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How Must a Polygon Be According to Hard of Hearing Students? An Investigation with a Semiotic Approach

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Abstract

This study explores how hard of hearing students decided whether the shape was a polygon and which semiotic sources were used when the students engaged in explaining geometrical concepts. It was defined how the students interacted with geometric shapes using semiotic sources and examined how such multimodal interactions with geometric figures displayed their reasoning. The study was a case study and carried out three hard of hearing students. The data was collected through interviews and analyzed with content analysis. It was detected that the students paid attention to edge, angle, and vertex of the shapes in the process of identifying polygon. It was seen that the students used gesture, speech, sign language, inscriptions which are semiotic sources and personal or mathematical definitions to express polygon concept. However, it has been determined that students have some misconceptions in the process of explaining concepts. It is suggested that the words used in the concept definition should be selected carefully by the teachers to teach the concepts correctly and the teachers use hand signs for concepts in their lesson.

Keywords: Gesture, Hard of hearing students, Polygon, Semiotic, Sign Language.

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¿Cómo Debe Ser un Polígono según los Estudiantes con Problemas de Audición? Una Investigación con Enfoque Semiótico

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Resumen

Este estudio explora cómo los estudiantes con problemas de audición decidieron si una figura era un polígono y qué fuentes semióticas se usaron al invoclucrarse en la explicación de conceptos geométricos. Se definió cómo los estudiantes interactuaban con formas geométricas utilizando fuentes semióticas y se examinó cómo tales interacciones multimodales con figuras geométricas mostraban su razonamiento. El estudio fue un estudio de caso y lo llevaron a cabo tres estudiantes con problemas de audición. Se detectó que los estudiantes prestaron atención a los lados, ángulos y vértices en el proceso de identificación del polígono. Se observó que los estudiantes utilizaron gestos, habla, lenguaje de señas, inscripciones que son fuentes semióticas y definiciones personales o matemáticas para expresar el concepto de polígono. Se sugiere que los maestros seleccionen cuidadosamente las palabras utilizadas en la definición del concepto para enseñarlos correctamente y que usen señales con las manos en su lección.

Palabras clave: Gestos, Alumnos con Dificultades Auditivas, Polígonos, Semiótica, Lenguaje de signos.

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National Council of Teachers of Mathematics (NCTM) expects young children to engage in mathematics learning. NCTM notes that mathematics helps them how to make sense of the world around them. The research, however, indicates that deaf and hard of hearing (HH) students often have a delay in their math performance and deaf children's mathematical achievement is inferior to that of their hearing peers (Kelly & Gaustad, 2006). In particular, research has demonstrated that low levels of achievement in various areas of mathematics involving computation and problem solving (Ansell & Pagliaro, 2006; Lee, 2010), in arithmetic and measure (Pagliaro & Kritzer, 2013), in fractions (Mousley & Kurz, 2015). One of the most important learning areas is geometry. Knowledge of geometric concepts is essential in many real-life contexts. It underlies design for everything from microprocessors to large structures and forms the basis for understanding and representing objects in the world around us and our geographic and spatial orientation. It provides the reasoning of spatial problems and situations, the properties of shapes, and the establishment of relationships between them (Battista, 1990). However, geometry activities contribute, sometimes greatly, to the development of other areas of mathematics. For example, building geometric and spatial abilities helps students build array models for multiplication and rectangular and circular area models for fractions. Geometric models also can contribute to learning about and solving problems in measure, graphing, ratio and proportion, probability, and algebra. High achieving students' numerical ability is connected to their spatial and measure ability. Furthermore, children who are poor achievers in mathematics generally show little growth in geometry. We feel that concurrently developing geometric concepts and relationships to give students tools necessary to successfully participate in a variety of professions in their future is vital. Geometric and spatial tasks can reveal, and often build on, unsuspected strengths of students with special needs. They can capitalize on student strengths in drawing or manipulating forms, thus offering alternatives for students with language and communication difficulties. For example, some studies asserted that deaf individuals show better visual-spatial skills than hearing individuals in some domains (Bavelier, Dye, & Hauser, 2009). Because of this, geometry with concepts and operations is quite important for hearing and HH students. Nevertheless, in literature, it was seen that less research has been conducted on deaf and hard-hearing students' achievement on geometric concepts. Scientists have focused primarily on numbers and operations, particularly work with whole numbers with little attention to geometry. Despite recommendations by the National Research Council in 2009 to focus research on “geometry, spatial thinking, and measure” in addition to number, no research has investigated performance in foundational mathematics and geometric concepts outside the area of number with deaf and hard-hearing students. Kritzer (2012) demonstrated that students were aware of geometric shapes (about sorting and labeling shapes such as squares, rectangles, and triangles) whereas children with relatively low mathematical scores were able to label only non-geometrical shapes such as stars and hearts. This study aims to determine the knowledge of HH students about polygon. The polygons are one of the most basic learning areas of geometry and all of the students, also HH students encounter with polygons almost at all class level. HH students also have knowledge about polygons and it was wondered students’ information in this subject. Various geometric shapes, including polygonal and non-polygonal shapes, were given to the students and it was revealed how the students determined what shapes were polygons, what they paid attention to while making this determination, how they expressed the concepts, the features used in this process, and what their misconceptions were. This may be a guide for teachers to implement in teaching. If a teacher knows how his/her students think, how much knowledge they have about the mathematical concept, he/she can provide opportunities them to learn geometry. It is important that teachers choose the appropriate adaptations of activity to meet the needs of the child (Stewart & Kluwin, 2001). A teacher can help students to optimize teaching environments for these learners by identifying any conceptual mistakes or situations that lead to misunderstandings and thus improve the level of students' perception of mathematical concepts (Goldin, 1998). Pagliaro (2015) expressed that “geometry concepts and skills for deaf children, can be developed sooner and/or more quickly than those of other areas, perhaps influenced by their visual access to information.”

