Conceptualizing and Exploring Mathematics Future Teachers' Learning of Didactic Notions

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Exploring how future teachers learn in a training program requires taking into account what they are expected to learn and what kind of learning opportunities they are exposed to in their learning. In this paper we focus our attention on training programs that expect future teachers to develop their competencies in using certain didactic notions for analyzing a mathematical topic. We propose three ideas —meaning, technical use and practical use— for exploring future teachers' learning.

Keywords: preservice mathematics teacher training; teacher learning; competencies

La indagación sobre el aprendizaje de los futuros profesores en un programa de formación requiere tener en cuenta qué es lo que se espera que ellos aprendan y a qué tipo de oportunidades de aprendizaje se exponen durante su aprendizaje. En este artículo, nos centramos en programas de formación que pretenden desarrollar las competencias de los futuros profesores en el uso de ciertas nociones didácticas con el propósito de analizar un tema matemático. Proponemos tres ideas —significado, uso técnico y uso práctico— para explorar el aprendizaje de los futuros profesores.

Palabras clave: formación inicial de profesores de matemáticas; aprendizaje de profesores; competencias

There is a well known concern on characterizing what secondary mathematics future teachers should learn in initial training and what they actually learn (e.g., Sfard, Hashimoto, Knijnik, Robert & Skovsmose, 2004). There is, as well, a developing concern about how these future teachers learn what they learn (Beijaard, Korthagen & Verloop, 2007). Understanding how future teachers learn is relevant. Characterizing future teachers' learning processes can help trainers and curriculum designers improve the curriculum design and development of the programs in which future teachers participate. We claim that in order to understand the *how* of future teachers' learning, we have to take to the front of the analysis what they are expected to learn and to what kind of learning opportunities they are exposed to in their learning (Strässer, Brandell, Grevholm & Helenius, 2003). In other words, we do not see future teachers' learning as a "general" learning problem. We claim that it has to be conceptualized and analyzed in terms of the specific training context in which that learning takes place (Putnam & Borko, 2000).

In this paper we consider the case of programs that are based on a functional view of future teachers' training (Gómez, González, Rico, Gil, Lupiañez, Marín *et al.*, 2008). These training programs have two features. First, what future teachers are expected to learn is determined on the basis of what a teacher is ideally expected to be able to do in practice. That is, they are based on a perspective of the competencies that the future teacher is expected to develop in order to perform properly in practice (Department of Education and Training, 2004; NBPTS, 2002).

Second, future teachers are expected to become competent in the use of certain conceptual and methodological tools that are supposed to help them analyze a specific mathematical topic in order to produce a curriculum design, put it into practice and asses its results. The didactic knowledge that future teachers are expected to develop is seen from a functional perspective: it is based on theoretical knowledge, but it has a practical purpose.

Our concern with future teachers' learning stems from a long term concern on future teachers' training and learning in our research group. We have been working for more than twenty years in preservice secondary mathematics teachers training. This work has enabled us to develop a functional model of future teachers' training based on two ideas: the curriculum organizers and the didactic analysis. Didactic analysis is a procedure that enables teachers to design, implement and assess curriculum designs for specific mathematical topics. This procedure organizes the use of a set of conceptual and methodological tools (the curriculum organizers) that enable the teacher to analyze the multiple meanings of mathematical topic from several perspectives (conceptual, cognitive, instruction and assessment).

In what follows, we describe the main features of the functional perspective on initial teacher training we work on and how we see future teachers' learning within this type of programs. In the second part of the paper, we describe the ideas of meaning, technical use and practical use of a curriculum organizer from two perspectives: (a) as a way of organizing the disciplinary knowledge that trainers expect future teachers to develop during training, and (b) as a means for exploring and characterizing future teachers' learning.

Future Teacher Training From a Functional Perspective

We think about mathematics teacher's knowledge from a functional perspective. According to this view, teacher's knowledge can be established from the analysis and description of the activities needed to plan, manage and evaluate a mathematics lesson. Thus, the problem of the teacher's knowledge can be considered as the integration of knowledge, abilities and attitudes for action. Instead of thinking on what the teacher should know, we ask ourselves what he should be able to do in a specific context of students' learning. Therefore, we start by adopting a functional view of school mathematics, and then we reflect on the teacher's activities that can promote students' learning in that context (didactic analysis).

