Received: 17/02/2022 ■ Approved: 16/08/2022 ■ Published: 19/11/2022

eISSN 2179-426X
10.26843/rencima.v13n5a17

Revista de Ensino de Ciências e Matemática

# Collaborative planning of ambitious Mathematics teaching practices: teachers' reflections on animations and simulations in exploratory teaching

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***Abstract:*** This article discusses actions and reflections of two Mathematics teachers when planning, in a collaborative way and mediated by a researcher, teaching practices considered ambitious, involving Exploratory Mathematics Teaching and animations and simulations in the GeoGebra software. We seek to understand what these teachers consider and what they mobilize or develop in this process. This is an intervention study, whose data derive from interviews and practice planning meetings, which constitute two units of analysis that focus, respectively, on highlighted aspects and aspects developed or mobilized by the teachers. Thus, expectations, beliefs and conceptions of teachers are evidenced, as well as actions and reflections oriented towards the integration of technology, which were enhanced by the assumed teaching perspective. It is concluded, therefore, that collaborative planning constitutes good opportunities for investigation and understanding of teacher professional development, and that Mathematics TPACK associated with Instrumental Genesis constitute relevant contributions to studies of this nature.

***Keywords****:* Mathematics Education. Planning. Technology. Instrumental Genesis. Mathematics TPACK.

# Planificación colaborativa de prácticas ambiciosas de enseñanza de Matemáticas: reflexiones de profesores sobre animaciones y simulaciones en la enseñanza exploratoria

***Resumen*****:** Este artículo aborda las acciones y reflexiones de dos profesores de Matemáticas al planificar, de forma colaborativa y mediada por un investigador, prácticas docentes consideradas ambiciosas, involucrando la Enseñanza Exploratoria de las Matemáticas y animaciones y simulaciones en el software GeoGebra. Buscamos comprender lo que estos docentes consideran y lo que movilizan o desarrollan en este proceso. Se trata de una investigación de intervención, cuyos datos derivan de entrevistas y encuentros de planificación de prácticas, que constituyen dos unidades de análisis que se centran, respectivamente, en aspectos destacados y aspectos desarrollados o movilizados por los docentes. Así, se evidencian expectativas, creencias y concepciones de los docentes, así como acciones y reflexiones orientadas hacia la integración de la tecnología, que fueron potenciadas por la perspectiva de enseñanza asumida. Se concluye, por tanto, que la planificación colaborativa constituye buenas oportunidades para la investigación y comprensión del desarrollo profesional docente, y que el *Mathematics TPACK* asociado a Génesis Instrumental constituye aportes relevantes para estudios de esta naturaleza.

***Palabras clave***: Educación Matemática. Planificación. Tecnología. Génesis

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instrumental. *Mathematics* TPACK.

## Planejamento colaborativo de práticas ambiciosas de ensino de Matemática: reflexões de professores sobre animações e simulações no ensino exploratório

***Resumo:*** Este artigo discute ações e reflexões de dois professores de Matemática ao planejarem, de forma colaborativa e mediada por um pesquisador, práticas de ensino consideradas ambiciosas, envolvendo o Ensino Exploratório de Matemática e animações e simulações no software GeoGebra. Busca-se compreender o que esses professores consideram e o que mobilizam ou desenvolvem nesse processo. Trata-se de uma pesquisa intervenção, cujos dados derivam de entrevistas e encontros de planejamento da prática, os quais constituem duas unidades de análise que focalizam, respectivamente, aspectos salientados e aspectos desenvolvidos ou mobilizados pelos professores. Evidenciam-se, assim, expectativas, crenças e concepções dos professores, bem como ações e reflexões orientadas à integração da tecnologia, as quais foram potencializados pela perspectiva de ensino assumida. Conclui-se, desse modo, que o planejamento colaborativo constitui boas oportunidades para investigação e compreensão do desenvolvimento profissional docente, e que o *Mathematics TPACK* associado à *Gênese Instrumental* constituem aportes pertinentes para estudos dessa natureza.

***Palavras-chave:*** Educação Matemática. Planejamento. Tecnologia. Gênese Instrumental. *Mathematics TPACK*.

### 1 Introduction

Teachers' pedagogical planning influences their actions, strategies, and decisions in the process of teaching and learning, which is intensified when this practice involves approaches that favor communication and the students' mathematical thinking (CANAVARRO, 2011), articulated with the integration of technology (GAFANHOTO; CANAVARRO, 2014). In this context, the planning designed by Mathematics teachers is being widely discussed (SULLIVAN; ZEVENBERGEN; MOUSLEY, 2005; SUPERFINE, 2008; SULLIVAN *ET AL*., 2013; KILPATRICK, SWAFFORD; FINDEL; 2001, STEIN *et al*., 2008; CANAVARRO, 2011; JAWORSKI *et al*., 2017), under the supposition that the decisions taken during classroom actions can impact on the learning opportunities provided to students.

Therefore, this article discusses the actions of two teachers while developing the planning of ambitious teaching practices, understood here, based on Oliveira (2020), as those that promote new ways (centered on students) and approaches (innovative and demanding) for Mathematical teaching, with ambitious learning goals, which favor communication and make the student's' mathematical thinking tangible. These practices are materialized in this investigation in classes based on the

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elaboration of animations and simulations in the GeoGebra software, in the perspective of Mathematics Exploratory Teaching (PONTE, 2005; CANAVARRO, 2011; CYRINO; OLIVEIRA, 2016; ESTEVAM *et al*., 2018). Thus, using as reference aspects related to the Mathematics Technological and Pedagogical Content Knowledge – Mathematics TPACK (NIESS *et al*., 2009) associated with Instrumental Genesis (RABARDEL, 1995; 2011), it aims to understand what these teachers take into consideration during this planning process and what do they mobilize or develop when planning this kind of practices in collaboration.

## 2 Teacher planning under different perspectives

Planning is one of the practices in the professional practice of teachers that relates content knowledge to what one intends to teach and to the way one intends to lead this practice. It is also a moment to define strategies and resources (material or instrumental ones) that will help during this process. To Superfine (2008), planning is an important phase in teachers' practice, during which they make decisions regarding many aspects of teaching that, ultimately, shape the students' learning opportunities. When interspersed with collaboration, planning involves teachers engaged in activities with common goals, which favor interactions and critical discussions, with an exchange of experiences for the implementation and resolution of challenging issues, in their professional context, leading them to reflect on their role at school. and in society (JAWORSKI *et al.*, 2017).

In this sense, Kilpatrick, Swafford e Findel (2001) believe that planning must reflect a *deep and complete consideration* of the mathematical content of a class, as well as of the students' thinking and learning. Instruction materials must support teachers in their planning, and teachers need time to plan.

In the case of the Mathematical teaching, it is indispensable to know the mathematical contents that will be taught due to a simple reason: teacher that do not know a subject well probably will not have all the knowledge needed to help students to learn this content. However, this is not enough to teach. What seems to be more important is to understand and be able to use the Mathematics that the teaching job demands (BALL; THAMES; PHELPS, 2008). It is fundamental to *know how to teach* (SERRAZINA, 2012).

