DEVELOPMENT OF SPANISH DOCTORAL STUDIES IN DIDACTIC OF MATHEMATICS

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The aim of this work is to show the current situation of Doctoral studies in Didactic of Mathematics in Spanish universities. These studies have gone trough considerable development since they began in the early 80's. The present work provides the general standards and legal requirements that must be met to submit a doctoral dissertation in Spain. In order to describe the process we have taken the Didactic of Mathematics doctoral program of the University of Granada as our model.

This work also describes diverse Mathematics Education research environments, the general components of a doctoral dissertation and the most relevant stages in writing it. We also provide criteria and recommendations that may serve as a guide for Didactic of Mathematics doctoral students. Lastly, we enunciate different criteria to assess a doctoral dissertation and we give a description of the recent production of research groups working today in the field and of doctoral dissertations written in Spain.

Institutional Frame

Didactic of Mathematics in the Spanish University

In the 60's, the interest in mathematics had considerably increased in Spain. New faculties were created these years to foster the study of Mathematics throughout the whole country, all of which exceed the original academic scope covered by the Universities of Barcelona, Madrid and Zaragoza. Early in the 70's, due to legal changes, the new Institutes for the Educational Sciences have promoted research in education within the university structure.

It is in this innovative context that the Didactic of Mathematics field is incorporated into the Spanish University in the 70's. Courses related to such discipline are offered in pre-service training courses for Basic Education (6 to 14 years old) teachers and in Mathematics. Spanish Mathematics Education research groups started working in the field in the 70's; nevertheless, institutional standards tend to be restrictive thus hindering the development of Didactic of Mathematics until the mid 80's.

In 1984, due to the new University Reform Law, the Spanish University went through administrative and academic reorganization. The academic ambit comprises different fields of knowledge. We can say that a field of knowledge is determined by: the homogeneity of the subject matter; the identification of its problems; its field of application; the existence of a historical tradition, and by professional groups finding of such problems along their professional practice and

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applying to them the studied solutions. The existence of international research communities is also an appropriate point of reference. The Spanish higher fields of knowledge are established by law (Royal Decree 1884/84, BOE 26 October 1984). Didactic of Mathematics became a field of knowledge in Spain ever since. (Rico and Sierra 1994).

From an administrative point of view the Spanish University consists of Departments that are focused on a certain field of knowledge, or on diverse related areas. Each of these Departments must be able to meet its teaching and research needs. In some Spanish universities (Granada, Madrid, Valencia and Sevilla), the Didactic of Mathematics Departments are created as a single entity; whereas in other universities, Didactic of Mathematics specialists belong to Departments of multidisciplinary Didactic together with specialists of other disciplines, or else, they are integrated with Mathematics Departments working together with other Mathematics specialists. Such grouping occurs on the basis of local tradition or previous relation with colleagues of other fields.

The current university Didactic of Mathematics Departments started their work in the mid 80's performing tasks such as:

Delimitating a discipline or field of knowledge, that of Didactic of Mathematics.

Developing and organizing research and as well as assignments that will belong to the Didactic of Mathematics field in the university context.

Proving the legal grounds and administrative structure for the group of university professors, lecturers and researchers dedicated to the Mathematics Education field.

Conducting first stage training in Didactic of Mathematics for Kindergarten, Primary and Secondary teachers.

Beginning and supporting the training of researchers in Didactic of Mathematics trough a doctoral program.

The outcome of the new situation beginning in 1984, has been the organization and development of doctoral programs in Didactic of Mathematics (Rico, 1999). We hereafter provide an outline of our doctoral programís general standards.

Doctoral Studies

In order to enroll in a doctoral program in Spain, one is required to have a second-cycle degree either as a graduate, an architect or an engineer. Due to this requirement, doctoral studies in the Spanish University are generically called third-cycle studies. As far as postgraduate studies are concerned, the third cycle is the means to obtain a doctor's degree. Such studies are regulated by the Royal Decree Num. 778/1998 (BOE 1th May 1998) and the doctor's degree is common to all Spanish universities.

