

# GRAPHICS CALCULATORS USE IN PRECALCULUS AND ACHIEVEMENT IN CALCULUS

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*Most studies on the effects of graphics calculators on students' achievement in precalculus use specially designed tests that are implemented immediately after the introduction of the technology. In many cases, the way the new technology is integrated into the curriculum is not taken into account. This study analyzed the achievement on calculus of students who took a curriculum innovation in a precalculus course that involved graphics calculators use. Even though no differences were found between the non-calculators and calculators groups at the adaptation phase, significant differences were found between these two groups at the consolidation phase, and between the calculators groups of the adaptation and consolidation phases.*

## **Introduction**

Current research on the use and effects of graphics calculators suggests mixed results (Penglase and Arnold, 1996). Some studies show that graphics calculators can enhance the learning of functions and graphing concepts and the development of spatial visualization skills. They can also promote a shift from symbolic manipulation to the graphical investigation and examination of the connections among the several representation systems associated to a given concept. Nevertheless, other studies show that graphics calculators use might not promote the development of some necessary skills and in some cases may result in some “de-skilling”. Most studies use specially designed tests to assess the effects of graphics calculators use. These tests are administered immediately after the experience and no follow-up is presented. Furthermore, in many cases it is difficult to distinguish between the effects of the graphics calculator as a tool and the effects of the instructional process in which its use was involved. There has been little attention paid to the effects of graphics calculators use depending on the level of integration of the tool into the curriculum.

In this study we were concerned about the effects on students' achievement on calculus of graphics calculators use in a precalculus course. We wanted to see whether students that had taken a precalculus course involving a curriculum innovation that included graphics calculators use could obtain better results in the second calculus course of the mathematics cycle (in which there was no graphics calculators use), when compared with other students who took the standard precalculus course. Additionally, we were interested in seeing whether the effects of graphics calculators use depended upon the phase at which the technology was integrated into the curriculum.

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## **Graphics calculator and students' "mathematical future"**

In this study we were not concerned about the effects on students' understanding of graphics calculators use. Research has shown that in most cases graphics calculators use can have enhancing effects on students' understanding of precalculus concepts. Even though this understanding is clearly important, it is meaningful if, for instance, it can help students succeed in their performance in the calculus courses that follow the precalculus one at the university level. Whether graphics calculators use has relevant "de-skilling" effects depends upon whether students will need those skills in the future. If students are allowed to freely use graphics calculators in all their mathematics activities through their career, then it might be possible that, even if this "de-skilling" takes place, it does not affect the students' "mathematical future". However, graphics calculators use is not generalized in all educational institutions and at all levels. This was the case of the university in which this study took place. Graphics calculators use in some precalculus courses was seen as an "experiment", and graphics calculators were (and are) not used in any other mathematics course. This meant that students taking the course that involved the curriculum innovation with graphics calculators were not going to be able to use graphics calculators in the two calculus courses that followed. This posed the question of whether, if there has been some "de-skilling" due to the curriculum innovation involving the graphics calculator, this "de-skilling" had any effects on the students' "mathematical future".

## **Graphics calculators integration into the curriculum**

Graphics calculators cannot be simply introduced into curriculum. They can be used at different levels and they can have different roles in curriculum design and implementation. The effects of graphics calculators use can depend upon how they are integrated into curriculum. Following the ideas suggested by Kissane, Kemp and Bradley (1996) for assessment and graphics calculators use, we introduce four phases concerning graphics calculators use in curriculum design and implementation: non-existent, introduction, adaptation and consolidation. We consider five elements of curriculum: students' use, teachers' use, tasks proposed, textbook, and assessment. Each of these elements can be in any of the four phases. The first phase is evident: graphics calculators are not used or mentioned at all. The table in the following page shows how each of the three other phases is defined on the basis of the curriculum elements considered.

The main difference between the adaptation and the consolidation phases concerns whether advantage is taken of the graphics calculator possibilities. This means whether graphics calculators are used in order to create new learning opportunities through promoting mathematical investigation and exploration and emphasizing relationships among representation systems. The above categories do not take into account the way graphics calculators are used by teacher and students when presenting an explanation or solving a problem.

These categories are proposed in order to classify curriculum innovations that involve graphics calculators use. It is clearly possible for a given curriculum innovation implementation to be located in different phases for different elements of the curriculum. This can be the case, for example, when teacher's use of the graphics