Didactic analysis is set up around a set of notions (like representation systems or learning goal) that we call *curriculum organizers* (Rico, 1997). The way we use these notions in future teachers training is coherent with the functional view we advocate: curriculum organizers are considered methodological and analytic tools with a didactic purpose. That is, we pinpoint our approach by postulating "a set of tasks, a set of conceptual tools and a subject that, when performing the task using the available tools [the curriculum organizers], put into play and set forth his/her competency in carrying out the processes involved" (Rico, 2007, pp. 49-50).

Didactic analysis can be used as a task planning procedure in preservice mathematics teacher training (Gómez, 2006). With it, the teacher can specify (and differentiate) the goals, content, methodology and evaluation scheme of each topic in planning. We claim that in the specific context of the planning of an hour of class or a didactic unit, the teacher can organise instruction based on four analyses (Gómez, 2007):

1. *subject matter analysis,* as a procedure by which the teacher identifies and organises the multiplicity of meanings of a concept;

2. *cognitive analysis,* in which the teacher describes his hypotheses about how the students can progress in the construction of their knowledge of the mathematical structure when they face the tasks that will make up the teaching and learning activities;

3. *instruction analysis,* in which the teacher designs, analyses, and chooses the tasks that will constitute the teaching and learning activities that are the object of the teaching; and

4. *performance analysis,* in which the teacher determines the capacities that the students have developed and the difficulties that they may have expressed up to that point.

We use *didactic analysis* to refer to a cyclical procedure that includes these four analyses, attends to the factors conditioning the context and identifies the activities that the teacher should perform to organise the teaching of a specific mathematical content.

Each analysis is performed with the help of a set of curriculum organizers. For instance, subject matter analysis is the procedure by which the teacher can identify, organise and select the meanings of a concept or mathematical structure within the school mathematics content. The procedure is performed by attending to three curriculum organizers : systems of representation, conceptual structure and phenomenology.

For instance, using the curriculum organizer systems of representation, a topic like the quadratic function can be analyzed to produce information like the one presented in Figure 1.

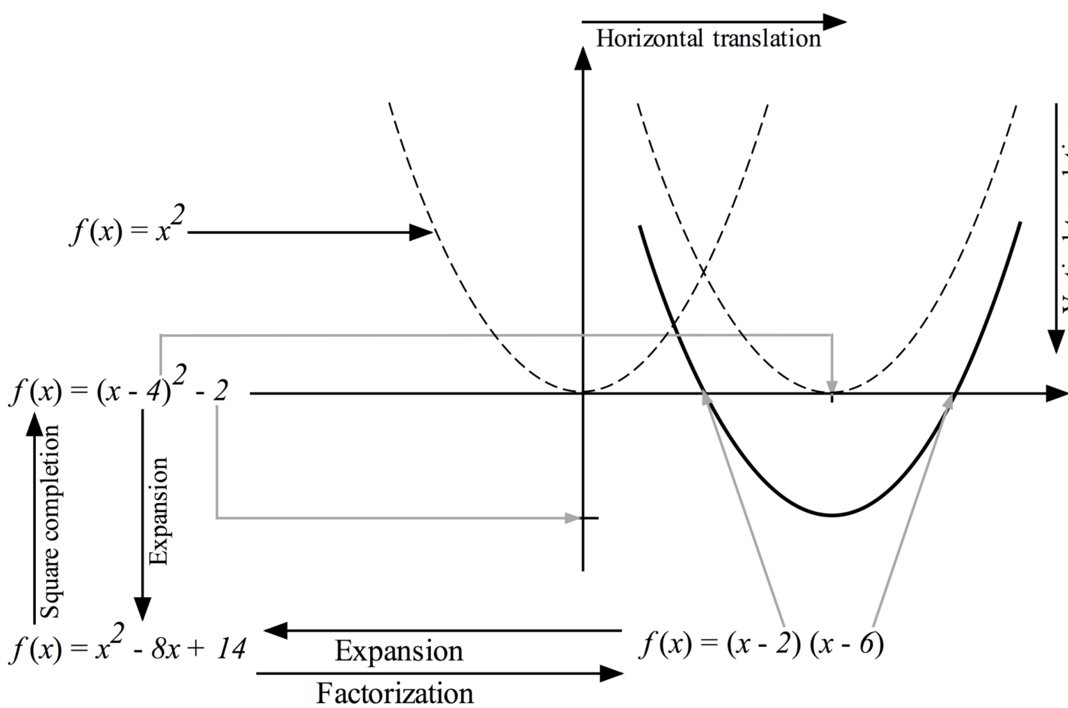

Figure 1. Operations in systems of representation

When analyzing a topic with a curriculum organizer one has to be knowledgeable of the topic and of the curriculum organizer. The purpose is to use the curriculum organizer as conceptual and methodological tool in order to produce "new" didactic information about the topic. It is a view of the topic from the school mathematics point of view. Curriculum organizers enable us to identify and organize the multiple meanings of a topic (Cooney & Wiegel, 2003; Rico, Castro, Castro, Coriat, Marín, Puig *et al.*, 1997).