Doctoral studies pursue four main goals:

* Providing a suitable framework to attain and transmit scientific work.

* Training new researchers and work teams that shall deal with the challenges posed by the new sciences, techniques and methodologies.

* Encouraging the training of new teachers.

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* Promoting the professional and scientific development of higher graduates.

The Spanish system forming researchers in different disciplines does not have an intermediate degree between the second cycle and the doctoral level, such as a master's degree in other countries. The common plan to form researchers in every discipline is based on a doctoral program that is mandatory pre-requisite to write a doctoral dissertation.

A Doctoral Program consists of several courses and some initial research work. These programs are updated once every two years and their main purpose is to assist the student in the development of his/her dissertation. Doctoral programs play an important role in the learning of specialized techniques and sciences, therefore, they play an important role in the training of new specialized researchers in a specific branch.

Enrollment period for a doctoral program in Spain is annual, usually in September. Applications are referred to the different departments thereafter, where they are assessed according to criteria established by each university. After selection, a name list of applicants who filled the requirements is published. Candidates to the Didactic of Mathematics Doctoral Program usually hold a degree in Mathematics; some have a degree in Physics or Education and have some experience in Mathematics Education. Many candidates are Latin American applicants with a degree of Mathematics Teachers. To be eligible to the Didactic of Mathematics doctoral program the candidate must have previously been related to Mathematics Education and to have previous experience in such a field.

Upon enrolling in a Doctoral Program, the student is assigned a Tutor who takes responsibility in his/her academic progress and research work and helps him/her throughout his/her academic program.

According to legal standards, a doctoral program must comprise:

* Courses or seminars on the main contents of the scientific, technical or artistic fields on which the Doctoral Program is focused.
* Courses or seminars in methodology and research techniques.
* Tutored research works.
* Courses or seminars on topics akin to the program and relevant to the doctoral dissertation project of the postgraduate student, including a maximum of five credits.

The training plan of each program consists of two different stages. The first is the teaching period, where the student must take a number of courses amounting to 20 credits (each credit is worth 10 teaching hours), each course must be worth three credits at least. This teaching period is focused on conceptual work and may be completed in one or two academic years. Once it has been completed, the student is awarded a Certificate showing that he/she has passed the third-cycle period. This certificate is valid in all Spanish universities and is a quantitative global valuation.

Each Department offers a variety of courses -some compulsory, some optional- so the student, together with his/her tutor, may select those he/she considers most appropriate to his formation orientation.

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The second stage is devoted to research; during this period the student must cover a minimum of 12 credits by doing at least one research work directed by a researcher of the program. The first stage must be completed before starting the second stage of the program, the first one should be completed. At the end of the second period, valuation is focused on the student's research skills.

When the teaching and researching periods have been completed with a minimum of 32 credits, a jury will do a public assessment of the doctoral student's knowledge which shall certify the student's research proficiency. Obtaining the research proficiency qualification is pre-requisite to submit and defend a doctoral dissertation. Third-cycle studies provide the student with the ability to write a dissertation and they have a minimum duration of two years.

