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Investigation of Pre-service Elementary Mathematics Teachers' Understanding on Solids

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Abstract

The aim of the present study is to determine the elementary pre-service mathematics teachers' understanding on solids. For this purpose, pre-service teachers' definitions and drawings of these objects were examined. Qualitative research method was used. A written questionnaire consisting of sixteen open-ended and multiple-choice questions was conducted with 127 elementary pre-service mathematics teachers chosen by convenience sample which is one of non-random sampling method. The collected qualitative data were analyzed by both descriptive and content analysis. The results revealed that pre-service teachers made insufficient connections among cylinder, prism, cone and pyramid. So, it can be said that their understanding about solids was weak and procedural.

Keywords: Solids, pre-service teachers, misconceptions

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Investigación de la Comprensión de los Futuros Maestros de Matemáticas de Primaria sobre los Sólidos

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Resumen

El objetivo del presente estudio es determinar la comprensión de los futuros profesores de matemáticas elementales sobre los sólidos. Para este propósito, se examinaron las definiciones y dibujos de estos objetos de los futuros maestros de primaria. Se utilizó el método de investigación cualitativa. Se realizó un cuestionario escrito que constaba de dieciséis preguntas abiertas y de opción múltiple con 127 futuros maestros elegidos por muestra de conveniencia, que es un método de muestreo no aleatorio. Los datos cualitativos recopilados se analizaron mediante análisis descriptivo y de contenido. Los resultados revelaron que los futuros maestros no hacían conexiones suficientes entre cilindro, prisma, cono y pirámide… Entonces, se puede decir que su comprensión sobre los sólidos era débil y procedimental.

Palabras clave: Sólidos, maestros de pre-servicio, conceptos erróneos

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Geometry is a branch of mathematics with abstract representations that help students to make sense of the world they live in and explain the universe (Baki, 2011). Geometry is in educational programs, starting from primary school, because it contributes to the critical thinking and problem-solving ability of students, helps teaching other branches of mathematics, is a major part of mathematics that is being used in daily life, is used in arts and sciences and helps student to get to know the world around them (Baykul, 2012). Geometry provides a natural environment for students to develop their thinking and querying abilities (National Council of Teachers of Mathematics, 2000). Also, it is closely related to many cognitive skills such as spatial and geometric thinking. Geometry which provides a bridge between daily life and mathematical concepts helps these other mathematical concepts to be understood better. For example, drawing a curve in analytical plane enables to look at the concept of slope in differentiation from a geometric perspective (Van de Walle, Karp & Bay-Williams, 2014). Geometry involves more abstract concepts than other parts of mathematics and especially geometric shapes forces students to think more complex by using their spatial thinking skills (Yıldız, 2009). Studies in the literature show that students in Turkey have difficulty in subjects of geometry. For example, Mullis, Martin, Gonzalez, Gregory, Garden, O'Connor, Chrastowski and Smith (2000) state that Turkish students get the lowest grades in five areas of mathematics from geometry based on the report of Trends in International Mathematics and Science Study (TIMMS). Similarly, in TIMMS report (year 1999), Turkish students have the lowest grade averages in geometric shapes and scaling (Oral & McGivney, 2011). These results show that the difficulties that the students face in geometry topics last through time. Among the subjects of geometry, geometric solids come first in topics that the students are having difficulties (Meng, & Idris, 2012; Hallowell, Okamoto, Romo & La Joy, 2015; Gökkurt, Şahin, Soylu & Doğan, 2015; Sarfaty, & Patkin, 2013). Studies in literature also state that pre-service teachers have difficulties in subject of geometric solid (Gökbulut, 2010; Gökkurt, Şahin, Başıbüyük, Erdem & Soylu, 2014; Koç & Bozkurt, 2011). Memorization of properties, use of prototypes and insufficient examples cause students to form limited and faulty structures and have difficulties in learning these concepts (Fujita & Jones, 2007). Teachers play a key role in overcoming these difficulties. This situation makes it necessary for teachers and pre-service teachers to determine what sort of understandings they form.

Concepts form as a result of an abstraction process of classifying the similarities between our experiences (Skemp, 1976). Two basic methods of understanding mathematical concepts are instrumental and relational (Skemp, 1978). Instrumental understanding means that students without understanding the concepts, by memorization of rules, use the mathematical structures, but by relational understanding it is meant that the mathematical algorithmic structures are constructed by discovering their meanings, through the learning process. In other words, in instrumental understanding concepts are transferred to students directly, but in relational understanding student is a problem solver that can use skills and intuition. Together with this, in the process of concept cognition examples regarding the concept and counterexamples are of great importance (Wilson, 1990). Especially, prototype examples are ideal examples that demonstrates strong visual qualities and important properties of the concept (Okazaki & Fujita, 2007). Prototype examples play a big role in formation of student's concept images (Levenson, Tirosh & Tsamir, 2011). Herskowitz (1990) states that prototype examples can cause students to have misconceptions. According to these students, can have different understandings for certain concepts based on certain prototype examples. For example, Deliyianni, Elia, Gagatis, Monoyiou and Panaoura (2010) demonstrates based on a work done in 1086 elementary and middle school student by investigating the role of perceptual, functional and lingual cognition that the educational life effects the understandings of geometric shapes. In this context, it is important that teachers, and pre-service teachers should understand the geometric concepts. The studies, however, show that pre-service teachers have difficulties in understanding of geometric concepts and have misconceptions (Gutierrez & Jaime, 1999; Cunningham & Roberts, 2010; Kabaca, Karadag & Aktumen, 2011; Marchis, 2012; Pittalis, Mousoulides & Christou, 2010)

In the literature, the main focus regarding cylinders, prisms and pyramids is mostly on middle school students (Avgören, 2011; Ergin & Türnüklü, 2015; Türnüklü & Ergin, 2016) and pre-service mathematics teachers (Bozkurt & Koç, 2012; Ertekin, et al., 2014; Gülkılık, 2008). Moreover, there are studies that examine the pedagogical knowledge of mathematics teachers (Gökkurt, 2014; Gökkurt, Şahin, Başıbüyük, Erdem & Soylu, 2014; Gökkurt, Şahin, Soylu & Doğan, 2015) and pre-service teachers (Gökbulut, 2010) regarding three dimensional figures. From the results of these studies show that pre-service teachers have difficulties in understanding three dimensional figures. There are a few studies that examine the misconceptions and misunderstandings of pre-service teachers regarding three dimensional figures. The future success of students in geometry is dependent on their early geometry education. For this reason, determining the understanding of pre-service mathematics teachers regarding three dimensional figures will give clues on the educational activities they will be giving in later years. The purpose of this study was to examine the elementary pre-service mathematics teachers' understanding of the solids. Moreover, the other aim of this study was to determine the effects of teacher educational program on the pre-service teachers' understanding of solids.