Future Teacher Learning From a Functional Perspective

We focus our interest in understanding how future teachers develop their teaching competencies when they are guided through the process of performing the didactic analysis of a mathematical topic. Since this a rather complex goal, we are focusing our attention, for the time being, on understanding how future teachers learn to use the curriculum organizers. That is, how they develop an understanding of each one of these notions (e.g., representation systems) and how they put into practice their understanding when analyzing a mathematical topic.

Our approach to conceptualizing and understanding future teachers' learning in a functional training context stems from several sources. On the one hand, we claim that working with a curriculum organizer implies three related steps: (a) understanding the curriculum organizer in order (b) to use it for analyzing a mathematical concept, producing information that, in turn, (c) can be used possibly in conjunction with others organizer's information, with a concrete didactic purpose. This is a way of interpreting how future teachers might integrate, tune and restructure theoretical knowledge to the demands of practical situations and constraints (Bromme & Tillema, 1995, p. 262). It is our interpretation of one of the multiple aspects of the theory-practice duality (Cobb & Bowers, 1999). On the other hand, we consider curriculum organizers as a particular type of concept when analyzed in a training context: they are concepts that have a meaning that can be put in practice for didactic purposes. They have to be learned (a) as theoretical notions applicable to a wide variety of situations, (b) as knowledge that is situation specific and related to the context in which they are applied (Korthagen & Kessels, 1999, pp. 8-9), and (c) as tools that enable the teacher to be productive (p. 13). This particular look of the concepts involved motivates us to see their learning in training from a Vygotskyan point of view. Concepts can be seen as instruments that mediate between the subject and his environment when the subject performs certain activities (Vygotsky, 1982). In the context of technology learning, the Vygotsky approach has been extended, giving rise to the theory of instrumental genesis. This theory has been mainly used for exploring how a subject can transform an artefact (i.e., a calculator) into an instrument when performing some activity (Rabardel, 1999, 2003). Nevertheless, it has also been adapted to situations in which the artefact is a concept (Vérillon, 2000). The development of this theory has highlighted the fact that the "the study of instrumented action schemes requires studying, beyond the techniques themselves, their epistemic, heuristic and pragmatic functions" (Trouche, 2005, p. 155). These three functions characterize the three steps that describe the activity of a future teacher when working with a curriculum organizer.

Three Ideas for Organizing Disciplinary Knowledge

These ideas has led us to postulate three dimensions of a curriculum organizer as a way for organizing the disciplinary knowledge that future teachers are expected to develop during their training and, hence, for exploring and characterizing their learning. We denote these three dimensions as meaning, technical use, and practical use of a curriculum organizer. They correspond to the three steps involved in the activity of using a curriculum organizer and represent its epistemic, heuristic and pragmatic functions. In what follows we first describe these three dimensions as a way of organizing the disciplinary knowledge that trainers expect future teachers to develop during training. In the next section, we describe them as a means for exploring and characterizing future teachers' learning.

Meaning of a Curriculum Organizer

In the context of preservice teachers' training, the *meaning* (M) of a curriculum organizer is the option that the trainers have taken for the formal meaning of the didactic notion to which it refers, from the multiple meanings that are proposed in the mathematics education literature. For instance, the following is the option we have taken concerning the curriculum organizer that we call "conceptual structure" (Gómez, 2007, pp. 44-50).

We use the expression "conceptual structure" to refer to three aspects of every concept of the school mathematics content:

1. *Interrelated mathematical structures.* We assume that every mathematical concept is related to at least two mathematical structures:
   - the mathematical structure that the concept configures and
   - the mathematical structures of which it forms part.
2. *Conceptual relations.* We emphasise the relations that are established between the concept and
   - the concepts of the mathematical structure that this concept configures (e.g., the relation between the quadratic function and the quadratic equation),
   - the objects that are specific cases of this concept (e.g., $f(x)=3x^2-4$ as a specific case of the quadratic functions of the form $f(x)=ax^2+c$), and
   - the concepts that belong to the mathematical structure of which the concept forms part (e.g., the relation between the quadratic function and continuous functions).
3. *Relations of representations.* Exploring the meanings of a concept requires systems of representation, since with them it is possible to identify the ways in which the concept is presented. On taking into account the systems of representation, we can point out the relations that arise from operations in the systems of representation: syntactical transformations and translation between systems of representation.

