FUTURE SECONDARY MATHEMATICS TEACHERS TRAINING FROM A FUNCTIONAL PERSPECTIVE

Pedro Gómez[1], María José González, Luis Rico, Francisco Gil, José Luis Lupiáñez, Antonio Marín, María Francisca Moreno, Isabel Romero

We describe a model that is being used in some Spanish universities for future secondary mathematics teachers training. This model is based on a functional view of students' learning, of how didactic knowledge is established on the basis of teachers' activities, and of how we see some mathematics education notions as conceptual and methodological tools with didactic purposes. This view gives rise to a procedure, didactic analysis, as a conceptualization of the teacher's activities needed for planning, implementing and assessing mathematics lessons. We present the general curriculum design of a methods course based on these ideas and procedures and mention some research results concerning the design and implementation of such courses.

In order to become a Spanish secondary mathematics teacher, a candidate has to satisfy three requirements: (a) have a Bachelors degree, (b) approve a short pedagogical course, and (c) pass a series of public exams. Even though a candidate needs not to have a mathematics degree, most of them do. The short pedagogical course gives students a broad survey of pedagogical and didactic ideas and methodologies. However, many universities have been offering additional training to their mathematics students, organized in optional methods courses. Those last year students who are interested in becoming secondary mathematics teachers take these courses.

The University of Granada has offered during the last 15 years two such methods courses: a theoretical one and a practical one. In this paper we describe some of the features of the first one. This course, which is being implemented in the same manner in other Spanish universities, is based on a functional approach to teacher training. In what follows, we describe our functional view of teacher training, depict the didactic analysis procedure that emerges from such a view, outline the curriculum design of a methods course based on these ideas and sum up some research results concerning its design and implementation.

FUNCTIONAL VIEWS OF TEACHER TRAINING

A usual approach for the design of a methods course resides in determining its contents by answering the question "what should a mathematics teacher know?" In other cases, this content is set up from analytical

[1] C/. Alisios 17, Albolote, 18220, Spain. Phone/Fax: (34) 958537304. Email: pgomez@valnet.es

classifications of the teacher's knowledge (e.g., Bromme, 1994; Shulman, 1987). This type of systematization is problematic for the design of teacher training programs, since they imply a separation (at least analytical) of the teacher's different kinds of knowledge. In practice, teachers put into play a coordinated implementation of their knowledge.

Simon (1995) partially solves these difficulties. He identifies the knowledge that is put into play when the teacher reconstructs a hypothetical learning trajectory based on the assessment of students learning in a constructivist setting. Simon's categorization of knowledge is functional: he takes a stance concerning students learning, proposes a teaching strategy coherent with such view, and identifies the kinds of knowledge that are needed to perform such teaching.

Having adopted a view similar to Simon's, we think about teacher's knowledge from a functional perspective. According to this view, teacher's knowledge can be established from the analysis and description of the activities needed to plan, manage and evaluate a lesson. Thus, the problem of the teacher's knowledge can be considered as the integration of knowledge, abilities and attitudes for action. Instead of thinking on what the teacher should know, we ask ourselves what he should be able to do in a specific context of students' learning. Therefore, we start by adopting a functional view of school mathematics, and then we reflect on the teacher's activities that can promote students' learning in that context (didactic analysis, see below). This approach allows us to establish the competencies that we expect future teachers to develop during their training. We suggest that, with this approach, it is possible to determine systematically and to organise in a structured way the capacities that contribute to the mathematics teacher's competencies. We develop this idea with respect to the planning competence of the mathematics teacher.

Didactic analysis is set up around a set of notions that we call "curriculum organizers" (Rico, 1997). The way we use these notions in future teachers training is coherent with the functional view we advocate: curriculum organizers are considered as methodological and analytic tools with a didactic purpose. That is, we pinpoint our approach by postulating "a set of tasks, a set of conceptual tools and a subject that, when performing the task using the available tools [the curriculum organizers] put into play and set forth his/her competency in carrying out the processes involved" (Rico, 2007, pp. 49-50).