HH students have difficulty understanding of abstract concepts (Nunes & Moreno, 2002) and tend to learn with the help of symbols or signs because they use visual knowledge well (Hall & Bavelier, 2010). The semiotic is a science that examines signs such as symbols, gestures, written signs and these signs are known as semiotic sources. With this study, it is desired to reveal how the students determine the polygon and use semiotic sources. Therefore, "How do HH students identify that a shape is polygon or not?" and "Which semiotic sources do HH students use in identifying polygon?" questions became the main problems in the current study.

Theoretical Framework

Semiotics

Learning is a multimodal phenomenon (Arzarello & Robutti, 2008), and fenemonologists who explain the learning process demonstrate that different sources are actively used by teachers and students in the production and transfer of knowledge in mathematics classes. These sources in semiotic sources included words (oral or written), extra-linguistic representations (gestures, glances …), different types of inscriptions (shapes, graphics, …) and various instruments (from pencil to computer) (Arzarello, Paola, Robutti, & Sabena, 2009). Arzarello and Robutti (2008) argue that these sources should be examined to reveal how mathematics learning effectuated. Students learn mathematics in activities where they interact with and judge the signs (Cook & Goldin-Meadow, 2006; Nemirovsky & Tierney, 2001). These sources show how people communicate with mathematical knowledge in the process they are engaged in with mathematics and how mathematical thinking occurs (Arzarello et al., 2009; McNeill, 2005). These semiotic activities are a holistic process where all the components of the semiotic resources are simultaneously active, intertwined with each other.

The gestures, one of the semiotic sources were classified according to their semiotic dimensions (McNeill, 1992). These are iconic gestures (IG), metaphorical gestures (MG) and deictic gestures (DG). Iconic gestures are gestures that refer to a concrete image or action and resemble the visual character of the image. For instance, the student draw a straight line in the air with her right and left thumb and index fingers to show line segment in Table 1. This line segment visual referred to edge of the polygon, that is a concrete image. The metaphoric gestures are pictorial but they emit an abstract idea not concrete image or action (McNeill, 1992). For example, the student also used metaphoric gesture for round concept while explaining that the shape in Fig. 5. Both iconic and metaphoric gestures are pictorial, but the iconic gesture refers to a concrete idea and the metaphoric gesture refers to an abstract idea.

Deictic gesture is a gesture pointing to an object, it is made more often with index finger or a pen and show objects and events in the concrete world (McNeill, 1992). In this study, it was revealed how the students used semiotic resources (gestures (iconic, deictic, metaphoric), languages and inscriptions) while identifying polygonal and non-polygonal shapes.

Hard of Hearing Students

Hearing loss is defined as the loss of functionality at the point of hearing or the inability to perceive and understand the sounds in the environment due to some reasons such as genetic reasons, illness and accident. A person who needs special education due to hearing loss is also defined as an individual with hearing impairment. People with hearing loss are generally referred to as deaf (deaf) or hard of hearing (difficult to hear) individuals. Tüfekçioğlu defines the deaf individual as an individual whose native language information is significantly hindered with or without a hearing aid, and the hard-of-hearing individual is defined as a person whose linguistic information is enabled to be processed successfully, usually with the help of a hearing aid. has defined. The participants of this study were also hard of hearing students.

Research Methodology

Research Design

In this study, case study model in qualitative research methods was used. It was investigated how HH students use Sign Language (SL) and gesture while defining mathematical concepts and identify whether a shape is a polygon using semiotic resources.

Participants

The participants in this study were three HH students selected from different grade levels in a Special High School for the HH. Deaf and HH students studied in this special school and this school at this high school level located in Ankara, one of the cities in the middle of Turkey is the only school in Ankara. The students were selected from ninth, tenth, and eleventh grade students who attended geometry course and had enough knowledge about polygons. These students who have taken or are taking a geometry course and have knowledge about polygons were selected using the criteria sampling method. The criteria were that the students had knowledge about polygon. In the study, the real names of the students were kept confidential, and the students were coded S1, S2, and S3.

Table 1
The characteristics of participants

Students	Disability situation	Grade level	The form of communication
S1	Innate HH	9th grade	Cannot hear and speak, can read lips and knows SL
S2	Not innate HH (lost the ability to hear after a feverish discomfort during the age of 2–3)	10th grade	can hear (through device in her ear), speak some sounds and read lips well and knows the SL
S3	Not innate HH (lost her ability to hear in the aftermath of a fever of 2–3 years)	11th grade	can hear (through device in her ear), speak some sounds, read the lip well and knows the SL.

Curriculum for the HH students is the same as the curriculum of hearing students. For all that, because HH students have lack of sense, they cannot learn all the subjects in the mathematics as targeted and some subjects are difficult for them. They were either not knowledgeable or had basic level knowledge in some mathematical subjects. Thus, these students learn more limited information in some subjects. For example, they have limited knowledge about the polygons. Because they have learning problems related to hearing and speaking, polygons and some basic properties are seen enough for these students by their teachers (although the curriculum and textbooks were the same as those of hearing students, teachers of the HH students provided more limited information to these students) in special high school for the HH. That is, it was not seen necessary to teach information which contains the perimeter and area calculation. Considering all of these, in the present study, students were selected from the most successful students in the mathematics in their class or even in the school.