Method

Study Group

The research group of the study consisted of 127 pre-service teachers who were enrolled an elementary mathematics education program at a state university in Turkey in the spring term of 2015-2016. Of these, sixty-two were freshman and sixty-five were senior. The sample of this study has been determined by convenience sampling, which is one of the non-random sampling methods. The reason for choosing the convenience sampling method in the study is that the group to be examined is accessible and practicable due to the limitations in terms of time, money and labor (McMillan & Schumacher, 2014).

Data Collection Tool

To determine the pre-service elementary mathematics teachers' understanding on solids, a written questionnaire which has 16 open-ended and multiple-choice questions was developed. The questions were adapted from the literature (Ertekin, et al., 2014; Van de Walle et al., 2014) relating to teachers' knowledge and images about solids. The questionnaire consisted of four sub-sections. In the first part, there were four questions to determine the understanding of pre-service teachers about the cylinder. In this section, the first question was about pre-service teachers' cylinder definition, the second question was to select which of the four given figures were a cylinder, the third question was to select which of the four given figures were not cylinder, and the fourth question was to draw a cylinder different from the given cylinder figures. As an example, the first part of the questionnaire was presented in Table 1.

In the second part, there were four questions to determine the pre-service teachers' understanding of the prism. In this section, the first question was about pre-service teachers' prism definition, the second question was to select which of the four given figures were a prism, the third question was to select which of the four given figures were not a prism, and the fourth question was to draw a prism different from the given prism figures. In the third part, there were four questions to determine the pre-service teachers' understanding of the cone. In this section, the first question was about pre-service teachers' cone definition, the second question was to select which of the four given figures were a cone, the third question was to select which of the four given figures were not a cone, and the fourth question was to draw a cone different from the given cone figures. In the last part, there were four questions to determine the pre-service teachers' understanding of the pyramid.

In this section, the first question was about pre-service teachers' pyramid definition, the second question was to select which of the four given figures were pyramid, the third question was to select which of the four given figures were not pyramid, and the fourth question was to draw a pyramid different from the given pyramid figures. The word questions in the questionnaire were based on the study of Ertekin, et al. (2014) and the cylinder, cone, prism and pyramid figures included in the questionnaire were selected from the "Geometric Thinking and Geometric Concepts" unit of Van de Walle et al. (2014). The cylinder, cone, prism and pyramid figures used in this study were non-traditional figures. The reason for using these figures was that these non-traditional figures can help to determine the pre-service teachers' images on solids. To determine pre-service teachers' understanding, non-traditional figures were important tools. For example, in the cylinder definition in some textbooks, it is not necessary that the bases of the cylinder are circular. Yet, many textbooks represent cylinders as circular bases.

Table 1.

The first part of the questionnaire.

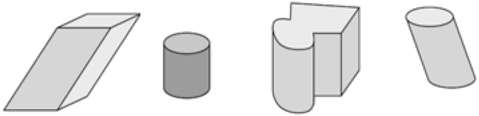
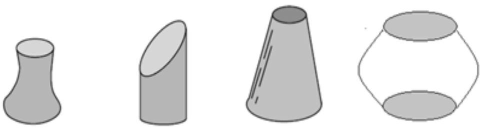
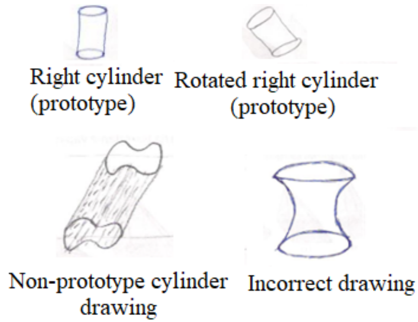
Questions	Aim	Examples
1. Define cylinder.	The aim of the first question is to determine pre-service teachers' formal definitions on cylinder.	• A cylinder is a solid that has two parallel closed curve bases (usually circular) connected by a curved surface • A cylinder is a solid with congruent circular bases that lie in parallel planes.
2. Select the figure(s) below which is(are) cylinder(s)?  (1) (2) (3) (4)	The aim of the second question is to determine pre-service teachers' identification on non-traditional cylinder figures.	All figures are cylinders
3. Select the figure(s) below which is(are) not cylinder(s)?  (5) (6) (7) (8)	The aim of the third question is to determine pre-service teachers' identification on non-cylinder figures.	All figures are not cylinders

Table 1. (continue)

The first part of the questionnaire.

Questions	Aim	Examples
4. Draw a cylinder which is different than the ones given above.	The aim of the last question is to determine pre-service teachers' concept images on cylinder.	Drawings are classified into four categories: prototype, non-prototype, incorrect and no drawing



For that reason, pre-service teachers may think that prism is not a special case of a cylinder. This prevents pre-service teachers to build connections among solids. Understanding can be defined as a measure of quality and quantity of connections with concepts (Van de Walle et al., 2014). The greater the number of connections, the better the understanding. Before administering the final form of the questionnaire, two mathematics educators and two mathematics teachers checked appropriateness of the figures included in the questionnaire. They suggested using more non-traditional solids figures to examine the pre-service teachers' understanding on solids. So, non-traditional solids figures included in the "Geometric Thinking and Geometric Concepts" unit of Van de Walle et al. (2014) were used mostly. Moreover, they checked the face validity of the questions and agreed that they were valid and appropriate for measuring pre-service teachers' understanding on solids. In addition, the instrument was administered to 25 junior elementary pre-service mathematics teachers as a pilot study. In the pilot study it was determined that the instrument was completed in about 50 minutes. Moreover, after the instrument was administered to 127 pre-service teachers, 8 of the participants as a volunteer were interviewed on solids figures in the instrument. In terms of ethics, pre-service teachers were coded as C1, C2, … and researcher was coded as R. The data from interviewed were presented as direct quotations in the finding.