According to the *National Council of Teachers of Mathematics* – Principles and

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standards for school mathematics (NCTM, 2000), when planning classes, teachers must make efforts to organize the Mathematics in a way that fundamental ideas compose an integrated whole. The encounter of big ideas in different contexts must be established with care, dealing with important elements, such as the terminology, definitions, notations, concepts, and skills that may emerge in the process. The sequence and coherence between school units and years is a challenge. Teachers need to be able to adjust and seize opportunities to lead the class in unforeseen directions.

Sullivan, Zevenbergen and Mousley (2005), by studying actions that teacher can take to improve their students' mathematical learning, especially those who face difficulties, have proposed a model with guiding aspects to be considered during Mathematical planning and teaching, namely: (a) *mathematical norms*, which cover principles, generalizations, procedures and tasks that ease the involvement of students with significant mathematics, as well as the sequence of tasks that allow for more efficient interactions; and (b) *norms of the mathematical community*, which include favoring and making the students' interpersonal interactions more dynamic, as well as focusing on the development of a mathematical learning community. Interviews with the involved teachers indicate that the model allows them to focus on the challenge of involving all students in productive mathematical explorations and providing paths, principles and strategies for their conduction, aspects that are characteristic of ambitious teaching practices (OLIVEIRA, 2020).

Superfine (2008), in its turn, when reporting an intensive study about teachers' planning, proposed a model that provides a way of understanding how teachers' planning practices change, or cease to change, throughout their careers. This model includes three elements: *curricular mathematics materials; teachers' conceptions; and planning problems*. The term *planning problems* refers to the relationship among teachers' experiences, teaching notions and mathematical learning, and the curriculum program used. This model aims to evince, in particular, how these elements interact among themselves and can affect teachers' planning decisions.

In their turn, Sullivan *et al*. (2013) have designed a report focused on curriculum-guiding documents, referring to a project that was especially interested in the way curricular documents are interpreted and used, their effects in teachers' action intentions, and, specially, their implications to the reconstruction of teachers'

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knowledge and of their teaching practices. *Knowledge* and the *curricular intentions* are considered key for the types of decisions teachers' make regarding their planning.

In their turn, Stein *et al*. (2008), when discussing five practices to orchestrate pedagogical practices based on inquiry and focused on students (therefore, ambitious ones), suggested that teachers will usually face challenges that go beyond identifying well-designed tasks and preparing them properly to be developed in the classroom. This happens because, as resolution paths are not specified in this type of practice, students tend to approach them in particular ways, which are sometimes unpredictable. For the researchers, the students must not only make efforts to understand how students are understanding/receiving the task. It is also necessary to start to align/organize students' ideas and create approaches with understanding/comprehension of the concepts and process about the nature of Mathematics. Therefore, Stein *et al.* (2008) presented a pedagogical model that specifies five practices so that teachers can learn to use students answers for these tasks in a more efficient manner in discussions, namely: anticipation, monitoring, selection, sequencing, and making connections among the students' answers. In this model, the *anticipation*, the first proposed practice, is directly related to teachers' planning and, consequently, it has implications directly related to the others. To anticipate demands that teachers at least *solve the mathematical tasks* they plan to present to their students, foreseeing and articulating different resolution strategies. They need to *put themselves in the students' shoes* when solving the task, *think on the different levels of mathematical sophistications* that the students will probably produce, and consider the ways through which they can *misinterpret* the problems or be confused with the situations throughout the path.

## 3 Planning in the Mathematics Exploratory Teaching involving digital technologies

Exploratory teaching (PONTE, 2005; CANAVARRO, 2011; CYRINO; OLIVEIRA, 2016; ESTEVAM *et al*., 2018) is opposed to traditional, directive, or expositive teaching since it aims to bring students to the center of mathematical activity, with cognitively challenging and demanding tasks (STEIN *et al.*, 2008). Orchestrating this type of action in the classroom is no simple task, since implementing ambitious practices mediated by inquiry, reflection, communication, and collaboration demand teacher actions directed toward foreseeing obstacles and challenges that may

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occur during the class (STEIN *et al*., 2008; CANAVARRO, 2011; CYRINO; OLIVEIRA, 2016).

Canavarro (2011) indicated the practices of anticipation as one of the most important components of the planning of a Mathematics class in Exploratory Teaching.

> By anticipating, teachers dedicate themselves to: predict the interpretation and the involvement of students in the task; list different strategies, correct and incorrect ones, that students may use with different levels of sophistication; relate these strategies with the concepts, representations or procedures that they wish students to learn and/or with the capacities they wish students to develop (CANAVARRO, 2011, p. 13).

This anticipation, at the same time it works as a planning of what teachers intend to develop, also works as a preparation for them. By designing a previous planning, teachers have the opportunity to reflect on certain ideas, strategies, procedures, and representations that students can use, be them right or wrong (CYRINO; OLIVEIRA, 2016). These reflections, in teachers' planning, must refer to actions in this teaching perspective with a class structure that usually happens in four phases (CANAVARRO; OLIVEIRA; MENEZES, 2014): (i) introduction of the task; (ii) performing the task; (iii) discussing the task; and (iv) systematizing mathematical learnings.

According to Ponte (2005), in this kind of practice, the teacher takes on the role of mediating the learning, and the students are the protagonist in their learning process. This happens because learning is related to making and reflecting on what one makes. Thus, the expected learnings are not reproductivist or essentially procedural, but rather focus on the students' reasoning and the attribution of meaning to what is done, on conceptual, procedural aspects, or even on the attitudes in relation to Mathematics. Therefore, it makes sense to admit this type of practice as ambitious from the didactic and pedagogical point of view.

Besides, teachers may and must use digital technological resources to plan their class in the perspective of Exploratory Teaching, which compose a strategy that may favor and foster students' learning (GAFANHOTO; CANANVARRO, 2014). In this context, GeoGebra has gained spotlight, which is a dynamic mathematics software for all levels and modalities of teaching, and combines geometry, algebra, calculus spreadsheets, graphics, probability, statistics, and symbolic calculus. Therefore, it allows for dynamic constructions, with the exploration of different mathematical objects, in different Mathematical fields.

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> The dynamic aspect of this software has enabled the interaction with mathematical objects in a dynamic way, that is, enabling their continuous and live modification. Moreover, this modification affects all representations of the mathematical object, and its apprehension involves considering its characteristic of dynamic multi-representation by GeoGebra (BASNIAK; PIZZORNO, 2020, p. 14).

Therefore, these possible modifications of mathematical objects with the use of software resources are promising (SANTOS; BASNIAK, 2021) and may favor planning, when articulated to Mathematics Exploratory Teaching. In particular, we are interested in the work involving *animations*, understood as modifications performed by using GeoGebra tools that allows to confer dynamicity to static objects; and in *simulations*, which consist of representations of a real phenomenon, from daily lives or not, developed in the software based on the available resources.

In this context, regarding teaching with technology, authors such as Mishra and Koehler (2006), and Koehler and Mishra (2009), based on studies conducted by Shulman (1987), discuss the connections, interactions, resources, conditions, and restrictions of the triad: content, pedagogy, and technology. Thus, it adds to the *content and pedagogical knowledge* that component of *technological knowledge*, which involves the knowledge of technological resources (analogical and digital ones), ways in which the content is or may be approached, besides skills to operate certain technologies. From the interrelation among these three knowledges, the authors have defined the Technological Pedagogical Content Knowledge – TPACK, which refers to the knowledge constituted by the merging of content, pedagogy, and technological resources.

