TWO-STEP ADDITION PROBLEMS WITH DUPLICATED SEMANTIC STRUCTURE.

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ABSTRACT: *This paper reports on a study made of 540 schoolchildren between 9 and 11 years old, engaged in two-stage addition problem solving tasks. Of the 64 different types of two-stage addition problems which can be identified by taking all possible pairs from semantic categories and crossing them with pairs of arithmetical operations, 16 problems were chosen for this study: those in which the semantic structure is the same in both stages of the solution. A multivariant analysis with repeated measurements (MANOVA) was used in two intersubject factors and the interactions shown were analysed. Furthermore, the indices of difficulty and discrimination for each of the problems were determined.*

I. INTRODUCTION.

At the end of the 70s and beginning of the 80s, researchers working on simple arithmetic word problems with addition structures obtained similar semantic categories, among them Vergnaud, Greeno and Heller, Carpenter and Moser, and Nesher. It is at this time when research into arithmetic problems divided into two large fields: addition structure problems and multiplication structure problems.

The semantic categories for addition problems, which represent alternative structures of quantitative information, were initially established by Heller and Greeno (1979) and, with successive improvements, are still with us today.

Fuson (1992-b) lists 22 structurally different addition problems, for which she bears in mind the four alternative semantic structures: Combine, Change, Compare and Equalize; two types of relation - increase or decrease - for the last three categories, or static and dynamic for the first category; and three possibilities for the unknown element in the structure of relations which the problem's statement establishes (only two in the Change structure).

A considerable number of research projects have tried to establish the level of difficulty of the addition structure problems in terms of the semantic categories and of the position of the unknown element in the framework of implied relations. The establishing of the knowledge structures employed to solve different types of problems according to the aforementioned clas-

sification still leaves a number of unanswered questions today (Fuson, 1991 and 1992).

Our project is placed within the general framework described previously: we study arithmetic problems of addition, classified according to the semantic structure four categories mentioned above. The studies into addition problems have centered on problems whose solution requires a single operation; these problems are called one-step problems and constitute the simplest category of addition problems.

Interest in the study of two-step arithmetic problems, which are those whose solution requires two consecutive arithmetic operations, has appeared recently (Shallin, 1985; Nesher, 1991). Two-step arithmetic problems of addition are those problems whose solutions involve only addition and subtraction and, in all cases, two of these operations are necessary. The study of arithmetic problems of addition is the aim of this research project.

We are going to consider problems in which two elements appear first and with which we must operate to obtain another number. This new number must be used in an operation with the third element to achieve the solution:

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ordered elements of the problem: a, b, c
order of operations to reach the solution:
a*b ------> d
c*d ------> solution

This study concentrates on those problems in which the semantic structure of the two operations is the same, thus obtaining 16 possible cases that we name *two-step problems with duplicated semantic structure*

I.1. Research Goals.

The following goals were proposed:

1. To compare the performance of schoolchildren in the 4th, 5th and 6th year of primary education (9-11 year olds) in relation to two-stage addition problems of arithmetic.

2. To determine if there are any differences of difficulty between the pairs of semantic structures and the operation sequences, as well as to study the interactions between these two factors.

I.2. Hypotheses to be Verified.

H_{01}. *There are no significant differences due to the "course" factor..*

H02. *There are no significant differences due to the "semantic structure" factor.*

H03. *There are no significant differences due to the "operation type" factor.*

H04. *There is no significant interactive effect between the variables "course" x "semantic structure".*

H05. *There is no significant interactive effect between the variables "course" x "operation type".*

H06. *There is no significant interactive effect between the variables "semantic structure" x "operation type".*

H07. *There is no triple interaction effect between the variables "course" x "operation type" x "semantic structure".*

II. METHOD.

II.1. Characteristics of the Sample.

The sample consists of a total of 540 pupils from six primary schools in the Province of Granada (Spain), five of which lie in the city itself and its surrounding districts, while the other is in a rural area. Three of the schools are state-run, while the other three are state-approved private institutions.

In each school three groups were chosen, from the 4th, 5th and 6th year respectively. The number of pupils in each group was as follows:

164, 4th year pupils.
175, 5th year pupils.
201, 6th year pupils.

Teachers were not previously made aware of the contents of the tests, nor did the children receive any type of specific instruction beforehand regarding the tasks proposed for the research project.

II.2. Instruments Used.

The 16 possible cases were classified into two groups of eight problems each.

The characteristics of these two tests were as follows:

Test A. This test included those problems in which the semantic structure of the two operations is the same, that is, the pairs (Ch, Ch); (Co, Co); (Cp, Cp); (Eq, Eq), and the two operations are also identical: (+, +) and (-, -). The resulting 8 problems were determined as being of duplicated structure.