	<b>Phases</b>		
	<b>Introduction</b>	<b>Adaptation</b>	<b>Consolidation</b>
<b>Students</b>	Students have restricted access during some classes. They do not have access outside the classroom.	Students have unrestricted access to graphics calculators.	
<b>Teacher</b>	The teacher has a basic knowledge about the graphics calculator operation. He/she does not use it during classroom activities at all, except for explaining how to use it.	The teacher uses the graphics calculator when it is necessary or when asked to do so by the students. His/her explanations do not take advantage of graphics calculator possibilities.	The teacher takes advantage of the graphics calculator possibilities for explanations and problem posing.
<b>Tasks</b>	The only tasks that involve the graphics calculator are those used to learn how to use it.	Few tasks take advantage of the graphics calculator possibilities.	Most tasks take advantage of the graphics calculator possibilities.
<b>Textbook</b>	Reference is made to graphics calculator as far as how to use it. Problems and exercises do not take advantage of the graphics calculator possibilities.	Some problems and exercises are specially designed for graphics calculators use. The way content is presented and learning is promoted do not take advantage of the graphics calculator possibilities.	Problems proposed and the way content is presented and learning is promoted take advantage of the graphics calculator possibilities.
<b>Assessment</b>	Graphics calculators are not allowed in tests.	Questions are “calculator-neutral”. There is no advantage to students with a graphics calculator.	Unrestricted calculator access. Students decide when and how to use the graphics calculator.

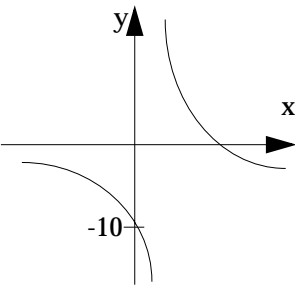
calculator remains at the introduction phase, whereas curriculum design imposes conditions for graphics calculators use at the adaptation phase on the other elements. In this sense, the teacher plays an important role in the process. This can also be the case concerning assessment. If assessment remains at the introduction phase, the effects of graphics calculators use might be curtailed even if other elements are at the adaptation

phase. Nevertheless, even if no curriculum innovation can be accurately classified in one level, it seems reasonable to think that most elements will adjust themselves so that they are approximately at the same phase.

### Context

In the university this study was done, first semester students of Engineering, Business Administration, Economics and Biological Sciences are classified according to their results in the mathematics section of the State Examination. Those students with best results enter directly to the first calculus course. The rest, approximately half of them, start their mathematics cycle with the precalculus course. The students who succeed in the precalculus course are supposed to take the first calculus course during the following semester. If they succeed in this course, they should take the second calculus course immediately thereafter. Students are allowed to drop any course before the mid-semester without getting a grade. Those who fail a course have to take the course again the following semester or during the summer holidays. The study considered only those students starting the mathematics cycle with the precalculus course who were able to succeed in the three courses comprising the cycle during the three consecutive semesters.

The established precalculus course is an introductory course to the study of functions in which some emphasis is given to the graphical representation and to problem-solving. Usually the teacher presents some theory at the beginning of the lecture, and the rest of it is spent solving exercises with some students at the blackboard. The curriculum innovation involving graphics calculators use introduced some changes to this precalculus curriculum. A stronger emphasis was given to the connections between the symbolic and the graphical representations and the concept of family of functions was introduced. Lectures were mainly developed around problem solving activities (Gómez et al., 1996) that followed the ideas of higher-order mathematical thinking (Resnick, 1987). As an example of some of the differences between the two courses, the table shows a question of the final exam from each course.

No calculators	With calculators
Solve: $\frac{x}{x-1} \leq x x $	$x^3 - k \geq g(x)$ has $[0, \frac{1}{4}] \cup [c, \infty)$ as solution set. It is known that $k$ and $c$ are positive numbers; $g(x)$ cuts the X-axis at $\frac{5}{4}$ ; if $h(x) = x + 2$ , the function $h(g(x))$ has $y=0$ as asymptote. The figure shows the graph of $g(x)$ . Find $k$ , $c$ and $g(x)$ . 

Graphic calculators are not allowed in the two calculus courses that follow the precalculus course. In these courses students are expected to develop operational skills for

symbolic manipulation. Lectures are taught in a similar way to the standard precalculus course.

The curriculum innovation involving graphics calculators use underwent the three phases (introduction, adaptation and consolidation) described previously. The three phases were developed during three consecutive semesters. Some results are already known concerning this curriculum innovation. Mesa and Gómez (1996) found no differences in some aspects of understanding between the students who took the traditional course and those who took the curriculum innovation at the adaptation phase. Gómez (1995) and Gómez and Rico (1995) found that the students of this group participated more actively in social interaction and in the construction of the mathematical discourse, changes that can partially be attributed to a different behavior of the teacher. Even though she changed her behavior, Valero and Gómez (1996) found that the teacher could not change completely her beliefs system. Finally, Carulla and Gómez (1996) found that the teachers and researchers who participated in the curriculum innovation (at the adaptation and consolidation phases) underwent significant changes on their visions about mathematics, its learning and teaching.

### **Problem**

We wanted to answer two questions:

- ▲ Were there any differences in the students' final grades in the second calculus course between those who took the traditional precalculus course and those who took the curriculum innovation involving graphics calculators use?
- ▲ Were there any differences in the students' final grades in the second calculus course between those who took the curriculum innovation involving graphics calculators use at the adaptation phase and those who took it at the consolidation phase?