Related to Mathematics and based on the ideas by Rogers (1995), Niess *et al.* (2009) have identified, in their investigations, five levels of teacher knowledge to integrate a certain technology in Mathematical teaching and learning, which is called Mathematics TPACK:

- *Recognition* (knowledge): indicate the ability to use and recognize the alignment of technology and mathematical content, but not to integrate technology into mathematical teaching and learning;
- *Acceptance* (persuasion): demonstrates a favorable or unfavorable attitude for teaching and learning Mathematics with a proper technology;
- *Adaptation* (decision): there is involvement in activities that lead the decision of adopting or rejecting Mathematics teaching and learning with a certain

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technology;

- *Exploration* (implementation): integrates actively the teaching and learning of Mathematics to a proper technology; and
- *Advance* (confirmation): evaluated the results of the decision of integrating Mathematical teaching and learning to a proper technology.

Therefore, *Mathematics TPACK* is being seen as a promising perspective for studies that discuss teachers' knowledge to deal with the integration of technology in diverse contexts (PALIS, 2010), and recognizing, within this context, the importance of planning with clear and achievable goals (BASNIAK; ESTEVAM, 2018), going far beyond approaches that treat technology as a mere complement in the classroom (KOEHLER; MISHRA; CAIN, 2013).

Based on the necessary teacher knowledge to teach with and about technology (MISHRA; KOEHLER, 2006; KOEHLER; MISHRA, 2009; NIESS *et al*., 2009; PALIS, 2010; KOEHLER; MISHRA; CAIN, 2013; BASNIAK; ESTEVAM, 2018), it makes sense to think on the way teachers insert technological resources in their pedagogical practices (BITTAR, 2010) and how do students deal with this integration (TROUCHE, 2005) in mathematics classes. In this context, aspects of the *Instrumental Genesis* (RABARDEL, 1995; 2011) have been shown to be adequate to interpret this process, since this perspective cover the *relationship* among *artifact, schemes,* and *instrument*.

Rabardel (1995; 2011) defines *instrument* as a mixed entity, from the psychological point of view, which understands the *artifact* and the association of one or more use *schemes*. The artifact is a material or symbolic media that subjects use to achieve their goals. In it turn, a use scheme covers the result of the adaptation of the artifact by the subject for a certain task, which created a possible instrument. Namely, Instrumental Genesis consist of the process of gradual transformation of an artifact into an instrument, in which the *instrumentalization* and *instrumentation* dimensions have special interest.

In instrumentalization, use schemes involve the process of understanding the artifact by teachers, when dealing with and appropriating the resources and tools. In instrumentation, use schemes happens when teacher employ these resources and tools intentionally to think on the pedagogical activity, that is, what they want to provoke in the students, what they wish them to perceive, think, and do. We thus understand that Instrumental Genesis is dynamic, sometime involving a use scheme, and other a

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utilization scheme, depending on the user and their intention related to the pedagogical process.

Under this point of view, to understand, investigate, and develop the Mathematics TPACK that is required for teachers to integrate technology into teaching, the process of Instrumental Genesis, in the instrumentation and instrumentalization dimensions, can help both to identify and to advance the level of teachers. On the other hand, teachers' levels of Mathematics TPACK can show evidence of processes they understand or which aspects they consider, related to their instrumentation and instrumentalization. Likewise, by thinking on the practice for their students, it can guide actions based on the dual Instrumental Genesis process, aiming to widen their knowledge.

## 4 Investigation Context: teachers and planning

This study resulted from the acceptance of the invitation made by the first author, in October 2019, to two Mathematics teachers that participated in a Practice Community coordinated by the second author, in which the first author also participates, to develop ambitious teaching practices in collaboration. These teachers are hereby identified by pseudonyms: Pedro, licentiate in Sciences and Mathematics; and Diego, licentiate in Mathematics. Pedro and Diego are experienced teachers and are committed to Basic Education, working as teachers in the final Elementary School years and in High School in the public network. Pedro has been working in the Paraná state network for 25 years and, during the conduction of the study, was working at a country school in the rural zone. Diego had been working in the Santa Catarina state network for 10 years, and in the federal network of the State of Paraná for at least 1 year (professional configuration possible due to residing in neighboring municipalities), and, during the conduction of the study, was teaching in urban schools.

After the teachers accepted the invitations, individual interviews were booked and conducted in March 2020 to identify their professional trajectories, what they knew, and their considerations about ambitious practices that, in our case, involve Mathematics Exploratory Teaching, technologies in Mathematical teaching, the GeoGebra software, animations, and simulations. These aspects served as base to forward the research actions.

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Due to the Covid-19 pandemic[3], in June 2020, the first meeting with the two teacher and the researcher was conducted via Google Meet and called *Planning Meeting*, with the goal of agreeing on some referrals based on the perceptions of the interview initially made. In this meeting, it was suggested the creation of a group chat (WhatsApp) to favor the communication, the sending of materials and discussions about the meetings. It was also agreed that the participants would read the text by Estevam *et al*. (2018), which approaches the Mathematics Exploratory Teaching and digital technologies in the context of elaboration of the law of sines, mediated by the GeoGebra software. During it, it was also presented some animations and simulations in the GeoGebra software, with initial representations to refer the interactions and subside subsequent dialogues.

All materials were forwarded in the group chat, and a new meeting was booked for the end of June. In this meeting, the participants discussed some ideas present in Valente's text (1993), who problematizes the computer and a *teaching machine* or a *teaching tool*.

Based on this discussion, the participants problematized the possibilities of using technologies in planning mathematical tasks using the GeoGebra software; the Mathematics Exploratory Teaching and GeoGebra; and the presentation and recognitions of animations and simulations, with discussions on their similarities and differences. This was associated to the discussion on the understanding and recognition of the objective of the socialized animations and simulations, as well was the approached mathematical concepts, leading to reflections on the possible tasks to be planned.

During July, through the WhatsApp group, the researcher forwarded messages to provoke and encourage the teacher to decide a specific content that would be approached in the planning while designing the task. The teacher Diego suggested spatial geometry, namely the calculus of the area and volume of prisms or round bodies; and teacher Pedro referred to analytical geometry, with focus on the approach to point, line, plan, and circumference. Having chosen the contents, they began thinking on possible animations and simulations to approach them. There was an initial

[3] The coronavirus is a family of common viruses (Sars-cov) that cause respiratory infections. A new agent (Sars-cov2) from this group was registered in December 2019, in China, and caused a hitherto unknown disease that came to be called Covid-19, causing a global pandemic with the need for social isolation that impacted the population. organization of space and production of research data.

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intention that the teachers would bring suggestions, but they did not feel safe, and asked for the researcher (first author) to present some possibilities. This process was conducted in collaborations (JAWORSKI *et al.*, 2017) so that the practice did not become instructional.