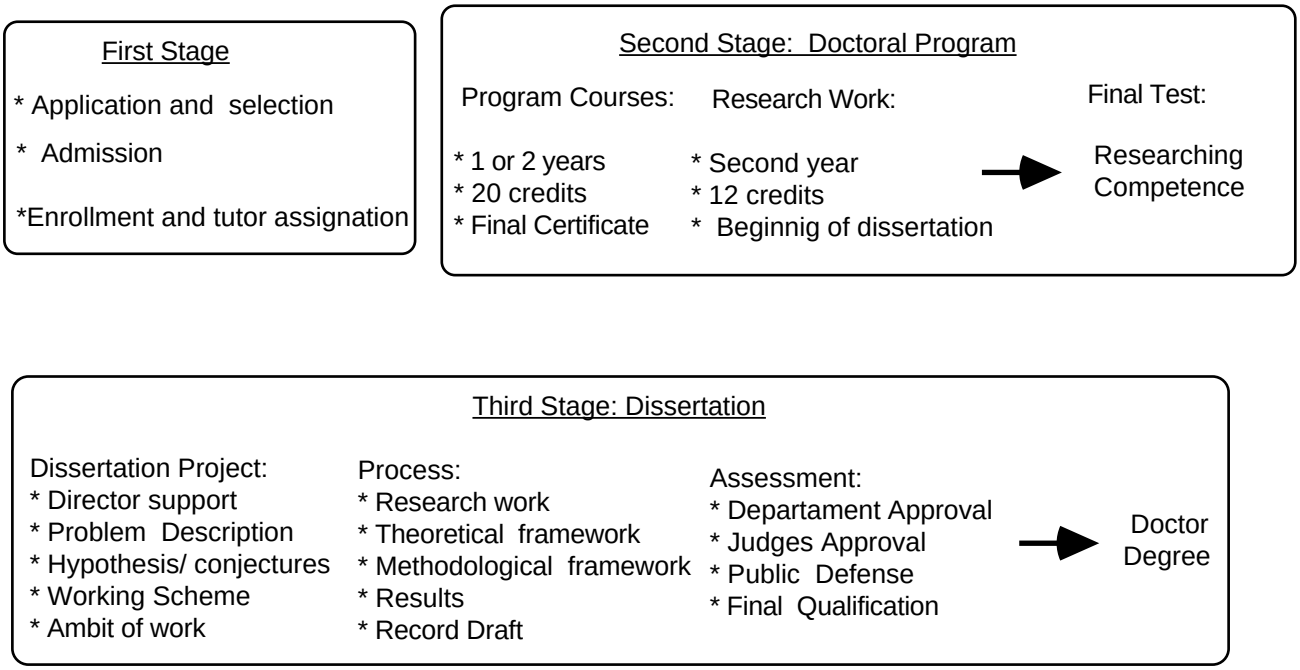
Once the student has finished his third-cycle studies, he/she must submit a doctoral dissertation project to the Department, supported by the researcher that will be his/her director. The director of the dissertation may or may not be the student's Tutor. The commitment to direct the dissertation is usually agreed upon during the program and is made official by the Department when it approves the Doctoral dissertation project. The contents of this formal commitment depend on the research lines of each Department and on the scientific interests of the postgraduate student. Setting up research lines leads to a better use of the limited number of personnel, material and time resources and facilitates team work, which is essential in doing research.

The dissertation consists of an original piece of research work related to the Didactic of Mathematics field, written by the postgraduate student under the supervision of his/her director. Talking in concrete terms, the dissertation is the writing of a research report written by the student, supported by his/her director and approved by a university Department.

The research report is sent to the Doctoral Program Committee who, in turn, submits it to public evaluation by the university community, within established deadline. During this period, it is possible for the doctors to put up arguments, suggest changes or even propose conveniently rationalized modifications. A copy of the dissertation is sent to each of the five members of the jury who will assess it. The director cannot be part of the jury. If there is no negative report against the research journal after 30 days, the public defense of the dissertation proceeds before the members of the jury who assigns the work a final grade and, if convenient, grants the Doctorís degree.

The Doctor's degree is granted by the Spanish State and is valid in the whole country. The Doctor's degree must coincide with the author's major studies, although it indicates the doctoral program it pertains to.

Hereafter is a chart where the main stages of the program and the general sequence of steps the student must follow in order to obtain his/her degree in Spain are outlined:



We can overall say that the first two stages of the above mentioned process can be considered the equivalent to the Masterís Degree that exists in other countries as a previous step to the Doctoral Dissertation.

Doctoral Program in Didactic of Mathematics

Doctoral programs in Didactic of Mathematics started running for the first time in Spain during the 1988-89 period, in the Autonomous University of Barcelona and the Universities of Granada and Valencia. Other Spanish universities have started offering full or partial courses in Didactic of Mathematics as part of their doctoral programs ever since. The Universities of Almería, Extremadura, Huelva, La Laguna, Málaga, Sevilla, Valencia and Valladolid are to be mentioned among them. The enrollment period takes place once every two years in Spanish universities. Information in detail about the programs offered by each university, their schedules and their structure can be obtained in each universityís web site.