Data Collection Processes and Instrumentation

In this research, the first author learned SL while taking a course to communicate with these students. She has a certificate in SL and was fluent in Turkish SL (TSL, 2014), but a non-native signer. The data of the study were collected through semi-structured interview and document review in qualitative research methods. The document consists of all kinds of inscriptions of the students on the paper (paper with open-ended questions), such as figures or writing. And these inscriptions were presented in the section of findings. Interviews were executed with the students as individual. Open-ended questions given in Appendix 1 were presented to the students in writing and asked the students to read the questions during the interview process. The students asked the questions using sign language, speech (not a clear speech), gesture or writing. The questions included polygonal and non-polygonal shapes and were formed considering to critical features of the basic elements of the polygons, such as edges, angles, and vertex. The questions were arranged in considering opinions of two experts in the field of mathematics education and one mathematics teacher of HH. According to opinion of the mathematics teacher of the HH students, the sentence questions were made simpler by avoiding long sentences and were asked in short sentences for HH students. The questions were given to the students in written form during the interview process and the students were allowed to read the questions and to distinguish polygon and non-polygon shapes. Then, the researcher explained questions in SL and talking loudly to the students again. The interviews were recorded with two video cameras, one of the cameras was installed to monitor students’ hand signs, gesture and SL, and the other was installed to monitor students' signs on paper. The interviews lasted an average of 60 minutes.

Data Analysis

The data obtained from the answers given by the students to the questions were analyzed using the content analysis. Firstly, students' comments on the figures given in the question were read one by one and various categorizes and sub-categorizes were determined by reference to their explanation during the identification process whether a shape was a polygon. In this process, properties of their (edge as polygon's properties, straight as property of edge, etc., …) and concepts (polygon, edge, angle, vertex, etc., …) was coded as categorizes and sub-categorizes while it was determining why a figure is a polygon or non-polygon by the HH students. For example, categories, features about edge belonging to polygon were determined as right, accurate, straight, regular and line segment and sub-categories about edge belonging to non-polygon were determined as broken, one point larger than the other and point. then it was also detected semiotic resources used in explaining process of the concepts in these categorizes. For example, while S2 was explaining edge, S2 used iconic gesture, oral expression and drew a figure. Semiotic sources have been categorized as speech, gesture, inscriptions in the literature and SL added to these sources by Gürefe (2015). Signs in Turkish Language Institution Sign Language Dictionary (TSL, 2014) were considered as SL. In addition, the signs that did not fall into the SL category were evaluated as gestures. In this coding, the researcher consulted to one expert's opinion to ensure coding reliability related to gesture and SL. The expert is a SL coder specialized in TSL classified all of signs as gesture or SL. In TSL, there are signs of a few mathematical concepts (see TSL, 2014). The researcher consulted to different one expert's (expert in mathematics education) opinion to ensure coding reliability related to mathematical concepts. At firstly, the researcher informed to the expert about the research topic and the data coding technique and then gave the data to the expert for coding. The consistency of the coding made by the researcher and the expert was calculated as 95%. "Consensus / (Agreement + Disagreement)" formula was used to calculate the consistency of coding. The disagreement among researchers has generally been in determining the type of gesture. After the researcher and expert have discussed on the codes that did not compromise, the consensus has been reached.

Findings

In this episode, it was examined situations that students consider when determining polygonal and non-polygonal shapes from different geometric shapes and the ways in which they are expressed. This seemed to direct the students' attention to the edge, angle, and vertex to decide whether or not figures were polygon.

Considering the "Edge"

Three students examined the edges of some of the shapes given to them and decided whether the shape was polygon edges according to some features which edges had. Students had stated that if the edge was right (straight), accurate, straight, regular or line segment, the shape was polygonal, if the edge was broken, one edge larger than the other, point (not line), curve, round, irregular or wave, it was not polygon. Data related to the categories obtained from student comments were given in Table 2.

Table 2 listed the categorizations obtained from students' explanations and semiotic sources which students used to describe edge concept and its properties. According to Table 1, for example, all three students stated that the edge could not be point and point (not line), but S1 expressed it using the metaphoric gesture and verbal explanation, S2 and S3 used the SL and the mouth to explain it.

In the excerpt below, it was given a figure and asked to determine whether the figure is polygon. So, as to understand how S1 decided whether given figure was a polygon, it was useful to consider the semiotic resources used by him. It was seen the semiotic activities of his as a holistic process where all the components of the semiotic resources (gestures, inscriptions, speech, and SL) were simultaneously active, intertwined with each other.

At the beginning of the task, he used inscriptions on the sheet without using speech or gesture (see Fig. 1b) to express the thing that hamper to be polygon on the figure. Then, he used both speech and SL to describe his general observation of his issue with the edge. He said that the shape is not a polygon because its edge is "broken."