Therefore, according to the chosen contents, two animations (Parallelepipeds and Bicycle) and two simulations (Airplane and Ferris Wheel) were presented, all available online. They were adapted and adjusted by the researcher, being presented along with protocols for construction on GeoGebra 5.0. The teachers visualized the animations and simulations and, despite considering them promising, highlighted difficulties to perform the discussion and the planning of the tasks remotely, considering the long workload of synchronous classes they had been conducting through computer and the context of family organization. Thus, after discussing the implications that the pandemic scenario was imposing, with no possibility of meeting at a school or even at the University (since in-person activities were suspended), the researcher suggested, with the agreement of the participant teachers, that the meeting should be conducted at his home, following the current health protocols. The following meeting happened then at Friday nights, during the time that the teachers were available. Table 1 describes the dates of the nine in-person planning meetings and the respective developed actions, with an average duration of two hours per meeting.

Table 1: In-person meetings and collaborative planning actions by the teachers

| **Date** | **Performed actions** |
|---|---|
| 14/08/2020 | ▪ Study: dimensions of the Mathematics Exploratory Teaching: inquiry, reflection, communication and collaboration, and the autonomous work by the students;<br>▪ Discussion of the *framework* proposed in Cyrino and Teixeira (2016), which associates teachers' actions to phases of a class in the exploratory perspective;<br>▪ Construction and discussion of the animations *Parallelepipeds*. |
| 21/08/2020 | ▪ Animation *Parallelepipeds*: resumption of the discussion on the possible actions by the teachers and students;<br>▪ Simulation *Airplane*: construction and discussion. |
| 28/08/2020 | ▪ Simulation *Ferris wheel*: construction and discussion. |
| 11/09/2020 | ▪ Designing the *Parallelepipeds* task: board of actions by the teacher and students. |
| 18/09/2020 | ▪ Designing the *Parallelepipeds* task: board of actions by the teacher and students. |
| 25/09/2020 | ▪ Teaching plan for the *Parallelepipeds* task. |
| 02/10/2020 | ▪ Designing the *Airplane* task: board of actions by the teacher and students;<br>▪ Watching the video and the discussion of the lecture *Mathematics Exploratory Teaching and the professional learning of (future) teachers*. |
| 16/10/2020 | ▪ Designing the *Airplane* task: board of actions by the teacher and students;<br>▪ Teaching plan for the *Airplane* task (WhatsApp group);<br>▪ Discussion of the *Ferris Wheel* task. |
| 30/10/2020 | ▪ Designing the *Ferris Wheel* task;<br>▪ Teaching plan for the *Ferris Wheel* task;<br>▪ Conversation with teacher: perceptions and challenges during task planning (Exploratory teaching, GeoGebra animations and simulations). |

Source: designed by the authors, 2020.

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The dynamics used in the 11 meetings – 2 remote ones (RM) and 9 in-person ones (PM) referred to in Table 1 – by the researcher and the teachers has promoted the engagement, the discussion of knowledge, conceptions, and doings of the teaching practice, related to the discussed theme, which resulted on the design of three tasks and their respective teaching plans[4]. It is also important to highlight that, to align ideas and discuss possibilities, the planning, design, and systematization of the tasks also involved discussions carried out in the group chat.

Therefore, in this article, we examine the data produced in the individual interviews and during the 11 meetings conducted. All meetings had their audio recorded, whose transcriptions are complemented by annotations of the researchers' field notebook.

It is an intervention-research (KRAINER, 2003), whose focus lies on developing and understand the practice of collaborating teachers through the interventions conducted by the researcher. The analyses were based on questions that allow to understand how it is or what do these teachers consider while planning these kinds of practices. At the same time, it evaluates what is developed or mobilized during this process of collaboratively planning ambitious practices, making use of aspects of the association with the Rabardel Instrumental Genesis (1995; 2001) and the Mathematics TPACK (NIESS *et al*., 2009) as theoretical lenses for the analyses.

Based on the data produced and considering the exploratory perspective involving animations and simulations in the GeoGebra software, two analysis units were constituted: i) aspects highlighted by the teachers during the planning process; and ii) aspects developed or mobilized by teachers while developing a collaborative planning action.

## 5 Aspects highlighted by the teachers during the planning process

During the meetings, the teachers were concerned with *changing the way they teach Mathematics*, and the possibility of integrating technological resources in their classes, based on reflections about the need for changes in traditional teaching. The following episode illustrates an example of this discussion.

> Diego: *Ah, traditional teaching, not that it is not good or that it does not work, it has worked for a long time, and it still does, but we believe that using it* [referring to the computer],

[4] The tasks and their respective plans can be access on https://prppg.unespar.edu.br/geptemate/material-didatico.

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*I at least believe, that using it as a teaching tool will lead to better results.*

Pedro: *Yes, indeed, the traditional needs, there are things that work and keep working, but we cannot be stuck in time […], before we had only a blackboard and chalk, today it is not only blackboard and chalk, we are going through a moment in which we are being forced to make use of other tools* [referring to the pandemic and the need to use technological resources]. (PM-26/06/2020)

The teachers' concern given the challenge of changes in their practice exposes the insecurity they feel regarding the use/integration of technology, but that they deem necessary to exercise their teaching, referring to *teachers' notions and planning problems* (SUPERFINE, 2008). Continuing the excerpt, the teachers discuss the possible uses of GeoGebra to think on tasks, expose their concern with the *time and knowledge* required in the planning. Pedro also refers to the difficulty in *accessing technology* in the reality he is inserted in and the enchantment of the students when he proposes its use.

Diego: *[...] I see that GeoGebra, the software, is very complete in this sense, for what I could see here, that, to manipulate it, it is possible to observe and relate geometric and algebraic variations, which would not be feasible using only pen, paper, and drawing tools. So, I think GeoGebra, for us to think on an activity, I think it is very complete, I don't know what you guys think of it.*

Pedro: *Yes, I think it is very interesting, it is a toll that must be used, we have to sometimes include it, I do not include it more due to lack of knowledge, lack of time, these things, technology. We talked a lot of reducing the workload, but it actually increases your workload, because you have to know so much more of the software to be able to teach, this happens even for normal contents, you always know more to be able to teach someone, otherwise you will hardly be able to […]. Sometimes, it is more pleasant when we vary, when we use not only the board, not only the traditional teaching, now we go to the lab, let's think differently, let's learn how to use GeoGebra to facilitate our teaching-learning, and they like it, I don't know if in Diego's reality, but there they do not have access, many of them do not have computers, so going to the informatic lab, for them, it is fantastic, many of them don't have cellphones, so, when we say we are going to the lab, their eyes sparkle [...].*

Diego: *But I also share this idea, this thinking of Pedro, because I also feel insecure about working with GeoGebra because, as you have said, we do not master the machine there, in this case, and then sometimes we get insecure. You don't know how to use that, what if you need to solve a student's doubt? Then, this is something that makes me insecure about using it more often, for example, in the classroom.* (PM-26/06/2020)

The teachers refer to the need for and importance of technology integration in their classes, even though they feel insecure in relation to *knowing how to teach* involving technological knowledge, expressed as "knowing more in order to teach" in less expositive classes, which is also evinced by Serrazina (2012). They also report the students' lack of access to material components (computer, cellphone), at the same time they indicate the school as a place for opportunities for the students to

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access these resources. However, Pedro seems to relate the use of technology as a motivation in his classes, without needing to explore mathematical concepts, for example.