DIDACTIC ANALYSIS

We have focused our work on task planning, as one of the teacher's most important competencies (Ball & Bass, 2003, p. 3; Van Der Valk & Broekman, 1999). We suggest that teacher's planning should take into account the complexity of the mathematical content from different points of view (Cooney, 2004; Timmerman, 2003). In fact, the negotiation and construction of the multiplicity of meanings of the mathematical concepts should be one of the central purposes of interaction in the classroom. Planning of a didactic unit or of an hour of class should be grounded in the exploration and structuring of the different meanings of the mathematical structures that are the object of that lesson plan (Rico, 1997).

Didactic analysis can be used as a task planning procedure. With it, the teacher can specify (and differentiate) the goals, content, methodology and evaluation scheme of each topic in planning. Our proposal approaches the meaning of the mathematical concept by attending to three dimensions: systems of representation, conceptual structure and phenomenology. We claim that in the specific context of the planning of an hour of class or a didactic unit, the teacher can organise instruction based on four analyses (Gómez, 2002):

1. *subject matter analysis,* as a procedure by which the teacher identifies and organises the multiplicity of meanings of a concept;

2. *cognitive analysis,* in which the teacher describes his hypotheses about how the students can progress in the construction of their knowledge of the mathematical structure when they face the tasks that will make up the teaching and learning activities;
3. *instruction analysis,* in which the teacher designs, analyses, and chooses the tasks that will constitute the teaching and learning activities that are the object of the teaching; and
4. *performance analysis,* in which the teacher determines the capacities that the students have developed and the difficulties that they may have expressed up to that point.

We use *didactic analysis* to refer to a cyclical procedure that includes these four analyses, attends to the factors conditioning the context and identifies the activities that the teacher should perform to organise the teaching of a specific mathematical content. The description of a cycle of didactic analysis follows the sequence described in Figure 1.

