

UNDERSTANDING RISK AND UNCERTAINTY: THE IMPORTANCE OF CORRELATIONS

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Abstract

There is general agreement today that students should leave school with a good grasp of all the basic mathematical ideas about how to deal with uncertainty. As adults, they will have to make important decisions about the probability of certain events and about the risk of a wide range of dangers. The understanding of uncertainty also plays a central role in modern science. Many scientific discoveries of great importance would have been impossible if scientists had only conceived of the world in terms of certainty. In many situations studied by scientists, and most certainly in all situations studied in social sciences, researchers can at best identify and measure imperfect associations between variables. This paper summarises research about understanding correlations, which assess whether, how variables and the degree to which variables show a mutual association. The cognitive demands of understanding these associations are analysed and a brief review of the literature is presented.

Keywords: association between variables, understanding correlations, understanding risk

Resumo

Atualmente existe um consenso geral que os alunos devem concluir a escolaridade básica com uma boa compreensão das ideias matemáticas básicas sobre como lidar com a incerteza. Como adultos, eles terão que tomar decisões importantes sobre a probabilidade de certos eventos e sobre o risco de uma ampla gama de perigos. O entendimento de incerteza também desempenha um papel central na ciência moderna. Muitas descobertas científicas de grande importância não teriam sido possíveis se os cientistas pensassem sobre os fenômenos no mundo apenas em termos de certezas. Em muitas situações estudadas por cientistas, e certamente em todas as situações estudadas em ciências sociais, os pesquisadores podem, na melhor das hipóteses, identificar e medir associações entre variáveis. Este artigo resume pesquisas sobre o entendimento de correlações, que avaliam se existem associações mútuas entre variáveis e a tipo e grau dessas associações. As

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bases cognitivas da compreensão dessas associações são analisadas nesse artigo e uma breve revisão da literatura é apresentada.

Palavras-chave: associação entre as variáveis, entendimento de correlações, compreensão de risco.

Between the complete certainty of determined events and the complete uncertainty of totally random events lies a world of imperfect, but nonetheless important associations. This is the world of associations between variables. Correlations are measures of the strength and the direction of the association between two variables. A correlation co-efficient of 1 tells us that two variables are perfectly and positively related and a co-efficient of -1 shows that they are perfectly but negatively related. Neither of these correlations leaves any room for uncertainty, but these perfect correlations are extremely rare. Correlations are greater or smaller than 0 and fall somewhere between 0 and 1 or between 0 and -1. They show that there is an association between the two variables but also indicate that we cannot be certain how the association will affect individual cases. We know, for example, that there is a relationship between how much people eat and whether their weight goes up or down, but we also know that the association between these two variables is not a perfect one, since the strength of the effect varies a great deal between people. The association, though less than perfect, allows doctors and dieticians to give good and worthwhile advice to people in danger of obesity, for example, but it is not strong enough for them to make precise predictions about what will happen to individuals as a result of changing their diet.

Many situations that confront us, both in science and in everyday life, involve at the same time associations between variables and some uncertainty about the effects of the association. Correlational reasoning is about the presence, nature and strength of a mutual relationship between two variables (Adi, Karplus, Lawson, & Pulos, 1978). This reasoning requires the recognition that relationships between variables are not absolute but exist in degrees (Ross & Cousins, 1993), and thus involve probabilistic reasoning.

Risk is an uncertainty that can be expressed as a number such as a probability or frequency on the basis of empirical data (Gigerenzer, 2002, p. 26). In everyday life the word risk is associated with negative outcomes, but this is not the use made of the word in the medical or psychological literature. To use a common example, in everyday life one would speak about the risk of having cancer (i.e. the probability of having cancer) if a test is positive but one would not speak of the risk of not having cancer (i.e. the probability of not having cancer) if the test is positive. However, the probability of not having cancer if the result of the test is positive is still important and must be considered when decisions are made about the subsequent course of action. According to Gigerenzer's definition, each of these probabilities define the risk of that event taking place.

The word risk carries another connotation, alongside the probability of an event, which is related to the seriousness of an outcome. This meaning of risk was explored by Pascal in the argument that is today known as Pascal's wager (Mlodinow, 2009). Pascal weighted the probability that God existed, if one did not know anything to prove it one way or the other, against the severity of the risks one could run by following or not the laws of God. Pascal's wager is presented here in a simpler form, focusing on the meaning of risk in terms of severity of the outcome. The risk of following the laws of God is that one might miss some pleasures in a life of limited duration. The risk of not following the laws of God is losing eternal life and happiness. So Pascal concluded that every reasonable person should obey the laws of God because the risk associated with not obeying them is clearly more serious than the risk of obeying them. Although this connotation of the word risk is important, the focus in this paper is on the first one, the probability of an event taking place.

