Revista de Educação Matemática e Tecnológica Iberoamericana
Universidade Federal de Pernambuco
ISSN 2177-9309
Pós-Graduação em Educação Matemática e Tecnológica
EDUMATEC

THE PRODUCTION OF DIGITAL PERFORMANCES ABOUT INFINITY: EXPLORING IMAGES OF MATHEMATICS IN PRE-SERVICE TEACHER EDUCATION

A PRODUÇÃO DE PERFORMANCES DIGITAIS SOBRE INFINITO: EXPLORANDO IMAGENS DA MATEMÁTICA NA FORMAÇÃO INICIAL DE PROFESSORES

Gabriel Souza Gregorutti
gabrielgregorutti@gmail.com

Ricardo Scucuglia Rodrigues da Silva
ricardos@ibilce.unesp.br

Abstract. *In this paper, we investigated the production of Digital Mathematical Performance (DMP) involving Public Image of Mathematics (PIM). DMP refers to the use of the arts and digital technology in teaching and learning mathematics. We report a qualitative case study regarding arts-based research in our analysis. From seven DMPs produced by pre-service mathematics teachers in one university knowledge mobilization course, we discuss two of them that explore the very notion of infinity. Specifically, the DMPs explored a convergent geometric series through a song and a video clip. Audio-visual recording of the course's sessions, video analysis of conceptual DMPs, and interviews compose the production of data. In terms of findings, we highlighted the exploration of alternative PIMs, which regards to mathematics as a human endeavour. We categorized these alternative images towards flexibility, engagement, creativity, multimodality, and collectivity. We also stressed that DMP environment offered ways to contribute to pre-service mathematics teachers' education, as participants were engaged in alternative use of digital media and the arts while developing mathematical activities, forming thinking collectives of humans-with-media.*

Keywords. Arts; Digital Technologies; Mathematical Performance; Convergent Geometric Series; Humans-with-media.

Resumo. Neste artigo, discutimos uma pesquisa que investigou a produção de Performance Matemática Digital (PMD) envolvendo Imagem Pública de Matemática (IPM). Basicamente, DMP refere-se ao uso das artes e tecnologia digital no ensino e aprendizagem de matemática. Relatamos um estudo de caso qualitativo envolvendo pesquisa-baseada-em-artes em nossa análise. A partir de sete PMDs produzidas por professores de matemática em formação inicial em um curso de extensão universitária, discutimos duas PMDs que exploram a noção de infinito. Especificamente, essas PMDs exploram uma série geométrica convergente. Por meio de uma canção e um vídeo clipe. Gravação audiovisual das sessões do curso, análise conceitual de PMDs e entrevistas compõem a produção de dados. Em termos de resultados, destacamos a exploração de IPMs alternativas, que consideram a

matemática como um empreendimento humano. Categorizamos as imagens alternativas em termos de flexibilidade, engajamento, criatividade, multimodalidade e criatividade. Também, enfatizamos que o ambiente PMD ofereceu meios para contribuir para a formação de professores de matemática em formação inicial, pois eles estiveram envolvidos no uso alternativo de mídias digitais e artes enquanto desenvolviam atividades matemáticas, formando coletivos de seres-humanos-com-mídias.

Palavras-Chave: Artes; Tecnologias digitais; Performance matemática; Série Geométrica Convergente; Seres-Humanos-Com-Mídias.

Introduction

In our research, we explored the very notion of *digital mathematical performance* (DMP), conceptualized as artistic mathematical digital narratives. Specifically, we theorized mathematical performance as the process of communicating mathematics using the (performance) arts and DMP as digital representation of mathematical performances (SCUCUGLIA, 2012). Particularly, in this article, we discuss aspects towards the *public image of mathematics* (PIM) regarding teaching activities developed with pre-service mathematics teachers, based on the collective production of DMPs in a knowledge mobilization course.

As it is supported by the theoretical framework on DMP (BORBA; SCUCUGLIA; GADANIDIS, 2014; SCUCUGLIA; GADANIDIS, 2013) and PIM (FURINGHETTI, 1993; LIM; ERNEST, 1999), this research also investigate the role of conceptual DMP production in pre-service mathematics teacher education, in regard of the (de)construction of images of mathematics. Thus, we investigated the process of construction and social dissemination of images of mathematics in a scenario in which pre-service mathematics teachers produce DMP. The DMPs produced in this research were created by pre-service mathematics teachers from a Brazilian public university within a knowledge mobilization course developed along twenty hours. From the seven DMPs created by the participants of the course, we discuss two of them, which explore a convergent geometric series through a song and a video clip created by the participants in order to learn the very notion of infinity.

