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SOCIOCULTURAL PERSPECTIVES ON RESEARCH WITH MATHEMATICS TEACHERS:
A Zone Theory Approach

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Abstract

Sociocultural theories view teacher learning as changing participation in social practices that develop their professional identities rather than as acquisition of new knowledge or beliefs that are internal to the individual. Although sociocultural research on mathematics teacher education has tended to focus on understanding teachers' learning, this paper argues that sociocultural perspectives can also guide more interventionist research involving changing classroom practice. The approach illustrated here uses an adaptation of Valsiner's zone theory to analyse teacher learning and development in two separate research studies. In one study the aim was to understand how teachers incorporated digital technologies into their practice, while the other study helped teachers implement an investigative approach to working mathematically consistent with a new curriculum. In both studies, productive tensions between teachers' beliefs, contexts, and goals were a trigger for learning and development.

Internationally, there is considerable interest in studying approaches to developing teachers' professional knowledge and practice. A relevant example within pre-service teacher education is the Teacher Education and Development Study in Mathematics, an international comparative study of the competencies of primary and secondary mathematics teachers in seventeen countries. The study collected data on prospective teachers' backgrounds, beliefs, opportunities to learn, and mathematical knowledge for teaching in order to investigate how they learned to teach mathematics effectively as a result of their pre-service program (Tatto and Senk 2011). Whereas this study emphasises the importance of both *knowledge* and *beliefs* for teaching, other research has concentrated only on analysing or changing teachers' *beliefs*. It has long been assumed that beliefs influence practice, and therefore changing teacher beliefs has been considered a necessary prerequisite to reform in mathematics education (Cooney 2001). However, studies investigating the relationship between beliefs espoused via questionnaires or interviews and beliefs inferred from observations of classroom practice have yielded contradictory results, with some finding consistency between beliefs and practices and others identifying mismatches. Within this research paradigm it is now acknowledged that the relationship between beliefs and practices is complex and dialectical in that they develop together and influence each other (Beswick 2005).

In his review of research perspectives on mathematics teacher education, Lerman (2001) went even further to argue that it makes no sense to examine teachers' beliefs and practices as if they are separate and stable entities. Beliefs are related to the context in which they are elicited, and specific situations are "*productive* of beliefs, practices, purposes, and goals, not *reflective* of them" (p. 44, emphasis added). Lerman pointed to the work of Vygotsky (1978) and followers in suggesting that research on teaching and teacher education needs to look at individual teachers through their social settings. In this approach, teachers' learning is understood as changing participation in sociocultural practices that develop their *identities* as teachers.

Lerman's (2001) review is indicative of the growing interest within the mathematics education research community in social and cultural aspects of learning. He identified the need for research in mathematics education to develop accounts of learning that explain how individuality and agency can be understood in the context of a person's social and cultural experiences, and he suggested that an appropriate unit of analysis might be the person-in-practice-in-person. The first part of this unit, person-in-practice, acknowledges that the object of study is more than individual cognition or affect because it highlights the notion of learning through social participation. The second part, practice-in-person, implies that participation develops identities as the practice becomes part of the individual.

The focus of this paper is on sociocultural studies that have taken a neo-Vygotskian approach, extending the well known concept of the zone of proximal development (ZPD) to incorporate the social setting and the goals and actions of the participants. The approach adapts Valsiner's (1997) zone theory of child development to study interactions between teachers and their professional environment: it permits interpretation of teacher knowledge and beliefs within a sociocultural framework, consistent with the person-in-practice-in-person unit of analysis.

The next section introduces my adaptation of Valsiner's (1997) zone theory of child development. This is followed by two case studies of research with teachers that has aimed either to understand or to change classroom practice. The final section offers reflections on how the use of sociocultural theories can inform research with teachers.

1. Applying Valsiner's Zone Theory in Educational Contexts

Valsiner (1997) viewed the ZPD as a set of possibilities for development that are in the process of becoming realised as individuals negotiate their relationship with the learning environment and the people in it. He proposed the existence of two additional zones, the zone of free movement (ZFM) and zone of promoted action (ZPA). The ZFM structures an individual's access to different areas of the environment, the availability of different objects within an accessible area, and the ways the individual is permitted or enabled to act with accessible objects in accessible areas. The ZPA comprises activities, objects, or areas in the environment in respect of which the individual's actions are promoted. The ZPA can include areas that are currently outside the ZFM as well as those that are inside; thus the actions being suggested, while possible, may seem "forbidden" at the present time. The ZFM and ZPA are dynamic and inter-related, forming a ZFM/ZPA complex that is constantly being re-organised by adults in interactions with children. However, a key claim of Valsiner's theory is that the process of development is neither completely random nor fully determined; instead, it is directed, or "canalised", along a set of possible pathways jointly negotiated by the child in interaction with the environment and other more mature people.