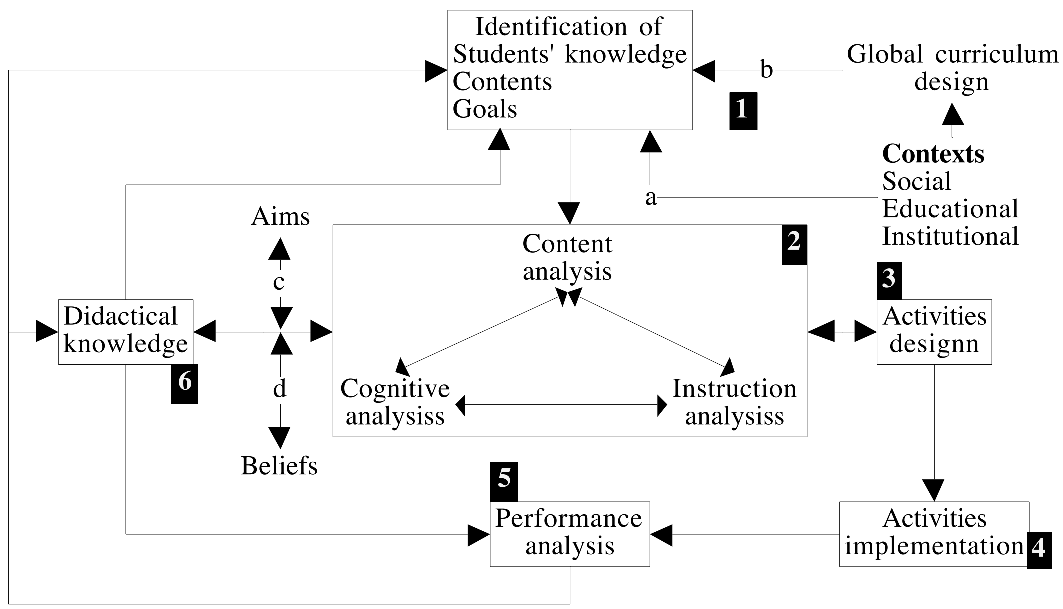

Figure 1. Cycle of didactic analysis

The cycle of didactic analysis begins with the determination of the content to be treated and the learning goals to be achieved. It starts from the teacher's perception of the students' understanding and is based on the results of the performance analysis in the previous cycle, taking into account the social, educational and institutional contexts that frame the instruction (box 1 of Figure 1). From this information, the teacher begins planning with subject matter analysis. The information that emerges from subject matter analysis serves as the basis for cognitive analysis, by identifying and organising the multiple meanings of the concept to be taught. The cognitive analysis can then give rise to a revision of subject matter analysis. This relation between the analyses is also established with instruction analysis. Its formulation depends on and should be compatible with the results of the subject matter analysis and the cognitive analysis; but at the same time, performing it can generate the need to correct the prior versions of these analyses (box 2). In cognitive analysis, the teacher selects some reference meanings and, based on these and on the learning goals that have been imposed, identifies the capacities that he seeks to develop in the students. The teacher also formulates conjectures on the possible paths by which students can develop their learning when they tackle the tasks that make up the lesson. The teacher uses this information to design, evaluate and select these tasks. As a result, the choice of tasks that compose the activities should be consistent with the results of the three analyses, and the evaluation of these tasks in the light of the analyses can lead the teacher to perform a new cycle of analysis before choosing the definitive tasks that compose the teaching and learning activities (relation between boxes 2 and 3). The teacher puts these activities into practice (box 4) and, in doing so, analyses the students' actions to obtain information that serves as the starting point of a new cycle (box 5). Didactic knowledge (box 6) is the knowledge that the teacher brings into play during this process.

From our functional perspective of teacher training, a future teacher learns by putting into practice a set of notions (the curriculum organizers) for analyzing a mathematical concept with didactic purposes. Therefore, the future teacher's activity is centred in the use of these conceptual and methodological tools for performing two types of tasks: (a) analyzing the mathematical concept and (b) using the information resulting from such analysis either in other analysis or in planning a lesson. Understanding the tool is a process that takes place while using it. The future teacher's actions while performing the task enhance his understanding of the tool. And this improved understanding enhances his performance of the task. This approach is rooted in the Vygostkian idea of mediation (Vygotsky, 1982). Curriculum organizers are seen as mediating instruments between the future teachers' action and their activity. Future teachers' design and selection of pupils learning tasks —the task planning activity— can be seen as a mediated activity when the future teacher uses curriculum organizers to produce and use information to propose solutions to this activity. We consider three dimensions of each curriculum organizer as an instrument: its meaning, its technical use and its practical use. Future teachers transform each curriculum organizer into an instrument through the interplay of these three dimensions.

Course Design

Our purpose in this section is to show the role of the previous ideas and procedures in grounding and conceptualising a methods course. Since the course evolves continuously, we describe here the version of the course delivered in 2000 in the University of Granada[2], with specific attention to three aspects: the context, its grounding and its curricular design.

Context

In 2000, the University of Granada had a study programme for initial training of high school mathematics teachers. This programme formed part of the Bachelor's degree in mathematics at the university. Nowadays, mathematics students have the option to take some of these courses. In what follows we will refer to a methods course called "Mathematics Education in High School".

Most future teachers who participate in this course believe that they have solid training in mathematics. Two thirds of the future teachers have teaching experience prior to the training plan, through work in private classes or in tutoring services for high school students.

Foundations

The notion of didactic analysis is central to the foundation of the second block of the course. In emphasising the role of didactic analysis in the teacher's activities and the initial training of teachers, we take sides: we start from a particular position on how students learn mathematics in the classroom and propose an ideal vision of how teaching should develop (didactic analysis). This establishes one of the two anchors of our conception of the training of high school mathematics teachers: to contribute to the development of the competences and capacities necessary to perform didactic analysis. Our view of the learning of future teachers provides the second anchor for our conception of the initial training of high school mathematics teachers, on which the design of the course is based. We have taken a socio-cultural position.