Analysis of the Data

Qualitative and quantitative analysis methods were both used in analyzing the data in this study. To evaluate the pre-service teachers' definitions related to solids a rubric was used (see Table 2). The qualitative analysis of the data led to the development of four categories of explanation. These categories were as "exactly correct definitions", "partially correct/incomplete definitions, "incorrect definitions" and "no answer". The rubric was prepared by using Karakuş's (2018) study.

Teaching geometric objects in Turkish education system

Teaching of cylinder

When the mathematics curriculum in Turkey is examined; it is observed that teaching of geometric solids is widely covered from primary education to secondary education. In the teaching of one of these, the cylinders, only the daily examples are included in the first grade. In the second grade, it is expected to form cylinders using shape figures and distinguish from other forms. Students can specify the basic elements of the cylinder in the third grade. At the secondary school level, the cylinder area and volume are given. The cylinder is defined as "the shape that occurs when a rectangular is turned around its edge" in the secondary school (MEB, 2010). At the high school level, students are expected to be able to classify the relationship of the cylinder with other geometric objects and to make applications about the surface area and volume of the cylinder. In high school mathematics textbooks, the cylinder is defined as; "Let a closed curve in the plane and a d-line that is not parallel to this plane be given. The surface formed by the gliding of a line which is parallel to the d-line on the closed curve is called cylindrical surface and cutting this surface with two parallel planes, the part left in between is called cylinder" (Hacısalihoğlu, 2006).

Teaching of the prism

The students meet daily samples of prism in the first grade; but teaching of it as a concept is included in the second grade. Students in the third grade are expected to be able to identify faces, angles, and edges of figures such as square, rectangular, triangular prism. In the fourth grade, they are in a position where you can draw the development of a prism and determine what prism is. In the secondary school mathematics program, the surface area and volume of the prisms are given. In a high school mathematics textbook, the prism is expressed as "the object whose base is a polygonal region and whose sides form from quadrilateral regions" (MEB, 2010). With this definition, when the secondary school mathematics textbooks are examined, it is seen that only prisms with smooth polygon in the base are allowed. At the high school level, students should be able to determine the relation of the prism with other geometric objects and to be able to apply the surface area and volume.

Teaching of the cone

Teaching of the cone just like cylinder and prism starts in primary school; students at this level recognize the basic elements of the right cone; building it and drawing it. In secondary school mathematics course book, an object that consists of all of the points of a circle merging with a point outside the circle." (MEB, 2010) At high school level, the purpose is to identify conics and basic elements and relate them to the environment they live in; define the basic elements of the special kind conical, write the standard equations and associate them with the environment they live in (MEB, 2010).

Teaching the pyramid

In the first grade, pyramids in daily life are also included but not named. In the second-grade students are expected to use the shape figures to create structures; draw these structures and to recognize and distinguish pyramids on the figures. The student in the third grade can specify the faces, angles, and edges of the pyramid. In middle school mathematics curriculum introduced the basic elements of the pyramids and constructing it. (MEB, 2010). The purpose of the high school is to define pyramid and its basic elements and relate them to the environment they live in (MEB, 2010).

Table 2.
The rubric for pre-service teachers' definition about solids.

Geometric Object	Categories	Evaluation Criteria	Expressions that are focused on definitions	Sample definitions
Cylinder	Exactly correct definitions	Complete and correct explanations about the cylinder	For an object to be cylinder: • There must be two bases. • These bases must be identical and parallel. • The bases must have a closed curve (it may or may not be rounded). • Lines cutting the curves at the base must be parallel to each other	A shape resulting from rotating a rectangle by 360^0 around an edge
	Partially correct / incomplete definitions	Incomplete or partially correct explanations about the cylinder		The shape in which upper and lower bases are circles
	Incorrect definitions	Incorrect definitions and explanations		The area formed by the circle and the inner region

Table 2. (continue)

The rubric for pre-service teachers' definition about solids.

Geometric Object	Categories	Evaluation Criteria	Expressions that are focused on definitions	Sample definitions
Cylinder	No answer	Not giving any explanation	• Rotating a rectangle 360^0 around an edge	
Prism	Exactly correct definitions	Complete and correct explanations about the prism	For an object to be a prism: • There must be two bases. • These bases must be identical and parallel. • The bases must have a closed rectilinear geometric shape. • Lines cutting the rectilinear figures at the base must be parallel to each other	The three-dimensional shape formed by mutual joining of points on identical and parallel polygonal bases.
	Partially correct / incomplete definitions	Incomplete or partially correct explanations about the prism		Geometrical shape whose bases are polygonal
	Incorrect definitions	Incorrect definitions and explanations		Three-dimensional type of smooth shapes other than circle
	No answer	Not giving any explanation		
Cone	Exactly correct definitions	Complete and correct explanations about the cone	For an object to be a cone: • Must have a base. • The base must be a closed curve (it may or may not be a circle). • It should be a fixed point outside the plane on which the base is located. • Each point on the closed curve must be linearly joined to this fixed point • 360^0 rotation of a triangle around an edge	Shape formed by union of line segments joining each point on a circle on a plane by a point that is not on the plane that contains this circle
	Partially correct / incomplete definitions	Incomplete or partially correct explanations about the cone		Three-dimensional object with pointed top and circle base
	Incorrect definitions	Incorrect definitions and explanations		Prism with circle base
	No answer	Not giving any explanation		

Table 2. (continue)

The rubric for pre-service teachers' definition about solids.

Geometric Object	Categories	Evaluation Criteria	Expressions that are focused on definitions	Sample definitions
Pyramid	Exactly correct definitions	Complete and correct explanations about the pyramid	For an object to be a pyramid: • It must have a base. • The base must be a polygon • There should be a fixed point outside the plane on which the base is located. • Each point on the polygon must be linearly joined to this fixed point	The geometric shape created by joining each point of the geometric figure in the base with a point outside the plane with line segments
	Partially correct / incomplete definitions	Incomplete or partially correct explanations about the pyramid		Shape with polygonal base and pointed top
	Incorrect definitions	Incorrect definitions and explanations		Prism with a non-circular base.
	No answer	Not giving any explanation		

Pre-service teachers ' answers to multiple choice questions were analyzed descriptively. Pre-service teachers' selection of given solid figures were determined by frequency and percentage. In addition, pre-service teachers' drawings of solids were analyzed using categories. First, the drawings were divided into three categories such as "correct", "incorrect" and "no drawing". Then the frequency and percentages of the drawings in each category were presented descriptively.