Doctoral programs represent the institutionalization of doctoral dissertations in Didactic of Mathematics and the academic promotion of teachers and researchers working in the field. The universities offering a doctoral program in Didactic of Mathematics is usually the ones where research in this field is done.

Course design for students' training is responsibility of the Department. In the case of the Didactic of Mathematics program, students usually have very little previous theoretical input when joining our program and it is focused on professional and practical issues. Therefore, going through doctoral training is essential in order for them to write a dissertation in the field.

Spanish Didactic of Mathematics doctoral programs have been focused on the achievement of the following general goals:

* Establishing and preserving a space for criticism, debate and communication in

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relation to the current status and recent development of Didactic of Mathematics research, as well as about its theoretical and methodological advances.

*Encouraging the delimitation of relevant problems regarding the teaching and learning of Mathematics so their systematic and thorough study may lead to the gathering of meaningful information, diagnosis and treatment, thus, giving place to the production of adequate materials and resources for the mathematics class.

* Consolidating research groups that may work methodically, orderly and steadily on specific research lines on Didactic of Mathematics that may serve as a reference for specialists and be coherent with the international trends.

* Producing qualified research, making original and specific contributions to the priority research issues of this field of knowledge; and submitting results regularly in forums and communication means of the Didactic of Mathematics community.

The emergence of doctoral programs has encouraged Spanish research in the Didactic of Mathematics field considerably. The consolidation of these programs provides an appropriate framework to achieve and transmit the scientific advances in Didactic of Mathematics, with qualified foreign researchers taking part in the orientation, tutorial and assessment of the dissertations. Doctoral programs have also fostered the relationship among researchers with different trends and universities. Thanks to these programs new researchers have been trained and new investigation groups have been consolidated. These groups have submitted their projects to different contests and international, national and regional public meetings. Finally, these programs have encouraged the training of new university lecturers and have supported the professional and scientific development of graduate students.

Research and Didactic of Mathematics

Mathematics Education

Today in societies mathematics is a core component of their basic training. That is why Mathematics Education is a key activity in the training of citizens within modern, democratic societies.

The traditional Mathematics teaching and learning processes are relevant issues in Education. The Mathematics Education field ranges from the first notions about quantity, shape and deductive reasoning taught to children to professional training at higher levels.

Mathematics Education involves intense intellectual activity of explanatory nature enhanced by the appreciation of formal beauty, proof and notions of argument, which is expressed through a great variety of actions, terms, symbols, techniques, attitudes and resources. Mathematics is a human creation focused on technical purposes leading to the modeling of our environment and applied in the solution of practical problems. (Rico, 1995).

Mathematics education comprises a range of concepts and procedures aiming at communicating knowledge, and organizing large areas of intellectual, scientific, economic, cultural and social activity. Mathematics is the language of science and

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gives concrete shape to a great number of problems, allowing for rational criticism before decision-taking (Skovmose, 1994).

From the specialistís point of view, we consider Mathematics Education as a set of ideas, knowledge and processes involved in the construction, representation, transmission and valuation of mathematical knowledge, all of which occur intentionally. Mathematics Education is developed by certain professionals at specific institutions through a variety of tasks; in all cases these professionals intend to provide an answer to problems and needs that arise from the teaching and learning of Mathematics (Rico, Sierra and Castro, 1999).

Research and Mathematics Education

Research is generally understood as an inquiry activity based on the scientific method, supported by a theoretical frame and aimed at transforming the human environment.

Mathematics Education is focused mainly on the problems encountered in its three general work frames:

Evaluation, development and design of the Mathematics curriculum;

Teacher training and professional knowledge;

Theoretical output and epistemological foundation.

Mathematics Education researchers select their problems out of these three areas following the standards of Education research which are: systematic inquiry within an theoretical framework aiming at the transformation of the educational environment.

