



Mathematics Education in the light of critical theory emerging from the Frankfurt School: discourses and meanings

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Abstract: This research realizes a mapping of the Theses and Dissertations present in the catalog of Theses and Dissertations of CAPES and the Brazilian Library of Theses and Dissertations, proposing a quantitative and qualitative analysis. Next, we use Wiggershaus (2002) as the central axis for obtaining the meanings produced by the Frankfurt School. Throughout the text, we use the theoretical and methodological bias of Discourse Analysis in the light of Orlandi (2012, 2017). The general objective of this research is to search for senses produced from the conceptions of critical theory in research in Mathematics Education and the Frankfurt School. As a result, the research converges to a problematization of the rationality potentially developed by the teaching of mathematics, when supported exclusively in formulas and relations, can gradually veto critical rationality.

Keywords: Curriculum. Emancipation. Dialektik der Aufklärung. Teaching.


La Educación Matemática a la luz de la teoría crítica surgida de la Escuela de Frankfurt: discursos y significados


Resumen: Esta investigación realiza un mapeo de las Tesis y Disertaciones presentes en el catálogo de Tesis y Disertaciones de CAPES y en la Biblioteca Brasileña de Tesis y Disertaciones, proponiendo un análisis cuantitativo y cualitativo. En la secuencia, utilizamos a Wiggershaus (2002) como eje central para obtener los significados producidos por la Escuela de Frankfurt. A lo largo del texto, estamos atravesados por el sesgo teórico y metodológico del Análisis del Discurso a la luz de Orlandi (2012, 2017). El objetivo general de esta investigación es buscar los sentidos producidos desde las concepciones de la teoría crítica en la investigación en Educación Matemática y la Escuela de Frankfurt. Como resultado, la investigación converge a una problematización de la racionalidad potencialmente desarrollada por la enseñanza de las matemáticas, que cuando se apoya exclusivamente en fórmulas y relaciones, puede ir vetando una racionalidad crítica.

Palabras clave: Reanudar. Emancipación. Dialektik der Aufklärung. Enseñando.

Educação Matemática à luz da teoria crítica emergente pela Escola de Frankfurt: discursos e sentidos

Resumo: Este artigo realiza um mapeamento das Teses e Dissertações presentes no catálogo de Teses e Dissertações da CAPES e na Biblioteca Brasileira de Teses e Dissertações, propondo uma análise quantitativa e qualitativa. Na sequência, utilizamos de Wiggershaus (2002) como eixo central para obtenção dos sentidos produzidos pela Escola de Frankfurt. Ao longo do texto, somos atravessados pelo viés teórico e metodológico da Análise de Discurso à luz de Orlandi (2012, 2017). O

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objetivo geral desta pesquisa reside na emissão de sentidos produzidos a partir das concepções de teoria crítica nas pesquisas em Educação Matemática e na Escola de Frankfurt. Como resultado, o texto converge para uma problematização da racionalidade potencialmente desenvolvida pelo ensino de matemática, que quando apoiado exclusivamente em fórmulas e relações, pode vetar gradativamente uma racionalidade crítica, gerando a aceitação de uma realidade em que os próprios alunos e alunas, como sociedade, constituem e sobre a qual nada podem.

Palavras-chave: Currículo. Emancipação. Dialética do Esclarecimento. Ensino.

1 Introduction

The present research constitutes a fragment of the master's degree dissertation in progress, carried out by the authors, selected from the incidence, influence and relevance of the critical theory promoted by the Frankfurt School in overlap with the investigations of Mathematics Education. With the main objective of highlighting the meanings produced from the conceptions of critical theory in research in Mathematics Education and by the Frankfurt School, in a direct way, converging on the following guiding question: What are the main meanings present in research in Mathematics Education and in the Frankfurt School that underlie the conceptions of critical theory?

In order to achieve the proposed objective, it was carried out, at first, a mapping of the Theses and Dissertations available in the Brazilian Digital Library of Theses and Dissertations (DLTD) and in the CAPES Theses and Dissertations Catalog (CAPES TDC) from prescribers that involve the critical theory and the teaching of mathematics. In the sequence, Wiggershaus (2002) was used as a central axis to establish the meanings produced directly by the Frankfurt School in the establishment of the first critical theory of society, projecting it in Mathematics Education.

Throughout this discussion, the curriculum is understood from the tensions that arise between nature and the form that knowledge assumes in the process (training course) of teaching and learning. We hope that this research contributes to the delimitation of the meanings that accompany the conceptions of emancipation and critical formation, so mentioned in the academic and curricular environment, which are often devoid of the direction or delimitation of the meanings that accompany these conceptions.

2 Methodological Path

We propose the use of Discourse Analysis from the perspective of the author

Eni Puccinelli Orlandi with a focus on Michel Pêcheux for theoretical and methodological foundations. This field of study is accompanied by four components that structure and relate to the articulation of Discourse Analysis in this text, namely: discourse, meaning, subject and ideology.

From the perspective of Orlandi (2012, 2017), discourse differs from the message as a transmission of information, comprising language beyond a code to be decoded. It is a process of meaning, identification of the subject, argumentation, subjectivation, construction of reality, incompleteness, among others. Understanding that “language serves to communicate and not to communicate. [...] Hence the definition of discourse: discourse is the effect of meanings between speakers” (ORLANDI, 2012, p. 19). This discourse, in the author's view, is taken from the relation of meanings, and under this notion, every discourse is related to other discourses, approaching this(these) discourse(s) for a broad and continuous discursive process. According to the author, “there is, therefore, no absolute beginning or end point for discourse. A saying is related to other sayings realized, imagined or possible.” (ORLANDI, 2012, p. 37).

This dialogue is constituted in its meanings, and with this Discourse Analysis approach “we can say that meaning does not exist in itself, but is determined by the ideological positions put into play in the socio-historical process in which words are produced” (ORLANDI, 2012, p. 40). In this way, the same word can vary in meaning according to the position of the speaker who uses it, and, in the author's point of view, it is up to the discourse analyst to understand the process of production of meanings, its relationship with ideology and the establishment of regularities in the functioning of discourses. For the author, this meaning is taken from the relationship between the sayings, “and it is through this relationship, this superposition, this transference (metaphor), that significant elements come to confront each other, so that they are clothed in meaning” (ORLANDI, 2012, p. 42).