Although Valsiner's (1997) theory is intended to explain child development, he noted that the ZFM/ZPA complex is observable in the context of education, both formal and informal. He also argued that zone theory is applicable to any human developmental phenomena where the environment is structurally organised, and so it seems reasonable to extend the theory to the study of *teacher* learning and development in structured educational environments.

My approach to the use of zone theory re-interprets Valsiner's zones from the perspective of teacher-as-learner. Thus the teacher's zone of proximal development becomes a set of possibilities for development of new knowledge, beliefs, goals and practices created by the teacher's interaction with the environment, the people in it, and the resources it offers.

The zone of free movement structures the teacher's environment, or professional context, so that elements of the ZFM could include:

- perceptions of students' social background, motivation, beliefs and attitudes, mathematics achievement, behaviour;
- access to resources and teaching materials;
- curriculum and assessment requirements;
- organisational structures, such as those governing timetabling, room allocation, and grouping of students; and
- organisational cultures, which influence what is accepted as "good" teaching.

While the zone of free movement suggests which teaching actions are *permitted*, the zone of promoted action can be interpreted as activities offered via teacher education programs, formal professional development, or informal interaction with colleagues that *promote* certain teaching approaches.

Valsiner (1997) regarded the zones as fuzzy abstractions without sharp or continuous boundaries. At any moment the zones are only quasi-defined and subject to further transformation, and this is precisely the feature that a zone theory analysis is intended to capture.

The following sections present two zone theory analyses, one from the perspective of understanding teacher learning and the other from the perspective of changing classroom practice.

2. Using Zone Theory in Research on Understanding Technology Integration

Zone theory informed a longitudinal research program in which I set out to identify factors influencing how secondary school mathematics teachers integrated digital technologies into their practice. The aim was to understand teachers' learning over time and in different contexts, and there was no formal intervention designed to change their pedagogical practice. This section illustrates the use of zone theory to understand the learning of one teacher (Brian) who participated in the study. It provides an expanded version of the analysis presented in Goos (2009).

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Several data collection methods were used. First, a semi-structured interview invited teachers to talk about their knowledge, beliefs, contexts, and professional learning experiences in relation to technology. The structure of the interview was based on the relationship of each zone to factors known to influence technology integration shown in Table 1. These interviews were conducted in each teacher's school; they were audio-recorded and later transcribed.

Additional data about the teachers' beliefs about mathematics, its learning and teaching was obtained via a Mathematical Beliefs Questionnaire consisting of 40 statements to which teachers responded using a Likert-style scale based on scores from 1 (Strongly Disagree) to 5 (Strongly Agree). Statements were paired to create positive and negative poles of a particular idea: in the literature, these poles are often referred to as representing beliefs about mathematics being transmissive versus child-centred (Perry, Howard, and Tracey 1999). The intention was not to evaluate teachers' beliefs or to examine the impact of these beliefs on their practice, but instead to understand the responses in terms of how individuals developed their pedagogical identities.

Table 1.
Factors affecting teachers' use of technology

Valsiner's Zones	Factors influencing teachers' use of digital technologies
Zone of proximal development (Possibilities for developing new teacher knowledge, beliefs, goals, practices)	Mathematical knowledge Pedagogical content knowledge Skill/experience in working with technology Beliefs about mathematics, teaching, and learning
Zone of free movement (Structures teachers' access to different areas of the environment, availability of different objects within an accessible area, ways the teacher is permitted or enabled to act with accessible objects in accessible areas)	Perceptions of students Access to resources Technical support Curriculum & assessment requirements Organisational structures & cultures
Zone of promoted action (Activities, objects, or areas in the environment in respect of which the teacher's actions are promoted)	Pre-service teacher education Professional development Informal interaction with teaching colleagues

The third source of data comprised observations of lessons in which technology was used to teach mathematics, with teacher interviews before and after each lesson. These interviews, which were audio-recorded and later transcribed, sought information about teachers' plans and rationales for the lessons and their reflections on factors that influenced their teaching goals and methods. Field notes taken during classroom observations were augmented by video-recording, and these two records were combined to produce written accounts of each lesson.

All interview transcripts were analysed to identify aspects of the teachers' zones of free movement and zones of promoted action, using the categorisations shown in Table 1 as a guide. As the zones are abstractions and subject to shifts in definition over time, this process focused on the particular circumstances under which the teachers "filled in" the zones with specific people, actions, places, and meanings. This overall analysis approach enabled exploration of how individual, social, and contextual factors came together to shape teachers' pedagogical identities as users of technology. The approach is illustrated via a case study of one teacher, Brian, presented below.