From an overall point of view, we believe that Mathematics Education may be seen from three different perspectives pertaining to different work spheres and different research fields.

First of all, Mathematics Education can be seen as a set of knowledge, arts, skills, languages, conventions, attitudes and values devoted to Mathematics and transmitted by the educational system. It refers here to mathematical knowledge as an object of teaching and learning. Mathematics Education is mainly aimed at enriching and structuring adequately the different meanings of mathematical concepts thus overcoming their apparently exclusive formal and deductive signification. It also deals with the training programs and curriculum needs enabling the transmission, learning and sharing of this knowledge. This is referred to as assessment, development and design of Mathematics curriculum (Rico, 1997).

Second, Mathematics Education can be seen as a social activity that takes place in specific institutions and is carried out by qualified professionals. In this case, Mathematics Education is understood as the whole set of actions and conditions that make the teaching of Mathematics possible. Therefore, it covers the set of knowledge, processes and conditions that allow the student-teacher interaction about mathematics topics to take place in the Mathematics class, thus, making the teaching and learning of Mathematics feasible. Mathematics Education refers, in this case, to an activity which is intentionally used to build, understand, transmit and assess

mathematical knowledge. In this area, it is related to the analysis and study of the conditions that are required to teach and learn Mathematics. Teachersí knowledge and professional development is a central issue so we refer to it as professional knowledge and the mathematics teachers training (Llinares, 1991; García, 1996).

Third, Mathematics Education can be seen as a scientific discipline, which is the same for Didactic of Mathematics. Mathematics Education is understood as the whole set of methodological and theoretical frames enabling the interpretation, prediction and work in the teaching and learning of Mathematics. Didactic of Mathematics is intended for the methodical and systematic inquiry of the Mathematics teaching and learning processes. This discipline also deals with the schemes used in the qualification of mathematics educators. Didactic of Mathematics intends to delimit and study the problems involved in the organization, communication, transmission, construction and valuation of mathematical knowledge. (Steiner, 1987; Biehler, 1994). The study object of Didactic of Mathematics involves the two above mentioned trends as well as a theoretical framework. Its recursive character within its own foundations may be occasionally miss-interpreted. As a consequence, when we wish to distinguish this ambit from the other two, we must refer to it as theoretical output and epistemological foundation.

Research in Didactic of Mathematics

From our epistemological position we claim that research in Didactic of Mathematics is characterized by three main features: its theoretical frame, its methodological frame and its work environments within the Mathematics Education field (Rico and Sierra, 1999).

Didactic of Mathematics research intends to have an impact on the social environments through transforming action. We therefrom envisage three general work bounds that deserve prior attention in this respect as we mentioned:

* Design, development and evaluation of Mathematics Curricula.
* Mathematics teachers training and professional knowledge.
* Theoretical output and epistemological foundation.

From our point of view, the theoretical foundations of Didactic of Mathematics implies a variety of disciplines such as:

Communication theories, curriculum theories, interaction theories, semiotics, educational technology and media, among others, when the focus is on teaching phenomena and processes.

Psychology of education, mathematics learning theories, constructivism, and social constructivism among other disciplines when the focus is on learning phenomena and difficulties.

Epistemological and conceptual analysis of mathematical knowledge, history of mathematics, the structure of the mathematical disciplines, mathematical processes, and phenomenological analysis, and others, when the focus is on the study of the complexity of evaluation of mathematical knowledge.

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Knowledge sociology, education sociology, cultural anthropology, axiology and teleology of the discipline, and educational institutions theory, when the focus is on the social processes involved in the creation and valuation of knowledge.

Taking these facts into account, we consider the Didactic of Mathematics theoretical frame comprises four dimensions: a pedagogical, a cognitive, a conceptual and a sociological one. (Rico, 1997).

Finally, to do research is to inquire through the scientific method. For this reason, research work entails mainly: a methodological orientation expressed by a paradigm, various approaches, techniques and instruments. Kilpatrick and Sierpinska (1993) considered three general methodological approaches in education research: a positivist approach, a phenomenological approach and a critical approach. After analyzing these patterns they adopted a holistic view as far as education methodology is concerned.