The senses are thus not predetermined by properties of language. They depend on relationships constituted in/by discursive formations. However, it is necessary not to think of discursive formations as homogeneous blocks functioning automatically. (ORLANDI, 2012, p. 42)

For the author, neither language, nor senses nor subjects are transparent; in general, “the subject of language is decentered, as it is affected by the real of language

and by the real of history, [...]. This amounts to saying that the discursive subject functions through the unconscious and through ideology” (ORLANDI, 2012, p. 22). In this conception, the very words of our daily life are already loaded with meaning that, in many cases, we are unaware of their constitution and, however, have meaning in us and for us. With this approach, we take the notion of subject when we consider a set of meanings that constitute a discursive process that is neither physical nor objective, and is determined from the discourses, meanings and interdiscourses that constitute it, that precede it and that are not present.

When we think of ideology in the midst of the three components already mentioned, “we can start by saying that ideology is part of, or rather, is the condition for the constitution of the subject and the meanings. The individual is challenged as a subject by ideology so that he can produce the saying” (ORLANDI, 2012, p. 44). From the author's point of view, the elementary ideological effect arises from the constitution of the subject; more than that, this ideological work is taken as a work of memory and oblivion that considers discourses and meanings in their historical materiality.

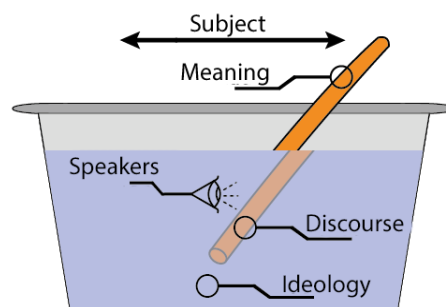
Ideology, in turn, in this way of conceiving it, is not seen as a set of representations, as a worldview or as a concealment of reality. Indeed, there is no reality without ideology. As a significant practice, ideology appears as an effect of the subject's necessary relationship with language and with history so that there is meaning. (ORLANDI, 2012, p. 46).

It is worth noting that Discourse Analysis does not work from the point of view of reflection, but of understanding, considering that “in the text itself, in its constitution, there are gestures of interpretation that show the position or positions of the subject who produced it. Understanding then means explaining the interpretation gestures made by the subject, gestures that are inscribed in the text.” (ORLANDI, 2017, p. 171).

The very establishment of the term Discourse Analysis suggests a meaning-relation which focuses on the analysis of a discourse that is not unique or regular, that is, it is not independently unified. Therefore, the analysis of this(these) discourse(s) goes through the constitution of a “memory” (ORLANDI, 2012, p. 29) intrinsic to the discourse, establishing interdiscursive aspects that can be defined as “what speaks before, in another place, independent. In other words, it is what we call discursive memory: the discursive knowledge that makes all saying possible and that returns in the form of the pre-constructed” (ORLANDI, 2012, p. 29). The author calls the elements

of discursive memory as figures of the “already said” (ORLANDI, 2012, p. 30), that sustain the possibility of saying everything. In other words, it is not a matter of analyzing “the discourse”, but the set of discourses that accompany and structure language in its various forms and in its various means of being present and absent. Inspired by Uchôa-Fernandes (2019), we propose the construction of a representation of the articulation performed by elements of DA in the bias of Orlandi (2017), shown in Figure 1.

Figure 1: Metaphor of articulation and functioning of five components of biased discourse analysis in Orlandi (2017)



Source: Authors (2022).

This figure positions the five elements involved in the DA process, explaining, through a metaphorical means, its working device. Ideology is represented by water and meaning is characterized by the object partially submerged in the container. The apparent distortion of the object when submerged in water, through the physical phenomenon known as refraction, illustrates the displacement in the meaning. The submerged part is displaced from the object which represents the discourse, which despite being apparently distorted by ideology, continues to have a certain alignment and coherence with the real object, that is, with the positioned meaning. The speech starts to be observed by the speakers, who are, metaphorically, submerged in the water, these speakers are under the effect of refraction (ideology), so that they are unable to observe the meaning directly. Furthermore, they are affected by different processes in this ideological atmosphere, according to their context in the environment. The subject is taken as responsible for the positioning and selection of the object (meaning) in the pool, also symbolizing the variation of its displacement in relation to the proposition of the sender of meaning (speaker/author). Thus, the discourse analyst uses, according to Orlandi (2012, 2017), the tools proposed by the Franco-Brazilian DA for the construction of meanings, so that meaning is not presented as something unified, fixed. In addition, it is worth noting that, in the DA bias, the analyst is also immersed in and under the effects of ideology, demanding tools built from paraphrase

and polysemy for the elaboration of meanings.

3 Analysis and discussion of research in Mathematics Education

In this topic, the mapping consists of an investigation based on the number of Theses and Dissertations available in the Brazilian Digital Library of Theses and Dissertations (DLTD) and the CAPES Theses and Dissertations Catalog (CAPES TDC), using these Platforms on August 30, 2021, from descriptors directed to the teaching of Mathematics, the curricular field and the use of critical theory as a teaching perspective or world view that permeate the first two descriptors. The terms used, followed by their quantities, can be found in Table 1:

Table 1: Curriculum Research in Mathematics Teaching with the incidence of critical theories in DLTD and CAPES TDC.

Searched terms	Number of productions found	
	DLTD	CAPES TDC
Critical theory, Mathematics and Curriculum	14	7
Critical training, Mathematics and Curriculum	11	9
Frankfurt School, Mathematics and Curriculum	0	0
Habermas, Mathematics and Curriculum	10	8
Adorno, Mathematics and Curriculum	6	5
Horkheimer, Mathematics and Curriculum	0	1
Marcuse, Mathematics and Curriculum	0	0
Critical thinking, Mathematics and Curriculum	23	16
Critical education, Mathematics and Curriculum	6	5
Critical mathematics, Curriculum	30	30
Total (with possible repetitions):	100	81

Source: Authors (2022).