Table 2

The Categorizations Obtained for Edge and Semiotic Sources Used

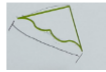
Categories-Students			Gestures			Languages		Inscription
Polygon			IG	MG	DG	Oral	SL	Figures
	Right (straight)	S1				x		
	Accurate	S1				x		

Table 2 (continue)
The Categorizations Obtained for Edge and Semiotic Sources Used

Categories-Students			Gestures			Languages		Inscription
Polygon			IG	MG	DG	Oral	SL	Figures
	Straight	S2				x		
		S3				x		

Table 2 (continue)

The Categorizations Obtained for Edge and Semiotic Sources Used

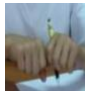
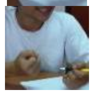
Categories-Students			Gestures			Languages		Inscription
Polygon			IG	MG	DG	Oral	SL	Figures
	Regular	S2				x		
	Line segment	S3	 			x		
Non-Polygon	Broken	S1				x	 	

Table 2 (continue)

The Categorizations Obtained for Edge and Semiotic Sources Used

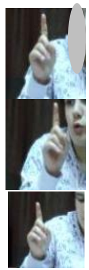
Categories-Students		Gestures	Languages	Inscription
One point larger than the other	S1		x	
Point and point	S1		x	
	S2		x 	

Table 2 (continue)

The Categorizations Obtained for Edge and Semiotic Sources Used

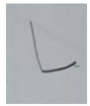
Categories-Students		Gestures			Languages	Inscription
Point and point	S3			x	 	
Curve	S1	 		x		
	S2			x		

Table 2 (continue)
The Categorizations Obtained for Edge and Semiotic Sources Used

Categories-Students			Gestures			Languages		Inscription
Non-Polygon			IG	MG	DG	Oral	SL	Figures
	Round	S2						
	Irregular	S2			 	x		
	Wave	S3				x		

Besides, he used gesture to construct the edge and signed a triangle using his fingers. At this stage, the following dialog executed between R and S1. “R” stands for “Researcher”; “S1” stands for “Student 1.”

R: Is this figure (Fig. 1a) a polygon?
S1: (Fig. 1b: S1 draws a line segment at the top of the shape.)
R: Is this figure (Fig. 1a) a polygon?
S1: No (Fig. 1c: points curved part in round).
R: Why?
S1: Broken (Fig. 2: signs for broken). (repeats same movement several times)
R: Is it broken? Why?
S1: (Fig. 3a: He uses SL and makes triangle) Here in triangle (Fig. 3b) is broken (Fig. 2).

a. b. c.

Figure 1. The Shape Had Broken Edge and Broken Edge

Figure 2. Broken in SL

a b

Figure 3. Triangle in SL and Edge in Triangle

S1 utilized SL, speech, iconic, deictic gestures, and inscriptions, together with his embodied experiences, to comment on whether the given figure was a polygon and to create an image of the corresponding edge on the polygon. He used both iconic to refer the edge on polygon and its physical forms resembling the visual characteristics of the image he attempted to portray with his left-hand index finger and deictic, pointing the left-hand index finger to represent the edge. The right-hand index finger is shown in Fig. 3b. However, he simultaneously utilized speech to verbalize and SL to visualize his interpretation of what the edge should not be. He also used the inscriptions drawing a line segment which is a figure what the edge should be on the sheet before he did not use the other signs. All these signs possibly allowed him to visualize the edge concept.

Initially, S1 drew a line segment at the top of Fig. 1a without saying something (Fig. 1c) while he reasoned that Fig. 1a is polygon. It was clearly seen that S1 tried to express that the edge must be line segment. On continuing of the conversation, he said that figure was not a polygon as he was encircling edge curved (Fig. 1c). In here, he tried to say that the part encircled obstructed that figure was a polygon. When he was asked why it was not polygon, he said that the edge was "broken", and he used SL for "broken" concept (Fig. 2). Even he tried to show that the edge had to be the line segment with figure drawn on the sheet, he said "broken" orally (but not clear in speech). He could said "kır.." in Turkish for "broken".

S1's oral expression was different from his written representation describing the edge. In fact, it was expected that shape was not a polygon because the shape's edge was not a line pigment. However, he used the "broken" concept. This situation showed that he did not know line segment. He profited by the "triangle" geometric shapes to show edge while explaining why "broken" edge was not. He expressed a sign called triangle combining the end of Right-Hand Index Finger (RHIF) with root of Left-Hand Index Finger (LHIF) and the end of Right-Hand Middle Finger (RHMF) with end of LHIF when RHIF, RHMF and LHIF was open, the others were close (Fig. 3a). S1's sign was the representation of triangle in SL.

He showed the LHIF represented as one of the edges of the triangle with the RHIF and he said that those edges were broken, and figure could not be a polygon. When S1 said "here on triangle," he was pointing LHIF with RHIF (Fig. 3b). In here, S1 meant to say that the edge should be straight like this figure without being broken. Even though he could not explain this with his mouth correctly, his gesture showed how the edge should be. In this situation, on the one hand, the gesture has become a sign of his mind. On the other hand, as Arzarello, Paola, Robutti and Sabena (2009) say, the gesture provided alternative ways of embodying and organizing information that the student is not able to express in purely verbal or formal ways. From this, it could be said that he did not have true knowledge about edge when it considered the concept of broken used by him. Nevertheless, signs made with his fingers and the figure drawn on the paper showed what he was trying to express was correct.

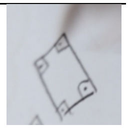
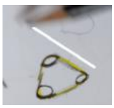
Considering the "Angle"

Only S2 has examined angle of some shapes given to them and decided whether shape was polygons according to the features of angle in this shape. S2 stated if the angle of the given geometric shape was regular, straight or formed straight edges, it was polygon and if the angle was round, it was not polygon. The data related to categories obtained from student comments were given in Table 3.