Theoretical framework

Mathematics and the arts are related throughout human history in various ways, such as mathematical communication through the arts or valorisation of culture (APROSIO, 2015; GERDES, 2010; ZALESKI FILHO, 2013). The notion of DMP has been researched since 2006 and its results are increasing the dialogical space and its range in the field of mathematics education (GADANIDIS; BORBA, 2008). DMP refers to the integrated and innovative use of the arts and digital media in order to communicate a mathematical ideas

(SCUCUGLIA, 2012). Actually, the expression *DMP* is employed in multiple meanings. It refers to a potential pedagogical methodology for teaching and learning mathematics, including teacher education, or a trend in mathematics education (BORBA; SCUCUGLIA; GADANIDIS, 2014).

The most common types of DMPs are the communication of mathematical ideas through videos created, performed or produced by students intending to portray digitally theatrical plays, poetry, dance, cinema, or other artistic expression. Based on it, DMP plays a semiotic role, as it can be conceptualized as multimodal artistic text/narrative utilized to represent mathematics through the arts and digital media (SCUCUGLIA, 2012). Those videos can be publicly watched through online platforms such as *YouTube* or *the Math + Science Performance Festival* (see www.mathfest.ca).

DMP can be aligned to the concept of knowledge in Ancient Greece, as in this environment, rational knowledge does not exclude pleasure, like those ancient poems or allegories (MARCONDES, 2001). We stress that DMP intends to encourage a different type of interaction between teachers and students, as the students should participate actively from the beginning of the creation process, following Pedagogy of the Oppressed concepts (FREIRE, 2011).

In regard these perspectives, we consider such learning environment as a didactical-pedagogical possibility towards pre-service mathematics teacher education (BORBA; SCUCUGLIA; GADANIDIS, 2014). In the midst of diversified aspects, literature on teacher`s Education addresses the lack of courses throughout undergraduate programs for Mathematics focusing on pedagogical aspects of technologies (GATTI; BARRETO, 2009). Literature also point to the exaggerated focus on specific content knowledge – Mathematics itself – over pedagogical content knowledge (NACARATO, 2006). We also explore the very notion of Humans-with-media, which refers to a theoretical framework based on the idea of thinking collectives (LÉVY, 1993) and reorganization of thinking (TIKHOMIROV, 1981). DMP can be discussed through humans-with-medias (BORBA; VILLARREAL, 2005), as:

> DMP production may form diverse thinking collectives such as teachers-students-with-arts-digital-technologies. Digital technologies in this context are those utilized to produce videos and video cameras, video edition software, cyberspace to its publication of digital representation, etc. (BORBA; SCUCUGLIA; GADANIDIS, 2014, p. 119).

Authors such as Furinghetti (1993), Lim (1999), and Gadanidis (2012) discuss PIM. They pose questions over the problems of this image as it commonly refers to

negative/stereotyped images people hold. Generally, most people see Mathematics as a cold, boring, inhuman, difficult science, course or content. Mathematics is also frequently related to aspects like ideology of certainty and absolutist conceptual nature of mathematical knowledge (LIM; ERNEST, 1999). Rarely, PIM is reported as positive, arousing feelings like love or happiness, or being lively with creativity and colourfulness. Literature also addresses the root of this problem as usually on the learning experience people have in school or other activities that involve Mathematics (GADANIDIS; SCUCUGLIA, 2010). It also includes the images disseminated by the media (TV shows, books, films, etc.) (FURINGHETTI, 1993).

According to Lim (1999), the term *image of mathematics* can be conceptualized*:*

> […] as a mental representation or view of mathematics, presumably constructed as a result of social experiences, mediated through school, parents, peers or mass media. This term is also understood broadly to include all visual, verbal representations, metaphorical images and associations, beliefs, attitudes and feelings related to mathematics and mathematics learning experiences (LIM, 1999, p. 1).