After the exclusion of searches with repetition between the Platforms and search terms, and the selection of works exclusively focused on Mathematics Education, of the 181 productions initially found, 81 remained, of which only one was unavailable for consultation on the Platforms and at the University of origin, making it impossible to read the abstract, and consequently, the analysis of some dimensions. In order to analyze the selected works, we initially carried out a floating reading of the abstract, identifying the basic information. Subsequently, we framed the information mobilized in a set of categories for the quantitative analysis, and later, in a set of statements in the light of Discourse Analysis in Orlandi's bias. (2012, 2017).

The quantitative analysis categories are constituted by: Region, Type (dissertation or thesis), Origin (Public University or other Universities), Dialogue with the curriculum (Sacristán Categories (2000)), Teaching modality (Basic Education, Graduation, etc.), Defense date and main theorists mentioned.

The first factor mentioned concerns the region of mapped research, which accounted for two works defended in the North region (2%), eleven in the Northeast region (14%), seven in the Midwest region (9%), fifty-one in the Southeast (64%) and, finally, nine studies defended in the South region (11%). Involving 61 Dissertations (76%) and 19 Theses (24%).

As for the origin of these researches, sixty-three come from the Public University (79%), while seventeen come from the other Universities (21%). Thinking about the approach of the works, the majority focuses on the theoretical field, diverging from pedagogical applications and executions, that is, sixty-four studies involved theorizations on analyzes of pedagogical practices (82%) and sixteen studies involved pedagogical applications and actions (18%).

Using the categories proposed by Sacristán (2000), we obtain six categories classified and arranged as follows:

- i. Prescribed Curriculum: Documentary sphere that orders, references, legalizes and controls teaching materials and teaching practices. Including 39% of searches.
- ii. Curriculum Presented to Teachers: Interpretation performed by teachers on the sphere of the Prescribed Curriculum and its implications. Engaging 4% of searches.
- iii. Curriculum Modeled by Teachers: Teacher's translation of the two previous categories, delimiting, from these, the boundary between theory and preparation for school practice. Comprising 15% of searches.
- iv. Curriculum in Action: Effectiveness of school practices. Does not involve any research.
- v. Realized Curriculum: Understands the effect of the previous category on educators, students and others. Circumscribing 39% of searches.
- vi. Evaluated Curriculum: It concerns the control and monitoring of both categories, with a certain focus on the previous category. Engaging 3% of searches.

We obtain a predominance of researches that focus on a theoretical field, through the Prescribed Curriculum, and proposing from theoretical constructions to didactic propositions for others to use, as well as there is a convergence of research directed to the impacts and implications of school practices, thinking about their effects and their purposes. Both the predominance and the convergence are in line, both in the Public Universities and in the other Universities.

Regarding the teaching modalities, fifty were directed to Basic Education (62%), eight to Youth and Adult Education (10%), nine to Graduation (11%), six to Continuing Education (8%) and seven works for the other training processes (9%).

The main theorists cited in the summaries during the mapping are configured, respectively, by: Ole Skovsmose (Basing Critical Mathematics Education), Paulo Freire (Basing dialectical relationships and ontological conceptions) and Jürgen Habermas (Basing mainly on the Theory of Communicative Action). Using these and other varied foundations, research often addresses expressions and factors such as: Reflection, Possibilities, Understanding, Problematization, Active Methodologies, Rationality, Analysis, Emancipation, Discourse, Co-authors, Dialogue, Critical Citizen, Contextualization, Daily Life, Re-signification, Integral formation, Heterogeneity and Power.

We now propose a qualitative analysis of the data. To do so, we list specific targeting statements that represent the research, that is, we use the metaphorical effect, proposed by Orlandi (2012), as a semantic phenomenon in textual substitutions, sliding meanings, in a process that synthesizes and produces meanings, from the titles, abstracts and objectives, for the constitution of a subject common to all the researches encompassed in the mapping, accompanied by its “uplifting speeches” (Orlandi, 2017).

Thinking about the interpretation, this effect points us to the “double and one speech”. This duplicity makes referring one speech to another speech so that it makes sense; in Psychoanalysis this involves the unconscious, in Discourse Analysis it also involves ideology. This duplicity, this misunderstanding is dealt as the fundamental ideological question, thinking about the material relationship of discourse to language and ideology to the unconscious. (ORLANDI, 2012, p.80-81)

Using this procedure, it was possible to identify five predominances, called: a) Problematization of learning — Critical of the traditional/technicist/classical/banking

model; b) Problematization of Teaching — Possibilities and teaching approaches; c) Decentralization of evaluation; d) Lack and need of guidance for teacher training; e) Critical Mathematics Education, based mainly on Ole Skovsmose, who makes a remarkable approximation of critical theories with practice.

Based on these predominances, we structured four general targeting statements, they are:

S1: Involvement of research that problematizes learning in traditional, technical, classical or banking models. Proposing reflections on the ideas of emancipation and resignification of educational processes from the perspective of students trained by this system. Comprising 20 surveys.

S2: Guided by research that problematizes teaching, thinking about possibilities and limitations from the perspective of the teacher involved in the school environment. Accompanied by the search for ruptures in the training process, having as one of the principles, the importance of teacher training. Involving 23 surveys.

S3: Directed by the emphasis on a curricular resignification, questioning the areas that, directly or indirectly, structure the training process for an optimized process, centered on evaluations, on the individualization of the individual and that deny the incompleteness of the human being in search of a totality that best meets to neoliberal precepts. This resignification can occur both through the curricular proposition and through the identification of the effects of the curriculum on the people involved in the training process. Directed by 21 surveys.

S4: It encompasses research that seeks didactic formulations, with the objective of theorizing didactic propositions aiming at the possibility of carrying out practices in the light of critical theories. Presenting formative possibilities along the lines of critical theories and that also seek to provide “examples to be followed” by the teaching community. These propositions are rarely realized in practice. Guided by 16 surveys.