2.1 Introducing Brian

Brian graduated from university in 1982 and since then has taught mathematics in secondary schools. For much of this time he was Head of the Mathematics Department at Matlock High School, a large urban school serving a socio-economically disadvantaged community. In the first year of the research project Brian had moved to a new position as Head of the Mathematics Department at Bancroft High School, also situated in a low socio-economic urban area.

2.2 Snapshot of Brian's practice

A vignette from a Grade 11 lesson illustrates key characteristics of Brian's teaching approach in that he used graphing technologies and probing questions to help students develop a general method for solving trigonometric equations. He explained that he preferred to first develop students' understanding of concepts to motivate the need for analytical methods involving algebra:

> The options are to give them heaps of algebra and watch them fail or try to get them to understand the concepts. If they're confident about what they're doing then I find the algebra's not such a task for them because there's a lot more meaning or reasoning behind it.

This lesson was also typical in the way that Brian was required to deal with constraints imposed by lack of resources and timetabling pressures. Other subjects took precedence over mathematics when it came to booking lessons in the school's computer laboratories, and so mathematics lessons usually took place in classrooms with no computers or data projectors. Brian overcame these difficulties by carrying his own laptop computer and portable data projector to the classrooms where he taught.

Brian started the lesson with a straightforward example of a trigonometric equation, $2\sin x + \sqrt{3} = 0$ for $0 \le x \le 2\pi$. He emphasised the critical importance of attending to the domain, "as this tells us how many solutions there are". He launched the *Autograph* program on his laptop computer and displayed the graph of $y = 2\sin x + \sqrt{3}$ shown in Figure 1.

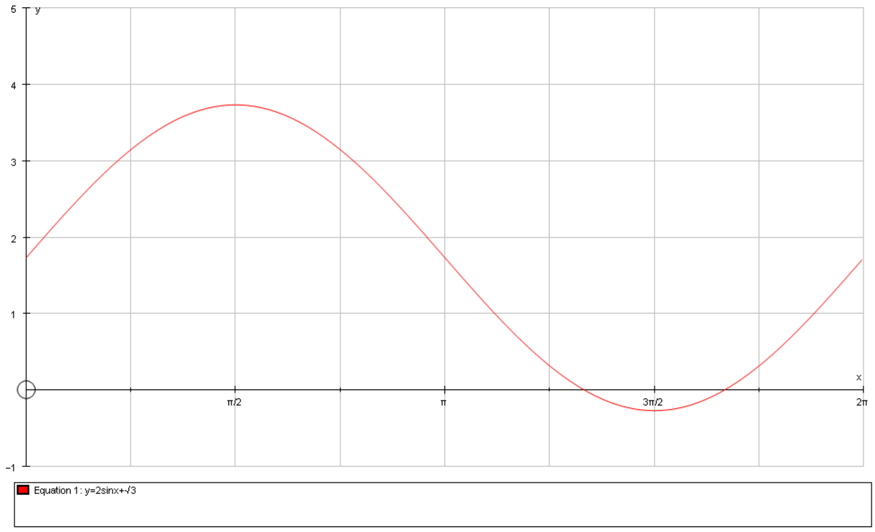

Figure 1. Graph of $y = 2\sin x + \sqrt{3}$ produced by AutoGraph software

The students also drew the graph using their graphics calculators, and observed that there are two roots. Brian then announced that they needed to "go into the algebra world", and through questioning he led the class through the algebraic process of "unwrapping" the equation. Upon reaching the conclusion that $\sin x = -\frac{\sqrt{3}}{2}$, the students were reminded that they needed instantly to recognise the exact trigonometric ratios for certain angles, in this case 60° or $\frac{\pi}{3}$ radians. Brian explained that "the negative sign tells us a story too", and he guided the students through sketching the unit circle and locating the relevant angles in the third and fourth quadrants as $\frac{4\pi}{3}$ and $\frac{5\pi}{3}$ respectively. The students then used the graphics calculator

TRACE function to give meaning to the solutions by entering them as *x*-values and observing that the corresponding *y*-values were zero in both cases: in other words, they had found the points where the curve cut the *x*-axis (see Figure 2).