Technical Use of a Curriculum Organizer

Besides, as a tool of the didactic analysis cycle, each curriculum organizer has a heuristic function that we call its *technical use* (TU). It refers to the set of strategies and techniques that, as trainers, we consider necessary for analyzing a secondary school mathematics topic and producing relevant didactic information about it. The following is an example of techniques that we propose concerning the conceptual structure of a topic.

When the teacher explores the conceptual structure of a concept in school mathematics, he should take into account three kinds of "elements" and two groups of relations between these elements. The elements are:

- the *objects,* as specific cases of a concept, forming its extension;
- the *concepts,* as predicates that are saturated by the objects and, in turn, form mathematical structures; and

- the *mathematical structures,* which are shaped by concepts.

We propose that the teacher use conceptual maps as a tool for gathering, organising, representing and sharing the information corresponding to the meanings of a mathematical concept. From the perspective of the mathematical content, in a conceptual map we can identify different kinds of connections:

- connections that establish relations between different elements of the mathematical structure (for example, between the different symbolic forms and their parameters),
- connections that associate different representations of the same element (for example, the parameters of the multiplicative form and the roots of the parabola),
- connections that associate the transformations of one element into another within one system of representation (for example, the procedure of factorization for transforming the standard symbolic form into the multiplicative symbolic form), and
- connections that show the relation between categories of phenomena and the substructures with which it is possible to organise them (for example, the relation between the properties of the focus of the parabola and the phenomena of optics that use these properties).

Practical Use of a Curriculum Organizer

The information that emerges from the technical use of a curriculum organizer can be used for didactic purposes. This is what we call its *practical use* (PU) and sets up the pragmatic function of the curriculum organizer. It refers to the set of strategies and techniques that, as trainers, we consider necessary for using the information produced with the technical use in other analysis of the didactic analysis procedure or in the design of a didactic unit on the topic at hand.

For instance, we suggest that the information produced with the subject matter analysis can be used to determine the capacities that the students should be expected to develop. If a the teacher wants to work with a learning goal like “To recognize and use the graphical meaning of the parameters of the symbolic forms of the quadratic function and communicate and justify the results of its use”, on the basis of the information outlined in Figure 1, the teacher can identify some of the capacities involved in this topic. Table 1 shows such a list. They have been obtained and classified taking into account the kind of representation involved (symbolic, graphical).

Table 1
Capacities of a learning goal

Perform, communicate and justify symbolic transformation procedures		*Identify, show and justify graphical elements*	
C1	Square completion	C8	Vertex coordinates
C2	Expansion	C9	Y-axis intersections
C3	Factorization	C10	X-axis intersections
		C11	Focus coordinates
		C12	Directrix equation
		C13	Symmetry axis equation

Identify, show and justify symbolic elements		*Perform, communicate and justify graphical transformation procedures*	
C4	Canonical form (a, h, k)	C14	Horizontal translation
C5	Focus form (p, h, k)	C15	Vertical translation
C6	Standard form (a, b, c)		
C7	Multiplicative form (a, r_1, r_2)	C16	Vertical scaling

Meaning and Uses of a Curriculum Organizer

From the disciplinary point of view, the ideas of meaning, technical use and practical use of a curriculum organizer can be related to each other as shown in Figure 2. First, the trainers establish the meaning of the curriculum organizer that will be promoted through instruction. This meaning grounds the strategies and techniques that can be used for analyzing a mathematical topic (technical use). As a result of this analysis, "new" information about the topic is produced which can be used for didactic purposes (practical use).