From the set of statements and the reading of the works, we project some conclusions. At the outset, we emphasize that research seeks *innovation in the educational process*, guided by the use of diversified teaching methodologies and processes, encompassing trends in Mathematics Education, such as: mathematical modeling, the use of digital technologies, problem solving methodology, among others. These trends are located in a theoretical dimension, even when they focus on the

school reality.

Next, we consider the predominance of research that highlights the importance of student *rationality developed throughout the educational process*, accompanied by teaching propositions based on dialogue and emancipation, to the detriment of inculcating rules.

Subsequently, the research points to a *resignification of the teaching and student figures for the use and experiences of educational approaches outside the traditional molds* in favor of the development of students' critical awareness and the appreciation of the teaching profession.

In the sequence, we have the *concern with education in a social bias*, projecting the school as a means responsible for the maintenance of social problems and, consequently, as capable of making ruptures. Subsequently, research focuses on *education as a means of developing political rationality*, understanding it as a right for all and distancing itself from neoliberal ideals and the exploitation of individuals.

Due to the presence of research in curricular incidences, according to Sacristán's (2000) categories, we emphasize that the research has a strong concern with the prescribed and carried out curriculum, in a way that *problematizes the curricular structuring and analyzes the effect of curricular applications in the school environment*, strongly guided by the theoretical bias. Due to the absence of curricular incidence, we also highlight the *gap between critical theories and the application of practices in the Brazilian reality*, due to the fact that no research fits strictly into the category of curriculum action.

4 Analysis and discussion of the Frankfurt School (FS) and Mathematics Education

At this moment, the central reference is constituted by a dialogue with the work of Rolf Wiggershaus (2002), which provides an important historical overview from a detailed perspective of the figures that were part of the FS movement, mentioning the central principles that converge these figures, and the political and cultural dynamics involved by the movement.

For the construction of the work, Wiggershaus uses his direct contact with Adorno, Horkheimer, Marcuse, as well as several interviews with Habermas,

Abendroth, Marie Iahoda, Leo Löwenthal, among other figures who were part of or crossed the history of the FS.

But he also wanted, with his first great essay on music, [...] to demonstrate the experience that in capitalism all roads were closed, that everywhere there was a wall of glass that prevented men from arriving at a true life. (WIGGERSHAUS, 2002, p. 38).

The excerpt above refers to a letter written by Adorno, highlighting the importance of thinking about the different (im)possibilities that we have throughout life. At the same time, we can think about the responsibilities that the educational field assumes in the ways we live, how we organize processes, and how citizens are formed. Such elements imply in a Mathematics Education attentive to the awareness of the students, in a teaching model that corroborates the perception of the limitations and problems existing within society, and of mathematics itself as a field of study. Charlot (2020) points out that the first pedagogical models were allied to an unquestionable knowledge and a teaching figure that represented the perfection of this standardized science, a paradigm that goes through a process of deconstruction along the social transformations, according to the author. Thinking about this paradigm shift in Mathematics Education leads us to a series of elements to be reviewed in the teaching and learning process, some of which are: teacher figure, organization of the school space, conception of mathematics as a science, teaching methodologies, evaluative models, conception of school, knowledge and the relationship between mathematics and society. We intend to revisit some of these factors throughout this text.

Among the existing divergences between the FS theorists, there are central points that unite them, such as “disobedience to tradition, a desacralization of naturalized knowledge as the only possibility to account for the real and which presents itself as the only possibility to constitute it” (WIGGERSHAUS, 2002, p. 10). Using these central points, the importance of suspending everything that is considered as absolute truth, as something normalized or even traditional, and starting to look for alternative possibilities, process improvements, interpretation of the world from other perspectives and even, “bets on the emphasis on contradiction and negativity, as a daily exercise in lucidity. [...] put in abeyance, sub judice, any judgment on the world” (WIGGERSHAUS, 2002, p. 11), observing the individual from two divergent extremes, that of

emancipation and that of exploitation. Thinking about emancipation and exploration within Mathematics Education leads us to consider a teaching model that encourages questioning and that is attentive to the process that is carried out in the classroom. One of the viable alternatives to highlight the process, understanding the formulation as a possible product, is through Mathematical Modeling in a socio-critical perspective and through Critical Mathematics Education.

Even at the beginning of the FS, the common experience was marked by the fact that “no assimilation was enough to be able to be sure of belonging to society” (WIGGERSHAUS, 2002, p. 36), also emphasizing that, in order to be convenient, these assimilations and these studies must necessarily be limited to precise social groups and specific times, in view of the specificity that permeates these issues.

We propose a look at the common experience demarcated by FS from metaphorical effects that “[...], in the discursive perspective, is “a word for another”. The metaphor is responsible for the sliding of meanings, for the drift, for the transference” (ORLANDI, 2017, p. 154). The author emphasizes that the exchange of words makes it possible to change the effect of meanings. Let's see the slips built under the common experience of FS in the excerpt: *No assimilation was enough to be sure of belonging to society*. Follow the buildings:

- i. *Knowledge is necessary to protect yourself from belonging to society;*
- ii. *Knowledge is necessary to avoid violations of social belonging;*
- iii. *Knowledge prevents violations of social belonging;*
- iv. *Knowledge creates ruptures in social belonging;*
- v. *Without knowledge, we maintain the social dynamics.*

The metaphorical effect constructed by the first slip of meanings (i) explains the attribution of knowledge as an instrument of social defense and, in addition, as a necessity for belonging to society, that is, it contemplates all those involved by society and that, inevitably, need to protect themselves from this interaction. The following slide (ii) shows that the need for protection carries the existence of a violation caused by society from social interaction, attributing to knowledge (iii), the social mission of protecting the violated individuals. Understanding as a possibility, the realization of social ruptures based on knowledge (iv). From the unsaid (v), we obtain that the absence of this knowledge results in the maintenance of the social system, that is, when we deny access to this type of knowledge, we facilitate social maintenance.