The Reliability of Study

After the instrument was administered, the researchers and a mathematics educator who was expert in solids were separately examine the pre-service teachers' responses to the instrument. In order to provide the reliability of the data collected from instrument, the rubric organized by researchers was given to the expert. Both researchers and the expert examined randomly 30 papers. The researchers matched each pre-service teacher's answer in suitable categories without staying any of them idle. Then expert's matchings were compared with researchers' matchings. Considering the comparisons, agreement and disagreement numbers were calculated according to Miles and Huberman's (1994) reliability formula, and the correspondence percentage was found as 95% for definition of solids and 96% for drawings. Since the consistency coefficients obtained are greater than 70%, it can be said that the analyzes obtained are reliable (Miles & Huberman, 1994).

Findings

Pre-service Teachers' Understanding on Cylinder

Pre-service teachers' answers about the first question of the cylinder sub-section of the instrument were presented in Table 3.

Table 3.
Pre-service teachers' definitions on cylinder

Category	Sub-category	Theme	Freshman f	Freshman %	Senior f	Senior %
Exactly correct definition		The shape obtained by rotating a rectangle around an edge	2	3	21	32
		A geometric object formed by matching the circular bases to each other	6	10	7	11
		A three-dimensional object created by connecting the bases that are discs with line segments	8	13	-	-
		Prism with base as a circle	2	3	1	2
		A geometric shape formed by rotating a rectangle around a circle	-	-	1	2
Partially correct / incomplete definition	Definitions focusing on the basic elements of cylinder	Geometric shape that has upper and lower bases as circle	5	8	5	8
		Closed shape with height and has circles as bases.	5	8	13	20
		Object with identical base and parallel sides.	1	2	-	-
		Geometric object with bases as circles and has height	-	-	3	5
		3 dimensional object	-	-	3	5

Table 3. (continue)

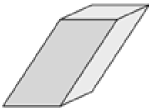
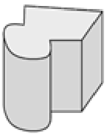
Pre-service teachers' definitions on cylinder

Category	Sub-category	Theme	Freshman f	Freshman %	Senior f	Senior %
	Definitions focusing on the open form of cylinder	Three-dimensional shape consisting of two circles and one rectangle	31	50	11	17
	Definition that use analogues	Toilet paper roll	1	2	-	-
Incorrect Definition		The object formed by joining two rectangles	1	2	-	-

According to Table 3, approximately 29% of freshman elementary pre-service mathematics teachers and 47% of the senior pre-service teachers gave correct explanations. While freshman pre-service teachers mostly used the expression "three-dimensional object formed by combining the circles that are bases with line segments"; senior pre-service teachers were focused on the definition of "the shape obtained by rotating a rectangle around an edge". It was found that very few pre-service teachers associate the prism to the cylinder, and they expressed that the prism was a cylinder with a circle base. Moreover, it was determined that approximately 70% of the freshman pre-service teachers gave partially correct / incomplete explanations for the cylinder. It was found that about 18% of these pre-service teachers were focused on the basic elements of the cylinder, such as the circular upper and lower bases, the height, and being a three-dimensional shape. Yet, the properties given were not sufficient to define a cylinder. It was seen that about 50% of freshman pre-service teachers who defined partly correct / incomplete focused on its open form when describing the cylinder. It was determined that these pre-service teachers mostly said, "two circles and a rectangle form a cylinder." In addition, it was determined that about 2% of freshman pre-service teachers gave incorrect definition of cylinder. Approximately 37% of the senior pre-service teachers who made partially correct / incomplete definitions were focused on the basic elements of the cylinder. In their definitions they expressed cylinder as "closed-shape with two bases as circle and has height". 17% of freshman pre-service teachers who made partially correct / incomplete definitions focused on open form of the cylinder. No senior pre-service teachers used analogy in the cylinder definition and did not gave any incorrect explanation.

The pre-service teachers' answers about the second question of the cylinder sub-section of the instrument were given in Table 4.

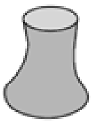
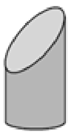
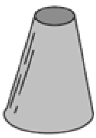
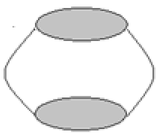
Table 4.
The frequency and percentages of the pre-service teachers' selection of cylinder figures

	Freshman		Senior	
Shape	F	%	f	%
 (1)	-	-	4	6
 (2)	62	100	64	98
 (3)	1	2	15	23
 (4)	60	97	62	95

In high school mathematics textbooks, a cylinder is defined as a solid that has two parallel closed curve bases (usually circular) connected by a curved surface. This definition shows that bases of the cylinder do not have to be circular. For that reason, all the figures given in Table 4 were cylinders. it was seen that all the freshman pre-service teachers selected figure (2) and 97% selected figure (4) as cylinder. Although most of the senior pre-service teachers selected the figure (2) and figure (4) as cylinders, 23% selected figure (3) and about 6% selected figure (1) as cylinder. This indicates that pre-service teachers accepted the cylinder based on more prototype examples. As a matter of fact, the freshman pre-service teacher C1 stated "2 and 4 figure is definitely cylinder. The other one is the oblique cylinder. But the others do not look like the cylinder we see. I did not choose it because of that." However, the senior pre-service teacher C7 expressed "We already say that shapes 2 and 4 are cylinders. R: Why? C7: Because we've seen these shapes in books for years. But when we examine the definition of the cylinder, it tells us that the other forms here that are given to us are cylinders as well. But as I said, if I looked at it without thinking, I would choose 2 and 4."

The pre-service teachers' answers about the third question of the cylinder sub-section of the instrument were given in Table 5.

Table 5.
The frequency and percentages of the pre-service teachers' selection of non-cylinder figures.

	Freshman		Senior	
Shape	F	%	f	%
 (5)	61	98	60	92
 (6)	24	39	22	34
 (7)	43	69	45	69
 (8)	56	90	49	75

Although all the figures given in Table 5 were not cylinders; it was seen that 98% of freshman pre-service teachers did not accept figure (5) as a cylinder. Similarly, 69% did not accept figure (8), 90% did not accept figure (7) and 39% did not accept figure (6) as a cylinder. A freshman pre-service teacher C3 expressed "I hesitated a bit in number (6); but none of these looks like the cylinders I have seen anywhere. So, I think that none of these shapes are cylinders." Likewise, 92% of senior pre-service teachers did not accept figure (5) as a cylinder. Similarly, 75% did not accept figure (8), 69% did not accept figure (7) and 34% did not accept figure (6) as a cylinder. Senior pre-service teacher C8 said "Shapes (5), (7), (8) are not cylinders, because they do not fit the definition of the cylinder. But I have accepted the number 6 as a cylinder. Because there are always expressions like truncated cylinders in the test books. When I look at the definition, I can now say that it's not a cylinder." The pre-service teachers' answers about the fourth question of the cylinder sub-section of the instrument were given in Table 6.