Correlations help people define the probability of a particular event taking place when something else is known. We can take, as an example, a committee carrying out an inquiry into the deaths of children undergoing a certain type of surgery in a particular hospital. The committee must consider the evidence using correlational reasoning. The question the committee needs to answer is whether children operated on in this hospital are more likely to die than those

who received the same surgical intervention in other hospitals. In other words, is there an association between receiving the treatment in this hospital and death? The relationship is unlikely to be absolute: not all children operated in the hospital will have died and not all operated elsewhere will have survived. But the question whether the chances of the children dying are increased by receiving surgery in this particular hospital can still be asked and answered by looking at the strength of the association.

The value and importance of these imperfect associations is now widely recognised. Even the most impressive discoveries in the history of science might have been dismissed if one were to expect a perfect association. When Florey and his colleagues ran the first experimental study on the effects of penicillin with mice (Lax, 2004), the outcome could be interpreted in deterministic terms: they infected 8 mice with the smallest dosage of virulent streptococci known to kill a mouse of average weight and then gave four of them penicillin. The four mice that did not receive penicillin died within a day. Of those that did receive the penicillin, 2 had received a single shot, and one died after 2 days and the other after 6 days. Of the 2 had received five shots over a period of time, 1 died after 13 days and the other lived on, presumably to a ripe old mouse's age. However, the trials with humans were by no means as successful: 2 of the first 6 patients treated with penicillin died, which suggested the possibility that penicillin may not be as great a success as the mice experiment had suggested. The question was whether this recovery rate was definitely better than a no-treatment condition, and Florey certainly wanted to seek more evidence before making these experiments known to the world.

Thus correlational reasoning is an essential element in scientific reasoning and scientific literacy (Gigerenzer, Swijtink, Porter, Daston, Beatty, & Kruger, 1989; Robinson, 1968; Ross & Cousins, 1993), and a means of controlling the present and predicting the future in order to maximise the desired outcomes in one's personal life (Alloy & Tabachnik, 1984).

The cognitive demands involved in understanding correlational reasoning

The cognitive demands in understanding correlational reasoning are various, and we focus here briefly on three. The first one is understanding randomness. If there is no relationship between two events, A and B, it is still possible that they might occur together by chance. The aim of analysing the correlation between two events is to establish whether they co-occur more often than one would expect by chance. Understanding randomness is therefore part of understanding correlations.

Correlational reasoning also involves understanding sample space. In order to examine whether two events are associated, we need to establish not only whether they co-occur but also what all possible the cases are: Did A happen? Yes or no. Did B happen? Yes or no. The sample space here is Yes-Yes, Yes-No, No-Yes and No-No. One could be tempted to think that only the cases Yes-Yes are relevant to the question of a correlation between the two events, but the probability of the events occurring together must be understood in the context of the events not occurring together as well.

If we think of the infected mice that did or did not receive penicillin, we have a slightly more complicated sample space. Some mice did not receive penicillin and did not survive for one day, so this is a No-No case (i.e. no penicillin, no survival). Some mice received a single shot of penicillin: one survived 2 and the other 6 days. If we simplify the survival criterion to, for example, surviving one week, these mice would also be Yes-No cases. Some mice received 5 shots of penicillin and survived for longer than one week: they would be examples of Yes-Yes cases. There were no mice that exemplified the No-Yes case (i.e. infected mice that did not receive penicillin and survived for at least one week).

This example hints at how sample space can be much more complicated than cases in four categories, because the cases may vary in more subtle ways than Yes or No. One could, for example, characterise the administration of penicillin by the number of shots the mice received and the survival of the mice by of the days they survived. This would create a much more complicated

sample space, which cannot be analysed as easily. However, most of the research on children's understanding of the association between variables has focused on the simplest sample spaces described by Yes or No on the two variables.

Once we have established the sample space, we need to move on to the quantification of probabilities in a proportional manner. Is the frequency of cases that support the existence of an association (the yes-yes and the no-no cases) proportionally really larger than the frequency of the cases that do not support the association, so that one can assume that this frequency departs from what one would expect by chance? If this is the case, we conclude that there is an association between the two events.

In summary, understanding the association between two variables makes at least three demands on children's reasoning: they need to understand randomness and the sample space, they must be able to recognise which cases support and which cases go against the idea of an association between the variables, and they need to be able to quantify the positive in comparison to the negative cases in order to assess whether the positive cases are frequent enough to suggest that the co-occurrence observed is not due to chance.

Past research on how people understand correlations

Studies of people's understanding of correlations can be best analysed if we separate them into groups, defined by the questions that they address. This is not to say that the questions are independent of each other; it is just a methodological step to help us identify the different pieces of the puzzle that constitute correlational reasoning. The studies are sorted in this paper into five types related to the questions they address:

(1) How do people react to contradictions of an expected relationship between two events?

(2) What sort of information do people seek when trying to find out whether two events are correlated?

(3) How does the presentation of information relate to our understanding of correlations?

(4) How do children and adolescents quantify the information that they are presented with and what inferences do they draw from information?

(5) How can we help students to understand correlations better?