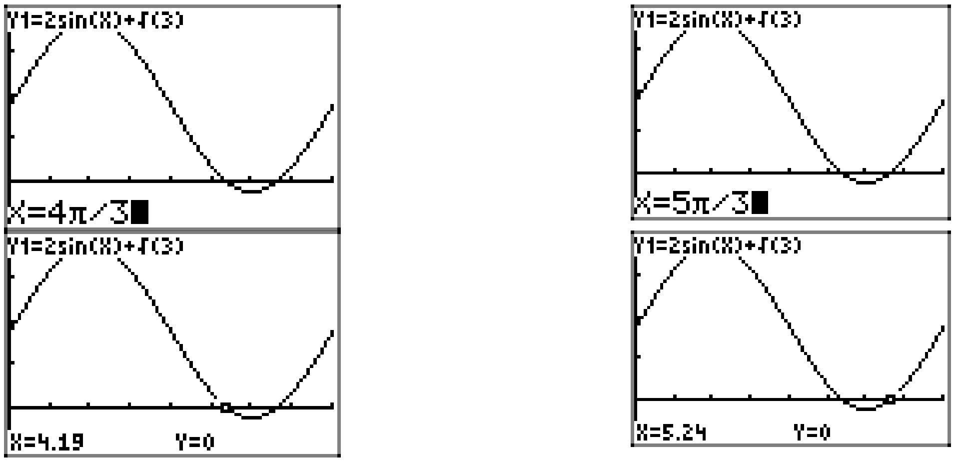

Figure 2. Using the graphics calculator TRACE function to verify the solutions to $y = 2\sin x + \sqrt{3}$

In this lesson, Brian coordinated two types of technology, a dynamic software program loaded onto his personal laptop computer and graphics calculators used by the students, to introduce his class to the standard algebraic method for solving trigonometric equations. Use of technology afforded a graphical approach that focused students' attention on the important underlying concepts and allowed them to check that their solutions were correct.

2.3 Brian's evolving ZFM/ZPA/ZPD system

Brian said that when he was a school student he was taught mathematics in a traditional, teacher- and textbook-centred manner, and he came to believe that this was how everyone learned mathematics. According to him, his beliefs about mathematics teaching and learning remained unchanged during his pre-service teacher education program and the early years of his teaching career. He described his teaching during this time as follows:

> I would present the mathematics as cogently and articulately and clearly as I could. I guess what I expected kids to do was learn what I'd done on the board, copy what I did and give it back to me on a test. Those that could were good at maths and those that couldn't weren't good at maths.

Brian’s early learning experiences suggest a limited zone of free movement with his teachers allowing a choice of activities, or zone of promoted action, only within allowed boundaries (e.g., textbook exercises, closed questions). His ZPD, the set of emerging personal meanings that could be constructed from this person-environment relationship, included understandings of how mathematics is learned and taught that he later actualised as a teacher, creating ZFM/ZPA complexes for his own students that limited their freedom of thought and action to reproducing the mathematical work he wrote on the board.

After more than ten years of teaching, Brian realised that the students he taught were not really learning mathematics. He was troubled by his observation that some students were able to pass a test but some months later had forgotten skills that they had previously demonstrated. Rather than blaming the students for their lack of ability or effort, he wondered whether his teaching was ineffective and he decided that he “needed to rethink what mathematics education was about, or get out”. He visited his former mathematics methods lecturer who was still teaching pre-service courses at the university. The lecturer suggested that Brian do some reading about current developments in mathematics education, and the ideas that Brian found to be most influential related to constructivist theories of learning. Brian took the initiative for his continuing development as he joined Internet discussion groups and undertook further reading. This led him to embark on a radical change to his teaching approach. As he read about constructivism and had contact with the broader mathematics education community he developed ideas and tried them in the classroom, and his students’ responses gave him impetus to explore this approach further.

Around the same time, Brian attended professional development workshops about graphics calculators, a new form of technology that was being introduced into secondary school mathematics. He also participated in conferences that introduced him to other types of technology resources. Apart from these instances Brian rarely sought out formal professional development to learn about digital technologies, preferring instead to “sit down and just work through it myself”. He emphasised that his reason for learning to use technology stemmed from his changed beliefs about how students learn mathematics:

> When my philosophy changed, it became a question of – what can I put in front of my kids to allow them to access the concepts? So then it didn’t really matter what it was, the outcome that I was after was them accessing the concept. So it became obvious over time that technology was a way that many students do access concepts that they couldn’t, wouldn’t normally access.

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Brian's responses to the Mathematics Beliefs Questionnaire were consistent with the constructivist principles that now guided his practice. For example, he expressed disagreement that doing lots of problems is the best way for students to learn mathematics, and he strongly agreed that the role of the mathematics teacher is to provide students with activities that encourage them to wonder about and explore mathematics.