Test B. As with the previous group, the problems in this test also have the same semantic struc-

ture in the two operations, but here the second operation is different from the first: (+, -) and (-, +). 8 solutions also result from this group, forming symmetrical pairs with respect to the operations.

Since the items are dichotomyc, the reliability as internal consistency of the test was calculated with the Kuder-Richardson's Kr-20 coefficient. The resultant reliability index was 0.86.

	Combine-Combine	Change-Change	Compare-Compare	Equalize-Equalize
(+, +)	Peter has 17 white wooden marbles inside a box, 9 red wooden marbles and 22 glass marbles . How many marbles has Peter got inside the box? **[Co-Co (+,+)]**	Peter had 23 video-games. On his birthday he bought 8 and his friend Joe gave him15 to . How many video-games has Peter got now? **[Ch-Ch (+,+)]**	This year, Peter had watched 23 films. Joe had watched 16 more films than Peter and Mary 6 films than Joe. How many films has Mary watched? **[Cp-Cp (+,+)]**	Peter has got 25 pesetas and needs 9 pesetas more to have as much money as Joe. For Mary to have as much money as Joe, she needs 17 pesetas more. How much money has Mary? **[Eq-Eq (+,+)]**
(-, -)	Kathy has 48 marbles inside a box, 22 of them are glass marbles and the rest are wooden marbles. Of the wooden marbles 9 are red and the rest white. How many white marbles are inside the box? **[Co-Co (-,-)]**	Kathy had 46 video-games and it broke 15 of them. Of the ones she has left, she give 8 to Susan . How many video-games does Kathy have now? **[Ch-Ch (-,-)]**	This year, Susan has watched 48 films. Mary has watched 8 less films than Susan and Kathy 16 less films than Mary. How many films has Kathy watched? **[Cp-Cp (-,-)]**	Mary has got 51 pesetas and if she spends 17 ptas, she will have the same amount of money as Susan. If Susan spends 9 pesetas she will have the same amount as Kathy. How much money has Kathy got? **[Eq-Eq (-,-)]**

Problems of the Primary session (Test A)

II.3. Application Procedures.

The tests were given to the children by members of the research team in two sessions, with a maximum interval of one week between sessions, approximately half-way through the first term of the 1993-94 academic year. The tests were all given on different days, leaving at least one day between tests, and also avoiding a weekend break falling between tests.

The tasks were carried out in the children's usual classrooms, using existing groups/ classes. The tests were applied in a large group format, the pupils having a maximum time of 30 minutes per session in which to solve their problems in silence.

Before the tests began each of the groups was given a series of verbal instructions to orient the problem-solving tasks. These included:

** Solve the problems in the order they are on the question paper. When you finish one problem go on to the next, and so on, until you finish.*

** Make all your notes and calculations in the space underneath each question. Don't just write in the answers.*

** Don't look at anyone else's work and keep silent so that everybody has a chance to do their best and solve the problems properly.*

The tests were marked by members of the research team, once agreement had been reached over the marking criteria to be followed. The missing subjets of the sample was under 1% of the total number of pupils, this corresponding to those pupils who only completed one of the tests as a result of their absence for one of the two sessions. Each of the answers to the problems was given a value of 1 or 0, according to whether its solution was correct or incorrect. Solutions were considered correct when they showed that the pupil had opted for the appropriate operations in order to arrive at a successful solution to the task. Calculation errors were ignored. In this paper only the correct solutions are analysed; research currently underway concentrates on an in-depth study of of the typology of errors according to empirically defined category frameworks.

A statistical design of repeated measurements has been applied to the matrix of data obtained from the 540 pupils' scores, with the year factor as an intersubject factor, considered as an independent variable with three levels of definition: 4th, 5th and 6th; and two intersubject factors, i.e. the operation type factor and the duplicated semantic structure factor, with four levels of definition, respectively: (+, +), (-, -), (+, -), (-,+) and Ch-Ch, Cp-Cp, Eq-Eq, Co-Co.

The statistical technique used for analysis of the data was a maultivariant analysis with repeated measurements in the two intrasubject factors (MANOVA).

The performance of each course, and the indices of difficulty for each problem according to the operation type and semantic structure variables appear in the following tables:

Structure Operations	Ch-Ch	Eq-Eq	Co-Co	Cp-Cp	Total
(+,+)	.890	.726	.927	.760	
(-,-)	.524	.598	.591	.732	
(+,-)	.811	.659	.610	.707	
(-,+)	.805	.634	.524	.659	
Total	.76	.65	.66	.71	

Table 1. Mean punctuation by course 4th

Structure Operations	Ch-Ch	Eq-Eq	Co-Co	Cp-Cp	Total
(+,+)	.937	.880	.977	.82	
(-,-)	.691	.789	.789	.61	
(+,-)	.903	.834	.789	.70	
(-,+)	.920	.840	.771	.66	
Total	.86	.84	.83	.71	