Table 6.
Cylinder drawings of pre-service teachers

		Freshman		Senior	
Category	Example drawing	f	%	f	%
Right cylinder		31	50	24	37
Rotated right cylinder		22	35	25	38
Non-prototype cylinder drawing		-	-	3	5
Incorrect drawing		9	15	10	15
No drawing	-	-	-	3	5

According to Table 6, half of freshman pre-service mathematics teachers drew right cylinders; for senior pre-service teachers this rate was 37%. However, 35% of the freshman pre-service teachers drew an oblique cylinder; 38% of senior pre-service teachers drew oblique cylinders. While none of the freshman pre-service teachers were able to draw non-prototype cylinders; only about 5% of senior pre-service teachers were able to draw a different cylinder. However, it was found that approximately 15% of both freshman and senior pre-service teachers drew the cylinder incorrectly. In addition, it was seen that about 5% of senior pre-service teachers did not draw any drawings.

Teacher Candidates' Understanding on Prism

Pre-service teachers' answers about the first question of the prism sub-section of the instrument were presented in Table 7.

Table 7.
Pre-service teachers' definitions on prism

Category	Theme	Freshman		Senior	
		f	%	f	%
Exact Definitions	Three-dimensional object that has identical, parallel polygonal bases; formed by connecting corresponding points of the polygons.	9	15	12	19
	Cylinder with polygonal bases	-	-	4	6
Partially correct/ incomplete definitions	Geometric object with identical and parallel, polygonal bases	18	29	11	17
	Three-dimensional object with identical and parallel bases and rectangular side surfaces	6	10	5	8
	Three-dimensional object with polygonal bases that has corners, sides and a height.	5	8	17	26
	Geometrical object that has identical polygons as bases and a height.	4	6	8	12
	Geometric object formed by connecting identical and parallel planes.	2	3	4	6
	Closed shape that has a geometric shape at the bottom, top and sides.	3	5	-	-
	Shape with quadrilaterals as base and side surfaces.	2	3	-	-
	Three-dimensional object named after its base.	2	3	-	-
Incorrect definitions	Geometric object	3	5	3	5
	Shape that has a polygonal base and connects at a certain point	4	6	-	-
	Three-dimensional version of the regular shapes other than the circle	1	2	-	-
	Three-dimensional object with six sides	1	2	-	-
	Shape with no free corners	1	2	-	-

In Table 7 it was seen that 15% of the freshman pre-service mathematics teachers and 19% of the senior pre-service teachers defined the prism as "The three-dimensional object that has identical and parallel polygons as bases and formed by connecting the corresponding points of these polygons." In addition, 29% of the freshman pre-service teachers gave the definition "The geometric object that has identical and parallel polygons." It was also observed that approximately 33% of the freshman and 49% of the senior pre-service teachers gave partially correct definitions for the cylinder; 8% of the freshman, 26% of the senior pre-service teachers focused on the definition "The three-dimensional object that has sides, corners and height, and has polygons as bases." When the wrong definitions were examined; it was seen that about 12% of the freshman pre-service teachers did not correctly defined the prism. From these incorrect definitions, the most repeated one (6%) was as "A shape that has a polygonal base and closes at a point."

Pre-service teachers' answers about the second question of the prism sub-section of the instrument were presented in Table 8.

Table 8.
The frequency and percentages of the pre-service teachers' selection of prism figures

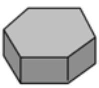
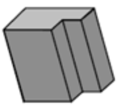
	Freshman		Senior	
Shape	F	%	f	%
 (9)	58	94	63	97
 (10)	24	39	35	54
 (11)	43	69	30	46

Table 8. (continue)

The frequency and percentages of the pre-service teachers' selection of prism figures

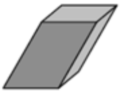
	Freshman		Senior	
Shape	F	%	f	%
 (12)	60	97	61	94

Table 8 showed that the vast majority of both freshman and senior pre-service teachers accepted figures (9) and (12) as prism. However, 39% of freshman pre-service teachers accepted figure (10) was a prism; this ratio was 54% for senior pre-service teachers. In addition, although about 69% of freshman pre-service teachers accepted figure (11) as a prism; 46% of senior pre-service teachers accepted it as a prism. While the freshman pre-service teacher C2 said: "I didn't choose number (11) because I was not sure of it, but I think the rest were prisms. I have seen these shapes before." A senior candidate C5 expressed his views as follows: "All of these are prisms; however, we were not shown figures like (10) and (11) before. When we take the definition of the prism into account, the bases are polygons, they are identical and parallel, and they were connected with line segments. Therefore, I think we can conclude that they are indeed prisms."

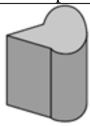
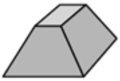
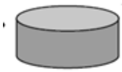
Pre-service teachers'' answers about the third question of the prism sub-section of the instrument were presented in Table 9.

Table 9.

The frequency and percentages of the pre-service teachers' selection of non-prism figures.

	Freshman		Senior	
Shape	F	%	f	%
 (13)	61	98	60	92

Table 9. (continue)
The frequency and percentages of the pre-service teachers' selection of non-prism figures.