Based on this conception, Lim e Ernest (1999, p. 55) states that:

> the results show that a person's image of mathematics is often unique and personal, and multifaceted and diffuse. There does not appear to be a typical image of mathematics. Perhaps, there are some commonalities among certain sections of the population, with certain common factors of influence, but no single overall pattern or typical image of mathematics seems to appear (LIM, ERNEST, 1999, p. 251).

Within this context, literature also discusses possible alternatives to disrupt negative attitudes and beliefs in matter of Mathematics and to construct alternative images of Mathematics related to positive aspects, like good feelings, or being lively (GADANIDIS; SCUCUGLIA, 2010; GREGORUTTI, 2016). For instance, Scucuglia (2014) explores these aspects in the matter of DMP production, investigating the educational use of the Arts and digital technology as it potentially offers ways to promote alternative images of Mathematics and mathematician. We must stress that PIM relates to the public image of mathematicians (PICKER; BERRY, 2000). Therefore, the public images of mathematicians refer to aspects such as authoritarian, anti-social human beings, nerds, monsters, and other negative stereotypes.

However, for the scope of this paper, we focus on the image of Mathematics. Considering such negative images, we discuss in this paper a research outline looking for aspects related to the process of construction of Mathematics images in a setting in which pre-service mathematics teachers produce DMP. Can DMP educationally act in order to create alternative images of mathematics? What educational aspects emerge from this setting? What

is the role of the arts and digital technology in this process? We explore two DMPs (a song and a video clip) aiming to discuss about (1) the nature of pre-service Mathematics teachers' and students' images; (2) DMP`s production as a didactic-pedagogical methodology in Mathematics Education, as an innovative use that integrates the Arts and digital technologies. In the next section, we describe the Methodology of this research.

Methods

In the matter of the research reported in this article, methodology is qualitative (BOGDAN; BIKLEN, 1994). Several perspectives were carried out in purpose of production and analysis of data, such as audio-visual recording of the sessions of the university mobilization knowledge course using qualitative case studies (STAKE, 2000), video analysis (POWELL; FRANCISCO; MAHER, 2004), analysis of conceptual DMP (SCUCUGLIA, 2012), and interviews (PATTON, 2002). We also used arts-based research to artistically communicate findings through poems (FINLEY, 2005). Specifically, we stress qualitative case study as a methodological perspective that offer ways to explore data in depth (STAKE, 2000). The case in this study is an investigation towards the production of two DMPs about infinity conducted by pre-service teachers in a knowledge mobilization course. Thus, Other relayed questions are: can DMP educationally act in order to create alternative images of mathematics? What are the educational aspects that emerge from this setting? What is the role of the arts and digital technology in this scenario?

Data for this study were produced in four sessions of five hours each of a university mobilization knowledge course for pre-service mathematics teachers of a Brazilian public university to produce DMP voluntarily as part of learning activities. Twenty participants produced seven DMPs and, in this report, regarding ethical issues, we use factious names for the participants. We had their consent to use their images. In five DMPs the participants explored the Four Color Theorem and in two DMPs, those discussed in this paper, they explored the following geometric series:

$$\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\cdots=\sum_{n=1}^{\infty}\left(\frac{1}{2}\right)^{n}=\frac{\frac{1}{2}}{1-\frac{1}{2}}=1.$$

In the following section of the article we present our discussions using analytic descriptions and transcriptions from the data analysis carried out.

Results and discussions

Pre-service teachers produced a song and a video clip about that geometric series, relating math and mathematical activity to feelings and sensations. Through analysing the data, we found specific images of mathematics in the participants' point of view. The following reports what literature addresses about PIM, regarding also aspects we identified in participants' views. Thus, we see the following poem as an artistic representation of our findings (FINLEY, 2005)

Here come the visions
About Mathematics
Cold disquisitions
Boring tactics
Hovering precisions
Oversupply frantic
Endless frustrations
As lack romantic
Vivid cognition
Missing fantastic
Gorgeous solutions
Image of Mathematics – First author

Cold, boring, frustrating, and inhuman are some of the stereotypes about mathematics commonly stressed in the literature (LIM, 1999). However, in this research, through the education activity involving the production of DMPs, we identified mathematics as: *flexibility, engagement, creativity, multimodality, and collectivity*. These are the PIM related to the production of DMP in this research involving pre-service mathematics teachers.