Reaction to information that contradicts an expected relationship between variables

Inhelder and Piaget (1958) carried out a variety of studies in which they analysed how children and adolescents react to information that confirms or contradicts the existence of an expected relationship between variables. In one well known study, they asked children to attempt to explain why things float or sink in water. This situation is a deterministic one, and not a matter of probabilities, and the researchers' interest was in the way children reacted to contradictions of their predictions. The problem is rather appropriate for examining reactions to contradictions because many people, including many adults, start out with the notion that heavy things sink and light things float, rather than with the idea of density, which involves a relation between mass and volume. The question is then how participants will react to the contradiction of their predictions. If children and adolescents cannot discard the hypothesis of a relationship between mass and sinking (i.e., if they cannot reject the hypothesis that heavy things sink and light things float) in a deterministic situation, they might find it even more difficult to interpret relationships that are probabilistic rather than deterministic. We focus here on the relevant aspects of this study, not on the details of whether and how the participants reached an understanding of density.

In the Inhelder and Piaget study, at the start of the session, the children are asked to classify the objects in two categories, those that will float when placed in a basin full of water and those that will sink when placed in the basin. If a child forms these two categories and provides a consistent explanation . for example, these float because they are light or small and those sink because they are heavy or large . the experimenter proceeds to ask for specific

predictions for each object and then notes the child's reactions to contradictions of these predictions. The experimenter chooses, for example, a large piece of wood, which is both large and heavy, and asks the child to make a prediction. To be consistent, the child should predict that it will sink. When the wood is put into the basin, it floats. Inhelder and Piaget noted three different types of reactions to this contradiction: (1) some children would ignore the contradiction, and continue to assert that heavy things sink and light things float and indeed attempt to make the piece of wood conform to their prediction by pushing it down into the bottom of the basin; (2) other children would modify their hypothesis, forming classes of objects that can float despite belonging to a class which is predicted to sink (e.g. heavy objects sink but wood normally floats because it has air inside, it is not very compact); (3) other children would note the contradiction and would no longer accept the simple association between mass and sinking (some of these actually give up seeking the solution, whereas others seem to go on to think of a relationship between mass and volume, constructing an understanding of density). The relevance of this study to correlational reasoning may not be immediately apparent but we hypothesise that it is not possible to think about correlations without understanding how expected relationships might be disconfirmed by evidence.

Inhelder and Piaget's (1958) work on propositional reasoning, exemplified in the study about the law of floating bodies and the elimination of contradictions, inspired a large number of subsequent studies on how children and adolescents interpret statements about causal relationships and how they interpret contradictions. It should be noted that studies on contradiction do not imply that children have no understanding of causality. For example, children know that if water is spilled, the floor gets wet, and if something is cut, it is no longer in one piece (Bullock & Gelman, 1979; Schultz, 1982, das Gupta & Bryant, 1989; Sobel & Kirkham, 2007). Inhelder and Piaget's studies were about how children re-examine their thinking about relationships if their thinking is contradicted by observations. As far as we know, other researchers have not disputed Inhelder and Piaget's central claim that children's ability to see the relevance of disconfirmation of a prediction about a relationship between two

events improves over time and that this ability is not observed among children in the early years of school. This important result has consequences for understanding correlations: if children cannot discard their explanations for events when these are contradicted in a deterministic situation, it will be difficult for them to do so in a probabilistic situation, in which both cases that confirm and cases that disconfirmation the prediction might be observed.

Subsequent research has not analysed reactions to contradictions in such detail but has shown that, in general, but has explored the judgements that people make about correlations depending on whether they hold beliefs about the association between the variables. There are several studies on the recognition of covariation between variables when participants have a certain bias (e.g. Alloy & Tabachnik, 1984; Jennings, Amabile, & Ross, 1982; Scholz, 1991) but the best controlled study was by Batanero, Estepa, Godino and Green (1996). Batanero and colleagues presented a large sample of last year secondary school students with tables (2x2, 2x3 or 3x3) that contained frequencies showing the co-occurrence of certain characteristics. For some of these, they expected the participants to have previous beliefs. For example, an association is expected between smoking and having a bronchial disease as well as between number of hours studied and results in an exam. For other characteristics, the students were not believed to have expectations: for example, leading a sedentary life and having a skin allergy. The students were asked to interpret the tables and answer whether there was an association between the variables. The level of difficulty of the problems, as defined by the size of the contingency tables and the direction of the association (direct relations are more easily recognised than inverse relations), was controlled across conditions of expectation. Batanero and colleagues found a strong association between the prior beliefs of the students and their interpretations of the contingency table. Even students who correctly analysed the proportions of confirming cases and the proportions of disconfirming cases often drew the wrong conclusion, either supporting the association when there was none according to the table or failing to detect an association when it should have been detected. Because these were secondary school students, these results

suggest that the interpretation of information about correlations is influenced by reactions to contradictions, even though the same participants might have reacted differently if the contradiction had been to a prediction in a deterministic situation.