Table 3 listed the categorizations obtained from students' explanations and semiotic sources which S2 used to describe angle concept and its features. According to Table 2, S2 used different iconic gestures when describing the angle, and she drew the figure of the angle on the paper while the angle was explaining by mouth. She explained that angle had to be regular and perpendicular with mouth and shapes drawn on the paper. However, she utilized iconic gesture and mouth while she explained that an angle should be formed with straight edges.

Table 3

The Categorizations Obtained for Angle and Semiotic Sources Used

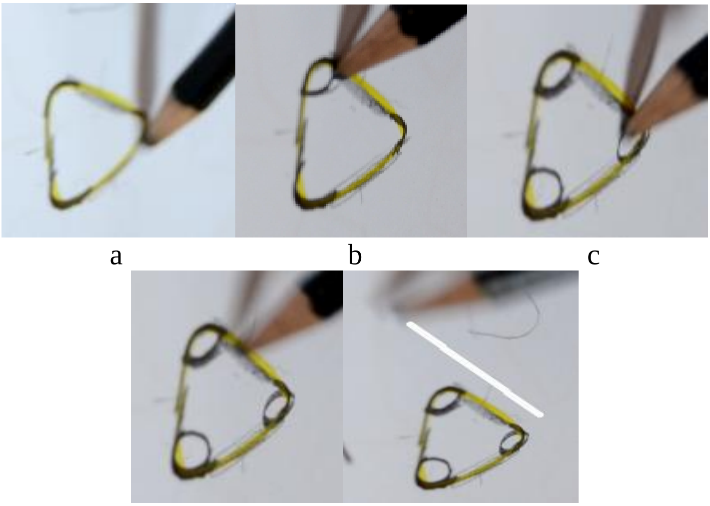
Component of polygon	Categories-Student	Semiotic sources					
		Gestures			Language		Inscription
		IG	MG	DG	Oral	SL	Figures
	S2				x		
Angle	Regular				x		
Polygon	Perpendicular				x		
	Formed straight lines				x		
Non-polygon	Round				x		

S2 expressed that shape was not polygon If the angle is round using deictic gesture, mouth, and drawing figure. He made imaginary oval lines on the air with the RHIF and LHIF to represent the angle. Thus, figure drawn by S2 on paper supported the gesture. S2 has drawn a rectangle on the paper and the four angles' measures of the rectangle have been shown on the rectangle as being perpendicular by her. Considering of gesture and figure on the sheet, it was determined that the angle is expressed in terms of angle measure. As a result of, it could be said that S2 confused the concepts of angle and angle measure and she had misunderstandings about this subject. Yet, S2 showed the two consecutive edges drawing on the rectangle while explaining that the angle should be regular, and it has been determined that. When this figure drawn by the student is examined, it can be said that she showed the angle correctly. However, she demonstrated angle measure as angle in gestures and other forms drawn. Because of this, it could be said that she had mind confusion about angle.

In the following dialog, S2 was given Fig. 4 and asked to detect whether it was polygonal. S2 paid attention to angle of shape, and she said that it could not be polygon because angle was "round." So, the following dialog between S2 and the researcher was executed.

R: Is there no edge in here?
S2: No (draws the dark places in Fig. 4a).
R: But when you say the edge you draw another place (draws the dark places in Fig. 4a). Are they (Fig. 4-a) edge?
S2: Angle. Iım, ımm … it must be something. Angle, for example angle (Fig. 4b). It does not angle.
R: Why?
S2: Because they don't look like each other. Round (Fig. 4c: draws rounds on all three vertex).
R: How should it be?
S2: The edges (Fig. 4d: points to the edge of the shape with pen) should be straight (Fig. 4e: makes gesture with pen on the paper).

S2 used speech, iconic, and deictic gestures, and inscriptions to comment whether the given figure was a polygon and to create an image of the corresponding angle on polygon. S2 used inscriptions, iconic and deictic gestures to express the edges forming the angle.



a b c

d e

Figure 4. The process of explaining which angle was non-round

While reconsidering whether shape in Fig. 4a was a polygon, S2 primarily stated that shape did not have an edge. At that moment, she indicated vertex of the shape with a pen while she explained the edge. The researcher asked whether the parts drawn by S2 were an edge and she has said angle after she had ephemeral confusion of the mind. However, she drew angle measure on all the vertex (Fig. 4b, c) while she wanted to demonstrate not to fit definition of angle. However, she declared that this part was not angle because this was round. In this process, the researcher asked how the figure must be to be angle and S2 indicated the edge of figure (Fig. 4d) with pen. She explained that the edge had to be straight (Fig. 4e). The situation of pointing the edge with the pencil was a deictic gesture. While she was explaining that the edge had to be straight to form the angle, she drew a straight line with pen on the paper doing gesture. This gesture used for straight concept was her metaphorical gesture.

Angle is set of the combination of rays [BA) and [BC) formed by the points A, B and C on the plane. [BA) and [BC) are the arms or edges of the rays. The graphical representation of the ray is a straight line with an arrow at one end (Argün, Arıkan, Bulut, & Halıcıoğlu, 2014). Therefore, the edges of the angle are a straight line. It could be said that she knew the angle concept correctly when it was paid attention her explanation that angle had to form straight lines.