[2] We do so because most of the research results we mention in the last section refer to this version of the course.

The characterisation of the procedures that compose didactic analysis and the meanings and uses of the notions involved in these procedures enable us to identify and structure the capacities needed for the high school mathematics teacher's planning competence and thus to specify the didactic knowledge that we wish future teachers to develop during the course. This functional view of the initial training of teachers grounds the goals and contents of the second block of the course. The methodological and evaluation plans in the design are based on our position with respect to the future teachers' learning.

To describe the design of the course, we follow a curricular structure and describe briefly its aims, goals, contents, methodology and evaluation plan.

Aims and Goals

The aim of the course is to contribute to the training of the future teacher in two dimensions: the beginning of his participation in communities of practice of mathematics educators and the development of the knowledge and capacities necessary for the planning of didactic units. In considering that the course, as a training plan in the processes of planning didactic units, is also a community of practice, we wish the future teachers to develop their capacity for participation in this community by constructing the knowledge and capacities needed to perform didactic analysis. The knowledge and capacities are specified in the social construction of the meanings of the notion of curriculum, the foundations of school mathematics and the curriculum organizers.

Contents

For the version we are describing, the contents of the course were organised according to the outline in Figure 2. The course began with analysis of and reflection on the history of mathematics and of mathematical education in Spain, which served as the context in which to discuss the antecedents of Spain's mathematics curriculum. The notion of curriculum was the foundation supporting the rest of the contents. We discussed the goals of mathematics education and reflected on the levels and dimensions of the curriculum. Using this conceptual reference, we analysed some Spanish and international curriculum projects, reflected on the antecedents of the mathematics curriculum in Spain, and studied the general organisation, levels of specificity and contents of the high school mathematics curriculum currently in effect.