The Institute's constitutions and research began to demand a collaboration between philosophers, sociologists, political scientists, among others, that "blinders need to fall, whether imposed by specialty or by a certain tradition of nation or school" (WIGGERSHAUS, 2002, p. 481). The author even points out that, in one of his speeches, Horkheimer would use critical theory as a deontological rule³ for sociologists, based on the theme of social change and guided by the "hope" factor. Thinking about the formative process of school Mathematics and the school site as an organ of social maintenance, what would make the blinders fall? This questioning begins the approach of this critical theory from an educational perspective, that is, from a formative perspective that expands visions, deconstructs unifications and exclusively reproductive processes.

In the midst of the existing conflicts between FS intellectuals and intellectuals adept to positivist sociology, different paradigms and models of science were established, directing the FS "towards critically oriented sciences, the picture [...] made us think that reality was captured under the aegis of the interest in the destruction of relationships of dependence whose action is objective, but which are, in principle, modifiable" (WIGGERSHAUS 2002, p. 611). The author also emphasizes that this science is motivated by an emancipatory interest of the individual, using these conceptions to oppose conservatism and to question normality and sexuality itself, putting in suspension, as in other philosophical spheres, the conceptions of normality and the behavior of people, assuming a critical position, demarcated by the renunciation of any doctrine that considers anthropological invariants.

The empirical research carried out by the FS "has the mission of clarifying vigorously and without sublimating it the objectivity of what social reality is, which is essentially foreign to the individual and even to the collective consciousness" (WIGGERSHAUS, 2002, p. 489), using a method attentive to social objectivities and subjectivities, which, through Adorno, was opposed to the concepts of infrastructure and superstructure, proposed by Marx, directing his studies to the relations between the economic, psychic and cultural spheres. Also highlighted is a

[...] concrete meaning of the resolution to "take into account the nature of man": the alliance of contemplation and drives. At the last moment, thought deviated from the path of domination of subjective reason over objective reason — the path of subjectivation, formalization, instrumentalization, desubstantialization of reason — and, as an organ of nature, it rose up against

the instrument of the dominating spirit. (WIGGERSHAUS, 2002, p. 378).

Understanding the subjective processes, formalized and even instrumentalized as tools of resistance against the processes of domination, thinking about these elements in the teaching of Mathematics, we begin to face two relevant dimensions. The first dimension, of formalization, of the rule and of the instrument, and the second a dimension that comprises subjectivations and the process of desubstantialization of reason, putting in abeyance everything that is true, absolute or unified.

One of the problematizations described by Wiggershaus (2002), when referring to Habermas' theorization, was the impasse of a process of industrialization and social constitution, which comprises an increasingly political society and, simultaneously, more apolitical citizens, establishing two main spheres that encompass a manipulative politicization and a true politicization, generating an apparent democracy and a true democracy, respectively. The author also highlights that Habermas emphasizes the ideal of democracy directly to critical theory, directing the importance of active political participation regardless of social inequalities. These problems lead us to think about the importance and relevance of an educational process, which, in a democratic and egalitarian perspective, presupposes a formation of law, which, within the possibilities, constitutes a person in the realm of true politicization, that is, of a citizen endowed with the ability to *(re)position themselves, argue and place themselves critically* at the forefront of the different forms of language that involve the society in which they are inserted. We can use these three elements to constitute an educational process that encompasses a teaching of mathematics that forms people capable of a democratic society, dealing with information in a critical and argumentative way, as this information becomes knowledge.

One of the main works of FS, pointed out by Wiggershaus (2002) as one of the cradles of critical theory, is given by the *Dialektik der Aufklärung* (DdA) (Dialectic of Enlightenment), written by Adorno in partnership with Horkheimer, published in 1947, in German. The work problematizes *the domination factor* under social individuals, to which, in a panoramic way, we will pay attention to the two terms that constitute the title of the text, seeking initial meanings.

DdA proposes a thought in action as a movement to free people from the fear of being emancipated and masters of themselves, presenting an initial thesis that "all

civilization until today was made of Lights imprisoned in mythical immanence, which, by themselves, suffocated at birth any possibility of escaping from mythical immanence.” (WIGGERSHAUS, 2002, p. 358). It is worth noting that the term Lights used by the author constitutes itself as a synonym for the word Enlightenment, coming from the translation of the term *Aufklärung*. Even with this expression, the very:

[...] DdA gave the impression of bringing together, forcibly, two concepts of *Aufklärung*: in one, the *Aufklärung* pursued a purpose, to place men at the level of masters, and once this end was achieved, it made the light of radical evil shine on the land completely dominated by the *Aufklärung*; in the other, the *Aufklärung* aimed to appease this claim to domination, and its realization meant the renunciation of power; in short, the first impression was that the Lights destroy themselves and can save themselves. But, after a second examination, the unconfessed thesis appears in the background: the false Lights prevents the victory of truths, a victory that would be the only one to be able to preserve the fatal consequences of the false Lights. (WIGGERSHAUS, 2002, p. 362).

Intending to further explore the DdA as a term *Dialectics of Enlightenment*, we first seek the definition of dialectic, in the sense of Hegel, which Charlot (2020, p. 50) defines as “when two concepts, at the same time, are contradictory and that is not possible to think of one without the other occurring, there is a dialectical relationship between them, in Hegel's sense”.

As for the definition of the term *enlightenment*, explained in the DdA, it is given by:

[...] enlightenment has always pursued the aim of ridding men of fear and investing them in the position of masters. [...] The enlightenment program was the disenchantment of the world. [...] To disenchant the world is to destroy animism. [...] What does not submit to the criterion of calculability and utility becomes suspect for enlightenment. [...] In advance, enlightenment only recognizes as being and event what is allowed to be captured by the unity. (ADORNO; HORKHEIMER, 1997, p. 5-6).