Seeking information about relationships between events

Among the numerous studies that were inspired by Inhelder and Piaget's work on propositional logic, one set of studies focused specifically on the analysis of how people seek information in order to test whether an association between two events affirmed in a proposition is true or false. Wason (1968) designed a task in which participants were asked to test whether there was an association between what was written on one face and on the other face of a set of four cards. The association was presented to the participants as a rule: "If there is a vowel on one side of the card, there is an even number on the other." The participants are asked to select only the necessary and sufficient pieces of evidence to test whether the rule is true. The cards that are on the table show a vowel, a consonant, an even number and an odd number. The necessary and sufficient information in this deterministic situation is to select the card with the vowel and the one with the odd number, because either could disconfirm the rule. The cards displaying a consonant and an even number are seen as irrelevant to the rule, because the rule does not state a mutual association between even numbers and vowels.

The commonest behaviour by children and adults in this task is to choose to verify what is on the other side of the card with the vowel and of the card with the even number. This choice is considered an error in testing the correctness of the rule because the card with the even number could not lead to disproving the rule. The participants' behaviour in this situation has been interpreted as revealing what has come to be known as a confirmation bias, i.e. a search only for information that would lead to confirming the rule without a realisation that other information could result in disconfirming the rule.

Subsequent research (e.g. Cheng & Holyoak, 1985; Cheng, Holyoak, Nisbett, & Oliver, 1986; Girotto, Light, & Colbourn, 1988) sought to provide an alternative interpretation to the behaviour of children as well as of adults in this task. Their behaviour was considered not to be adequately described by a logical analysis but rather by pragmatic schemas, which determined the relevant cases to be analysed. If the *if-then* statement was interpreted as a permission or a prohibition, rather different behaviour in the testing of rules was observed.

These studies suggest that children and adults evaluate the relationship between events differently depending on what they expect the nature of this relationship to be. Therefore, in studies about children's understanding of correlations, which are mutual relationships between events, one must ascertain whether they understand what a mutual relationship means when they test its existence. If their behaviour seems to indicate, for example, a confirmation bias, as in the Wason four-card problem, we need to consider what consequences this bias has for the understanding of correlational reasoning.

Confirmation bias is a term used to refer to seeking evidence that can only support the existence of a presumed association between two events. This bias does not have to be intentional or explicit: Nickerson (1998) defines confirmation bias as *unwitting selectivity in the acquisition and use of evidence* (p. 175). Evans (1989) considers this as *perhaps the best known and most widely accepted notion of inferential error to come out of the literature on human reasoning* (Evans, 1989, p. 41) and Dawes (2001) suggests that professionals may be prey to this bias as a consequence of their professional experience: for example, if a clinical psychologist asserts that child sex abusers do not recover from this condition without professional help, this assertion is often based only on the cases that the professional has seen, namely those who seek professional help. Disconfirming cases, i.e. those who recover from their condition without professional help, are usually not part of a psychologist's experience.

Loren Chapman and Jean Chapman analysed confirmation bias in a series of studies with psychologists and undergraduate psychology students

who were given information about patients and also about their performance in psycho-diagnostic, projective tests. In one study (Chapman & Chapman, 1967), for example, the participants were presented with drawings of human figures supposedly produced by patients who had one of six symptoms (e.g. he is suspicious of other people, he is worried about how manly he is). The pairings of the drawings with the symptoms had been carried out randomly. However, the participants supposedly discovered relationships between characteristics and symptoms as a consequence of remembering only confirmatory cases: for example, 80% of the participants %discovered+ that a figure drawn as muscular, with broad shoulders, indicated that the patient was worried about his manliness. Chapman and Chapman referred to this as illusory correlation, which they describe as the erroneous reporting of co-occurrence of symptoms and signs in a diagnostic test (see also Chapman, 1967; Chapman & Chapman, 1975).

In summary, research with adults and children has suggested that they are influenced by the nature of the relationship that they expect to exist between two events in the way they search for information to test whether the relationship exists. Some researchers have described a confirmation bias or illusory correlation in a number of situations and by a variety of participants . but note that these are simply terms and do not constitute an explanation for why information is selected in a particular way. Chapman (1967) attempted to explain this phenomenon as a consequence of stronger memory for associations between phenomena that were previously associated in one's experience. However, the information does not have to be committed to memory for this bias to be observed. In subsequent descriptions of how children and adolescents deal with information about the relationship between two events, we will consider the possibility that the confirmation bias stems from cognitive demands made by tasks and the difficulties that we have in dealing with such tasks.

Presentation of information and understanding correlations

In order to assess whether two events are correlated, people must be given information. It could be argued that the best way to analyse correlational reasoning is to provide information to participants about individual cases because organising the information can be seen as part of understanding how to assess whether two events are related. Mlodinow (2002) actually suggests that historically the analysis of probabilities only became possible when people developed better means of recording the occurrence of events.

Inhelder and Piaget (1958) briefly mention, in their study of adolescents' correlational reasoning, that the participants performed better when the classes of events were presented to them in 2x2 tables, which organised the information according to the sample space . Yes-Yes, Yes-No, No-Yes, and No-No.