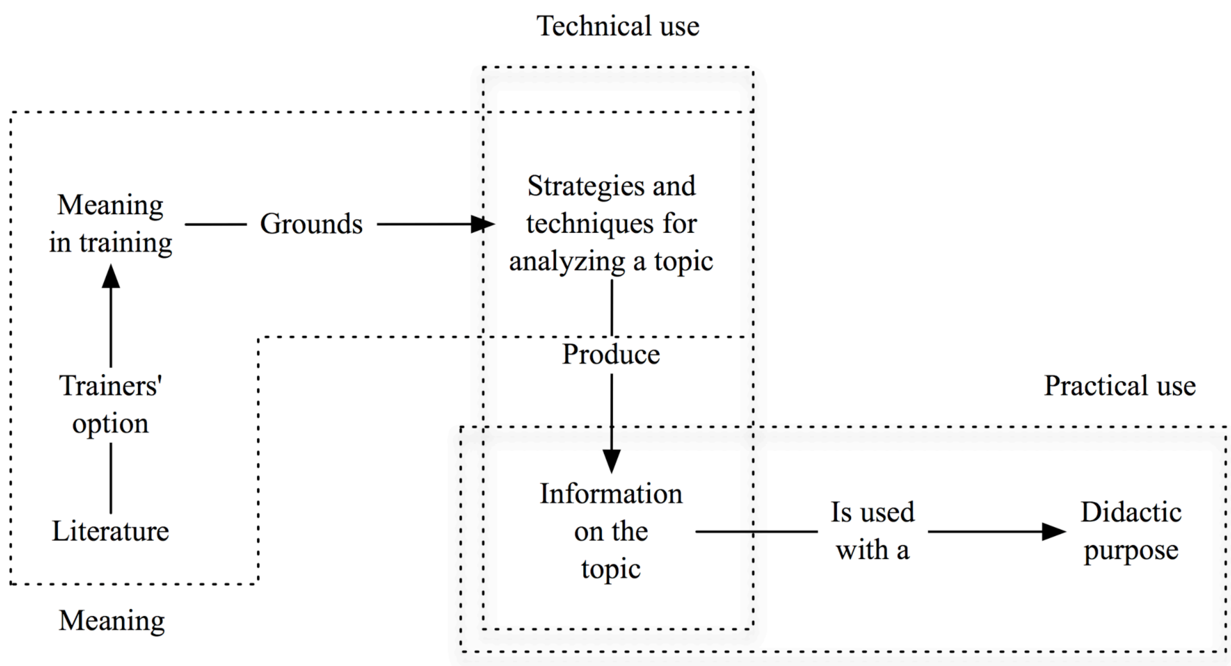

Figure 2. Meaning and uses in teacher training.

Exploring and Characterizing Future Teachers' Learning

We claim that it is possible and relevant to explore and characterize future teachers' learning of curriculum organizers in terms of the ideas of meaning, technical use and practical use. In what follows, we describe how we have operationalized the disciplinary meaning of these three ideas in order to use them for analyzing and coding the transcriptions of the audio recordings of the interaction of a group of future teachers when performing the tasks required in a methods course that followed the principles described above. For each idea, we first establish its operational definition, and then we present an example of an episode and its corresponding coding. We recorded in audio the interaction of a group of future teachers working on the tasks given to them in a methods course in a Spanish university. This methods course was based on the functional perspective mentioned above. The transcriptions refer to the period of the course during which the future teachers were analyzing their topic (plane figures' area) with the curriculum organizers of the cognitive analysis.

Meaning of a Curriculum Organizer

When performing tasks during training, we say that a future teacher *develops the meaning* of a curriculum organizer, if he proposes examples of it, or declares, discusses or reflect on its properties, definition or relationships with other notions. A future teacher or a group of future teachers develop the meaning of a curriculum organizer when there is evidence that they are putting into play the meaning they have constructed so far, but its clear enough that they can improve such construction. It is not an issue of putting in practice their meaning in order to analyze a topic (this is part of the technical use, see below). It is a question of making progress in the individual and social meaning of the curriculum organizer, by putting it into practice and questioning whether such an interpretation aligns with their interpretation of the disciplinary meaning proposed by instruction.

In the transcriptions we found that the future teachers made, from the start, an effort for understanding and reinterpreting the disciplinary meaning of the learning goal notion. This effort was done while performing the tasks proposed by instruction: they were asked to establish the difference between the notions of capacity and learning goal. These tasks lead them to discuss about the properties and the relationships between these notions, as seen in episode 39:

> *Future teacher 1: OK, for the learning goal you give all of them [the capacities]; in order to fill up the learning goal, they have to have all the capacities…*
>
> *Future teacher 2: Each topic has some capacities, doesn't it?, each learning goal will have some capacities that you have to fill up or that you have to…*
>
> *Future teacher 1: The capacities that you have… When you label a learning goal, you label some capacities as well. Another thing is that if the other group is worse, then they will not be able to give the capacities…*
>
> *Future teacher 3: OK, but the capacities of a learning goal are a list, isn't it?*

The fact that they are developing the meaning of the learning goal notion (and the capacity notion as well) is highlighted by phrases like "doesn't it?" and "isn't it?" as evidence of a situation in which the future teachers are putting forward some conjectures based on their current meaning with the purpose of testing them and deciding whether that meaning aligns with their interpretation of the disciplinary meaning.