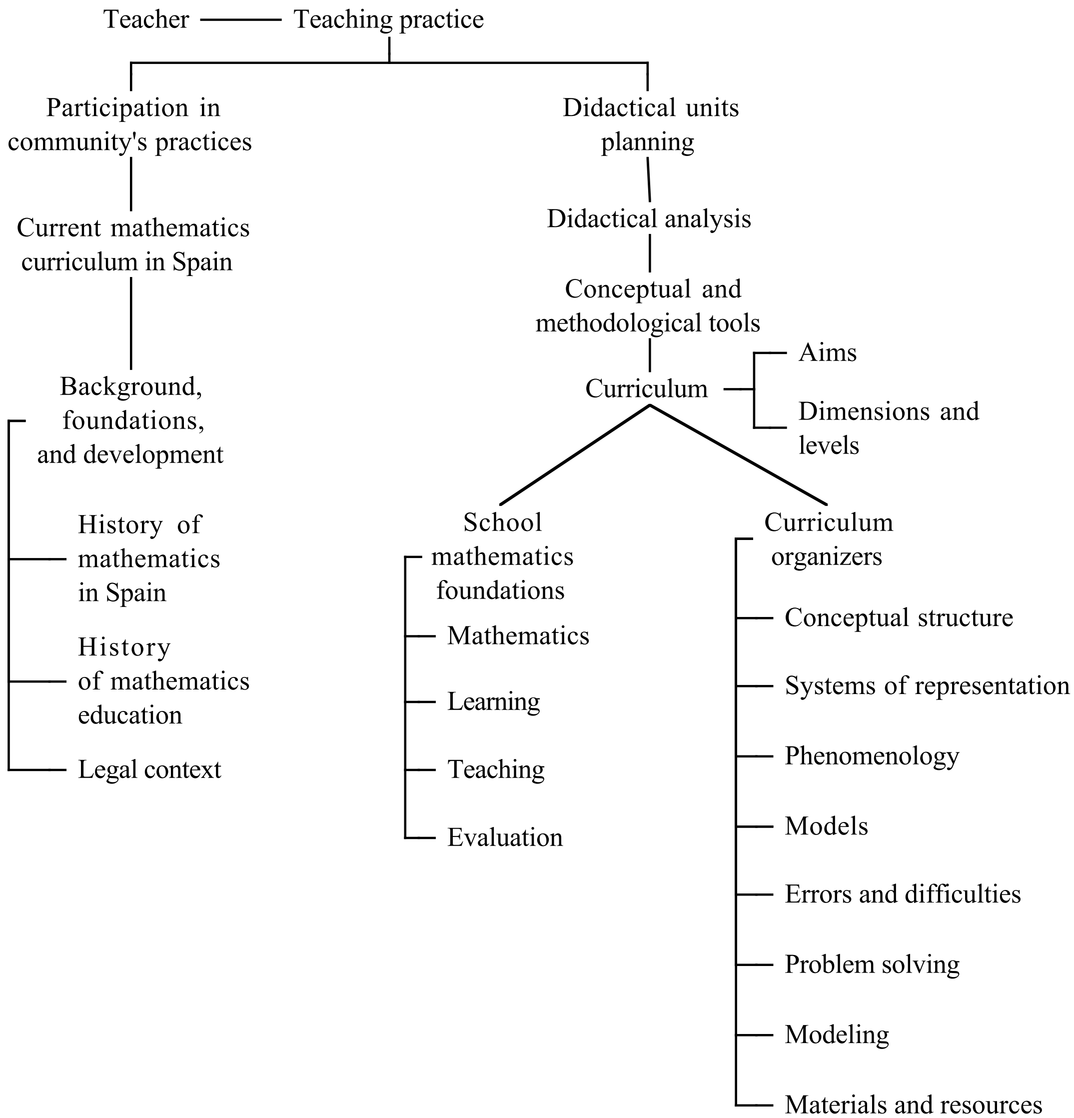

Figure 2. Content structure of the course

Didactic analysis organised the treatment of the curriculum organizers. We developed a general theoretical analysis of each of the curriculum organizers but also studied the ways that these notions acquire technical and practical use when they are used to analyse specific mathematical structures. The course thus had a specific mathematical content that is shown in the mathematical structures for which the didactic analysis is performed.

Methodology

In the course, we used different methodological plans. We will now describe the plan used systematically in the simulation of the process of planning a didactic unit. Each group of future teachers chose a mathematical topic on which to perform the didactic analysis and design a didactic unit. The plan was cyclical. Each cycle corresponded to a curriculum organizer. The sequential order in which the curriculum organizers were treated follows the plan shown in Figure 2. Figure 3 shows the cycle of methodological treatment of each curriculum organizer.