Other researchers have shown that children may have difficulties in sorting out the information in order to construct the relevant classifications for a table (Adi, Karplus, Lawson, & Pulos, 1978) and that providing children with information already organised in tables improves their performance in tasks in which they are asked to assess whether there is an association between two events (Carvalho, 2008). Ross and Cousins (1993) showed that students can be taught how to organise information about individual cases in 2x2 tables, which can then be scrutinised in order to assess whether the events are associated. They also showed that students can be taught to organise information even in more complex, multivariate situations, in which a relationship between variables only exists under one condition but not under another (e.g. the relationship between a treatment and recovery may be conditional on the amount of medication used). Ross and Cousins found that the ability to organise the information in tables can be considered part of the skills necessary for correlational reasoning because some students cannot even start to organise the information. However, this ability may improve without a similar improvement in the ability to make correlational inferences from the information. Thus organising information can be seen as an important step in assessing correlations, but distinct from the process of making inferences about

whether a correlation does or does not exist. Tables still have to be analysed in order to assess whether there is a correlation between the variables.

Although tables are very often used in correlational reasoning studies in which children and adults are asked to assess whether there is a correlation between events, information about correlations is often presented in the media and in scientific papers, not in tables but either in conditional probabilities or in ratios. Probabilities can be stated as percentages and proportions or as ratios, referred to by some researchers as frequencies. Scientific and media reports seem to resort to proportions or percentages because these figures are easier to compare than frequencies: for example, if we are told that 62 people in a sample of 243 from city A had a particular illness and that 93 people in a sample of 329 from city B had the same illness, it is difficult to know whether the illness was more frequent in city A than in B. If we were told, in contrast, that approximately 25% of the people in each sample had the illness, we would quite easily conclude that the incidence was very similar in the two cities. The ease with which we compare figures in this example does not extend to correlational reasoning, in which the percentages or frequencies are related to conditional probabilities. When information is presented in frequencies or ratios rather than percentages or proportions, both children and adults seem to find the information easier to interpret.

Correlations can be presented in 2x2 tables, when the events are discrete (of the Yes-No type) and, when the variable are continuous, they can be presented either in tables or in graphs or actually in both formats at the same time. Carvalho (2008) analysed secondary school students' inferences about specific data points or trends in co-variation situations and found that they did not appear to use information from graphs by looking at the spatial characteristics of the graphs alone: when they explained their answers, they did not refer to slopes, for example, but to values that they read from the graphical representation. Their performance in problems presented in tables or both graphs and tables did not differ significantly, probably because they relied on numerical information. When the graphs represented a negative correlation, secondary school students found it rather difficult to draw the appropriate

inferences from the graphs, although they could note the negative relationship between the variables when the information was presented in tables.

The quantification of probabilities to determine whether there is a mutual relationship between two events

The initial work on how children use information to decide whether two events are related was carried out by Inhelder and Piaget (1958). They asked the adolescents aged 12 to 14 years who participated in their study to ascertain whether there was an association between hair colour (blond hair versus brown hair) and eye colour (blue eyes versus brown eyes) in a set of cards. The researchers made it clear to the participants that the question referred simply to the set of cards presented to them, in which faces with these attributes were drawn, and not to their experience outside the cases they were considering.

The researchers were initially interested in examining the responses from the standpoint of propositional logic, and not in the quantification of the relationship. The sample space (or possible cases) was thus definable as Blond-Blue eyes (p and q), Blond-Brown eyes (p and not q), Brown hair-Blue eyes (not p and q), and Brown hair-Brown eyes (not p and not q). The researchers asked the participants whether there was a relationship between hair colour and eye colour in this set of cards and later to subtract or add cards that would make this relationship stronger or weaker.

Inhelder and Piaget noted that participants had a problem right from the start with establishing how the four classes that define the possible cases (or the sample space) relate to the question of whether there is a relationship between eye and hair colour. They tended to establish one class . for example, the class Blond-Blue eyes (p and q) . and base their answer on this class only, thereby thinking only of the probability of having blond hair and blue eyes in that sample. When the researchers made the distribution of cards such that the relationship was actually perfect . 6 cards with blond hair-blue eyes (p and q), 6 with brown hair and brown eyes (not p and not q) and zero cards in the other categories . some adolescents dissociated the two categories from each other in their answers. The blond hair-blue eyes cases indicate that you have more

chance to have blue eyes if you are blond; when asked about the brown hair-brown eyes cases, they thought this was not relevant, these cases only indicated that you are more likely to have brown hair if you have brown eyes.

Other adolescents realised that both of these classes are confirming cases and the remaining classes are disconfirming cases, but did not combine the confirming cases in order to compare them to the disconfirming cases as a group. When they were asked to compare two sets of cards and say in which set they were more likely to find cases that follow the rule of a relationship between hair and eye colour, they did not use all four classes in their answers. For example, when comparing two distributions with the same number of cards with blond-blue eyes, the same number of errors according to the rule, and different numbers of cards with brown hair-brown eyes, they thought that the relationship between hair and eye colour in the two sets is the same: there were 5 chances of being right in both sets (ignoring that the brown hair-brown eyes increased the chances of being right in one of the sets).

