LEARNING THE NOTION OF LEARNING GOAL IN AN INITIAL FUNCTIONAL TRAINING PROGRAM

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In an initial functional teacher-training program, future teachers are expected to develop their competences for using mathematics education notions in order to analyze a mathematics school concept and use the information emerging from such analysis for the design and implementation of didactic units. In this paper we propose a set of conceptual and methodological tools for exploring and characterizing future teachers learning of those notions. These tools are based on the ideas of meaning, technical use, and practical use of a notion. We exemplify the use of these tools for the case of the notion of learning goal.

What and how future teachers learn depend on the kind of training they get involved in. If the training follows a functional model (Gómez, et al., 2008), future teachers are expected to develop their teaching competences, where "competence is related to the process of activating resources (knowledge, skills, strategies) in a variety of contexts, namely problematic situations" (Abrantes, 2001, p. 130). In our case, we expect them to learn how to use a set of notions in order to solve didactic problems. Exploring and characterizing future teachers learning implies then describing how future teachers interpret the meaning of those notions, how they use them for analyzing the subject matter, and how they use the resulting information for solving the didactic problems at hand. In what follows, we explain what we mean by initial functional training, introduce the ideas of meaning, technical use and practical use of a mathematics education notion, and present a detailed example of the use of these ideas for describing a group of future teachers' learning of the notion of learning goal.

TRAINING WITH A FUNCTIONAL MODEL

A functional model of teacher training rests on the idea that the competences of the mathematics teacher can be characterized in terms of what he should be able to do in a specific context of students' learning. Future teacher training under this approach postulates "a set of tasks, a set of conceptual tools and a subject that, when performing the task using the available tools, put into play and set forth his/her competency in carrying out the processes involved" (Rico, 2007, pp. 49-50). In particular, the planning competence of the mathematics teacher can be characterized from the analysis and description of the tasks, conceptual tools and activities needed to plan a specific mathematics lesson.

Under this functional view, Gómez (2007) has produced a detailed conceptualization of the ideal process by which a teacher designs and implements a mathematical lesson, also attending to the factors conditioning the context. This conceptualization, called didactic analysis, is based on a cyclical procedure that can be used in training

courses for developing future mathematics teachers' planning competence (Gómez, 2006). Four analyses compose the didactical analysis procedure: (a) *subject matter analysis,* as a procedure by which the teacher identifies and organises the multiplicity of meanings of a concept; (b) *cognitive analysis,* in which the teacher describes his hypotheses about how the students can progress in the construction of their knowledge; (c) *instruction analysis,* in which the teacher designs, analyses, and chooses the tasks that will constitute the teaching and learning activities; and (d) *performance analysis,* in which the teacher determines the capacities that the students have developed and the difficulties they may have expressed up to that point. In fact, the planning competence is mainly linked with the three first analysis just described.

The four analysis set up around a set of notions called *curriculum organizers* (Rico, 1997). Representations, errors and learning goals are examples of these notions. Each one of the curriculum organizers captures a face of the complexity of the mathematics curriculum and serves as theoretical support to structure the variety of meanings of a mathematical concept that have to be negotiated in a mathematics classroom. According to the functional view we advocate, curriculum organizers are considered methodological and analytic tools with a didactic purpose. They are the basic pieces that support the future teacher decisions when they are involved in the task planning activity.

LEARNING A CURRICULUM ORGANIZER

From the future teacher learning perspective, we assume that teachers develop their knowledge as a product of action, through the integration, tuning and restructuring of theoretical knowledge to the demands of practical situations and constraints (Bomme & Tillema, 1995, p. 262). When the future teacher takes his first contact with didactic notions-the curriculum organizers-with the perspective of using them in practice, he develops a particular understanding of them that depends on the actions he performs to solve a particular activity. At the same time, the proposed solutions and actions are affected by the developed understanding of the notions. This learning approach is rooted in Vygotsy perspectives and his consideration of instruments as mediators in the individual psychological activity. These instruments undergo complex processes of appropriation-development by users when they are involved in activities. Vygotsky (1982) describes processes that, together with developments of Vérillon and Rabardel (1995), have been recently used to develop the instrumental genesis theory that characterizes learning with artefacts in CAS environments (Guin, Trouche, & Ruthven, 2005). As Trouche (2005, p. 155) has claimed, "the study of instrumented action schemes requires studying, beyond the techniques themselves, their epistemic, heuristic and pragmatic functions". In this paper we will focus on the epistemic, heuristic and pragmatic functions of the curriculum organizers. These three functions characterize the three aspects of the use of a curriculum organizer by a subject: the subject (a) needs some understanding of the curriculum organizer in order (b) to use it for analyzing a mathematical concept, producing useful information that, in turn, (c) can be used possibly in conjunction with others organizer's information, with a