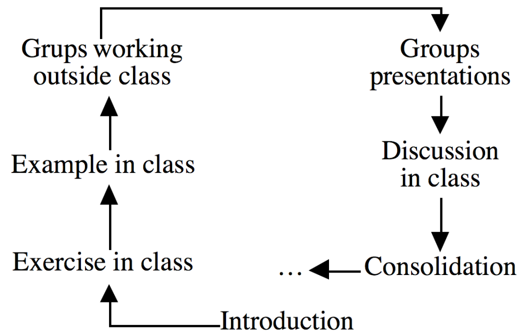

Figure 3. Cycle of methodological treatment of didactic analysis

The cycle starts from the discussion that ended the previous cycle. In general, this discussion (for example, of systems of representation) leads to the introduction of a new curriculum organizer (for example, the notion of phenomenology). From this introduction, we propose an in-class exercise that consists of using this notion for a predetermined mathematical structure or the mathematical structure on which each group is working. The groups present their proposals and discuss possible meanings of the curriculum organizer in its practical application. Then, the trainers present an example of how the notion can be used for a specific mathematical structure (different from those assigned to the groups). For the next class, the students are to apply this curriculum organizer (and those considered so far) to a mathematical structure. In the next session, each group presents the results of its work to the rest of the class. Classmates and trainers discuss and critique each presentation. Finally, the trainers moderate a discussion in which we seek to formulate questions and activities that tackle the errors and difficulties we found in the presentations. On some occasions, the trainers suggest aspects of the reference meaning of the curriculum organizer being used. The end of the cycle has two parts. First, the trainers use the previous discussion to motivate the introduction of a new curriculum organizer. Second, one of the trainers reviews each of the productions and produces a document with his comments and suggestions. The future teachers receive this document at the next session.

Evaluation

The evaluation of the work of the future teachers is the result of the evaluation of all of their productions and of the trainers' appraisal of the way in which each future teacher progresses in his participation in the classroom's community of practice. We pay special attention to the work and the final presentation in which each group presents and justifies the design of a didactic unit on its topic.

SOME RESEARCH RESULTS

We have studied several aspects of this teacher training model. On the hand, we have analyzed several of the methods courses that follow it from the point of view of their relevance (Gómez, González, Gil, Lupiañez, Moreno, Rico *et al.*, 2007). In this study we assessed the degree to which the courses' syllabus fulfilled the expectations that society places upon them and characterized them in terms of those expectations. These social expectations were reflected in a set of competencies, the Itermat list, agreed by several agents concerned with mathematics teacher training in Spain (Recio & Rico, 2004). In this study, we found that even though the courses design was aligned to the list of competencies, several objectives could be revised to improve the course fulfilment of those competencies.

One of the courses has been the object of study of the doctoral dissertations of three of the authors (Lupiáñez & Rico, 2006; Marín, 2005). One of these projects has recently been finished (Gómez, 2007). Its purpose was to describe the learning processes of the future teachers that participated in the version of the course we have described. It aimed to characterise the development of didactic knowledge in the groups of future teachers with respect to the notions of subject matter analysis and to propose some conjectures to explain this process.

Gómez found that the didactic knowledge of the groups of future teachers who participated in the course evolved gradually, heterogeneously, and out of synch with the instruction. The groups of future teachers faced difficulties when they analysed their topic with each of the curriculum organizers of the subject matter analysis. These difficulties were the product, among other things, of the complexity of the curriculum organizers as instruments to be mastered. Furthermore, it was found that future teachers develop their competence in task design when using the curriculum organizers through a dynamic interplay between the process of meaning construction of each notion and its technical and practical use. For instance, bringing the practical use of a curriculum organizer into play enabled the groups to succeed in reifying its technical use.

Gómez established four states of development of the future teachers didactic knowledge. These states characterize their learning processes along the course. In this sense, the construction and negotiation of the meaning and uses of a curriculum organizer within a group was an evolving process. In the process of transforming a curriculum organizer into an instrument, the analysis of the mathematical structure and the construction of the technical use of the notion interacted dynamically. As the group advanced in the analysis, they constructed more complex meanings (of the curriculum organizer and the concept) that in turn enabled new and deeper analyses.

The groups also advanced in the construction of the technical use of each curriculum organizer when they tried to put the information that emerged from their analysis into practice. The technical and practical uses of a curriculum organizer interacted in two ways: first, practical use was put into play when the information that emerged from the analysis of the topic was made explicit (technical use); second, the groups advanced in materialising the technical use of the curriculum organizer when they performed its practical use. Nevertheless, that a group developed and materialised the technical use of a curriculum organizer did not necessarily mean that it advanced in the performance of its practical use.

These results hint at several conjectures concerning future teachers learning in terms of how they are able to transform the curriculum organizers into instruments. In a study currently underway, González and Gómez (forthcoming) are exploring more deeply this process. They have found so far that the interplay between the meaning and technical and practical uses of a curriculum organizer can take different configurations depending, for instance on the previous knowledge brought by the future teachers to task.

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