Inhelder and Piaget suggested that, with further comparisons, as the experiment proceeded, some adolescents were able to reach the understanding that they needed to relate the sum of the sets of confirming cases to the sum of the sets of disconfirming cases. They also suggested that only exceptionally the adolescents anticipated the need to combine the two types of cases in their analyses. Thus, although some adolescents were able to reach an understanding of how cases confirming and disconfirming the relationship could be taken into account quantitatively, they did not come across many that demonstrated this understanding from the outset of the experiment.

It should be pointed out that in this study, as in the study on quantification of probabilities, Inhelder and Piaget's clinical method may lead to a more positive assessment of adolescents' understanding of probabilities than other methods, in which the participants are asked questions but not presented with conflicting approaches to the problem or asked to increase or decrease the relationship between the variables by manipulating the number of cards in the different cases.

The study by Inhelder and Piaget inspired some further research, both with adolescents and with adults. Much of the work with adolescents consisted in developing measures to assess correlational reasoning (e.g. Tobin & Capie, 1981), using such measures to predict students' success in science courses or assessing whether science courses had an effect on correlational reasoning (Lawson, Adi, & Karplus, 1979).

Studies with adults focused on whether adults who did not take many mathematics courses showed competence in correlational reasoning in their domain of work. Smedslund (1963), for example, interviewed nurses and student nurses (N= 96; in Denver and in Oslo) using a task which was very similar to the one used by Inhelder and Piaget, but the relationship to be analysed was relevant to their work: it concerned the correlation between a symptom and an illness. They were instructed to concentrate only on the information in the cards, which were meant to be about patients, numbered from 1 to 100 in the order in which they were admitted to the hospital. The cards contained four letters, representing different specific symptoms, and four other letters, representing specific diagnoses made by the hospital. The nurses were asked to focus on each of the associations, one at a time.

Smedslund expected that participants that understood correlational reasoning would use a selective strategy, organising the cards into the four categories relating to presence or not of the symptom and presence or not of the diagnosis would count or estimate the frequencies in each category; would attend to all four categories; and would compare the frequencies of the sum of confirming cases with the sum of disconfirming cases. Some of the participants were also shown the frequencies for the different combinations of presence or absence of the symptom and the diagnosis and asked to estimate the strength of the relationships between symptoms and illnesses.

Smedslund found some of the same behaviours described by Inhelder and Piaget in these participants as well: some nurses seemed to think that a relationship may exist when the symptom and the diagnosis co-occur and at the same time not exist when neither is present in the cards; they did not see both cases as examples supporting the mutual relationship between the symptom

and the diagnosis. If a symptom, such as a headache, appeared in many illnesses, it was considered an important factor for a diagnosis of a particular illness if it appeared on the card sometimes. Smedsland reported that not a single participant gave an indication of having understood that the degree of the relationship between symptom and illness depended on the ratio of confirming to disconfirming cases. This study, as some of those referred in the section on quantification of probabilities, suggests that the clinical method used by Inhelder and Piaget in their interviews may have led to a positive view of what adolescents achieve which is not replicated in other studies in which less interactive methods are used.

The concern with lack of evidence of correlational reasoning in secondary school and university students documented in different studies (e.g. Karplus, Adi, & Lawson, 1980; Lawson, 1982) inspired efforts to develop ways of promoting this reasoning, exemplified by studies summarised in the subsequent section.

Improving students' understanding of correlations

Researchers interested in promoting the development of correlational reasoning, such as Nours and Raven (1973) and Ross and Cousins (1993) seemed to approach correlational reasoning as involving a set of interrelated skills, such as organising data in tables or graphs, articulating predictions (e.g. in a graph, if the value of variable A increases, does the value of variable B increase, decrease or stay the same?), locating data, synthesising data, and drawing conclusions. The results suggested, as briefly mentioned earlier on, that it is possible to improve the skills of organising and synthesising without improving the inferences regarding the correlations. Students in the taught groups were able to organise and synthesise the data better than those in the untaught groups; however, they continued to answer the correlation questions not by using the information but on the basis of their preconceived ideas. For example, one of the questions was whether taller people were faster swimmers. Students in the taught groups could say %yes+ or %no+, even though they had

organised and synthesised the data appropriately; when drawing their conclusions, their arguments were not based on the data. The actual intervention in this study was carried out by classroom teachers, and Ross and Cousins (1993) report that a strong relation between the commitment of the different teachers and the effect of the intervention on the students' achievement. They also found that their programme led to similar results with younger, grade 7 students (about 13 years of age), and older, grade 10 students (about 16 years).

Other approaches to teaching correlational reasoning also used lessons on correlations, but these were combined with teaching other formal operations schemes, such as control of variables and proportional reasoning (e.g. Lawson & Snitgen, 1982). Although these are relevant and important studies, it is difficult to relate the improvements in correlational reasoning to a specific aspect of the teaching programme.