Borba and Skovsmose (2001) addresses that "mathematics curriculum officials usually deal with problems that have one and only solution, what stresses that mathematics is free of human influence" (BORBA; SKOVSMOSE, 2001, p. 130, our translation). Thus, flexibility may offer significant aspects for critical mathematics education and its public image. In this research, students pointed to a different way of doing mathematics while producing DMPs, stressing that it offer ways to make mathematics more interesting and flexible. One participant expressed the following view in a interview:

> **Leticia:** I think that, firstly, DMP shows that mathematics goes beyond paper and pen. Several teachers, even in college, say that the only way to study Mathematics is with a textbook, as you are sit, writing. If you analyze other courses, like Pedagogy or Biology, they are watching a movie while

> studying. I think that DMP shows to students as they produce it, that Mathematics can be done in different ways. [Mathematics] being more attractive, indeed. Because if you ask a student whether to what a movie or to solve a problem in a textbook, surely it will be more attractive to watch a movie. […] So, as we produce DMP, Mathematics becomes more flexible.

Mathematical discussion during the process of DMP production was also engaging, while the students advanced in their study and learning about the convergent geometric series, approaching concepts like dimension. They were thinking in terms of embodiment (GEROFSKY, 2006), because they were representing the movement of the series walking from the back part of the classroom until the door, that is, walking one half per time. The following dialogue, constructed from the video analysis of one session of the course, reveals this engaging component in learning math through DMP:

> **Professor:** What do you think? Walking one half of one half? If you keep doing that infinitely, will you reach the end or not?
> **Rodolfo:** It depends.
> **Letícia:** If you think visually, it will take forever. So, I'll never reach what I want. I will always walk one half and one half, missing the next last one half. I never walk past it all. Always walking one half of it.
> **Henrique:** But at some point, you`ll not be moving, because your space will be really small.
> **Professor:** Do you all agree that is a paradox? If you are following this thought, you will never get there. There will be always one half left. But if you stand up and go, you will go out the door.
> **Letícia:** Can we prove the series is equal one?
> **Professor:** Why don't you guys represent the series as 2S? It will help in terms of algebra.
> **Pedro:** That is a convergent geometric series! We can prove that in Analysis.
> **Professor:** Good. But we can use another approach, right?
> **Henrique:** If you name the series as S, then 2S will be 1 + ½ + ¼ and so on. Thus, 2S – S will be equal…
> **Henrique e Letícia:** … 1. Then, S = 1.
> **Professor:** Great. We are algebraically convinced, but, empirically, are we? How can we solve the paradox?

The dialogue above reveals a pedagogic engagement in learning math through the production of DMP. It actually offers students the opportunity to explore mathematics in terms of language diversity, since they used "traditional" math symbols in combination with body language, orality, sounds, and other modes of communication. Thus,

> A learning environment that encourages students to come up with and share their questions, as well as describe and discuss their discoveries with others, would facilitate the action of communicating. The more these actions are available to children, the more they learn (SINCLAIR, 2001, p. 26).

Thereby, we think that DMP, as they involve imaginative and metaphoric thought (GADANIDIS, 2007), offered ways to shared experiences and inspired reflections, critics and dialogue (GADANIDIS; BORBA, 2008) following a mathematical exploratory environment (SCUCUGLIA; GADANIDIS; BORBA, 2011), which keep ideology of certainty away (BORBA; SKOVSMOSE, 2001), as mathematics is flexible. Engagement and flexibility emergent in DMP scenarios contrast negative public images of mathematics:

> Performances are typically shared experiences, and mathematical performance challenges us to find ways of sharing with one another the rare events of mathematics education, to make them accessible for reflection, critique and dialogue, and as models on which to base the mathematics experiences we stage in our classrooms (GADANIDIS; BORBA, 2008, p. 50).

Such aspect can blast creativity. In such way, the DMPs analyzed in this paper are colorful or grim? Are they lively, active, or are they wilt? Are they ludic or are they dismal? Although several participants of the course reported mathematics as commonly seen as cold, boring, difficult, as reported in the literature (FURINGHETTI, 1993), plenty of them stresses the possibility of positive images when participating in a DMP environment. In one of the interviews, a participant stated:

> **Carolina:** Among Elementary School students, Mathematics is something difficult, as it is inaccessible, indeed. It is related to the way it is presented in classrooms: mechanic, cold, following exercises resolutions. Exercise to apply some formulas, to apply the content. That is mechanic and inaccessible. Mathematicians have also a negative image. Some of them are arrogant, as they think they own knowledge. So, [through DMP], I observed different images. Following DMP, the mathematician can be creative and this image never appears, nobody has it publicly. Nobody sees a creative mathematician, an enthusiast.