This conception of enlightenment is taken from the problematization about the incompleteness of the human being, in its historical, scientific, political, religious aspects, ..., considering that the superiority of the human being is found in knowledge. This highlights the difficulty of dealing with the uncertainties of life, which are often attributed to the religious sphere and gradually attributed to the scientific field itself. This science, when taken in the form of enlightenment, is directly affiliated with mathematics itself and in its forms of homogenization of factors and processes, this leads us to think of the place occupied by the teaching of mathematics as an element,

which in many cases, performs an erasure from the dialectical proposition, that is, from the existing tensions to obtain relationships and standardizations. One of the consequences of this thought leads us to highlight the evaluative factors that involve the teaching and learning of mathematics, emerging the importance of valuing these processes, the tensions of mathematics and its relationship with society.

Authors Adorno and Horkheimer (1997, p. 7) claim that “the awakening of the subject has as its price the recognition of power as the principle of all relationships”. Thinking about this awakening of the subject, from the perspective of Discourse Analysis, reminds us of the importance of training attentive to constitutive silencing, that is, to the erasure of words, since “power relations in a society like ours always produce the censorship, in such a way that there is always silence accompanied by words” (ORLANDI, 2017, p. 82). With this approach, we can think about the place of investigation,

When, in the mathematical procedure, the unknown becomes the unknown factor of an equation, it is characterized by this as something that has been known for a long time, even before any value is introduced. Nature is, before and after quantum theory, what must be learned mathematically. (ADORNO; HORKHEIMER, 1997, p. 14).

This excerpt from the authors problematizes the developed rationalities; even claiming the importance of the universality of knowledge, they emphasize that the restriction and validation in all fields of science, through Mathematics, need to be reviewed. This also leads us to think about the relationship between Mathematics Education, that is, the way in which this knowledge is taken and approached in the formation of individuals, and the role played by this sphere of knowledge as something that unifies, validates and defines the knowledge taken as scientific and irrefutable. Boosting the fact that “men wait for this dead-end world to be set on fire by a totality that they themselves constitute and on which they can do nothing” (ADORNO; HORKHEIMER, 1997, p. 16).

Next in DdA, the authors still claim that:

Only the mediation, by which the empty sensory data brings thought to all the productivity of which it is capable and by which, on the other hand, thought abandons itself without reservation to the impression that overwhelms it, overcomes the morbid solitude in which it is trapped whole nature. *It is not in the certainty unaffected by thought, nor in the preconceptual unity of perception and object, but in their reflected opposition, that the possibility of*

reconciliation is shown. *The distinction occurs in the subject who has the external world in his own consciousness and yet knows it as another.* That is why this reflection, which is the life of reason, takes place as a conscious projection. (ADORNO; HORKHEIMER, 1997, p. 89, our italics).

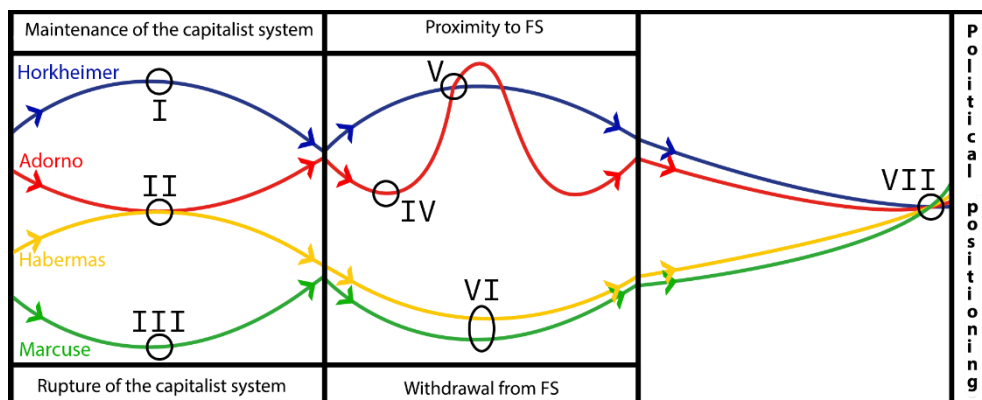
Thinking about Mathematics Education from these points, we converge on reflection as the basis of reason and mediation as fundamental for the establishment of a configuration of thought, of a *modus rationem*, that overcomes unity, that is, the overcoming of a set of closed and *taken-for-granted* definitions, generalized and universal, for a *modus rationem* directed by the *search for truth*, that is, by the oppositions themselves reflected and by the establishment of relationships that direct the individual to an awakening of conscience.

Considering the possibility of using other symbolic approaches that refer to the four outstanding theorists of FA in the light of DA, we propose a new approach “because an event that says an event, with more reason, they refer to the same “fact”, but do not build the same meanings.” (ORLANDI, 2017, p.58)

However, we propose the use of Figure 2 as a vehicle of analysis that comprises a fragment of the discourses of the theorists addressed, since “an event does not stop producing meanings” (ORLANDI, 2017, p. 58). Dealing with these processes of signification, we can say that:

It is linguistically describable as a series of possible drift points offering room for interpretation. It is always susceptible of being/becoming another. This place of the other utterance is the place of interpretation, manifestation of the unconscious and ideology in the production of meanings and in the constitution of subjects. (PÉCHEUX, 1990 apud ORLANDI, 2012, p. 57).

Figure 2: A clipping of the production of discourses and interdiscourses by FS theorists



Source: Authors (2022)

With the constitution of Figure 2, we propose to understand and analyze the

speeches of the four outstanding theorists from the FS, followed by the way in which the speeches behave in each scenario, starting from these speeches in relation to the economic system, their relations with the FS, and from possible convergences. To do so, we will analyze based on the points (I, II, III, ..., VII) located in the image.

The first point (I) to be highlighted comes from Horkheimer's position against the capitalist system, who, outraged by exploitation and injustice, believes that only a partial revolution of this system is necessary, that is, he believes that "most men, at birth, enters a prison" (HORKHEIMER, 1934 apud WIGGERSHAUS, 2002, p. 80). Let us take this passage from Horkheimer for purposes of analysis: if most men, at birth, enter a prison, we can think of an unsaid implicit in the sentence: "This can be thought of as the breath of signification, a place of retreat necessary to that can be meant, so that the sense makes sense" (ORLANDI, 2012, p. 81), in which we obtain that: *the minority of men, at birth, remain free*. Thinking about the group of people, framed by the author, as men from the perspective of the capitalist social pyramid, the minority is represented by the group of economically privileged people, which we will call upper class, while the minorities; are represented by the economically underprivileged people, which we will call the lower class.