The teachers' intentions, by glimpsing possible explorations with the artifact, considering the dynamism it can provide to the construction, different from a static environment (blackboard and chalk), represent *understandings and reflections* regarding the strategies and procedures that favor students' learning and are thus promising to planning. This aspect is related to what Sullivan, Zevenbergen and Mousley (2005) indicate as *norms of the mathematical community*, which guide decisions made during planning that seek to favor students' learning (Superfine, 2008).

At the same time teachers deal with GeoGebra, they discuss the attention needed to develop it with students in their groups, especially aiming to promote their participation in this practice, to which they are not used, as expressed by Pedro *"And even more because we want them to participate, we want to instigate them, there is this also, this barrier that they want, as they say, to do math only. Then we have to instigate them, make them think, discuss, so there is this process"* (PM-25/09/2020).

Regarding the tasks that help developing the students' mathematical thinking, based on different resolution strategies and on favoring their autonomy (SULLIVAN; ZEVENBERGEN; MOUSLEY, 2005), Diego points out this situation in a positive way: "*Maybe you have seen there that the student though in a way you didn't even think, and then he can explain"* (PM-25/09/2020). In this context, Pedro refers to his perceptions regarding the planning, the development of the practice involving the Mathematics Exploratory Teaching and the animations and simulations, and what is expected from students.

> Pedro: *[...] Of course, it may be that it has a different posture and develops more the questions of thinking, the student's autonomy. We can see that for sure, because, in traditional teaching, there is so much less of it, although I was a teacher and today I am another one, I make more room for students to talk, I question more, I am a different teacher from who I used to be. I like students to talks, I value their mistakes. Why did you do like that? Because sometimes they did not do wrong, but understood it wrong, they developed a thinking […} When you talk, you instigate students to tell you, it flows differently. I think that the traditional teaching is very limited in that, not that it cannot be, but it is limited.* (PM-02/10/2020)

Reflections such as these offer opportunities for teacher to analyze the way they are working, and how this change impacts their professional practice, as well as the

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opportunities that a student-centered approach promotes for mathematical discussions in the classroom, mediated by inquiry, reflection, communication, and collaboration, as in the indications by Ponte (2005), Canavarro (2011) and Cyrino e Oliveira (2016).

In the simulation that originated the *Airplane* task, when discussing the mathematical content that would be approached with students, Pedro glimpses possible explorations, according to the following excerpt.

> Pedro: *The concepts, I think the basic ones: point, plane [...]. The point, ordered pair, the question of knowing that, moving here it is positive, moving there it is negative, that (1,2) is one thing and (2,1) is another, these basic things really. That there is a predetermined order, that is why it is an ordered pair, it follows a certain order. I think it is these basic things, initial things, as you said. The student did not have that, we will introduce axis, abscissa, ordinate.* (PM-02/10/2020)

The concern with decisions regarding the mathematical content and the sequence in their approach suggests the teacher's attention to the curriculum (SUPERFINE, 2008) and to the *guiding documents for Mathematical teaching* (Sullivan *et al*., 2013), which are very present in teachers' practice.

Another point highlighted by Diego refers to the dialogue among teachers, the experience of collaborative planning that favors teacher's practice, in agreement to Sullivan, Zevenbergen and Mousley (2005): "[...] *experiencing these moments was good in the sense of performing this collective information exchange; planning alone is complicated, so, if we can plan in group, I think thing would go on fine"* (PM-30/10/2020). During teachers' discussions, the collaborative planning was seen as a propeller of communication, interaction, and cooperation, and was considered a conditioning factor for the improvement of professional practice, despite not happening in daily teaching practices, be it due to the curricular organization, be it due to professional valorization policies.

The change in teacher practice, when they aim to perform ambitious practices, is a challenge for teachers:

> Diego: *Leaving this comfort zone and entering a risk zone, because going to what is new, leaving this comfort zone, which is working with the traditional teaching, in which teachers are the ones who know everything, and students only obey, is to give voice to students. Leaving the comfort zone and entering a risk zone is a challenge for most of us, so Exploratory Teaching makes the teacher move a little more.*
>
> Pedro: *Yes, because even the logistics is different, we were even organizing the time, I will arrive, they will be in the classroom, I will take attendance or do I make it in the informatic lab, how will it be? Will it be organized, who will organize it, or do I go there*

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> *before to organize the equipment, otherwise the time will me shorter. While in the previous one, I go with my chalk box, my book under the arm, and that is it* [laughs]. (PM-30/10/2020)

In this sense, the anticipation that occurs during planning, foreseeing the necessary orchestration of the class (STEIN *et al*., 2008), is evinced as an element that makes difference in this process. However, the time needed for this process is considered a limiting factor. According to Pedro's view, "*we do not have enough time to reflect about things we need, we have already discussed that we should have more activity-hours to be able to think and reflect more about the planning*" (PM-30/10/2020). The *need for proper time* for planning is also seen in the indications by Kilpatrick, Swafford and Findel (2001).

The designed tasks and plans have led to reflections by the teacher, by thinking the anticipation in their actions and intentions with possible questions by the students, understanding that bringing students to the center of the process contributes to their learning, even with the difficulties that may happens during the class, be it caused by the adaptation to the new approach or by the communication between teacher and students. For Pedro, "*it is essential for students to task, to express themselves, not only the teacher"* (PM-30/10/2020), so that we can identify their difficulties and the referrals to be made. In this context, anticipated planning teachers' actions favors the execution of practice and the processes of communication.

Therefore, teachers, by thinking about actions during practice planning, highlight the need to instigate the participation of students, to foresee the communication and interaction during class, their expectations, impressions, beliefs, and notions. These aspects are indications of their intention of providing conditions for the learning and the development of mathematical thinking by students, which guided the collaborative planning of the studied practice. In the next section, the aspects developed or mobilized by teachers during the collaborative planning will be discussed.

## 6 Aspects developed or mobilized by teachers during planning

The aspects that were mobilized (those that already existed and were put into action) or developed (those that had no indications of previous existence) are analyzed in light of the process of their Instrumental Genesis, in the instrumentation and instrumentalization dimensions, associated with the Mathematics TPACK, focused on the advance of the teachers' level of knowledge.

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Provoking teachers to think possibilities to design tasks, their actions and intentions has enabled the first reflections about planning, and the use of the GeoGebra software (artifact) for the proposal of animations and simulations, as it is portrayed by the following episode.

> Pedro: *You have a much wider range of possibilities in a short period of time, from the moment you have constructed, and then you will explore it with students, and when you draw it on the board, you do it once, it is a static and fixed drawing… it is there, but there is a series of variations. We could test this with the groups …*
>
> Diego: *I see that this task's planning has to be very well thought, and how we will make to put GeoGebra along with it.*
>
> Pedro: *I think that any contents we explore will be good.* (PM-26/06/2020)

The teachers show different levels of Mathematics TPACK, as described by Niess *et al*. (2009), by measuring the integration of the artifact in their practice and highlighting their objectives to promote students' understanding and learning. While Pedro indicates acting on the level of *knowledge*, referring the use only to facilitate representations and speed up processes, without showing concern with the concepts to be approached with GeoGebra, Diego shows this concern, suggest that he is in the level of *decision*.