From a zone theory perspective, Brian had changed his interpretation of one aspect of his ZFM – the mathematical abilities of his students – leading to a new person-environment relationship and opening up of a new set of possibilities for his development. Because there was no ZPA within his school that mapped onto his ZPD, he looked outside to his former lecturer and professional reading to advance his personal goal of thinking differently about mathematics education. But this external ZPA needed to be brought within the school's ZFM so that the actions promoted were seen to be "permitted" rather than "forbidden". Some elements of the ZFM did not seem to afford a change in teaching practices; for example, the school had few teaching resources apart from textbooks, and many teachers assumed that students' low socio-economic backgrounds and challenging behaviours reduced their ability to learn. However, as Brian was by now Head of the Mathematics Department he was in a position to modify some aspects of his professional environment as well as to influence change in curriculum and teaching practices in his department. He began by creating a new mathematics curriculum for Grade 8 that would allow him to experiment with teaching approaches informed by constructivist theories of learning. This strategy required a great deal of ingenuity because Brian had to buy, collect, borrow, or make teaching materials that were not readily available in this poorly resourced school. Support from the school leadership, another important element of his ZFM, was valuable in redirecting budgeted funds towards resourcing the Mathematics Department. Brian implemented the curriculum with his Grade 8 class, overcoming the scepticism of other teachers in his department by inviting them to observe his lessons and witness the students' increased levels of engagement and understanding. His strategy eventually led to the adoption of a social constructivist pedagogy throughout the Mathematics Department, and significant improvement in mathematics achievement for students in all grades where previously low achievement had been the norm.

In the seventeen years that Brian spent at Matlock High School he was able to fashion a ZFM/ZPA complex that created a set of possibilities for development of new beliefs,

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knowledge, goals and practices (ZPD). To implement the new teaching approaches promoted by his professional reading and interaction with colleagues outside the school (ZPA), Brian had to change his environment (ZFM), thereby constructing a developmental pathway (ZPD) that was coherent with his goal of improving student learning and his changed beliefs about how students learn. During this time his person-environment relationship was constantly being re-organised as he extended his constructivist teaching approach into the senior secondary years, negotiated the curriculum and assessment constraints that applied at these year levels, and inducted new staff into the organisational culture he had established in the Mathematics Department.

When Brian arrived at Bancroft High School he experienced a different ZFM/ZPA/ZPD system that presented challenges for introducing the mathematics staff to his teaching philosophy and obtaining sufficient resources to put his philosophy into practice across the department. There was little in the way of mathematics teaching resources. Mathematics students in this school were not accustomed to technology, even though the use of computers or graphics calculators was now mandated by the senior secondary mathematics syllabuses. At the start of the year there were no class sets of graphics calculators and only a few students could afford to buy their own. Because of timetabling and room allocation issues it was also difficult for mathematics classes to gain access to the school's computer laboratories. Exacerbating the problems of limited access to technology resources was an organisational culture that Brian diplomatically described as "old fashioned". Almost none of the mathematics teachers were interested in learning to use technology; they had fallen into stereotyping their students as incapable of learning; and it appeared that an atmosphere of lethargy had pervaded the mathematics department for many years. Students demonstrated a similarly passive approach to learning mathematics, expecting that the teacher would "put the rule up and example up and set them up and away they go".

At Bancroft High School, Brian's ZFM was structured by a lack of resources, negative teacher perceptions of students' motivation and abilities, and organisational structures and cultures that worked against innovative use of digital technologies. These constraints may seem similar to those he faced, and overcame, at his previous school. A key difference, however, was the additional challenge of promoting in his fellow mathematics teachers the same kind of productive tension he had experienced in his own thinking and actions that ultimately led to the change in his teaching philosophy. Because he was new to the school, none of the mathematics teachers there had witnessed the foundational change that profoundly transformed his identity and embedded the *practice-in-person* (see Chapman and Heater 2010 for a discussion of teacher change). Thus the “present-to-future possibilities” (Valsiner 1997, p. 208) that defined Brian’s zone of proximal development at that time were not accessible to colleagues who could only observe his present identity and practices without understanding how these emerged from past possibilities.

3. Using zone theory in research on changing classroom practice

Zone theory offers a useful framework for research that aims to *understand* the complexities of teachers’ learning and development. But another important line of inquiry within mathematics teacher education research is concerned with *changing* classroom practice, and in this section I argue that zone theory can also provide guidelines for designing such projects and interpreting the outcomes. It is common for educational change projects to seek to challenge teachers’ beliefs and build their mathematical and pedagogical content knowledge so that they can more effectively implement innovative teaching practices. But even if teachers do gain knowledge or reconsider their beliefs, they might still regard the new teaching practices promoted by these interventions as “forbidden”, or at least not feasible to implement within their school environments. For example, teachers might claim that a lack of teaching resources, pressures to “cover” curriculum content and prepare students for mandated assessments, or students’ low ability levels are barriers to innovation. From a zone theory perspective, this means that the ZFM/ZPA complex created by interaction between the professional environment and the reform project may not map onto teachers’ ZPDs in a way that canalises development along the pathway intended by the researchers. This analysis suggests that the design of educational change projects needs to take into account possible person-environment relationships and anticipate how these might constrain (but not fully determine) teachers’ pathways of development and the formation of new professional identities. Such an approach would be consistent with a focus on the person-in-practice-in-person, rather than only the decontextualized “person” and their knowledge and beliefs.