As far as we know, only one recent teaching study by Vass, Schiller and Nappi (2000) conceived of the process for promoting correlational differently from the earlier studies. The early studies approached the teaching of correlational reasoning as involving a set of skills, as described in the Ross and Cousins study. Vass, Schiller and Nappi concentrated on the conceptual basis on which correlational reasoning rests. Inhelder and Piaget (1958) and Karplus, Adi and Lawson (1980) advanced the hypothesis that understanding correlations depends on two cognitive schemes: understanding probabilities and understanding proportions. Vass and colleagues taught one group of teacher-education students in three lessons about proportionality and probabilities only and a second group, also in three lessons, about proportionality, probabilities and correlation. The first two lessons were the same for both groups but the third one differed. The members of the first group were given a review of the concepts of proportionality and probabilities correlations in the third lesson. The second group was taught about correlations in this lesson. Vass et al. reasoned that, if proportionality and probabilities are building blocks for understanding correlations, the group taught only about

these two concepts should make progress in understanding correlations, but perhaps not as much as the group that was also taught about correlations.

Both intervention groups made more progress from pre- to post-test than the control groups. The means attained in the correlational reasoning measure by the two groups were almost identical at post-test, and for both groups significantly better than their pre-test performance. Vass et al. concluded that teaching them about the building blocks gave them the necessary schemes to reason about correlations even without further teaching. This is an impressive demonstration of how helping students to meet the separate demands of a complex concept can promote significant advances in the students' understanding of that concept.

Summary and Conclusions

We argued at the start of this section that correlational reasoning involves the co-ordination of three schemes of reasoning: understanding randomness, the sample space and quantification of probabilities. It goes beyond each of these on their own and provides the basis for much of modern science and for understanding a large variety of situations in everyday life. Many of the relationships between events and between variables are not deterministic, but are probabilistic in nature. In order to assess whether there is a mutual relationship between events, we must test whether the association that we note between their frequencies is one that departs from what could be expected by chance. We must also understand the sample space that allows for scrutinising the relationship: in a typical discrete variable situation, we can characterise this space as the combination of Yes and No for each of the events: for example, a plant was treated with a pesticide (Yes-No) and it is no longer infested (Yes-No), gives a table with four cells: Yes-Yes, Yes-No, No-Yes, and No-No. In order to draw inferences from the frequencies in this table, we must understand the relevance of the different cells to a mutual relationship between the variables and draw conclusions accordingly. Two of the cells represent confirmations of the mutual relationship (Yes-Yes and No-No) and the

other two represent disconfirmations. We then need to know how to deal with these cells quantitatively.

Reasoning about a contradiction to a hypothesis is not simple in deterministic situations; it is also not simple in correlational situations. Some of the studies we reviewed, including for example the Smedslund study with adults, show that contradictions might not be noted by them when considering correlational evidence: they might focus only on the cases that seem relevant to them (the Yes-Yes cell in the 2x2 table) or they might think that if two events co-occur sometimes, then they must be associated in some way. Organising the information is important for scrutinising a relationship between events but even when this is not necessary, because the cases are already organised and quantified in the categories, drawing inferences is still a difficult matter.

The quantification of the confirming and disconfirming cases in order to test for a mutual relationship between two variables was rarely observed by Inhelder and Piaget at the outset of their experiment. Even their clinical method, which tended to guide participants to think about problems from different perspectives, did not suggest that correlational reasoning is easily attained. Subsequent research indicated that many secondary school and college students might not demonstrate high levels of correlational reasoning. These results motivated the search of methods to improve correlational reasoning, which was seen as particularly important in biological and medical sciences, and which indeed predicted students' success in courses in these domains.

Teaching studies that aimed to improve secondary school and college students' correlational reasoning were sometimes conceived in terms of the skills necessary for correlational reasoning and sometimes were part of a larger programme designed to improve formal reasoning in general. We found a single study that was conceived differently: a control group was compared to two taught groups, one that had received instruction only on the schemes necessary for correlational reasoning (probabilities and proportionality) and one that had received instruction on these two schemes plus instruction on correlational reasoning. The striking similarity in results in the two taught groups provides strong support for the notion that, if people master the cognitive demands of

correlational reasoning, they can use these resources in order to understand correlational problems. It is, however, too soon to reach a firm conclusion, on the basis of a single study. A combination of longitudinal, predictive studies showing that these particular demands predict learning of probabilities (even after controlling for other intellectual measures) and further teaching studies is required to support our hypothesis that correlational reasoning is the reward that one can reap in the scientific domain from understanding randomness, sample space and proportional quantification in the mathematics classroom.