By thinking on approaches for the *Parallelepipeds* animation, the teachers were familiarized with the GeoGebra software, instrumentalizing themselves to explore its tools. By discussing the construction guide, manipulating GeoGebra, its commands and their implications, when they confer parameters to the sliding controls and the intended learning objectives, they begin their instrumentation, as shown by the following episode.

> Diego: *In the controls, the minimum and maximum can be -5 and 5, and the increment can be 0.1?*
>
> Pedro: *With negative we can't, with negative values.*
>
> Researcher: *Yes, I even put an observation over there.*
>
> Diego: *Observe the minimum?*
>
> Researcher: *Yes, why do we have to observe the minimum? Because I am working with area, with length, with measure.*
>
> Pedro: *That's when it doesn't build* [referring to GeoGebra]*, if it is up to zero, it* [referring to the parallelepiped's dimensions] *will disappear.*
>
> Diego: *You have to pay attention to the minimum!*
>
> Researcher: *So, you think about it, this is something we have to think with the students too when they are working on it.*

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Diego: *And the increment can be 0.1?*

Pedro: *It can, if you wish so.*

Diego: *Yes, these are things that we will have to pay attention to. Ah, I have already done something wrong here, let me see.*

*[...]*

*Ah, but the students will put negative, most of them will. This is an action that we have to think already.* (PM-14/08/2020)

The collective discussion of the use schemes (by manipulating the sliding control tool) and of the usage (by reflecting on the intention of changing the control parameters) indicated the artifact adaptation to construct an instrument that is capable of encouraging learning, as described by Bittar (2010). It is possible to see the enthusiasm and engagement of the teachers in thinking about the mathematical content involved. These actions are important to go beyond approaches that deal with technology as a mere complement in the classroom, as highlighted by Koehler, Mishra and Cain (2013). Anticipating how to discuss the mathematical content with students, by testing possibilities and identifying possible obstacles for the students, evinced advances in the teachers' levels of Mathematics TPACK. This aspect contributes to foresee actions that must be performed in the construction – in this case, the parameters of the sliding control – to be evinced and problematized during the task, and that can mobilize the understanding of the concept of measures of length and area. This is a necessary journey to experience the process of Instrumental Genesis and advance the Mathematics TPACK. We highlight, however, that the answer given by the researcher may have hampered the possibility of exploring the question more deeply, being a complex aspect that demands more care. In other words, it is necessary for teacher that will plan in collaboration to be constantly alert to not lose sight of the investigation dimension and take on ideas without the proper discussion.

In the simulation that originated the *Airplane* task, the researcher provoked teacher to think on approaches, which led to indications and ideas that were complementary, which is considered a favorable aspect for the teachers' Instrumental Genesis.

Researcher: *How will we approach it, let us think… We will begin by building, then, they will open GeoGebra and will build, they will start to think on the construction. They follow the guide, the questions come after building this simulation of a moving Airplane. Then, my question would be: what should we approach? What would be the questions related to the airplane?*

Diego: *Because this is different from the other, it has a landscape behind, a scenario really.*

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Researcher: *[...] it is a simulation that will have a scenario.*

Diego: *Yes, that is the thing.*

*[...]*

Diego: *Yes, how can we create some questions?*

Researcher: *That's the idea.*

Diego: *Why, what happens, there with the scenario ready, how will they visualize it?*

Pedro: *But they will build!*

Diego: *Yes, maybe…*

Researcher: *As Pedro said, they will build, so they will...*

Pedro: *Have visualized.*

Diego: *I think that, maybe, for us to come up with some questions, we would have to look at the sliding controls.*

Pedro: *Relate it?*

Diego: *Yes, like, move it there …*

Pedro: *See what happens, move control a or control b.*

Diego: *Yes, because what do we want with this?*

Pedro: *What relationship you see with the positions of the airplane, of the object....*

Diego: *What is our goal with this task?*

Pedro: *What changes when you change the sliding control? Try to make them related, make them see that that is the ordered pair, their coordinates.*

Diego: *Yes, something like that. What we want with this task is to show that …*

Pedro: *The analytical geometry, the initial one, there.* (PM-02/10/2020)

To see the questions and ideas present in this except alone is no simple task, which shows the prominence of the collaborative dimension of the practice to the teachers' learning. This seems to have been enriched by having involved the Mathematics Exploratory Teaching and technology, in an intentional and pedagogically articulated manner, and the particular aspects of animations and simulations, which constitute specific artifacts that give rise to instrumentation and instrumentalization processes and are articulated in a collaborative action that demonstrates potential advances in the levels of knowledge of the involved teachers.

By foreseeing actions that would be performed and the resolution of the *Airplane* task, the teachers and the researcher moved back to discussing what they had thought after the performed planning, showing enthusiasm, and comparing it with the initial difficulty of the first task. By developing possibilities for using the software to treat a certain mathematical object, they indicate that knowledge is constituted in actions. It is possible to learn how to work with GeoGebra during the actions.

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Pedro: *It is very well detailed.*

Researcher: *And to develop it with the students, what do you think?*

Diego: *I think it will be good.*

*[...].*

Diego: *I guess, in terms of difficulty, that students will face more difficulty in the first task than in this one. I am putting myself in their shoes, because, before we started making the simulation....*

Pedro: *Maybe because now we know it a bit more.*

Diego: *Yes, we are used to it.*

Pedro: *After the first one, something issues are gone, even dealing with GeoGebra.*

Diego: *Knowing to. Apparently, it seems it will be like that*. (PM-16/10/2020)

The excerpt shows the expectation of how the students will deal with the artifact, comparing it with the difficulty faced by the teachers and that they overcame by mobilizing use schemes, as well as with the discussions of approaches with pedagogical intentionality, developing use schemes. This enabled the teachers to reflect on the diverse strategies that can be used by students during the task in the classroom, enriching the anticipation process.

Something similar to that was expressed in the process of designing the *Ferris Wheel* task and its respective anticipation board, a moment when the teachers highlighted that they were more used to dealing with GeoGebra, and that this would be evinced with the students as well. These affirmations indicated that they have developed in their instrumentalization, by knowing how to handle GeoGebra, and in their instrumentation, by thinking on other applications with pedagogical intentionality, favoring the development of their Mathematics TPACK levels.

Diego: *I think it is very good.*

Pedro: *I think it will be as we have built, they will be more comfortable to do it.*

Diego: *It is the third time already.*

Pedro: *We had that* [referring to the instrumentalization and instrumentation] *due to being more adapted to the software, we had already done the first two, I guess it will be easier for them to do so. Even to handle GeoGebra, especially not the content issues, but the construction, I guess it is be easier, And even to observe somethings, because we will be instigating them in the first ones, and the tendency is that, in the last one, they will be more attentive to the way of working, with the annotations, we will have to insist on the first ones, but, after that, they will be more connected to these things.*

Diego: *For sure. I think it is very didactical, easy to observe when visualizing the equation. I think it is very clear, overall*. (PM- 30/10/2020)

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This action that occurred is not spontaneous or natural, it does not come only from the teachers' willingness and availability, but from a collective construction process, of thinking their actions in collaboration, reflecting, going and coming back, until a path in which they felt minimally safe was found, making it possible to advance. The teachers started to glimpse possibilities, overcame the initial apprehension and fear, widened their perceptions to envisage richer and more complex tasks, focused on understanding and on the mathematical processes related to the possibilities provided by the GeoGebra software. Therefore, at the same time it creates favorable conditions for more ambitious teaching practices, the development of teacher instrumentation and instrumentalizations indicated advances in the Mathematics TPACK, with indications of the level of *confirmation*.