We also highlight PIM in DMP in terms of multimodality. The combination between arts and the use digital technology offer ways to communicate and represent mathematics using multiple modes of communication. It is an alternative way of doing mathematics because it combines the objectivity of math with the subjectivity of the arts. It is also "subversive" in terms of language, because, traditionally, math is written-base and the use of videos, for instance, offer ways to use different modes of communication. In terms of the arts, in this case, we regard music production and cinema, because participants created a song and a vide clip for the song. The use of music in math classrooms offers ways to construct an artistic multimodal learning environment (WALSH, 2011). Students with background on music, with skills to sing and/or play a music instrument may lead these pedagogic sessions

and teach music theories or practices to other students. Figure 1 shows a screen-captured image from the video recording of one session of the course.

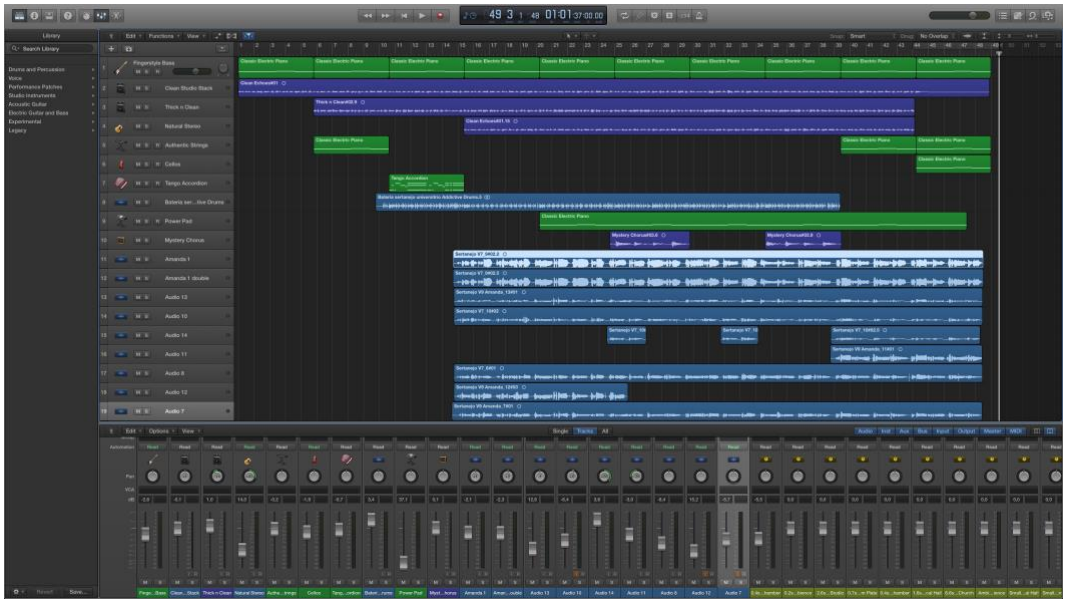
Figure 1 - Students creating the song

Students created a first version of the song in classroom. Many of them highlighted how difficult is to compose a song about math, because one has to consider harmony and metric for the lyrics, usually involving rhymes as well. They also aimed to express feeling and emotions, which are not common aspects of scholarly mathematics discourses. The lyric in Portuguese created by the participants during the knowledge mobilization course and a translation of it to English is presented following.