In addition, we can analyze the passage that says: *they enter a prison*: if people, at birth, deprived of their own conscience or any knowledge about the world, enter a prison, this implies that these people do not enter by free will, enabling the use of paraphrase with a metaphorical slip, so that "paraphrase works by repetition and metaphor, in the discursive perspective, is 'a word for another'. It is metaphor that is responsible for the slippage of the senses, for the drift" (ORLANDI, 2017, p. 154). The meaning effects of the initial expression (enters a prison) and the attribution to men, derive for the assertion: *most people, at birth, are placed* in a prison. And then we have the question: who puts them in this prison? We have previously highlighted that, from the perspective of Wiggershaus (2002), a good part of Horkheimer's theorizations are aimed at privileged, upper-class people; in other words, the discursive subject directs meanings to mobilize the privileged minority for the liberation of the majority, which is, metaphorically, imprisoned.

The second point (II) concerns the intersection of discourses and meanings between Adorno's and Habermas' positioning in relation to the capitalist system. Both,

affiliated with Lukács, considered the awareness of individuals important. Adorno, on the one hand, had a discourse that synthesized meanings towards a cultural and everyday dimension of the population in general. Habermas, on the other hand, incorporated discourses on awareness in a bias that approached the political dimension. Both problematized elements of the social structure, however, without proposing a revolutionary break with the capitalist system. One of Adorno's problematizations was given by:

What sense does it make for the subject that there are no longer those windows that opened, but panes that are brutally pushed together, that there are no longer door latches, but knobs that turn, landings and vestibules, in addition to a wall around the garden? And what driver was not tempted by the power of his engine to drive, with risks and dangers through the busy streets, in which pedestrians, children and cyclists circulate? Movements and machines impose on those who handle and contain them the violence, the brusqueness, the inexorable jolts of fascist brutality. (ADORNO, 1944 apud WIGGERSHAUS, 2002, p. 548).

In this passage, with the concern between the transition from *opening it to pushing it brutally*, Adorno is concerned with the attribution of the movement, in which, metaphorically, the opening refers to the opening of means while the pushing brutally refers to breaking violently, directing meanings that are aimed directly at the insertion of *aggressiveness into the movement*. Next, thinking about the assignment of the wall around the garden, we can link the wall element to functions directed at it, such as: fence, separate and protect. If we pay attention to the last two verbs, the separation refers to the individualization of the garden, or rather, of my garden. While we can think of the verb protect by the unsaid, that is, if I am protected with a wall, then I am unprotected without a wall, an element that strengthens *individualization* from collective insecurity. The third and final question refers to the driver who, tempted by the power of his engine, drives through the city amid the circulation of pedestrians, children and cyclists, making direct reference to the *risk of the community*. With this, the senses lead us to a subject concerned with the normalization of violence, with the individualization of citizens and with the aversion to collectivity. We can use this discursive subject to think about a Mathematics Education attentive to the individualization processes that, when projected under the teaching figure and under the place occupied by the students, manifest the importance of dialogue, communication and collective work in the school space.

Habermas dialogues directly with these ideals and believes that, when directed to scientific fields, they work by “radicalizing specialization to self-reflection. Each specialized science should reflect on its foundations and, at the same time, on its relationship with social reality. (WIGGERSHAUS, 2002, p. 582).

Still dealing with discourses and meanings under the economic system, Wiggershaus (2002) highlights that Marcuse was the theorist who came closest to revolutionary ideals, which perhaps, in part, is due to his affiliation with Marx, believing that “scientific and technical progress it not only legitimized the dominant production relations, showing that they were adapted to their function, but was itself conceived for domination” (WIGGERSHAUS, 2002, p. 672). Referring to a different knowledge from that mentioned by Habermas, from the perspective of the industrialization process, Marcuse states that

Against this great opportunity for non-capitalist industrialization is, unfortunately, the fact that most of these developing countries are dependent, for initial capital accumulation, on the developed industrialized countries, for better or for worse, whether they are from the West or the East. (MARCUSE, 1964, p. 121 apud WIGGERSHAUS, 2002, p. 647).

The excerpt above highlights Marcuse's desire for a non-capitalist industrial system. The next point (IV) deals with the first approach that relates the authors' proximity to the FS, in particular, with one of Adorno's dimensions, which, before converging his discourse with Horkheimer's, presents some divergences. The first of these is pointed out by Wiggershaus (2002) when he highlights the existence of two poles of critical theory, one taken by Adorno, the other by Horkheimer.

As discussed in point (II), Adorno was concerned with the interpretation of social movements and compositions, a dimension that departs from Horkheimer as it deals with the awareness of individuals in a process of social submission. In fact, Adorno presented “[...] enthusiasm for the “primacy of conscience”, for a globalizing concept of rationality. He interpreted the concept of the unconscious sometimes as a mark of consciousness, sometimes as the name of the unconscious states that could bring to the conscious.” (WIGGERSHAUS, 2002, p. 113).

About point (V), which configures the junction of the position of Adorno and Horkheimer, strongly represented by research, such as the DdA, which theorizes rationality and knowledge under a kind of “camouflage” for its philosophical core:

In the reduction of thought to a mathematical apparatus, the ratification of the world as its own measure is implied. What appears as the trump card of objective rationality, the submission of every being to logical formalism, has as its price the obedient subordination of reason to the immediately given. (ADORNO; HORKHEIMER, 1997, p.15).