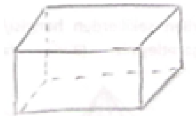
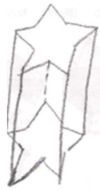
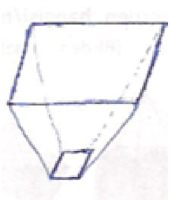
	Freshman		Senior	
Shape	F	%	f	%
 (14)	24	39	22	34
 (15)	43	69	45	69
 (16)	56	90	49	75

None of the figures given in Table 9 were prisms. 98% of the freshman pre-service teachers didn't select figure (13), 90% didn't select figure (16), 69% didn't select figure (15) and 39% didn't select figure (14) as a prism. Similarly, 92% of the senior pre-service teachers didn't select figure (13), 75% didn't select figure (16), 69% didn't select figure (15) and 34% didn't select figure (14) as a prism. The interview of a senior pre-service teacher C7 was following:

> C7: I think shapes (13), (15) and (16) are not prisms. But figure number (14) confused me. The bases are identical and parallel.
> R: Is it enough for the bases to be identical and parallel for a prism?
> C7: I am trying to remember its definition. How did we define it, was it supposed to be a polygon?
> R: Try to remember, think again.
> C7: If you think like the closed shape, it is prism but not if not.
> R: If there was a closed curve, would it be necessary to define the prism? Wouldn't we call all of them as cylinders?
> C7: Hm, I do not know (thinks). If there is a polygonal statement in the description, we can say there is no prism."

Pre-service teachers' answers about the fourth question of the prism sub-section of the instrument were presented in Table 10.

Table 10.
Prism drawings of pre-service teachers

Category	Example drawing	Freshman		Senior	
		f	%	f	%
Right prism		52	84	50	77
Rotated right prism		3	5	1	2
Non-prototype prism drawing		-	-	5	8
Incorrect drawing		6	10	3	5
No drawing	-	1	2	6	9

According to Table 10, approximately 89% of freshman pre-service teachers drew a right prism, this rate was 79% for senior pre-service teachers. In addition, none of the freshman pre-service teachers drew a different prism while 8% of the senior pre-service teachers drew non-prototype drawings. Moreover, 10% of the freshman and 5% of the senior pre-service teachers drew incorrect drawings. It was seen that 2% of freshman and 9% of senior pre-service teachers did not draw any prism.

Pre-service Teachers' Understanding on Cone

Pre-service teachers' answers about the first question of the cone sub-section of the instrument were presented in Table 11.

Table 11.
Pre-service teachers' definitions on cone

Category	Subcategory	Theme	Freshman f	Freshman %	Senior f	Senior %
Exact definitions		Shape formed by the line segments that connect every point on a circle, with a point that does not lie on the plane of the circle.	14	23	24	37
		Shape formed by revolving an equilateral triangle around one of its equal sides.	2	3	4	6
		Pyramid with a circular base	2	3	3	5
		Geometric object with 1/3 of a cylinder's volume	-	-	2	3
Partially correct / incomplete definitions	Definitions that focus on the basic elements of the cone	Geometric object with circular base	9	15	4	6
		Three dimensional object with circular base and a height	6	10	11	17
		Closed shape with a circular base and pointy edge.	6	10	4	6
		Connecting regular polygons at a vertex point	2	3	-	-
		Three dimensional object	-	-	1	2
	Definitions that focus on the open cone	Three dimensional object formed by folding a circular slice around a circular base.	5	8	4	7
		Three dimensional object with a circular base and triangular surface	5	8	1	2
	Definitions that use analogies	Witch hat	1	2	1	2
		Ice cream cone	2	3	-	-
Incorrect definitions		Shape with circular base and rectangular surface	2	3	1	2
		Shape formed by combining shapes that are not parallel.	1	2	-	-
No answer			5	8	5	8

According to Table 11, approximately 29% of the freshman and 48% of the senior pre-service teachers defined the cone correctly. Among those freshman pre-service teachers, the most prominent definition was "The shape formed by connecting every point on a circle with a point that does not lie on the same plane, with line segments." It was also noted that among the correct definitions, pre-service teachers linked the cone with the pyramid. 59% of freshman pre-service teachers and 45% of the seniors gave incomplete definitions. When partially correct / incomplete definitions were examined; it was seen that approximately 38% of the freshman and 34% of the senior gave definitions focusing on the basic elements of the cone. It was also noted that about 5% of the freshman and 2% of the senior used analogies while defining the cone. Also 5% of the freshman and 2% of the senior defined the cone incorrectly. In these incorrect definitions, they identified the surface of the cone as a rectangle shows their incorrect understanding of the cone. It was also identified that 8% of both freshman and senior pre-service teachers gave no definition for the cone.

Pre-service teachers' answers about the second question of the cone sub-section of the instrument were presented in Table 12.

Table 12.
The frequency and percentages of the pre-service teachers' selection of cone figures.

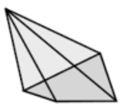
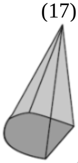
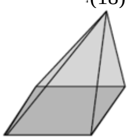
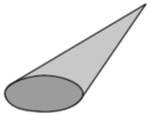
	Freshman		Senior	
Shape	F	%	f	%
 (17)	4	6	4	6
 (18)	7	11	6	9
 (19)	4	6	6	9

Table 12. (continue)
The frequency and percentages of the pre-service teachers' selection of cone figures.

Shape	Freshman F	Freshman %	Senior f	Senior %
 (20)	56	90	63	97

Although all given shapes in Table 12 were cone figures, about 90% of freshman and 97% of the senior pre-service teachers identified figure (20) as a cone. It was seen that 11% of the freshman and 9% of the senior pre-service teachers accepted figure (18) as a cone. It was also seen that very few both freshman and senior pre-service teachers identified figure (17) and figure (19) as cones. C1 pre-service teacher expressed: "I have never seen the figures other than figure (20) being called a cone, therefore I think only that one is a cone.", the C6 pre-service teacher expressed "I have identified figure (18) and figure (20) as cones. Figure (20) is already a cone. Figure (18) is a cone, sliced with a plane." When asked why the remaining shapes were not cones, both pre-service teachers explained that the figures (17) and (19) were pyramids, therefore they were not cones.

Pre-service teachers' answers about the third question of the cone sub-section of the instrument were presented in Table 13.

Table 13.
The frequency and percentages of the pre-service teachers' selection of non-cone figures

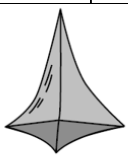
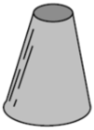
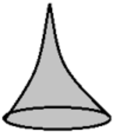
Shape	Freshman F	Freshman %	Senior f	Senior %
 (21)	58	94	61	94

Table 13. (continue)

The frequency and percentages of the pre-service teachers' selection of non-cone figures

Shape	Freshman		Senior	
	F	%	f	%
 (22)	22	35	24	37
 (23)	58	94	59	91
 (24)	40	65	35	54

None of the shapes given at Table 13 was a cone; 94% of the freshman pre-service teachers and most senior pre-service teachers selected figure (21) and figure (23) as not cones. Similarly, 65% of the freshman and 54% of the seniors was not selected the figure (24) as a cone. However, it was seen that figure (22) was interpreted as a cone by vast majority of the pre-service teachers. For example, a freshman pre-service teacher C4, said "This is a truncated cone". A senior pre-service teacher C8 said: "There was a cone, but it is cut by a plane. But I did not select it. Because I remember a similar expression in our lectures or in our textbooks. So, I accepted this shape as a cone. Yet actually this is another shape formed by truncating a cone. But, as I said, I answered it according to the books that I remember."