Juntos somos um	**Together we are one**
O que fazer para ate conquistar?	What should I do to attract you?
Se metade do caminho eu alcançar	If one half of the way I have achieved
Caminhando, passo-a-passo	Walking, step-by-step
Já andei mais um quarto	I have walked one quarter more
Minha razão é você	You are my reason (ratio)
Mas ½ é a da PG	But ½ the common ration of the GP
Com a soma vou chegar	With the sum I will arrive
Convergir par te amar	I'll converge to love you
Progredindo ao infinito	Progressing to infinity
Essa série vai convergir	This series will converge
Temos parte em comum	We have similar parts
Nós dois junto, somos um	Together we are one
Temos parte em comum	We have similar parts
Nós dois junto somos um	Together we are one

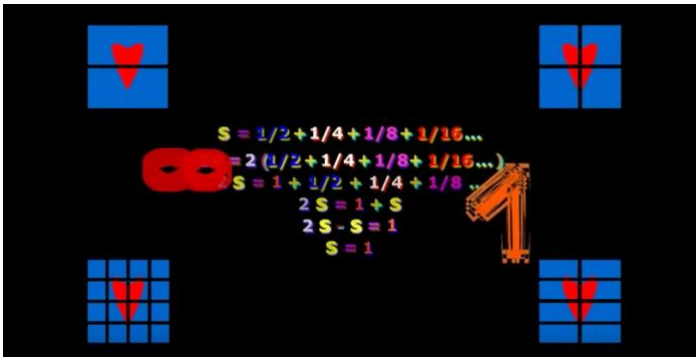
For the recording of the song, the professor and two students leaded the process after the draft of the song created in classroom. That is, the lyrics, genre, and the melody of the song "Together we are One" were collectively created by the participants. In Figure 2 we present an image of the song recorded using Logic Pro X for Mac.

Figure 2 – The recording of the song



To produce the video clip used a trial version of "Adobe Flash Player". We focused on representing visually what the lyrics. Thus, we explored the series displaying a square with a heart being divided by ½, and ¼, and so, as the terms of the series. We also showed in synchrony with the division/terms the algebraic representation of the series (see Figure 3). Artistically, a "love arrow" reaches the square, starting the division process. When the lyrics say "Walking, step by step", synchronized steps show up on the video. Moreover, hearts flip on the scene all the time, symbolizing the love portrayed by the lyrics. The video ends with a square representing the whole square generating a square representing half of it, on and on, at the same time a formal proof for the convergent geometric series appears in an animated way, showing numbers and letters glowing. Thus, we have both the visual and the algebraic proof for the series. The last action on the video is a number 1 emerging from the square that is representing 1/16, along with an infinite symbol, as it is an infinity series. Up to the end of the music, the number 1 and the infinite symbol keep rolling on the screen, while the proof keeps being developed. We sent the video clip to the Math + Science Performance Festival and it was one of the Top 5 DMP within the mathematicians' Community that served as judgers.

Figure 3 – A scene of the DMP



The video clip stresses a human component of mathematics as it is associated to love, in contrast with the public strict vision of hatred to this science (FURINGHETTI, 1993; LIM, 1999; LIM; ERNEST, 1999; PICKER; BERRY, 2000). Although it has a black background, the video is animated with several colors, indicating a lively aspect of mathematics, also opposing with common visions of it ("black/white"). This aspect is heighted by one of the participants in an interview

> **Marcela:** The video is interesting, as it shows several colors and varied forms, what is attractive to the audience. It is also concise, a great didactical tool. I would not change anything in it.

So far, we approached DMP as a:

> Differentiated possibility, with innovative potential to teach and learn Mathematics, as an alternative to transform negative image of school Mathematics and mathematicians. It is also a way to take the pleasure of a Mathematics activity to non-scholar contexts (BORBA; SCUCUGLIA; GADANIDIS, 2014, p. 106–107).

In this regard, we also found from the interviews some evidence towards a significant role of DMP production in pre-service mathematics teachers learning.

> **Letícia:** I think that it added up (to participate in DMP`s production) as a Mathematics professional, even if it is not in basic education, I mean, in college. It added up not only for me, but for everyone, because Mathematics, even to us (future mathematics teachers) it is boring, heavy, hard. So, if we deconstruct that in here, it will help a lot. Because sometimes a mathematical concept can be explored in a lighter way. That was great

Besides to the contribution of DMP environment to pre-service teachers' education, it offered ways form collectivity towards both participants and media. Thus, the role of technology was significant in term of mathematical thinking and knowledge production towards (artistic) learning of geometric series. From the interviews we highlight:

> **Letícia:** About the collective process, it is really important following other`s people point of view. Because when I study Mathematics, I study alone. I am going to read all the content, make exercises, and take my test. However,

> when I am studying in a group, I can realize in a different way. When the teacher is prepared to that and show us lots of contents, you and the others will discuss following each one`s responses, because it is different the way each was gets it, accordingly to what he knows, this discussion cause a sharing experience.
>
> **Henrique:** We composed lyrics, we filmed a Harlem Shake, recorded a song. Obviously, technology was really important for the pre-service teachers produce DMP: camera, musical production. Each technology used in the musical production, the cameras and the videos and video edition. Everything happened collectively, when the teacher wanted to suggest some changes, he asked our opinions. The song and the video were created collectively with technology.