Let us take the term *pragmatic mathematical rationality* as a junction between the mathematical apparatus that constitutes objective rationality and logical formalism. We can restructure the passage by Adorno and Horkheimer and affirm, through paraphrase and metaphor, that *pragmatic mathematical rationality subordinates individuals to a rationality based on obedience to the immediate*. If we use the expression School Mathematics as a representation of a formative process based on a pragmatic Mathematics from a logical formalism, and if we re-signify the expression prison, previously mentioned by Horkheimer, from the limitation of the individual subjected to a rationality based on obedience of the immediate, we can once again restructure the expression and state that: *people subjected to school mathematics are imprisoned*.

Regarding point (VI), Marcuse's departure, also from Habermas, is crossed by Adorno's advice, when he suggests "to act like Adorno and Horkheimer, to remain masked and not present as an understandable and certain foundation the core of his philosophical activity. For Marcuse, Adorno's criticism was incomprehensible." (WIGGERSHAUS, 2002, p. 545).

Marcuse's departure from the FS also happens due to friction with Horkheimer, due to his problematization of an industrialization system that, according to Marcuse, is based on the exploitation and violation of the freedom of workers who, when exploited, need to react.

Meanwhile, Habermas also distances himself from FS due to friction with Horkheimer, mainly due to problematizations focused on the political structuring of society, pointing out the emergence

[...] of apolitical citizens in a society that is itself political. With the retreat of the class struggle in broad daylight, the contradiction has changed its form: it now has the appearance of a depoliticization of the masses in a city that was itself increasingly politicized. (HABERMAS, 1961, p. 34 apud WIGGERSHAUS, 2002, p. 584).

If we consider the political from the point of view of *power relations* (ORLANDI,

2012), we can restructure the first passage of Habermas, obtaining: *of citizens reserved for power in a society, in itself, driven by power*. We would have, as a consequence, that citizens are subjected to social power. Returning to the individual, in Hegel's conceptions, we have its representation in three dimensions: economic, cultural and psychological, if we frame the domain of the individual under this political perspective, we have: *in society, people are subjected to economic, cultural and psychological domination*. These concerns with the political dimension from the perspective of a weakening of the masses “[...] which had led Horkheimer to insistently advise him to remove Habermas from the institute” (WIGGERSHAUS, 2002, p. 589).

As a seventh and final point, we have the junction of discourses, which, despite their divergences, represent the position of a subject that constitutes meanings projected in the experience of individuals and in the concern with human beings, recognizing their completeness and incompleteness, taken as centrality is the so-called empirical research, which, based on a series of experienced approaches, gradually constitutes a series of methodological tools that structure what we currently call *qualitative research*. In dialogue with the discursive aspect, “This position that continues to frequent the political concerns of our time: the idea that man is an “influenceable animal” of great plasticity imposes itself as evidence of the 20th century [...]” (ORLANDI, 2017, p. 110).

5 Final considerations

The Frankfurt School played a fundamental role in the constitution of quantitative research fields, moving concepts in several areas of knowledge that, when crossed by the social bias, end up explaining the problematization of the function of scientific pragmatism from a positivist perspective, which makes it possible to think about the education in the midst of several existing tensions between the (un)certainities of science today.

This positivist perspective has always had a close relationship with the approach to mathematics in science and its processes of universalization and comprehensive interpretation of phenomena, consequently, this affiliation becomes intrinsic to something that we understand as important to be reviewed within the teaching of mathematics itself, making it important a teaching that makes it possible to explain the impasses of the scope of the universalization of phenomena, understanding

mathematics as something that seeks truth and standardization, however, is permeated by activities that often limit universalization. Considering the approach, we highlight a Mathematics Education that is concerned with the process that permeates this field of study, understanding the formulations as a possible product of this process, such as Mathematical Modeling in a socio-critical perspective, Critical Mathematics Education and Paulo Freire's Critical Pedagogy.

Such a proposal makes it possible to project a rejection of social processes as natural and intrinsic processes to citizens, understanding that there are several aspects that can be improved, when we refer to the way the social system develops. Taking into account that the knowledge and positioning of individuals, who live in a society, are fundamental for the maintenance or improvement of this same society.

The highlight of the central elements of the critical theory carried out by the FS leads us to a set of tensions taken from the constitution of knowledge as a factor of social protection in a society that brings citizens closer to an individualistic logic, which problematizes the incompleteness of the human being and the (in)capacity to deal with the unknown. This projection on Mathematics Education leads us to problematize the place and role occupied by the teaching of mathematics as a trainer of individuals under the unification, validation and definition of scientific knowledge.

When we think about the role of dialectics itself under the ontology of the false state of human knowledge, the role of rules within the teaching of mathematics is established as a central element of discussion, since this teaching becomes relevant, in a dialectic bias, as a formative process that has unification and validation as a point of arrival, still problematized. This dynamic becomes latent when we think about the place of education in the face of social achievements and tragedies. It is worth noting that this chapter moved by Wiggershaus (2002) points out a set of possible deepening, which can be carried out both by the direct study of the figures involved in the chapter and also by the due deepening of theorists such as Hegel, Kant, Lukács, Freud, Marx and others.

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References

ADORNO, T. W.; HORKHEIMER, M. **Dialética do Esclarecimento: fragmentos filosóficos**. Translation by Guido Antonio de Almeida. Rio de Janeiro: Jorge Zahar, 1997.

CHARLOT, B. **Educação ou barbárie? Uma escolha para a sociedade contemporânea**. São Paulo: Cortez, 2020.

KANT, I. **Crítica da razão prática**. Translation by Valerio Rohden. São Paulo: Wmf Martinsfontes, 1788.

ORLANDI, E. P. **Análise de discurso: princípios & procedimentos**. Pontes, 2012.

ORLANDI, E. P. **Discurso em análise: sujeito, sentido, ideologia**. 3. ed. Campinas: Pontes, 2017.

SACRISTÁN, J. G. **O Currículo-: Uma Reflexão sobre a Prática**. Penso Editora, 2000.

UCHÔA-FERNANDES, J. A. **Webativismo e as Jornadas de Junho: primavera de um acontecimento discursivo**. 2019. Thesis (Doctorate in Letters). Universidade de São Paulo.

WIGGERSHAUS, R. **A Escola de Frankfurt: história, desenvolvimento teórico, significação política**. Difel, 2002.