When designing the tasks, teachers' actions in relation to the artifact that because an *instrument* have favored planning and enabled the process of instrumentalization associated with the instrumentation, by manipulating the software, using its tolls, and their relationship with the mathematical knowledge and content they wished to explore with the students. By mobilizing different use schemes, individual or collective ones, by proposing and evaluating strategies, and thus discussing possible obstacles to be faced, considering the possible limitations of the software, the process of instrumentation happened in association with the instrumentalization.

This happened throughout the whole planning process, having considered the possibilities of continuous improvement, resulting om three tasks and their respective teaching plans. By understanding and operation and the resources of the GeoGebra software, with its intentional use to approach spatial and analytical geometry, use schemes were developed. On the other hand, the teachers' level of knowledge evolves by dealing with the software, especially by developing use schemes that give them the conditions to glimpse pedagogical practices that can be developed with their students. However, to advance alone in these levels of knowledge is not simple, and it ratifies the potential of collaborative contexts of widening possibilities, perspectives, views, experiences, and thus enable advances in the teachers' Mathematics TPACK levels, which will be observed in the processes of instrumentation and instrumentalization experiences and seen as promising for students in their pedagogical practices.

## 7 Concluding and Final Remarks

Throughout this article, we have discussed the actions and reflections of two

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Mathematics teachers while planning ambitious teaching practices in collaboration, involving the elaboration of animations and simulations on the GeoGebra software from the perspective of the Mathematics Exploratory Teaching, with the intention of understanding what do these teachers take into consideration and what do they mobilize or develop while planning these practices in collaboration. We have made use of the Instrumental Genesis (RABARDEL, 1995; 2011) and the Mathematics TPACK (NIESS *et al*., 2009) to analyze this trajectory.

In this sense, the teachers have highlighted the need to rethink their practice, leave the comfort zone, change, as gradual as it may be, the traditional teaching approaches, with less expositive classes so that students can be involved, develop their autonomy and their mathematical thinking. They have also highlighted the attention that must be given to the curriculum, being concerned about the mathematical contents that will be approached. In relation to the integration of technology, the teachers have reported that they do not use it that much, due to the knowledge they deem necessary, to the insecurity, and to the time required for the planning. This is caused by the lack of professional valorization policies, and it demonstrates the need for continuous and effective professional formation. To propose (and perform) ambitious teaching practices, the teachers deem necessary to instigate students to participate, lead them to reflect, instead of prioritizing mechanical and memorizing processes, which reflect a Mathematical notion based on performing calculations. On the other hand, they have reported that the collaborative planning has constituted an experience that favor dialogue, interaction, and cooperation. These aspects highlighted by the teachers are in line with the discussing that are being conducted about planning (SULLIVAN; ZEVENBERGEN; MOUSLEY, 2005; SUPERFINE, 2008; SULLIVAN *et al*., 2013; KILPATRICK, SWAFFORD; FINDEL, 2001, STEIN *et al*., 2008; CANAVARRO, 2011) with integration to technology (BITTAR, 2010; GAFANHOTO; CANAVARRO, 2014; BASNIAK; ESTEVAM, 2018) to implement teaching and learning processes.

In addition, by planning in collaboration, teachers mobilize the perception of the need for tasks of exploratory nature with technology, with clear and tangible objectives, in which teachers play the mediating role and students are the protagonists. In this process, they have developed the understanding of the need of anticipating their actions during planning, with discussions and exploration of strategies to involve the

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students during the class, as well as to facilitate the identification of difficulties and referrals to be done, which is a characteristic of ambitious teaching practices (OLIVEIRA, 2020).

With the design of simulations and animation in the GeoGebra software, during the process of their Instrumental Genesis, the teachers have developed schemes and started to visualize possible explorations and pedagogical uses for this kind of practice in Mathematics teaching. In the instrumentation and instrumentalization dimensions, they have shown enthusiasm and highlighted the simulations and animation as promising for understanding mathematical concepts and relations, as well as for the engagement of students. The have pointed out the initial difficulty they faced, but which they overcame as they got familiar with the artifact (by developing use schemes), adapting and glimpsing resolution strategies with pedagogical intentionality (developing usage schemes). They have thus started to consider the software intentionally, based on their possibilities for exploration, advancing in their Mathematics TPACK levels, showing evidence of *confirmation* levels.

Within this context, the perspective of the Mathematics Exploratory Teaching mediated by inquiry, reflection, communication, and collaboration has contributed to the reflections and actions performed by the teachers in the collaborative planning. Therefore, it constitutes an ambitious teaching practice by focusing on the student's thinking and aiming at complex and diverse learning, associated to the integration of technology. The dimensions that support Exploratory Teaching have favored the integration of technology, since the teachers had to anticipate referrals to be done with students, prioritizing investigative, communicative, reflexive, and collaborative aspects. Therefore, the animations and the two simulations developed in the GeoGebra software have been constituted as pedagogical proposals, with the potential to allow/promote mathematical learning. The reflections and actions, during the collaborative planning, have enabled the articulations of the knowledge components: content, technology, and pedagogy, and have promoted diverse possibilities regarding the artifact itself, as well as usage schemes.

Thus, by advancing in the knowledge levels and rethinking their practices, the teachers have evinced, in association, the dimensions of instrumentation and instrumentalization throughout the entire process, especially regarding the usage schemes related to the artifact (GeoGebra software), which went from instrument to a

certain mathematical (spatial and analytical geometry) and didactical-pedagogical content (elaboration of tasks and teaching plans).

Therefore, we conclude that, even though planning ambitious practices is a complex process, especially for teachers that are strange to this dynamic, it is possible to make them real, considering the interaction and cooperation that have occurred during the collaborative planning, so to overcome challenges and go beyond beliefs and notions related to the pedagogical process. By focusing on the integration of technology, thinking, and anticipating actions to be employed makes it possible to understand what is being considered and to observe the development and advance in the teachers' knowledge level of the Mathematics TPACK, associated with the instrumentation and instrumentalization, which lead teachers to (re)think their practices, considering it whole multifaced complexity.

**Acknowledgement:** We thank the participating teachers, the National Council for Scientific and Technological Development – CNPq, for the support granted (Proc. 440517/2019-2), and to PRPGEM and Capes for supporting this publication (PROAP/Capes/PRPGEM).

## References

BALL, D. L.; THAMES, M. H.; PHELPS, G. Content Knowledge for Teaching: What Makes It Special? **Journal of Teacher Education**, v. 59, n. 5, p. 389-407, nov. 2008.

BASNIAK, M. I.; ESTEVAM, E. J. G. Conhecimento tecnológico e pedagógico de matemática revelado por professores quando relatam suas práticas. **Amazônia**, Belém, v. 14, n. 31, p. 3-21, mar./out. 2018.