However, she mentioned the edges of the shape, not the edges of the angle, while saying edge, and never used the concept of arms/edges of the angle. In addition, she drew the angle measure as an angle in Fig. 4b-d. The angle measure is the distance of rays which form angle (Van De Walle, 2004). S2 had marked the distance among the rays as angle in Fig. 4c, this situation indicated that she confused the angle and angle measure concepts. S2 could not explain exactly how to express the concept of angle but she mentioned that the edges which formed angle was straight lines and the angle could not be round.

Considering the "Vertex"

Three students examined the vertex of some of the shapes given to them and she decided whether the shape was polygons according to some features they had. Students stated that if the vertex was right (straight), straight or occurred straight two lines line segment, the shape was polygonal, if the vertex was round, ovoid or point of combination of the irregular two edges, it was not polygon. Data related to the categories obtained from student comments were given in Table 4.

Table 4

The Categorizations Obtained for Vertex and Semiotic Sources Used

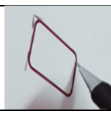
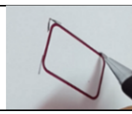
Component of polygon	Categories-Student	Semiotic sources					
		Gestures			Language		Inscription
		IG	MG	DG	Oral	SL	Figures
Vertex	S1				x		
	S2				x		
	S3				x		

Table 4 (continue)
The Categorizations Obtained for Vertex and Semiotic Sources Used

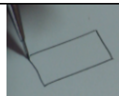
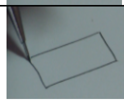
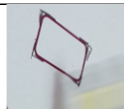
Component of polygon	Categories-Student		Semiotic sources: Gestures IG	Gestures MG	Gestures DG	Language Oral	Language SL	Inscription Figures
Polygon		S1		 		x		
	Right	S3				x		
	Regula	S3				x		
	Point of	S2				x		
Non Polygon	Round	S1		 		x		
	Ovoid	S3						
	Point of	S2				x		

Table 4 explained the semiotic sources used in processing the explanation the vertex concept and its properties and categories derived from the comments of S1, S2 and S3. According to Table 3, S1 used deictic gesture, oral expression, and inscriptions, S2 and S3 only utilized speech. S1 explained that the vertex was right with metaphoric gesture, oral expression, and SL, S3 explained with speech and inscriptions. S3 declared that the vertex had to be regular with speech and inscription, S2 also expressed that the vertex was the point of the combination of the straight two edges with speech and inscription. However, S1 mentioned that the vertex was not round using metaphoric gesture, speaking, and using inscriptions and S3 explained that the vertex was not ovoid and point of combination of the irregular two edges speaking and using inscriptions.

In the following expert, Fig. 4a was given S1 and it was asked whether this shape was polygon. S1 paid attention to vertex of the shape, and she said that the vertex could not be round. Because of this, she mentioned that the shape was not polygon. At that moment, following dialog was materialized between S1 and R below.

> R: Is this (shows Fig. 5) polygon?
>
> S1: (points Fig. 5 with pen) (makes signs in Fig. 6) It could be, but must not be (Fig. 7a,b: shows vertex of the shape). (Indicates that vertex must be as in Fig. 7c without speech) This (makes sign in Fig. 6) is, but it is round (using gestures in Fig. 8a, b) not be. Right (makes gesture in Fig. 8c, d and makes sign in Fig. 8e).

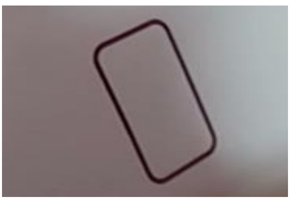


Figure 5. Shape Given

a b

Figure 6. Gestures for Rectangle

a b c

Figure 7. Perpendicular Vertex

a b c

d e

Figure 8. The Process of Explaining Which Vertex was Round

S1 used oral expression, iconic, metaphoric, and deictic gestures, and inscriptions while he was explaining that Fig. 5 was not a polygon. He made an iconic gesture referring to the rectangle to express the shape of the polygon that had to be. He also used metaphoric gesture for round concept while explaining that the shape in Fig. 5 was not a polygon because it had round corner. In addition to, he elucidated that corner had to be line using the SL and metaphoric gesture.

When the researcher asked S1 whether Fig. 5 was a polygon, S1 wanted to explain that the shape was not polygon. He used deictic gesture pointing the shape in Fig. 5 with the pencil and stated that it should be like Fig.10. He made iconic gesture to represent rectangle concept in Fig. 6. In this gesture, while the RHIF, LHIF, and Right-Hand Little fingers (RHLF) were open, the others were close and palm was downward, end of the LHIF with end of the RHLF and end of the RHIF with root of the LHIF were combined. Then, LHIF was moved the forward being removed from RH. He used fingers to embody edges of shape. The S1's movement was an iconic gesture made for rectangle. He explained that Fig. 5 was not polygon because Fig. 5 did not appear like Fig. 6. In the continuation of the conversation, to illustrate why Fig. 5 was no polygon, he drew the shapes in Fig. 7, in which S1 rearranged the corners of figure in Fig. 5 as forming the perpendicular angles. He showed gestures in Fig. 8a, b that corners were not round. In the gesture of Fig. 8a, S1 made fingers as in Fig. 6a, and then he did avoid the left hand, sharp part of the left hand put into the corner of the shape in Fig. 6. In Fig. 8b, S1 put into left hand in ovoid position on the any corner of the shape in Fig. 6b. The signs in Fig. 8a and b were not metaphorical gestures referring to the concept of "round" and but an iconic gesture which indicated the corner because it was placed at the corner point. Similarly, to explain that the corner must be right, S1 made fingers as in Fig. 6 and then LHIF was moved starting from the root parts of the middle finger throughout the ring finger and a perpendicular (Fig. 8c, d) angle was occurred. He stated that corner consisted of combination of the straight edges as forming perpendicular angle. The signs in Fig. 8c, d were metaphoric gestures that explained the right concept. After doing gesture, S1 immediately expressed the correct concept with the SL as in Fig. 8e. It could be said that the gesture came before the SL and simplified to make the SL.

