*" (CAUCHY, 1989, p. 26). Before Gödel's refutation of Hilbert's project, Frege was already looking for a non-numerical absolute quilting. He tried to ground quilting on logic, something apparently surer than numbers. Every proposition would define a class, namely, the class of objects for which the proposition applies. Since classes are identified by propositions, the apparently simple idea of a class of classes implied the existence of propositions about propositions. Thereby emerges the possibility of a signifier to signify itself, contradicting the identity $A = A$ as in the example: "I have three brothers, John, Bob and I", where "I" is not "I". When Frege's main work was already in press, Bertrand Russell warned him of the paradox into which he had run: the barber who shaves all men who do not shave themselves. As we understand, from then on logicians made great efforts to free the identity $A = A$ from paradoxes and preserve it as a fundamental law of thought. Frege was looking for *absolutely quilted* speech, which certainly included numerical quilting as well as semantic quilting. Bertrand Russel showed that this was not possible, but grounding quilted speeches on the identity $A = A$ persisted as an ideal of 20th century mathematics (M20). We call these *identity-quilted speeches*.

We know that PP stumbled upon square root two, but the intellectual *jouissance* of quilting speeches that it inaugurated persists; today, deeds stemming from this kind of *jouissance* are rewarded with Nobel Prizes and Fields Medals. In Baldino and Cabral (2013, 2015) we consider this form of intellectual *jouissance* as connected to the value of qualified-labor-power produced in the school apparatus. Despite the deep changes in the concept of number throughout history (KLEIN, 1968), from the point of view of *jouissance*, Hilbert's second problem may be considered as a continuing PP. Hilbert first reduced Euclid to real numbers (HILBERT, 2015), then in 1900 he proposed to ground mathematics on arithmetic (SCOTT, 1900). In 1933 Gödel (1992) proved that this project also fails. Starting with Frege's disaster, "mathematics" built anti-monster barriers until the discipline that we are supposed to teach stabilized itself as M20.

In summary, in the 5th century BC intellectual *jouissance* began to be staged in the *agora* under the form of debates between philosophers and sophists. Protagoras (490-415 BC) is known as the first sophist; Socrates, twenty years younger, became the most famous philosopher. The nature of quilting operations diversified. In the 3rd century BC it became geometric. Dialectics assumed the form of quilting/un-quilting speeches. Philosophers organized themselves into communities according to different quilting criteria, resulting in one

pole of dialectics. The Pythagorean School persisted in its mystic numeric quilting. Plato's academy was founded in 387 BC. The most famous and durable quilting community was that of Euclidean geometers. They had to diversify their statute and divide themselves when non-Euclidean geometries appeared. Galileo postulated a statute that originated the community of physicists; their statute suffered two radical changes, one with Einstein and another with quantum mechanics, still in course. Kant proposed a very broad statute of quilting speeches grounded on human faculties (synthetic a priori judgments) about space and time. This statute had to be revised after the 1919 eclipse that confirmed Einstein's general theory of relativity.

The pole of this dialectics opposite to the pole of the philosophers was the movement of the sophists, who lost their initial identity as mercenary nomad teachers because, as un-quilters of all statutes, they could not have one to unite them without contradicting themselves. Their heirs can be recognized as un-quilters throughout history. Their latest form is post-modernism, with its rotating machine gun against all narratives. In the political debate, generally, but not always, quilters are found at the right, un-quilters at the left. The most remarkable exceptions are *Marx* and *Althusser*, two typical leftist quilters. Insofar as post-modernism raises the deconstruction of Marxist narrative, it can be seen as a *rightist un-quilting movement*; we may conjecture whether this was not actually its primary target. There are also quilted speeches directed against the rightist ideology based on the absolute certainty that stems from “mathematics” as totalizer of an ideological field (ERNEST, 1998, 2016). We will endorse this view below.

Diversification of quilting speech communities around their respective statutes, determining what counts as quilting, resulted in what we call the *exact sciences*. Individual *jouissance* preference determines allocation of people into such communities. According to what we have developed thus far, we can say that a historical dialectical process resulted, on one hand, in a very particular form of speech and, on the other hand, in a community of builders of a statute of quilting, the so-called mathematicians. The *community of speech*, organized around the statute of identity-quilted speech deserves the name Mathematics, with a capital M and no quotations marks. Inside this community, what counts as quilting is called *proof*. Therefore, according to their logical genesis, there is no difference between Physics, Chemistry, Biology, and Mathematics. There is no reason to use the expression “mathematics and sciences” in many university programs. Mathematics education is a detachment of Mathematics which takes up an interdisciplinary object: the student.