concrete didactic purpose. We denote these three processes *meaning*, *technical use*, and *practical use* of a curriculum organizer, corresponding to its epistemic, heuristic and pragmatic functions.

In the context of preservice teachers' training, the *meaning* (M) of a curriculum organizer is the option that the trainers have taken for the formal meaning of the didactic notion to which it refers, from the multiple meanings that are proposed in the mathematics education literature. Besides, as a tool of the didactic analysis cycle, each curriculum organizer has a heuristic function that we call its *technical use* (TU). It refers to the set of strategies and techniques that, as trainers, we consider necessary for analyzing a secondary school mathematics topic and producing relevant didactic information about it. The information that emerges from the technical use of a curriculum organizer can be used for didactic purposes. This is what we call its *practical use* (PU) and sets up the pragmatic function of the curriculum organizer. It refers to the set of strategies and techniques that, as trainers, we consider necessary for using the information produced with the technical use in other analysis of the didactic analysis procedure or in the design of a didactic unit on the topic at hand. Figure 1 shows a schematic representation of these ideas.

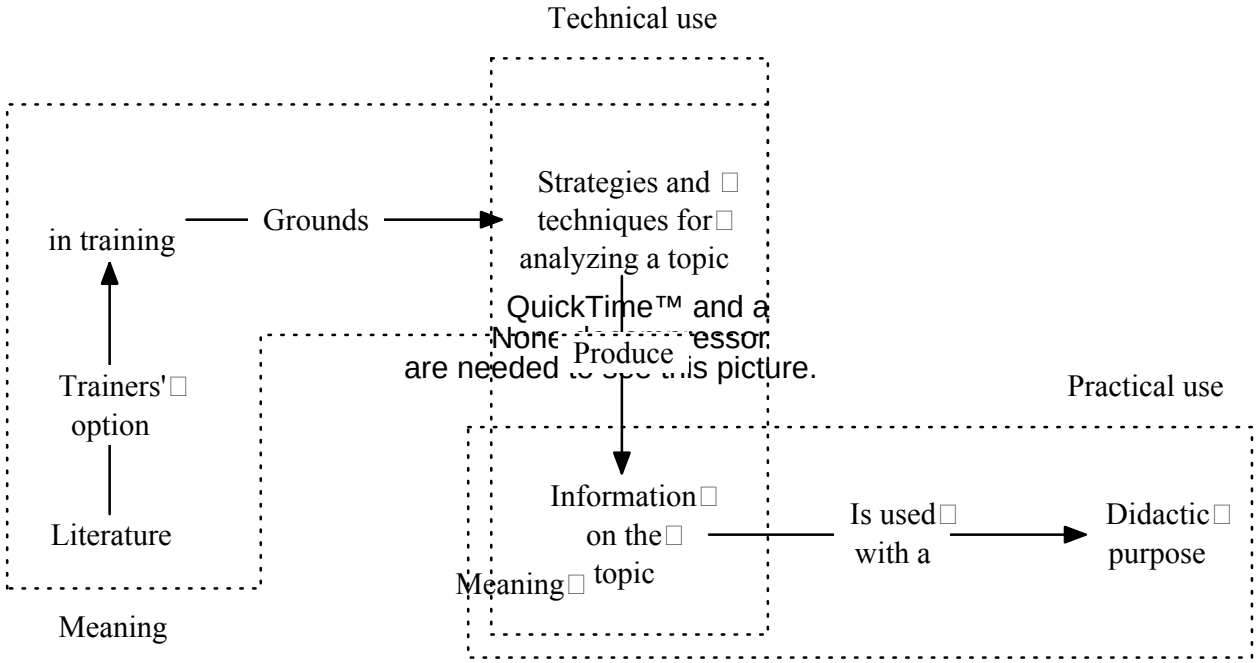


Figure 1. Meaning and uses in teacher training.