In this example, I discuss how zone theory was used to inform the design of an action learning project to help secondary school mathematics teachers implement a new syllabus (see Goos, Dole, and Makar 2007, for a report on the overall impact of the project). For teachers, the main implementation challenge involved designing learning experiences and assessment tasks that took an investigative approach to working mathematically (Queensland Studies Authority 2004). Mathematical investigations are “contextualized problem solving tasks through which students can speculate, test ideas and argue with others to defend their solutions” (Diezmann, Watters, and English 2001, p. 170). Although investigative approaches are consistent with established mathematics education reform movements (e.g., Australian Education Council 1991; National Council of Teachers of Mathematics 2000), in Australia there is research evidence to suggest that investigative teaching practices are rare in secondary school mathematics classrooms (e.g., Stacey 2003).

Four pairs of teachers from four schools within the same geographical region volunteered to participate in the project. Project activities took place over five months spanning the end of one school year and the start of the following year. The research team visited the regional city to work with the whole group of teachers on three occasions, with each visit lasting two days. Between these visits the teachers were expected to implement investigative curriculum units with their classes and collect evidence of student learning and engagement. The intentions of the project were to help develop teaching approaches that start from real world problems or questions that engage students in significant mathematics, and to broaden teachers’ assessment repertoires so they base their judgments about students’ learning on a comprehensive range of evidence rather than only on timed, supervised, pen-and-paper tests.

In the first research visit, two methods were used to seek information from the teachers about their knowledge and beliefs and their professional contexts. First, all teachers completed the Mathematical Beliefs Questionnaire described earlier. Second, the researchers and teachers shared their mathematical autobiographies by talking about their own experiences of learning mathematics at school, the teachers who made a different to them, the decisions and events that led to them becoming mathematics teachers, and significant moments that shaped their practice. As part of this oral account (audio-recorded for research purposes) the teachers went on to describe their professional contexts, and here they typically referred to elements of the zone of free movement that constrained their teaching actions, such as student characteristics, access to teaching resources, and their schools’ organisational structures and cultures. The researchers encouraged teachers to identify any aspects of their professional environments that could be changed, or suggested ways of interpreting differently some of the constraints within their environments. For example, one pair of teachers regarded their students' apparent lack of interest in learning as their main obstacle because they claimed that poor motivation and negative self-perceptions of ability led to disruptive behaviour in class. In response to the frustration expressed by these teachers, the researchers proposed that, if students were not learning any mathematics now, then there was nothing to lose by trying an investigative approach. This suggestion led the teachers to take some risks in making changes to their practice and allowed them to observe the positive effects on students' learning and engagement.

Another main purpose of the first visit was to immerse teachers in investigative activities suitable for use in lower secondary mathematics classes: this modeling of investigative pedagogies was part of the zone of promoted action offered by the project. Information from the mathematical autobiographies and questionnaires was used to start "filling in" each teacher's zone of free movement and to anticipate the sets of next possible states (their ZPDs) given the current configuration of their ZFM/ZPA complexes.

In the second research visit, one month later, teachers evaluated their implementation of a mathematical investigation and discussed successes and any problems they encountered with their classes. Their evaluations were informed by various types of student data they had collected, such as work samples, photographs, and student attitude surveys. The evaluation assisted teachers to set new goals for planning a subsequent unit of work with the new class they would be teaching from the start of the following school year. The researchers continued to promote change by engaging the teachers in authentic assessment tasks and demonstrating ways of creating assessment rubrics; these activities constituted another part of the ZPA offered by the project.

The final research visit occurred about a month after the start of the following school year. On the first day, the researchers observed and video-recorded lessons taught by the project teachers and then discussed the lessons with teachers as they watched the video. The discussion was audio-recorded for later review and analysis. On the second day of the visit, the whole group gathered once more to evaluate the outcomes of the project as a whole and reflect on successes and challenges.