ABC angle is set of the combination of rays [BA) and [BC) formed by the points A, B and C on the plane. B also corner of the angle (Argün et al., 2014).

In the above dialog, he had never uttered the corner concept, but, in the light of the corner definition, it was understood that he referred to the corner concept by examining his gesture, SL, and inscriptions.

Results and Discussion

In this study, it was investigated how the students determined the polygons, what they paid attention while determining, how they expressed the concepts and their features used in this process and what their misconceptions. We also examined how students utilized semiotic resources in developing and explaining their geometrical reasoning. It was detected that students focused on edges, angle and vertex in the given figures when determining the polygon.

First, in the study, it was determined that while HH students in the current study who focused on the basic elements of polygons determined polygons, even if figures of some concepts could draw, they did not detail information about the concept and have limited information. For example, the shape which had broken edge according to S1 is a concave quadrilateral. S1 perceived the two edges twisted inward as a single edge and stated that this edge was broken. For this reason, he explained that it was not a polygon. This situation has shown that participants did not have knowledge about polygons such as concave and convex. In addition, it was determined that the participants who draw as a regular bar correctly had difficulty in explaining the critical features of the edge. It was also determined that participants used the concept of "right" instead of "regular bar." It was obtained that participants confused "line segment" concept with "line" concept and any regular bar was distinguished as edge by students, non-smooth bar was distinguished as non-edge by them. This may be because participants focused on the visual features rather than critical properties of concepts. This result supported the view of Tall and Vinner (1981) that formal directional are more dominant than conceptual direction in recognizing the concept. Likewise, S2 and S3 have stated that shape had edge given was not polygon, but it was polygon if points were merged. When the reason (*why*) was asked, they could not explain why it was not polygon. Even though pupils could draw shape of polygon, they had difficulty while identifying the polygon. This situation may have been based on the fact that students focused on more visual characteristics of the shape. This is like the results of Fujita and Jones (2006). They too have reached the conclusion that prospective teacher was more successful in subject of drawing polygon than subject of the defining. In this studying, HH students explained that the edge could be “accurate, regular,” not be “broken, irregular, wave,” angle could be “regular,” not be “round,” vertex could be “regular,” not be “round, ovoid,” polygon could be “regular, naturel, beautiful image,” not be “round, zero.” In here, it was detected that the definitions they occurred themselves often took from in the context of their perceptions while they were explaining polygon and its properties. That is, they usually used personal expressions rather than formal expressions (formal language) in their definitions. That is, HH students utilized personal definitions which students constructed their definitions and filtered these definitions in their mind (Tall & Vinner, 1981). This result showed what the use of personal language-definition led to the students' perceptions about concepts and gave information about the perceptions of students. This finding supported the findings in researchs of De Villiers (1998), Tall and Vinner (1981), and Türnüklü, Gündoğdu-Alayli, and Akkaş (2013). The distinction from formal mathematics definitions has shown that students did not sufficiently pay attention to critical features that objects possess is not given sufficient attention (Türnüklü & Ergin, 2016).

Second, gestures, linguistic resources and inscriptions were employed that students were engaged in geometric shapes. Students used what McNeill (1992) calls iconic gestures, to bear a relation of resemblance to the semantic content of discourse that also referred to as representational (Kendon, 1988), metaphoric gestures, to provide an image of something invisible, an image of an abstraction, deictic gestures, to point to objects that were visually available on the figures and draw the audience’s attention. HH students’ gestures represented virtual mathematical objects (e.g., angle, edge) or mathematical relationships (e.g., smoothness of polygons) and animated the mathematical concepts. The HH students gestured using the sharp part of hands and index finger to represent the edge. The edge is a straight line because it is line segment. In this sense, we can say that the HH students’ gestures correctly reflect the concept. HH students who expressed that the edge is right, straight, regular, line segment used all these concepts in the same sense drawing a straight line on the paper. Because the gestures or shapes drawn for all of these concepts were similar. One student stated that the edge could not be broken, and the broken edge represented with a slight curved finger. The HH students have stated that the edge could not be curved, trapezoidal, and wavy in appearance, and for all three concepts, the gestures of the students were in the form of a curve in the air. Goldin-Meadow and Singer (2003) stated that gestures strengthened the information spreading in the conversation. In this context, it can be said that student's gestures support their speech. It was seen that there were various gestures about the characteristics of the vertex where the students did not make any pictorial gesture for the vertex concept. Considering the S1's gesture and shape drawn by him, it was seen that he showed the angle which expressed the combination of the two edges as vertex, not the vertex points of the polygon. S2 and S3 also demonstrated vertex points as vertex. While the students were explaining that the vertex should be right or regular, S2 told that the edges were surely straight, and the figure was not vertex if the edges were curved. Students also stated that the vertex could not be round and ovoid. The story of the signs described in the example showed the nature of the semiotic bundle. The semiotic bundle is a theoretical structure that allows students to handle multi-activities and a model that shows the complex relationship between speech, gesture, and inscriptions in learning mathematics (Arzarello, 2006). The entire components of the semiotic bundle are consistent and in a harmony. In this context, all the signs used in here are formed in harmony and one is transformed into another is a powerful example for the semiotic bundle.