References

- Adi, H., Karplus, R., Lawson, A., & Pulos, S. (1978). Intellectual development beyond elementary school UI: correlational reasoning. *School Science and Mathematics*, 675-683.
- Alloy, L. B., & Tabachnik, N. (1984). Assessment of covariation by humans and animals: The joint influence of prior expectations and current situational information. *Psychological Review*, 91, 112-149.
- Batanero, C., Estepa, A., Godino, J. D., & Green, D. R. (1996). Intuitive Strategies and Preconceptions about Association in Contingency Tables. *Journal for Research in Mathematics Education*, 27, 151-169.
- Bullock, M., & Gelman, R. (1979). Preschool children's assumptions about cause and effect. *Child Development*, 50, 89-96.
- Carvalho, L. M. T. L. d. (2008). *O papel dos artefatos na construção de significados matemáticos por estudantes do ensino fundamental (The role artifacts play when elementary school students construct mathematical meanings)*. Universidade Federal do Ceará, Fortaleza, Ceará, Brazil.
- Chapman, L. J. (1967). Illusory correlation in observational report. *Journal of Verbal Learning and Verbal Behavior*, 6, 151-155.
- Chapman, L. J., & Chapman, J. P. (1967). Genesis of popular but erroneous psychodiagnostic observations. *Journal of Abnormal Psychology*, 72, 193-204.
- Chapman, L. J., & Chapman, J. P. (1975). The Basis of Illusory Correlation. *Journal of Abnormal Psychology*, 84, 574-575.
- Cheng, P. W. & Holyoak, K. J. (1985). Pragmatic reasoning schemas. *Cognitive Psychology*, 17, 391-416.
- Cheng, P. W. Holyoak, K. L., Nisbett, R. & Oliver, L. (1986). Pragmatic versus syntactic approaches to training deductive reasoning. *Cognitive Psychology*, 18, 293-328.
- Das Gupta, P., & Bryant, P. E. (1989). Young children's causal inferences. *Child Development*, 60, 1138-1146.

- Dawes, R. M. (2001). Probabilistic thinking. In N. J. Smelser & P. B. Baltes (Eds.), *International Encyclopedia of the Social & Behavioral Sciences* (pp. 12082-12089). Amsterdam: Elsevier.
- Evans, J. S. B. T. (1989). *Bias in human reasoning: Causes and consequences*. Hillsdale, NJ: Erlbaum.
- Gigerenzer, G. (2002). *Reckoning with risk*. London: Penguin Books.
- Gigerenzer, G., Swijtink, Z., Porter, T., Daston, L., Beatty, J., & Krüger, L. (1989). *The empire of chance: How probability changed science and everyday life*. Cambridge: Cambridge University Press.
- Giroto, V., Light, P., & Colbourn, C. (1988). Pragmatic schemas and conditional reasoning in children. *The Quarterly Journal of Experimental Psychology, Section A*, 40, 469-482.
- Inhelder, B., & Piaget, J. (1958). *The growth of logical thinking from childhood to adolescence*. New York: Basic Books.
- Karplus, R., Adi, H., & Lawson, A. E. (1980). Intellectual Development Beyond Elementary School VIII: Proportional, Probabilistic, and Correlational Reasoning. *School Science and Mathematics*, 80, 673. 683.
- Lawson, A. (1982) The relative responsiveness of concrete operational seventh grade and college students to science instruction, *Journal of Research in Science Teaching*, 19(1), pp. 63-77.
- Lawson, A. E., & Snitgen, D. A. (1982). Teaching formal reasoning in a college biology course for preservice teachers. *Journal of Research in Science Teaching*, 19, 233-248.
- Lawson, A. E., Adi, H., & Karplus, R. (1979). Development of correlational reasoning in secondary schools: Do biology courses make a difference? *The American Biology Teacher*, 41, 420-425.
- Lax, E. (2004). *The mould in Dr. Florey's coat*. London: Little Brown.
- Mlodinow, L. (2009). *The Drunkard's Walk*. London: Penguin.
- Nickerson, R. S. (1998). Confirmation Bias: A Ubiquitous Phenomenon in Many Guises. *Review of General Psychology*, 2, 175-220.
- Nous, A. & Raven, R. (1973) The effects of a structured learning sequence on children's correlative thinking about biological phenomena, *Journal of Research in Science Teaching*, 10(3), 251-255.
- Robinson, J. (1968). *The Nature of Science Teaching*. Belmont: Wadsworth.
- Ross, J. A. & Cousins, J. B. (1993) Enhancing Secondary School Students' Acquisition of Correlational Reasoning Skills, *Research in Science & Technological Education*, 11: 2, 191-205.
- Scholz, R. W. (1991). Psychological research in probabilistic understanding. In R. Kapadia & M. Borovnick (Eds.), *Chance encounters: Probability in education* (pp. 213-249). Dordrecht (The Netherlands): Kluwer.

- Schultz, T. (1982). Rules of causal attribution. *Monographs of the Society for Research in Child Development*, 47.
- Smedslund, J. (1963). The concept of correlation in adults. *Scandinavian Journal of Psychology*, 4, 165-173.
- Sobel, D. M., & Kirkham, N. Z. (2007). Bayes nets and babies: infants' developing knowledge of statistical reasoning and their representation of causal knowledge. *Developmental Science*, 10, 298-306.
- Tobin, K. G., & Capie, W. (1981). The Development and Validation of a Group Test of Logical Thinking. *Educational and Psychological Measurement*, 41, 413-423.
- Vass, E. Schiller, D. & Nappi, A. J. (2000). The Effects of Instructional Intervention on Improving Proportional, Probabilistic, and Correlational Reasoning Skills among Undergraduate Education Majors. *Journal of Research in Science Teaching*, 37, 981-995.
- Wason, P. C. (1968). Reasoning about a rule. *Quarterly Journal of Experimental Psychology*, 20, 273-281.