We claim that it is possible and relevant to explore and characterize future teachers' learning of curriculum organizers in terms of the ideas of meaning, technical use and practical use. When performing tasks during training, we say that a future teacher *develops the meaning* of a curriculum organizer, if he proposes examples of it, or declares, discusses or reflect on its properties, definition or relationships with other notions. He *develops the technical use* of a curriculum organizer, if he puts it into play in order to analyse a mathematical topic. A future teacher's technical use of a

curriculum organizer is usually based on his interpretation of the meaning of the notion and can involve specific methods or other notions of the didactic analysis procedure. Finally, we consider that a future teacher *develops the practical use* of a curriculum organizer when there is evidence that he uses the information emerging from its technical use for didactic purposes.

SERIES AND DIAGRAMS

The notions of meaning, technical use and practical use can be used, for instance, for coding and analyzing the protocols of the interaction of a group of future teachers during their training. Episodes along time in the protocols can be identified and coded according to the criteria we proposed above. With this coding procedure, it is possible to organize the sequence of episodes over time that characterizes the future teachers' learning process. This sequence can be depicted graphically in a *series*, as shown in Figure 2 for the learning goal curriculum organizer. The four horizontal zones in Figure 2 under the thick line, show those episodes that have been coded for meaning development (M), technical use (TU), and practical use (PU) of the learning goal notion on other notion A, or practical use of a notion B on the learning goal notion. These other didactic notions (type A or B) have been grouped in the three top horizontal zones over the thick line, depending on whether they belong to subject matter analysis, cognitive analysis or task analysis.