Data from the mathematical autobiographies, Mathematical Beliefs Questionnaire, teacher planning documents, lesson videos and field notes, teacher interviews, student work samples, and student attitude surveys were organised and interpreted in terms of the analytical categories represented by the ZFM, ZPA and ZPD. This approach made it possible to identify how different ZFM/ZPA complexes mapped onto teachers' respective ZPDs to set up possible pathways for development. A case study of one pair of teachers, Val and Shanti, illustrates the impact of this research project on their classroom practice. The case study provides an expanded version of the analysis presented by Goos, Dole, and Makar (2007).

3.1 Introducing Val and Shanti

Val and Shanti taught mathematics at Seaside State High School. Val had been teaching for 12 years and Shanti for 18 years, both of them in several different schools. Val was a high achieving mathematics student at school and she decided to study engineering at university. She later gained formal teaching qualifications, specialising in junior secondary mathematics and physics.

Shanti was qualified to teach science and art, but was often assigned to teach junior secondary mathematics classes as well. (In Australia, teaching "out of field" is common because of continuing shortages of formally qualified secondary school mathematics teachers.) She did not consider herself to be a "real" mathematics teacher "because I don't know all the little tricks". Shanti's science and art background led her to explain problems in different ways, emphasising visual, oral and physical approaches.

3.2 Snapshots of Val and Shanti's mathematical investigations

When discussing their mathematical autobiographies, both teachers explained that it was important for them to connect their students' learning to the real world. However, they felt disillusioned and marginalised by the traditional teaching and assessment practices expected at their school. They claimed that their preference for authentic learning and assessment tasks met with disapproval from the other mathematics teachers. This disapproval could be subtle; for example, when Val took her class outside for mathematical activities, "I get kind of eyebrows up as if to say 'You should be in there, you know, doing textbooks'". Sometimes the censure was more direct, as when Shanti relayed the comment from another teacher: “My kids can see you out there playing, you know, why are you taking them out? It’s disrupting the other kids’ learning when you go outside”.

Val and Shanti said that their main concern was the narrow focus on testing and textbooks that encouraged passive learning and limited students’ engagement in meaningful mathematics. Therefore the focus for their mathematical investigations was on authentic and practical assessment tasks.

The first investigation, planned for Grade 9 students, involved concepts of area and volume. It asked students to design a container for the supermarket shelf that would hold exactly one litre of rice. Students were also to consider aspects of aesthetics and practicality in designing a container appropriate for a shop shelf. Learning activities served as formative assessments, and these included examining nets to investigate surface area and volume, measuring the capacity of various sample containers, designing and constructing 1 litre models, and using Excel to explore relationships between surface area and volume data collected from all completed models. Assessment for this unit of work consisted of a portfolio that included the net of the completed model, notes on its design and justification of its fitness for purpose, calculations of surface area and volume, and interpretation of any data-based relationships between surface area and volume found from Excel graphs. The assessment rubric for this task gave attention to students’ mathematical calculations, explanations, problem solving strategies, capacity to work with others, clarity of diagrams and sketches, and use of mathematical terminology and notation.

For the second investigative curriculum unit, Shanti modified an activity previously developed by Val that asked students to examine their own health and fitness and develop a personal health plan. The assessment for this investigation required students to complete a series of tasks in which they collected and analysed data to inform development of their individual health plans. The tasks included measuring various parts of their body (e.g., height, waist, average pace length), measuring each other’s resting and active heart rates and from this information estimating their level of fitness, and assessing their activity levels and food intakes for the previous day to determine the balance between the kilojoule values of food eaten and exercise undertaken.

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3.3 Val and Shanti's ZFM/ZPA/ZPD system

Although the ZPD is usually conceptualised in terms of an individual's development, the examples given by Valsiner (1997) to illustrate the organisation of classroom ZFM/ZPA complexes show that it makes sense to consider how these structures set up ZPD possibilities for groups of learners within the same educational setting. Similarly, one can analyse the ZFM/ZPA/ZPD systems negotiated by a pair of teachers who work together to change their teaching practices.

Val and Shanti differed in their formal qualifications to teach mathematics, but their common desire to move away from textbook-based teaching was evident in their mathematical autobiographies (discussed earlier) and their responses to the Mathematical Beliefs Questionnaire. Both agreed or strongly agreed that there are many ways to interpret something in mathematics or to solve a mathematical problem, that cooperative group work and discussions are important aspects of good mathematics teaching, and that students should be encouraged to build their own mathematical ideas and use their own interpretations. They disagreed or strongly disagreed that in mathematics something is either right or wrong, that mathematics is about learning to get the right answers, that good mathematics teachers taught only what is important for mathematics tests, or that solving a mathematics problem usually involves finding a rule or formula. On the surface, then, it might appear that any educational change project promoting innovative teaching, such as through mathematical investigations, would succeed with these two teachers because they already held appropriate beliefs. In other words, their beliefs were not an obstacle to change. But this person-centred approach ignores the settings in which teachers develop their identities – it does not consider the *person-in-practice*.