Pre-service teachers' answers about the fourth question of the cone sub-section of the instrument were presented in Table 14.

Table 14.
Cone drawings of pre-service teachers

Category	Example drawing	Freshman		Senior	
		f	%	f	%
Right cone		49	79	33	51
Rotated right cone		9	15	20	31
Non-prototype cone drawing		1	2	3	5
Incorrect drawing		1	2	4	7
No drawing	-	2	3	5	8

According to Table 14, about 96% of the freshman and about 82% of the senior pre-service teachers drew right cones or oblique cones. Although senior was more successful in drawing non-prototype cones than freshman. Yet, senior drew more incorrect drawing than freshman pre-service teachers.

Pre-service Teachers' Understanding on Pyramid

Pre-service teachers' answers about the first question of the pyramid sub-section of the instrument were presented in Table 15.

Table 15.

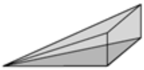
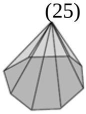
Pre-service teachers' definitions on pyramid

Category	Theme	Freshman f	Freshman %	Senior f	Senior %
Exactly correct definitions	The geometric object that is created by combining the line segments which are formed by drawing from an every point of the polygon at the bottom to a point outside the plane	37	60	39	60
Partially correct/ missing definitions	A cone which has a polygon at the bottom	-	-	3	5
	Special case of a cone	1	2	3	5
	A three-dimensional object which has a polygon at the bottom and which has triangles at the side faces	10	16	8	12
	A geometric object which has a polygon at the bottom, and named according to the shape of its bottom	5	8	4	6
	A geometric object	3	5	2	3
	A shape obtained by combining a plane with a point on top of it	-	-	1	2
Incorrect definitions	A prism which does not have a circle shaped bottom	2	3	-	-
No answer		4	6	5	8

Table 15 showed that approximately 78% of freshman and 82% of senior pre-service teachers made correct definition of a pyramid. It was seen that the pre-service teachers who defined correctly focused on the definition of "the geometric object that is created by combining the line segments which are formed by drawing from every point of the polygon at the bottom to a point outside the plane". It was also noteworthy that among the correct explanations, the relationship between the cone and the pyramid was established by pre-service teachers. In addition, it was seen that approximately 11% of the freshman and approximately 13% of the senior pre-service teachers made partially correct definitions. Also, while approximately 3% of freshman made incorrect definition of the pyramid and none of the senior pre-service teachers made incorrect definition. Moreover, about 6% of the freshman and about 8% of the senior pre-service teachers didn't make any definition for pyramid.

Pre-service teachers' answers about the second question of the pyramid sub-section of the instrument were presented in Table 16.

Table 16.
The frequency and percentages of the pre-service teachers' selection of pyramid figures.

Shape	Freshman F	Freshman %	Senior f	Senior %
 (25)	49	79	54	83
 (26)	50	81	60	97
 (27)	42	68	50	77
 (28)	57	92	61	94

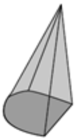
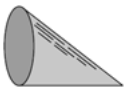
All the figures at Table 16 were pyramids; it was seen that the vast majority of the freshman and senior pre-service teachers selected them as a pyramid. For example, a freshman pre-service teacher C3 expressed: "The main feature of a pyramid is a shape at the bottom is connected to a point outside. Because of this, all of them are pyramids". Another senior pre-service teacher C7 expressed: "There is a polygon at the base, and each point of this polygon is connected to a point at top linearly. So, they are all pyramids".

Pre-service teachers' answers about the third question of the pyramid sub-section of the instrument were presented in Table 17.

Table 17.
The frequency and percentages of the pre-service teachers' selection of non-pyramid figures.

Shape	Freshman F	Freshman %	Senior f	Senior %
 (29)	50	81	52	80

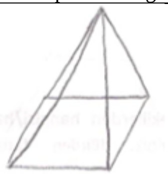
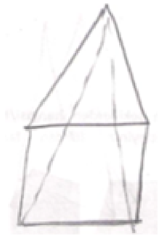
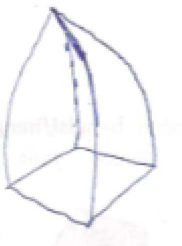
Table 17. (continue)
The frequency and percentages of the pre-service teachers' selection of non-pyramid figures.

	Freshman		Senior	
Shape	F	%	f	%
 (30)	49	79	42	65
 (31)	47	76	33	51
 (32)	48	77	36	55

All the figures at Table 17 were not pyramids. The vast majority of the pre-service teachers selected them as pyramids. Moreover, the freshman pre-service teachers selected more these figures as pyramids. For example, a freshman pre-service teacher C1 expressed the reason of not to select these figures as pyramids: "The first two shapes have top points, but the bottom shape is connected with a curve to that top point. Because of this, I think those shapes cannot be pyramids. And the last two shapes are cones. So, there is curvature on the bottom. But there must be edges on the bottom of the pyramids."

Pre-service teachers' answers about the fourth question of the pyramid sub-section of the instrument were presented in Table 18.

Table 18.
Pyramid drawings of pre-service teachers.

Category	Example drawing	Freshman f	Freshman %	Senior f	Senior %
Right pyramid		12	19	32	49
Rotated right pyramid		44	71	20	31
Non-prototype pyramid drawing		1	2	4	6
Incorrect drawing		2	3	2	3
No drawing		3	5	7	11

According to Table 18, 90% of the freshman and 80% of the senior pre-service teachers drew prototype shapes. The senior pre-service teachers drew more non-prototype / different pyramid than freshman. While 3% of both freshman and senior pre-service teachers made incorrect pyramid drawings. It was seen that the ratio of senior pre-service teachers who did not draw any pyramids were more than freshman.