Table 2. Mean punctuation by course 5th

Structure Operations	Ch-Ch	Eq-Eq	Co-Co	Cp-Cp	Total
(+,+)	.965	.915	.990	.945	
(-,-)	.826	.920	.881	.965	
(+,-)	.955	.886	.896	.910	
(-,+)	.955	.896	.876	.945	
Total	.93	.91	.91	.94	

Table 3. Mean punctuation by course 6th

Structure Operations	Ch-Ch	Eq-Eq	Co-Co	Cp-Cp	Total
(+,+)	.993	.846	.967	.95	
(-,-)	.691	.780	.768	.90	
(+,-)	.894	.800	.774	.91	
(-,+)	.898	.798	.735	.92	
Total	.87	.81	.81	.91	

Table 4. Global Mean punctuation by 4th & 6th

Results of the contrast of hypotheses according to the variables considered are given in the following table:

	F	p (α=0.01)	Level of signification
H01	55.96	p<0.01	Differences by course
H02	14.49	p<0.01	Differences by semantic structure
H03	53.95	p<0.01	Differences by operations
H04	3.26	p<0.01	Differences by course x semantic structure
H05	7.42	p<0.01	Differences by course x operacion
H06	31.17	p<0.01	Differences by operatios x semantic structure
H07	2.14	p<0.01	Differences by course x operations x structure

Table 5. Hypothesis & Levels of signification by course, semantic structure and operations

The global interaction between operation type and semantic structure variables appear in the following figure:

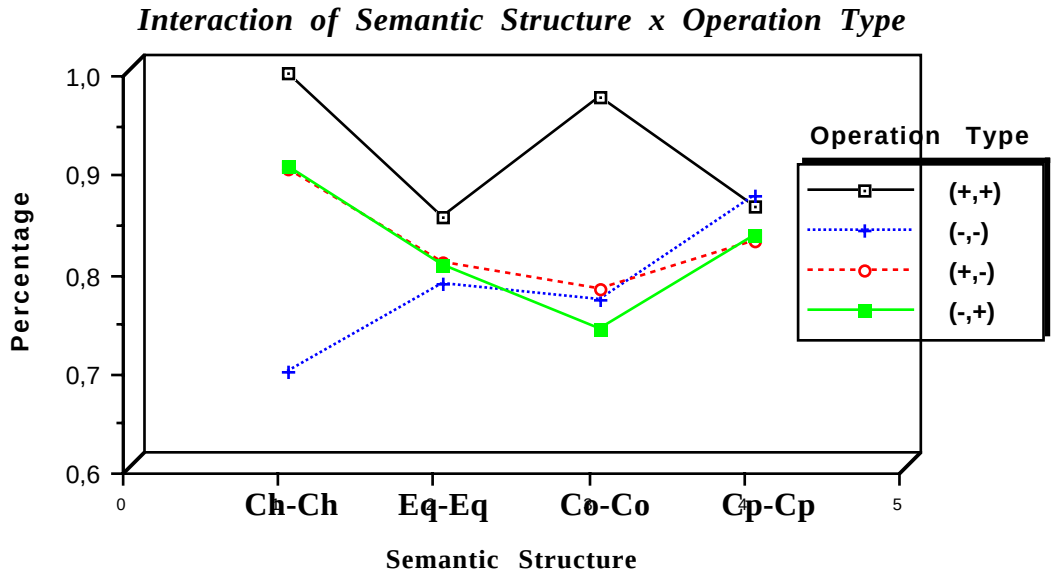

Figure 1. Percentages of appropiate solution for different operations types for de four semantics structures.

IV. CONCLUSIONS.

Globally, following analysis of the variance of the total mark for the 16 items of the two two-stage, duplicated semantic structure addition tests, significant differences between school years were found (F=57.813; p=0.0). Subsequently multiple comparisons between year pairs were carried out according to Scheffe's method at 5% significance level, and significant differences between the school years were found: between 4th and 5th, 4th and 6th, and 5th and 6th.

Analysis of the variance in the two intrasubject factors (semantic structure and operation type) showed significant differences in the following cases: due to the "semantic structure" factor (F=14.49, p=0.000), due to the "operation type" fcator (F=53.95, p=0.000) and due to the mutual interaction between both factors (F=3.26, p=0.000). Finally, significant differences were also found due to the triple interaction of the factors "year" x "semantic structure" x "operation type" (F=2.14, p= 0.003).

Global comparisons made for the variable "semantic structure" showed the existence of significant differneces only between the following pairs: Ch-Ch with Eq-Eq, and Ch-Ch with Co-Co.

For the variable "operation type", significant differences were obtained for with the following pairs, by year:

4th year: (+,+) with (-,-); (+,+) with (+,-) and (+,+) with (-,+).

5th year: (+,+) with (-,-) and (+,+) with (+,-).

6th year: (+,+) with (-,-).

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