Val and Shanti described a professional environment, or zone of free movement, characterised by textbook-dominated teaching practices, an assessment regime limited to frequent tests that were used to place students in ability-streamed classes, no access to teaching resources that might support more innovative approaches, and an organisational culture that seemed resistant to change. Thus their ZFM defined narrow boundaries for action, and the actions that were promoted by other mathematics teachers and the Head of Department (ZPA) lay firmly within these boundaries. Because the two zones functioned in a mutually reinforcing fashion, this ZFM/ZPA complex worked to maintain the status quo with little apparent hope that Val and Shanti could renegotiate the boundaries. For the researchers it was vital to the design of the project to allow time for discovering the teachers' epistemological and pedagogical beliefs, understanding their institutional contexts, and anticipating how this initial zone configuration might constrain possibilities for development. Teacher beliefs do not unequivocally determine the impact of an educational change project; nor is the professional environment simply the backdrop for practice. Beliefs and contexts both matter, but not in ways that deny individual agency or possibilities for change. The role of the teacher educator-researcher in these circumstances is to analyse the ZFM/ZPA complex currently shaping teachers' practice and find ways of intervening in the person-environment relationship. This may involve promoting actions that teachers believe to be feasible in their current environment and/or identifying aspects of the environment that can either be changed or reinterpreted as affording new practices. In Val and Shanti's case, the researchers took advantage of the productive tensions that existed between their student-centred beliefs, the rigid assessment regime promoted in their school, and the new curriculum's support for investigative approaches to mathematics to help them extend their repertoire of assessment practices. This offered Val and Shanti an alternative ZPA that legitimized their preference for rich assessment tasks.

4. Conclusion

In this paper I've argued that sociocultural perspectives on research with mathematics teachers can be useful both for understanding teacher learning and for promoting their development. Within this perspective, learning is conceptualised as participation in social practices that develop teachers' professional identities, and the appropriate unit of analysis is neither the individual nor the social setting but the person-in-practice-in-person. Focusing on the person-in-practice allows for interpretation of knowledge and beliefs within teachers' professional contexts, while refocusing the lens on the practice-in-person shifts attention to identity formation as practice changes the person. An adaptation of Valsiner's zone theory from the perspective of teacher-as-learner and incorporating teachers' knowledge and beliefs, professional environments, and opportunities to learn was created to analyse teachers' learning and development in two separate projects, one that sought to understand how

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teachers used digital technologies and another with the more interventionist aim of changing classroom practice.

Although the two case studies presented in this paper were chosen to highlight differences between two major research themes in mathematics teacher education – one predicated on understanding and the other on change – the similarities between them are also of interest. One point of similarity is that both studies involved elements of understanding and change. It was necessary for the researchers to understand Val and Shanti’s previous professional histories and perceptions of their environment in order to intervene appropriately by promoting alternative assessment approaches. In Brian’s case, it was only possible to understand his practice by analysing a previous transformational change in his professional identity. Even though it may seem that this change was self-initiated and self-determined, a zone theory perspective highlighted the social resources for change offered by his interaction with colleagues outside the school.

Another issue common to both case studies was the significance of productive tensions within a teachers’ zone system as a potential trigger for learning and development. For Brian the initial tension was between the beliefs that underpinned his rather traditional teaching actions and the unsatisfactory learning outcomes of his students, which eventually led him to embrace a new teaching philosophy. Further tensions between this new set of beliefs and elements of his professional environment led him to work on changing the organisational culture of the Mathematics Department in order to achieve the goal of learning with understanding that he now held for his students. The task that remains for Brian is to create similarly transformative tensions for the mathematics teaching staff in his current school. For Val and Shanti, it was the tensions between their student-centred beliefs and their school’s rigid assessment regime that led them to formulate the goal of creating rich and authentic assessment tasks in keeping with the investigative flavour of the new mathematics syllabus. The possibilities for individual agency are implicit in both case studies: for neither Brian nor Val and Shanti were environmental constraints regarded as insuperable obstacles. Instead, they looked for things they could change, such as access to teaching resources, and they aligned their practice with an external zone of promoted action provided by professional reading, colleagues, or participation in a research project.

The stories of Brian, Val and Shanti are unfinished and although there is evidence that Brian developed his professional identity the same cannot be said of Val and Shanti. The educational change project in which they participated was of short duration; they changed some of their teaching practices but the extent to which the practice "became the person" is not known. Future research from a sociocultural perspective might usefully investigate the sustainability of teacher change with person-in-practice-in-person as the unit of analysis.

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