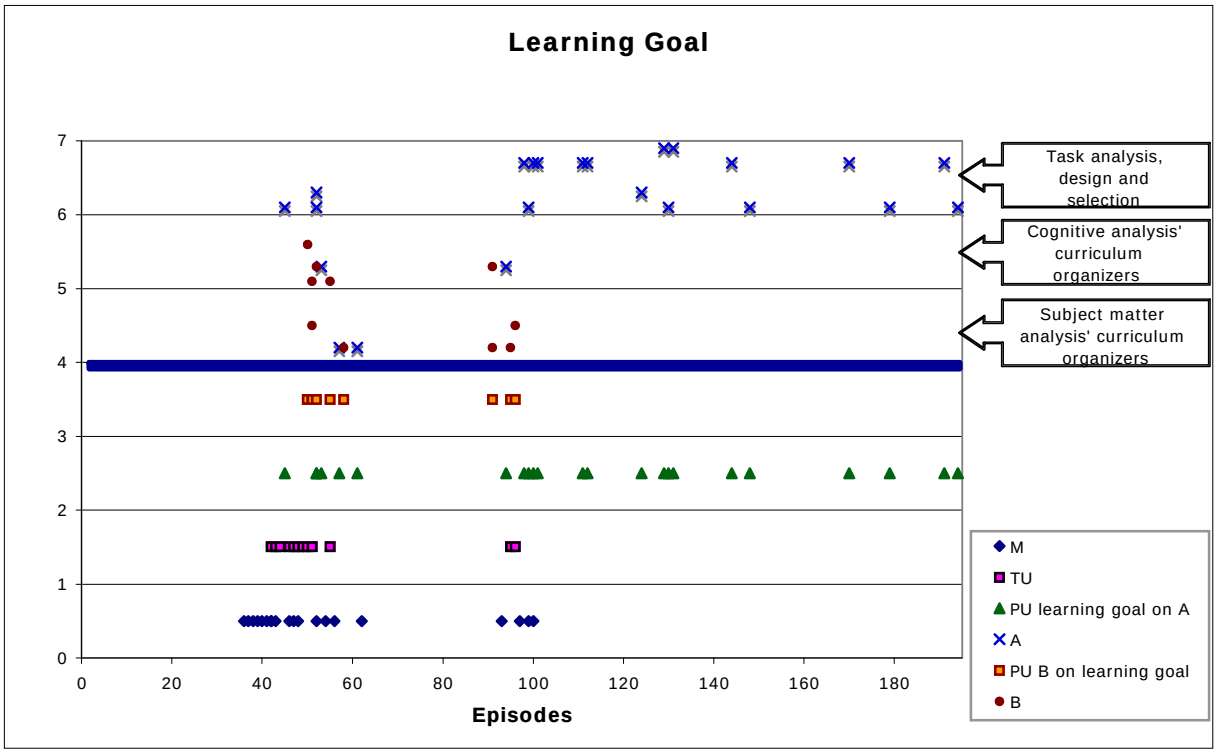


Figure 2. Episodes series for the notion of learning goal.

The series depicts the relationships between the three dimensions (M, TU, PU) of this process and provides information for identifying patterns of use of the curriculum organizer considered. For instance, the analysis of the learning goal series in Figure 2

shows that the meaning and the technical use develop simultaneously: there are two periods in which there is meaning construction at the same time that there is technical use development. One can also see that the information emerging from the technical use is used frequently, mainly in task analysis and selection. This practical use takes place very early in the sequence and keeps developing along it. The diagram of Figure 3 represents this overall learning pattern.

Figure 3. Diagram for the notion of learning goal.

The learning process of a curriculum organizer does not have to follow a unique pattern. Over time, future teachers might follow different patterns in their learning process. We see a curriculum organizer diagram as a representation of the future teachers learning process of that curriculum organizer. It reflects how, in practice and over time, the future teachers transform the curriculum organizer into an instrument. In particular, diagrams show how the curriculum organizers mediate in the process of performing the tasks assigned to them.

LEARNING THE NOTION OF LEARNING GOAL

During the academic year 2006-2007, the University of Cantabria implemented an optional methods' course designed according to the functional model described above. Three female future teachers participated in the course. They were third year mathematics students. They worked as a group and chose the topic "area of plane figures" to perform the didactic analysis on. The methodology used in the course was the same for each curriculum organizer. The trainer started with the presentation of some of the disciplinary meanings of the curriculum organizer. Then, she described with examples how a given secondary school mathematics topic could be analyzed with that curriculum organizer, producing the corresponding information. Then, the group was asked to produce the technical analysis of their topic with that curriculum organizer. Once the information from the different curriculum organizers comprising the didactic analysis was produced and organized, the group was asked to use that information as a whole for designing and justifying a didactical unit for their topic.

The learning goal notion is one of the curriculum organizers involved in the cognitive analysis (together with the notions of error, difficulty, capacity and learning path). The group of future teachers used the information emerging from the subject matter analysis of the topic in order to identify the learning goals of the lesson. Then, they formulated the capacities related to each learning goal and the types of tasks involved. This information enabled them to characterize the corresponding learning paths, to locate students' errors and difficulties and, therefore, to analyze, compare and select those tasks that, in their opinion, could better promote students' learning goals' achievement.

We recorded 35 hours of audio corresponding to the cognitive analysis period of the course. In the transcription of the recordings, we identified 220 episodes, which were coded according to the procedure described above.

A detailed analysis of the episodes series in Figure 2 shows three different periods. In the first one, between episodes 35 and 60, there are passages of different types. On the one hand, there is an effort in meaning construction. Most of these episodes are simultaneous with the development of technical use. This technical use seems to produce a later practical use mainly in task analysis and selection. In this period, the notions from cognitive analysis and subject matter analysis are used in the technical use of the notion of learning goal. The following is episode 48 of this period:

> Future teacher: Learning goal, that is, for instance, let us see if we can say something like: "strategy development". We could start a learning goal like that, couldn't we? OK, "strategy development for calculating unknown magnitudes"… And there, we could finish the learning goal, couldn't we? We could extend it, that is what I was saying, or include more things in the learning goal.