Discussion and Conclusion

The aim of this study was to reveal the understandings of elementary pre-service mathematics teachers about cylinder, prism, cone and pyramid. While the majority of pre-service teachers had difficulty in defining solids; it was seen that senior pre-service teachers were more successful in defining each given solid. This shows that teacher training program has positive effect on pre-service teachers' knowledge on solids. In comparing the responses according to grade level, the freshman pre-service teachers' answers were mostly weak and incorrect. This shows that freshman pre-service teachers had inadequate instruction about solids in their past educational experience. Likewise, previous studies have reported that pre-service teachers at the beginning of their university education generally have rule-bound mathematical understanding, and they have difficulty in giving appropriate mathematical explanations for a given mathematical concept (Toluk-Uçar, 2011). However, in the senior level, the correct explanations and non-prototypal drawings increased. One reason for this may be that, at the sophomore and junior levels, the participants had taken courses in geometry, teaching geometry and special teaching methods in mathematics teaching where they had frequently encountered the solids. Moreover, senior pre-service teachers had taken some courses such as school experience and teaching of elementary mathematics. In these courses, they have a chance to interact with students and to practice elementary school mathematics topics that they have previously encountered mainly on theoretical. Therefore, it can be said that these courses are effective in improving the pre-service teachers' pedagogical content knowledge; and thus, variety in their correct and non-prototypical drawings may be the result of these courses. Except for the correct definitions, pre-service teachers focused on the basic elements of solids, when they define a solid. In literature (Bozkurt & Koç, 2012; Ertekin et al., 2014; Gökbulut, 2010; Gökkurt, 2014; Karakuş, 2018), it is stated that the pre-service teachers give less correct definition for the geometric objects and they mostly prefer to give general features of them.

It was seen that pre-service teachers who make the correct definition or explanation, try to describe the solids given in a way similar to the definitions found in elementary and secondary school mathematics textbooks. On the other hand, pre-service teachers gave less correct definitions similar to the definitions in high school or university mathematics textbooks. Ertekin et. al. (2014) say that cylinders, cones and prisms are taken from high school mathematics courses at the end of the semester. Therefore, students do not meet these topics adequately. This may be a reason why pre-service mathematics teachers give insufficient definitions in solid. Another reason may be the pre-service teachers' concept images on solids. In traditional mathematics teaching from primary school to university in Turkey, pre-service teachers are frequently provided with examples which have common specific characteristics, of these concepts and these examples become prototypes in time (Ertekin et al, 2014). When the textbooks are examined; it is seen that example of solids given are always prototype such as right cylinder or right cone. Ertekin et al. (2014) also state that the formal definitions are not effective at altering the students' understanding of solids. In the literature (De Villers, 1998; Türnüklü and Ergin, 2016), it is stated that personal definitions are preferred more than formal definitions.

When a student forms a mathematical concept in his mind; definitions, examples and counter examples have an important role (Wilson, 1990). Pre-service mathematics teachers meet definitions and examples of many different cylinders, prisms, cones and pyramids both verbally and visually since the primary school. For this reason, they are expected to have a correct understanding of these concepts and to create rich concept images. However, the vast majority of pre-service teachers have identified solids as partially correct / incorrect. When defining the cylinder, pre-service teachers repeated some features of solids such as two circle bases, height, parallelism, rectangle and three-dimensional shape. This indicates that teacher candidates have the following image for the cylinder. Two bases for cylinder, these bases must be a circle, having height and being a three-dimensional shape. The definitions for the prism are similar to cylinder. In the prism, they repeat some features of it like polygonal bases, quadrangular lateral face, and height. When defining the cone; they give some features of it like disk/circle base, corner point, triangular shape and three dimensional. pre-service teachers have image for cone circle base, corner point and three-dimensional shape. Similarly, when defining the pyramid, their explanations include polygonal base, triangular faces, a corner point, and a three-dimensional shape. Although, senior pre-service teachers were more successful defining solids than freshman, majority of both freshman and senior pre-service teachers were not sufficient to use mathematical language. Bozkurt and Koç (2012) express that pre-service teachers' solid definitions are insufficient and the mathematical language also has great deficiencies. Moreover, pre-service teachers who give more properties regarding the solids make more accurate definitions.

When the pre-service teachers' selections for solids were examined; freshman were more successful in determining non-solid figures; and the senior were more successful in determining solid figures. However, in the case of different drawing examples for solids, the majority of them drew the prototype drawings. Moreover, senior drew more different solids which were not prototype. Of course, the past experiences of pre-service teachers for solids are one of the important factors. As a matter of fact, past experiences are influential in the formation of students' different understandings (Bingölbali & Monaghan, 2008; Vinner, 1991).

The result of the study showed that pre-service teachers found little common relationship among solids. Van de Walle et al. (2014) suggest that prisms are also cylinders and pyramids are cones at the same time. However, in many textbooks such relations are either never mentioned or implied. For example, it is seen that the cylinder definition in the high school mathematics textbooks covers the definition of the prism at the same time and the definition of the cone also includes the pyramid definition. However, the relationship between these definitions is not clearly revealed. In the textbooks there is no example, picture or figure showing that a prism is a cylinder at the same time. Monaghan (2000) states that prototype examples can limit the concept by creating limited visual perceptions. For example, the idea that the base of the cylinder or the cone must be a circle; keep pre-service teachers away from thinking that the bases may be any closed curves or that the objects may not be right. Similarly, the idea that the prism or pyramid base should be a regular polygon; away from the idea that the base may be any polygon. This, in turn, prevents different drawing to be made for the concept (Avgören, 2011).

Recommendations

In this research, the understandings of pre-service mathematics teachers for solids were examined. Pre-service teachers encounter geometric objects in many teaching stages from primary school to teacher training program. For this reason, they are expected to give rich definitions and images for these concepts. On the other hand, the results show that pre-service mathematics teachers have difficulties in defining solids and frequently prefer prototype drawings for these concepts. This makes it necessary to examine the effects of the definitions, examples and drawings for teaching solids at different grade levels.

In the teacher training program, mathematics teacher candidates take, Geometry, Special Teaching Methods I and Special Teaching Methods II courses. The fact that teacher candidates have difficulty in defining the solids and the fact that they mostly prefer and draw prototypes It is suggested that the contents of these courses in the teacher training program need to be reconsidered.

In the future research, the conceptual understandings of geometric objects for students and teachers at different grade can be examined. Thus, how the understanding of these concepts forms and changes at every level of education can be examined.

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