Technical Use of a Curriculum Organizer

We say that a future teacher or a group of future teachers *develop the technical use* of a curriculum organizer if they put it into play in order to analyse a mathematical topic. A future teacher's technical use of a curriculum organizer is usually based on his interpretation of the meaning of the notion and can involve specific methods or other notions of the didactic analysis procedure. The purpose of the technical use of a notion is to produce information about the topic.

This information characterizes the topic in terms of the notion (see the example of the quadratic function above). Future teachers need not to be conscious that they are putting into play the meaning they have developed in order to analyze the topic. Furthermore, they need not to be conscious of having developed a concrete technique for analyzing the topic. Their attention is usually focused on the tasks at hand; in particular, in producing the results they are expected to obtain.

In the following episode (46) the future teachers develop the technical use of the learning goal, when they make proposals for how to write the statement for a learning goal.

> *Future teacher: Leaning goal would be, for instance, let us see if we can say something like: "to develop strategies", we can start a learning goal like that, can't we? OK, "to develop strategies for calculating unknown magnitudes", this is the learning goal we want to establish.*

While making their conjectures they express their doubts about whether the solution is appropriate. In doing so they are questioning their interpretation of the meaning and the technical use of the notion. At the end, they decide this interpretation is correct and they establish a statement for the learning goal.

Practical Use of a Curriculum Organizer

Finally, we consider that a future teacher *develops the practical use* of a curriculum organizer when there is evidence that he uses the information emerging from its technical use for didactic purposes. Within the context of the procedure of didactic analysis, the information emerging from the analysis of a topic with a curriculum organizer can be used with mainly two purposes. On the one hand, it can be used in other analysis of the didactic analysis. For instance, the information emerging from the analysis of the topic form the perspective of its representation systems, can be used in the phenomenological analysis or in the identification of the capacities corresponding to a learning goal (see the examples above). On the other hand, this information can also be used, often in conjunction with information from other curriculum organizers, in the design of the didactic unit. That is, for analyzing and selecting the tasks that they would propose for the teaching and learning activities.

In the following episode (170) the future teachers develop the practical use of the learning goal notion by establishing the relationship between the learning goal proposed and the tasks they are analyzing. In this case, they realize that they need to look for an additional task in order for the didactic unit to promote the students' development of the learning goal.

> *Future teacher 1: We need one more on the measuring unit dependency.*
>
> *Future teacher 2: Yes, we have none of those.*
>
> *Future teacher 3: Say, in measuring unit dependency, can we put one of those exercises about how many pounds of painting and all of that?*

Teacher Learning Complexity

The ideas of meaning and uses of a curriculum organizer can be useful for exploring and characterizing future teachers' learning of the curriculum organizers. One can describe the learning process over time in terms of the series of episodes that are labelled with each of the three codes corresponding to the three ideas. This information can show when and how future teachers develop the meaning, the technical use and the practical use of a curriculum organizer (Gómez, González, Rico & Lupiañez, 2008).

In the preliminary analysis that we have done of the transcriptions, we have found that future teachers' learning is more complex than what we described in the above paragraph. Figure 1 represents what could called the canonical learning process. It is a process that assumes that the future teacher should first develop his/her meaning of the curriculum organizer, then put into practice that meaning in order to analyze a topic and produce some information, and finally use that information for a didactic purpose. We represent this process as M $\rightarrow$ TU $\rightarrow$ PU. But, we have found that this canonical process seldom appears in practice. Meaning, technical use and practical use development appear in different manners. In some cases, as seen in episode 46, the relationship is twofold. In this case, the future teachers put into play their meaning of the curriculum organizer in order to produce some information (technical use) giving rise to a situation that can be labelled as M $\rightarrow$ TU. But, at the same time, the effort of analyzing the topic with the curriculum organizer leads to a questioning about the meaning they are using in such a task. That is, a situation that can be labelled as TU $\rightarrow$ M.

Describing and characterizing future teachers' learning imply identifying in the transcriptions, for each curriculum organizer, these relationships. For instance, in the case of episode 46 one can see that the technical use can promote meaning development. We claim that this information is relevant, both for research (as an understanding of the future teachers' learning process) and for the curriculum design and development of training programs. They give clues of how instruction can be improved in order to promote future teachers' learning.

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