Following those observations, we think DMP environment created for this university mobilization knowledge course which served as setting of this research contributed to the formation of thinking collectives (LÉVY, 1993) of teachers-with-arts-and-digital-technologies (BORBA; SCUCUGLIA; GADANIDIS, 2014), such as: teachers-students-music-instruments-software. Therefore, in this section, we discussed emergent images of mathematics when pre-service teachers produce DMPs.

Conclusions

In terms of PIM, DMP plays a role in its transformation from a cold, boring and inhuman science or school subject to a creative, human, regarding this research data. From the session of the course, from interviews and from the DMPs, we identified five main alternative images of mathematics. They are *flexibility*, *engagement*, *creativity*, *multimodality*, and *collectivity*. Thus, the process of producing DMP in this study was identified as an engaging learning scenario regarding artistic and digital aspects in doing mathematics. Overall, it was conceived as a multimodal-learning environment for mathematics teaching and learning in teacher education. In terms of teachers' practice and humans-with-medias, DMP plays an important role in the production of pedagogic knowledge, as it forms thinking collectives, reorganizing knowledge.

In the specific case of this research, the DMPs played roles in pre-service teacher education, as the student`s learned Mathematics by producing digital Arts collectively. In contrast, we identify limitations in this research such as a potential lack of Mathematics learning to those who listen to the music, accordingly to several students` quotes. However, the richness of the mathematical ideas in combination with reasoning that involves aesthetics, emotions and sensations should be explored in researches involving DMP and PIM.

References

APROSIO, A. P. *Pinóquio no país dos paradoxos: uma viagem pelos grandes problemas da lógica*. Tradução Isabella Marcatti. Rio de Janeiro: Zahar, 2015.

BOGDAN, R.; BIKLEN, S. *Investigação Qualitativa em Educação: uma introdução à teoria e aos métodos*. Porto, Portugal: Porto Editora, 1994.

BORBA, M. C.; SKOVSMOSE, O. A ideologia da Certeza em Educação Matemática. *Educação Matemática Crítica: a questão da democracia*. Campinas: Papirus, 2001.

BORBA, M. C.; VILLARREAL, M. E. *Humans-With-Media and the Reorganization of Mathematical Thinking: information and communication technologies, modeling, experimentation and visualization*. New York: Springer, 2005. v. 39.

BORBA, M. C.; SCUCUGLIA, R. R. S.; GADANIDIS, G. *Fases das Tecnologias Digitais em Educação Matemática: sala de aula e internet em movimento*. 1. ed. Belo Horizonte: Autêntica, 2014.

FINLEY, S. Arts-based inquiry: Performing revolutionary pedagogy. In: DENZIN, N. K.; LINCOLN, Y. S. (Org.). *Handbook of Qualitative Inquiry*. 3ª. ed. Thousand Oaks, CA: Sage, 2005.

FREIRE, P. *Pedagogia do Oprimido*. 50. ed. São Paulo: Paz e Terra, 2011.

FURINGHETTI, F. Images of Mathematics outside the Community of Mathematicians: Evidence and Explanations. *For the Learning of Mathematics*, v. 13, n. 2, p. 33–38, 1993.

GADANIDIS, G. The Pleasure of Attention and Insight. *Mathematics Teaching*, n. 186, p. 10–13, 2007.

GADANIDIS, G. Why can't I be a mathematician? *For the Learning of Mathematics*, v. 32, n. 2, p. 20–26, 2012.

GADANIDIS, G.; BORBA, M. C. Our lives as performance mathematicians. *For the Learning of Mathematics*, v. 28, n. 1, p. 44–51, 2008.

GADANIDIS, G.; SCUCUGLIA, R. R. S. Windows into Elementary Mathematics: Alternate public images of mathematics and mathematicians. *Acta Scientiae (ULBRA)*, v. 12, p. 8–23, 2010.