BASNIAK, M. I.; PIZZORNO, S. R. (Orgs.). **Perspectivas teórico-metodológicas em pesquisas que envolvem tecnologia na Educação Matemática**: o GeoGebra em foco. São Paulo: Pimenta Cultural, 2020.

BITTAR, M. A incorporação de um software em uma sala de matemática: uma análise segundo a abordagem instrumental. In: JAHN, A. P.; ALEVATTO, N. S. G. (Org.). **Tecnologias e Educação Matemática**: ensino, aprendizagem e formação de professores. Recife: SBEM, 2010. p. 209-226.

CANAVARRO, A. P. Ensino Exploratório da Matemática: práticas e desafios. **Educação e Matemática**, Lisboa, n. 115, p. 11-17, dez. 2011.

CANAVARRO, A. P.; OLIVEIRA, H. M.; MENEZES, L. Práticas de ensino exploratório da Matemática: Ações e intenções de uma professora. In: PONTE, J. P. (Org.). **Práticas Profissionais dos Professores de Matemática**. Lisboa: IEUL, 2014. p. 217-236.

CYRINO, M. C. C. T.; OLIVEIRA, H. M. Ensino Exploratório e casos multimídia na formação de professores que ensinam matemática. In: CYRINO, M. C. C. T. (Org.). **Recurso multimídia para a formação de professores que ensinam matemática:** elaboração e perspectivas. Londrina: EDUEL, 2016. p.19-32.

CYRINO, M. C. C. T.; TEIXEIRA, B. R. O Ensino Exploratório e a elaboração de um framework para os casos multimídias. In: CYRINO, M. C. C. T. (Org.). **Recurso multimídia para a formação de professores que ensinam matemática:** elaboração e perspectivas. Londrina: EDUEL, 2016. p.81-100.

ESTEVAM, E. J. G. *et al*. Ensino Exploratório de Matemática e 0Tecnologias Digitais: a elaboração da lei dos senos mediada pelo software GeoGebra. **Acta Scientiae**, Canoas, v. 20, n. 3, p. 342-358, mai./jun. 2018.

GAFANHOTO, A. P; CANAVARRO, A. P. A. adaptação das tarefas matemáticas: Como promover o uso de múltiplas representações. In: PONTE, J. P. (Org.). **Práticas Profissionais dos Professores de Matemática**. Lisboa: IEUL. 2014, p. 113-134.

JAWORSKI, B. *et al*. Mathematics teachers working and learning through collaboration. In KAISER, G. (Ed.). **Proceedings of the 13th International Congress on Mathematical Education**. ICME-13 Monographs. Hamburg: Springer, 2017. p. 261-276.

KILPATRICK, J.; SWAFFORD, J.; FINDELL, B. (Eds.). **Adding it up**: Helping children learn mathematics. Washington, DC: National Academy Press, 2001.

KOEHLER, M. J.; MISHRA, P. What is technological pedagogical content knowledge? **Contemporary Issues in Technology and Teacher Education**, v. 9, n. 1, p. 60-70, 2009.

KOEHLER, M. J.; MISHRA, P.; CAIN, W. What is technological pedagogical content knowledge (TPACK)?. **Journal of Education**, Boston, v. 193, n. 3, p. 13-19, 2013.

KRAINER, K. Team, communities & networks. **Journal of Mathematics Teacher Education**, v. 6, n. 2, p. 93-105, jun. 2003.

MISHRA, P.; KOELHLER, M. J. Technological pedagogical content knowledge: a framework for teacher knowledge. **Teachers College Record**, Malden, v. 108, n. 6, p. 1017-1054, 2006.

NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS. **Principles and standards for school mathematics.** Reston, VA, 2000.

NIESS, M. L. *et al*. Mathematics teacher TPACK standards and development model. **Contemporary Issues in Technology and Teacher Education**, v. 9, n. 1, p. 4-24, 2009.

OLIVEIRA, H. M. **A prática e as representações da prática na formação do (futuro) professor de Matemática**. Publicado no Canal CIEspMat no Youtube. Disponível em: https://www.youtube.com/watch?v=Zl5_Njv0X10. Acesso em: 15 out. 2020.

PALIS, G. L. R. O conhecimento tecnológico, pedagógico e do conteúdo do professor de Matemática. **Educação Matemática Pesquisa**, São Paulo, v. 12, n. 3, p. 432-451,

2010.

PONTE, J. P. Gestão curricular em matemática. In: GTI (Ed.). **O professor e o desenvolvimento curricula**r. Lisboa: APM, 2005. p. 11-34.

RABARDEL, P. **Les hommes et les technologies:** aproche cognitive des instrumentns contemporains. Paris: Armand Colin, 1995.

RABARDEL, P. **Los hombres y las tecnologías**: visión cognitiva de los instrumentos contemporáneos. Tradução de Martín Eduardo Acosta Gempeler. Colombia: Universidad Industrial de Santander, 2011.

ROGERS, E. M. **Diffusion of innovations**. New York: Free Press, 1995.

SANTOS, L. P.; BASNIAK, M. I. Construção de cenários animados por alunos com indicativos de altas habilidades/superdotação como atividade Matemática. **Revista de Ensino de Ciências e Matemática**, São Paulo, v. 12, n. 3, p. 1-20, 2021.

SERRAZINA, M. L. M. Conhecimento matemático para ensinar: papel da planificação e da reflexão na formação de professores. **Revista Eletrônica de Educação**, São Carlos, v. 6, n. 1, p. 266-283, mai. 2012.

SHULMAN, L. S. Knowledge and teaching: Foundations of the new reform. **Harvard Educational Review**, Cambridge, US, v. 57, n. 1, p. 1-22, 1987.

STEIN, M. K. *et al*. Orchestrating productive mathematical discussions: five practices for helping teachers move beyond show and tell. **Mathematical Thinking and Learning**, v. 10, n. 4, p. 313-340, 2008.

SULLIVAN, P. *et al*. Process and priorities in planning Mathematics teaching. **Mathematics Education Research Journal**, v. 25, n. 4, p. 457-480, Dec. 2013.

SULLIVAN, P.; ZEVENBERGEN, R.; MOUSLEY, J. Planning and teaching mathematics lessons as a dynamic, interactive process. In: CHICK, H. L.; VINCENT, J. L. (Eds.). **Proceedings of the 29 th Conference of the International Group for the Psychology of Mathematics Education**. Melbourne: PME, 2005. p. 249-256.

SUPERFINE, A. C. Planning for mathematics instruction: a model of experienced teachers' planning processes in the context of a reform mathematics curriculum. **The Mathematics Educator**, Georgia, v. 18, n. 2, p. 11–22, 2008.

TROUCHE, L. An instrumental approach to mathematics learning in symbolic calculators environments. In: GUIN, D.; RUTHVEN, K.; TROUCHE, L. (Eds.). **The didactical challenge of symbolic calculators:** Turning a computational device into a mathematical instrument. New York: Springer, 2005. p. 137-162.

VALENTE, J. A. Diferentes usos do computador na educação. **Em Aberto**, Brasília, v. 12, n. 57, p. 1-16, jan./mar. 1993.