In this episode we recognize how the future teacher is constructing the meaning of the notion of learning goal. She is not sure whether a statement is a learning goal or not and whether a statement can be regarded as a complete learning goal ("couldn't we?"). This construction of meaning is not done from a formal definition of the notion. It is done through its technical use: the future teacher proposes two versions of a learning goal for the topic at hand and looks for confirmation of her conjectures from the trainer and her fellow future teachers.

In the second period, from episodes 91 to 100, the situation is similar to the first period. Nevertheless, there is less technical use development. Meaning and technical use of this notion do not appear to be simultaneous.

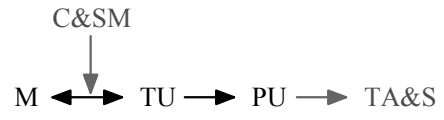
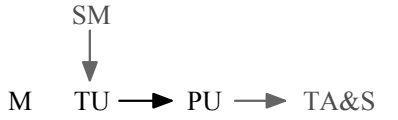
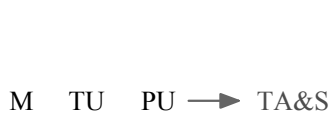
Finally, the third period is concerned with the notion's practical use, mainly for task analysis and selection. The following episode (108) shows the practical use of the notion of learning goal.

> Future teacher 1: For our learning goal, the goat problem is perfect. All the items, except one. There is only one question that refers to other types of learning goals. The others are all about our capacities.
>
> Future teacher 2: What are we looking for? Only one problem for each learning goal? Because I like the perimeter problem.
>
> Future teacher 1: The perimeter problem is a good one, but it does not refer at all to this learning goal.
>
> Future teacher 2: OK, OK.

In this episode, we see how the group of future teachers makes a practical use of the notions of learning goal, capacity and learning path with the purpose of selecting a task: they assess whether the sequence of capacities that are put into play by the task correspond to the given learning goal. They recognize that the goat problem puts into play the capacities they are interested in, whereas the perimeter problem does not. In

the same episode, they develop the meaning of the learning goal curriculum organizer: they ask themselves how many tasks were necessary in order to assess it.

Table 1 shows the diagrams representing the patterns identified in these three periods. They give a more detailed explanation of the patterns depicted in Figure 2.

Periods		
1	2	3
		

CandSM: Cognitive and subject matter analysis; SM: Subject matter analysis; TAandS: Task analysis and selection

Table 1. Learning diagrams (periods)

DISCUSSION

One could expect that the process of learning a curriculum organizer should correspond to the sequence followed in the course instruction: first to construct the meaning of the notion, then interpret this notion in practice in order to develop strategies for analysing a topic with it (technical use), and finally use the information emerging from the technical use for other analysis or the design of a didactic unit. The diagram M $\rightarrow$ TU $\rightarrow$ PU can represent this sequence. But this was not exactly the case for the notion of learning goal that we have presented. In this case, meaning and technical use were developed interactively in the first period. Future teachers had an informal meaning of the notion of learning goal. When, in the technical use phase, they tried to identify learning goals of a lesson, they realized that such informal meaning was not enough, and was not necessarily the same as the meaning proposed by the instruction. They progressed in the meaning construction as a consequence of the requirements of the technical use of the notion. In the second period, technical use did not appear directly linked to meaning development. Now, future teachers made a practical use of subject matter analysis curriculum organizers to produce learning goals of the lesson and use them in practice. This method constituted their particular strategy to produce and use didactic information. In the third period, the absence of arrows in the diagram shows that future teachers did not made explicit references to meaning nor technical use while they performed the notion's practical use.

We have found in some preliminary explorations with other curriculum organizers of the cognitive analysis, that preservice teachers do in fact enact different sequences for different curriculum organizers (González and Gómez, Forthcoming). In some cases, they do not have an informal meaning of the curriculum organizer, and they have to construct it before putting it into play. In other cases, they do not see the need for checking their informal meaning against the meaning proposed by the trainers. We

have not yet explored if there are characteristics of particular curriculum organizers that promote some specific patterns, or under what circumstances some patterns are more frequent than others.

We claim that answering in detail the questions of what patterns appear and why, can help us, as researchers, understand how learning takes place in a methods course based on a functional perspective of teacher training and learning. It can also help us, as trainers, in assessing the design and development of our training programs.

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