GATTI, B. A.; BARRETO, E. S. S. *Professores do Brasil: impasses e desafios*. Brasília: UNESCO, 2009.

GERDES, P. *Da etnomatemática a arte-design e matrizes cíclicas*. Belo Horizonte: Autêntica Editora, 2010.

GREGORUTTI, G. S. *Performance matemática digital e imagem pública da Matemática: viagem poética na formação inicial de professores*. 2016. 165 f. Dissertação (Mestrado em

Educação Matemática) – Universidade Estadual Paulista "Júlio de Mesquita Filho", Rio Claro, 2016.

LÉVY, P. *As tecnologias da inteligência: o futuro do pensamento na era da informática*. Rio de Janeiro: Editora 34, 1993.

LIM, C. S. *Public Images of Mathematics*. 1999. 366 f. Tese (Doutorado em Educação) – University of Exeter, Exeter, 1999. Disponível em: <http://people.exeter.ac.uk/PErnest/pome15/lim_chap_sam.pdf>.

LIM, C. S.; ERNEST, P. Public Images of Mathematics. *Philosophy of Mathematics Education Journal*, n. 11, p. 44–56, 1999.

LINCOLN, Y. S.; GUBA, E. G. *Naturalistic Inquiry*. London: Sage Publications, 1985.
MARCONDES, D. *Iniciação à história da filosofia: dos pré-socráticos a Wittgenstein*. 6. ed. Rio de Janeiro: Jorge Zahar Ed, 2001.

NACARATO, A. M. Formação do Professor de Matemática: pesquisa x políticas públicas. *Revista Contexto e Educação*, v. 21, n. 75, 2006. Disponível em: <https://www.revistas.unijui.edu.br/index.php/contextoeducacao/article/view/1114>.

PATTON, M. Q. *Qualitative evaluation and research methods*. 3rd ed. NewBury Park, CA: Sage, 2002.

PICKER, S. H.; BERRY, J. S. Investigating pupils' images of mathematicians. *Educational Studies In Mathematics*, v. 43, n. 1, p. 65–94, 2000.

POWELL, A. B.; FRANCISCO, J. M.; MAHER, C. A. Uma abordagem à Análise de Dados de Vídeo para Investigar o Desenvolvimento das Ideias Matemáticas e do Raciocínio de Estudantes. *Bolema. Boletim de Educação Matemática*, v. 17, n. 21, p. 81–140, 2004.

SCUCUGLIA, R. *On the nature of students' digital mathematical performance*. 2012. Tese (Doutorado em Educação) – University of Western Ontário, London, 2012.

SCUCUGLIA, R. R. S. Narrativas Multimodais: a Imagem dos Matemáticos em Performances Matemáticas Digitais. *Bolema. Boletim de Educação Matemática (in press)*, v. 28, n. 49, p. 950–973, 2014.

SCUCUGLIA, R. R. S.; GADANIDIS, G. Performance Matemática: Tecnologias Digitais e Artes da Escola Pública de Ensino Fundamental. In: BORBA, M. C.; CHIARI, A. S. S. (Org.). *Tecnologias Digitais e Educação Matemática*. 1. ed. São Paulo: Livraria da Física, 2013. p. 325–363.

SCUCUGLIA, R. R. S.; GADANIDIS, G.; BORBA, M. C. Lights, Camera, Math! The F Pattern News. In: WIEST, L. R.; LAMBERG, T. (Org.). *Proceedings of the 33rd Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education*. Reno: University of Nevada, 2011. p. 1758–1766.

SINCLAIR, N. The aesthetic is relevant. *For the Learning of Mathematics*, v. 21, n. 1, p. 25–33, 2001.

STAKE, R. Case Studies. *Handbook of Qualitative Research*. 2ª. ed. Thousand Oaks, CA: Sage Publications, 2000. p. 435–454.

TIKHOMIROV, O. K. The psychological consequences of computerization. In: WERTSCH, J. V. (Org.). *The concept of activity in soviet psychology*. New York: M. E. Sharpe. Inc, 1981. p. 256–278.

WALSH, M. *Multimodal literacy:* researching classroom practice. Australia: Primary English Association, 2011.

ZALESKI FILHO, D. *Matemática e Arte*. 1. ed. Belo Horizonte: Autêntica, 2013.