The term “speech community” is already in use, connected to sociolinguistics (PATRICK, 2008). However, in these studies the community is first discerned from the outside,

then the “use” of language by this community is analyzed. Our approach goes in the opposite direction: we identified a speech together with a community of speakers and followed their movement in history. Briefly, “speech community”, implies the primacy of community over language whereas in *community of speech* we intend to assign primacy to the historical dialectical process whose results are the community *and* the speech.

6. Words for the future: a political agenda for mathematics education

We have shown how identity-quilted speech together with a community of speech called Mathematics emerged from the dialects of history. We have argued that the operation of buying/selling is an exchange of lumps of human work, called value. We have argued that this operation requires comparison and equalization of values that Sohn-Rethel calls real abstraction. We have argued that this real abstraction predisposes people to hear and reproduce a form of speech called quilted speech. We have discerned a form of quilting that stems from payment with gold coins, called absolute quilting. We considered the situation where gold coins were replaced by all sorts of pieces of paper representing amounts of gold stored under the responsibility of an authority, the most famous being Fort Knox. This form of real abstraction has been present for millennia in the everyday life of all individuals in societies that use metal-based money.

In the second half of the last century, the abandonment of the gold standard produced a major change in this real abstraction; it affected the *moment of quilting*⁵ *inherent to exchange*. During the time from the operation of selling to the moment of its spending in the next purchase, the value of a bill will depend on a collective will called *the market*. Credibility replaced accumulated human work as financial guarantee. The capital, as an authorization to buy and use labor power, was liberated from the constraint of being measured in terms of simple human work, whose basic unit lies in gold prospection. Credit could produce capital at will: prime, sub-prime, leverage, etc. Large bulbs of capital explode from time to time. Generalized participation of individuals in the stock market subjected them to an entirely new real abstraction, a kind of *fluctuating quilting* of the exchange operation. People opened their ears to un-quilting discourses. Philosophies came to the fore that seek to deconstruct all narratives and relativize all forms of certainty (ERNEST, 2016). People lost the ground for any experiences with quilted speech; political debate becomes a clash of monologues; legal proofs

⁵ We believe that we could say “moment of truth” and show how this concept too, is grounded on the real abstraction inherent to coinage, but this would be a much longer philosophical project.

are never ultimate.

In this frame, mathematics education research and classrooms assume paramount importance as perhaps the last *bastion of quilted speech*. Our urgent mission as Mathematics educators is not only to initiate the students into the practice of identity-quilted speech, but also to show them that this form of dialogue is also important outside the classroom. Mathematics promotes the ability to listen to the other's discourse and to point out its quilting gaps *before presenting one's own argument*, an ability that we wish our politicians had.

Identity-quilted speech emerged from the necessity to make summative evaluation precise in our calculus classrooms. At this universal evaluation moment, teachers materialize the consequences of the universal school credit system into students' lives (CABRAL; BALDINO, 2019). We are constantly concerned with this universal moment because part of the mathematics education community rejects rewarding effort as a strategy to combat inequalities. We have been using this strategy in association with Solidarity Assimilation Methodology (SAM) as explained in Cabral, Pais and Baldino (2019). The concept of quilted speech applied under the circumstances of mandatory remote teaching in 2020/2 allowed our effort-rewarding grades to be based on a clear concept and be as checkable as the accepted right/wrong criterion.

We must show people that quilted speech does not mean political oppression; on the contrary, it can be a valuable weapon to *fight back the attempts to confiscate our acquired rights*, especially in the aftermath of the COVID-19 pandemic. These precepts should make a political agenda for mathematics education in the years to come.

We are aware that many such agendas have been proposed and criticized (PAIS, 2017; PAIS; VALERO, 2012). Their impact in classrooms seems to have been small. To avoid this dismaying outcome, we have already been implementing this agenda in our calculus classroom practice as teacher-researchers (TABACH, 2006). Under remote teaching, with "eye-to-eye" communication captured in video – thanks to the circumstance of the pandemic –, students are required to individually justify randomly assigned passages from worksheets they solved previously, thereby entering into a dialog with teachers and other students. “The effort invested by the student to prepare for this moment becomes evident to all. Close eye-to-eye conversation eliminates any questions about the authenticity of the answers. No written homework is required” (BALDINO; CABRAL, 2021). In the emergent remote teaching circumstances, we have discovered the natural habitat of our thirty-year-old solidarity assimilation methodology (CABRAL; PAIS; BALDINO, 2019).

Nevertheless, we are also aware that school will return to its stable equilibrium as soon as our action ceases. An attempt to explain this resilience may be found in Baldino and Cabral

(2015). Perhaps school, as we criticize it today, is already perfect for capitalist society. That is why it does not change.

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