### **Design**

Two groups of students starting the precalculus course during two consecutive semesters were taken into account. The first group was divided into two subgroups. The first one (**G1C**, with 134 students and five different teachers) took a precalculus course in which the curriculum innovation was implemented. The second sub-group (**G1NC**, with 111 students and five different teachers) took the established precalculus course without calculators. A different group of students starting the precalculus course the following semester were divided in the same way: those taking the precalculus course in which graphics calculators were used (**G2C**, 58 students and two teachers), and those who took the traditional precalculus course (**G2NC**, 125 students and four teachers). The graphics calculators sub-group of the first semester (**G1C**) followed a curriculum innovation that was at the adaptation phase. The curriculum innovation for the graphics calculators sub-group of the second semester (**G2C**) was at the consolidation phase. Students were randomly assigned to each teacher.

This was a longitudinal comparative study. Students' achievement was measured on the basis of the students' final grades in the second calculus course of the mathe-

matics cycle. The comparisons between groups and between graphics calculators adaptation and consolidation phases were established on the basis of the difference of sampling means of the final grades of the second calculus course. The parameter analyzed was of the form  $\mu_A - \mu_B$ . The statistical significance of the difference of sampling means was measured with a two tails p-value associated to the t-test of comparison of two independent populations. In order to analyze the possibility of confusing factors, the teacher's effect at the adaptation phase was taken into account. In the first group there were ten teachers. Five of them implemented the curriculum innovation. Only two of these five teachers implemented the curriculum innovation at the consolidation phase.

Three comparisons were made: between the calculators and non-calculators groups corresponding to the adaptation phase (G1C and G1NC); between the calculators and non-calculators groups corresponding to the consolidation phase (G2C and G2NC); and between the students of the two teachers that implemented the curriculum innovation at the consolidation phase and the students from these two teachers at the adaptation phase (G2C and G1C(2T)). Since the proportion of students who succeed the precalculus course differs from one teacher to another, in order to establish appropriate comparisons, we considered the 25% of students who obtained the best grades in the second calculus course from each group.

## Results

The table presents the grades' mean and standard deviation and the percentage of students considered for each of the five groups mentioned above, together with the results for the three comparisons proposed. We observe that, for the first comparison (G1C and G1NC), even though the difference was negative, it was not significant ( $p=0.14$ ). Nevertheless, when we look at the other two comparisons, we observe that there were very significant differences. In the case of the two groups corresponding to the consolidation phase (G2NC and G2C), the difference favors the calculators group ( $p=0.0034$ ). In the case of the comparison for the same two teachers (G1C(2T) and G2C), the difference favors the group corresponding to the consolidation phase ( $p=0.00057$ ).

	G1NC	G1C	G2NC	G2C	G1C(2T)	G2C
X	3.43	3.22	3.56	4.04	3.42	4.04
s	0.6	0.44	0.5	0.4	0.385	0.4
%	24.3%	24.6%	25%	24.1%	26%	24.1%
Dif	-1.53		3.09		3.94	
p	0.14		0.0034		0.00057	

## Discussion

We cannot assure that the results obtained in this study are valid for other circumstances except for a hypothetical situation in which similar students take the same courses with the same teachers and curriculum implementation. The statistical analy-

sis refers to that hypothetical population.

The results show that the effects of graphics calculators use in this study depended directly upon the phase at which graphics calculators were integrated into the curriculum. While no significant difference was observed between the calculators and non-calculators groups when the curriculum innovation was at the adaptation phase, significant differences were found between these two groups at the consolidation phase, and between the calculators groups of the adaptation and consolidation phases. This might be due to the fact that during the consolidation phase, graphics calculators were used to create new learning opportunities through the promotion of mathematical investigation and exploration and the emphasis given to the relationships among representation systems. Furthermore, these differences (specially those concerning the two teachers that participated at the two phases) might also be explained by the change that teachers and researchers had of their visions about mathematics, its teaching and learning as a consequence of the way graphics calculators were integrated into the curriculum (Carulla and Gómez, 1996). These results show that, at least as far as achievement is concerned, graphics calculators effects cannot and should not be studied independently of the way the new technology is integrated into the curriculum. Furthermore, it might be possible, as it was the case for the experience reported here, that the use of graphics calculators needs to go through an “integration process” in which in order to attain a given phase, the previous phases have to be completed. It remains to be seen whether a successful consolidation phase (as far as achievement is concerned) can be attained without a change in teachers’ visions.

The results obtained in this study do not support the “de-skilling” argument that is sometimes presented against graphics calculators use. The two calculus courses that follow the precalculus course considered in this study do not allow graphics calculators use and follow a traditional curriculum in which students are expected to develop operational skills that emphasize symbolic manipulation. If, in fact, some “de-skilling” took place, then either it was not relevant, or its negative effect was overcome by other skills and knowledge developed by the students who used graphics calculators.

Even though this study did not analyze the new skills and knowledge developed by the students who used graphics calculators, it showed that graphics calculators use had positive effects on their “mathematical future”.

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