In the study, S1 showed LHIF with the RHIF to represent the edge. In the set of the gesture did by S1, S1 used iconic gesture as LHIF represented the edge and deictic gesture as RHIF pointed LHIF. In a different example, S1 used metaphoric gesture to refer the "round" concept in Fig. 8a, b and iconic gesture to represent the vertex as putting into fingers on the vertex point. In this example, the same gesture set sequence has more than one gesture in it. This result strongly supported the view that Parrill and Sweetser (2004) pointed out that it could be in the form of another gesture when a gesture might belong to the any gesture. Similarly, McNeill (2005) stated that gestures could typically be loaded more than one dimension and revised gesture classifications.

Based on the findings, it was determined that the gestures had important roles. The gesture allowed alternative information organization where analytical thinking or speech could not be easily obtained. Even though S1 could not explain that the edge must be straight by his mouth orally correctly, his gesture showed how the edge should be. Arzarello, Paola, Robutti and Sabena (2009) also expressed that gesture provides alternative ways of embodying and organizing information that the student is not able to express in purely verbal or formal ways. However, gesture reflected the model in the students' minds. S2 stated by the gesture and figure drawn that openness between the rays meant the angle. This movement was a sign of the angle measure. The angle measure is the openness between rays while the angle is the set of the combination of the rays (Argün et al., 2014). Thus, S2 stated that the openness between the rays was the angle by the gesture and figure drawn. She pointed out that the angle should be regular only on the figure drawn and showed the combination of the edges as angle. In this way, she showed the angle correctly. However, this figure drawn by S2 was not enough to show that S2 knew the angle. Considering the S2's speech and gesture, it can be said that S2 stated the "openness, gab" as angle. Indeed, the S2's gesture gave us an important clue as to what he thought. In the literature, some studies also point out that gestures are a way of showing students' thoughts in their mind and an embodied form used to transfer to the environment knowledge in their cognitive (Edwards, 2009; Presmeg, 2006). This study strongly supported these research in the literature.

Recommendation

In the study, it has been revealed how the three HH students determined polygon and non-polygon. For this case, it was seen that the participants of this study used more personal expressions than mathematical expressions in the process of expressing the physical appearance of the polygon features of edges. The mathematical expressions used can generally be said to be incorrect, for example, that edge is line, circle is a polygon, etc. The knowledge students had can be influenced by the way teachers handle the lesson. Therefore, it is suggested that the words used in the concept definition should be selected carefully by the teachers to teach the concepts correctly. It has been observed that students often need to make a sign when expressing concepts in this study. These signs were often seen as gestures. Although the use of SL in the education of HH students is challenged, that students needed these signs showed how important the sign was for the students. For this reason, during the mathematics teaching in class, it is suggested that the teachers use sign for concepts in their lesson. These signs may be signs or gestures. Apart from some basic concepts (triangle, square, rectangle, etc.), many mathematical concepts are not available in SL. In this study, some geometric concepts used by HI students were investigated. In the future, the other concepts in mathematics can be investigated and it can be detected whether the signs will be used and how they will be used while the mathematical concepts are being explained.

In this case, teachers can also teach mathematics with the gestures that they produce. The use of gestures has been associated with the enhancement of mathematical learning (Goldin-Meadow & Singer, 2003) and is seen as a useful pedagogical resource for classroom instruction (Arzarello et al., 2009). Gestures present an embodied form (Arzarello et al., 2009). In other words, when the gesture of the student was examined and his/her thought about the concept could be understood. By carefully examining the gesture of the students about the concept, it can be detected that teacher can have an idea about the misconceptions and can remove the misconceptions.

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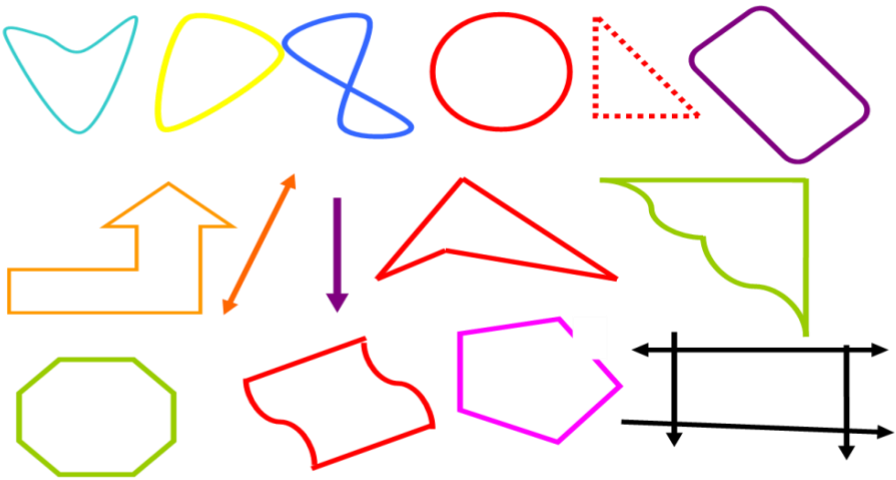
Appendix 1- Questions



In the above, there are various geometric shapes. I want you to detect which shape is polygon or non-polygon.

1 … are polygons. Because ….

2.… are not polygons. Because ….

3. Can you explain how you detected this and what you paid attention to?