On the basis of their fecundity and their frequent use, we have only considered the positivist, the interpretive and the critical as the most usual methodological frames used in Spanish Mathematics Education research.

Doctoral Program of the University of Granada

General Data and Research Lines

The Didactic of Mathematics Department of the University of Granada has offered a biennial doctoral program since 1988. Today, after six biennial programs, this field has become quite a well-defined course for the training of researchers in Didactic of Mathematics in Spain. For further information, see the Web site of the Department of Didactic of Mathematics:

http://www.ugr.es/(dpto_did/.

We show supporting data as follows:

Biennium	Enrolled students	Dissertation projects approved	Completed dissertations
88- 90	17	16	14
90- 92	14	10	6
92- 94	8	6	3
94- 96	6	3	-
96- 98	12	3	-
98- 00	12	-	-
Totales	69	38	23

The first doctoral dissertations were defended during the 94-95 period and, as can be seen in the table above, 33% of the students that joined the program have so far finished their dissertations. The minimum time to complete it, from the beginning of their studies to their conclusion, has been five years. Fifty five percent of the students have submitted their dissertation projects that have been approved

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by the Department. Dissertation projects may be submitted from the beginning of the program but within the first two years.

It is worth mentioning that the number of Latin American students registered in the program, so far amounts to 12 people (17% of the total), mainly in recent years.

The program has received financial support from the Spanish government as a quality doctoral program. It is included in postgraduate studies offered by the European Commission; it has received students on scholarship from the Spanish government, as well as from six Latin American countries (Argentina, Brazil, Chile, Colombia, Mexico and Venezuela). It also receives funds from the Alfa Program of the European Union and is a member of the FIEMAL Network (Training of Researchers on Mathematics Education for Latin America) which includes three EU countries (Italy, United Kingdom and Spain) and three Latin American countries (Colombia, Guatemala and Mexico).

We do not have accurate data at hand concerning the output of doctoral programs on Didactic of Mathematics of other Spanish universities; at present, the number of doctoral dissertations produced in the University of Granada is almost 50% of the Spanish production.

The course of initial training for researchers provided by the Department of Didactic of Mathematics of the University of Granada is based on research activity. In order to organize and develop that activity, the Department comprises five research lines:

* Didactic of Mathematics: Numerical Thinking.
* Didactic of Statistics and Probability.
* Design, Development and Assessment of the Mathematics Curriculum.
* Training of Teachers of Mathematics.
* Theory and Research Methodologies in Mathematics Education

In general terms, these five lines specify the way researchers of the Department deal with the three fields of work we have mentioned in the section above.

The Didactic of Mathematics research lines -Numerical Thinking and Didactic of Statistics and Probability- determine research priorities in curricular innovation. Courses that are offered by the Didactic of Mathematics Department of the University of Granada are: Algebra, Analysis, Statistics and Probability, the first two being focused on numerical structures and processes.

The training of Teachers of Mathematics research line is devoted to professional knowledge and the mathematics teachersí training.

Finally, Design, development and assessment of the Mathematics curriculum and Research Theory and Methodology in Mathematics Education, are the lines that see to the development and foundation of the Didactic of Mathematics field.

It is clear to see here that research lines have not been determined on the basis of an epistemological nor a methodological frame but rather to their activity scope within the Mathematics Education field. Each line is founded on its own theoretical and methodological frames but they are rather determined by their activity scope.

Program Structure

According to the rules regarding doctoral studies, these should include:

Courses or seminars on the main contents of the scientific, technical or artistic fields to which the doctoral program may be devoted to

Courses or seminars on methodology and research techniques

Tutored research works

Courses or seminars akin to the program and relevant for the doctoral dissertation project. These courses can cover up to 5 credits.

The Didactic of Mathematics doctoral program of the University of Granada complies with general rules and its structure has improved as its personnel have gained further expertise and training.

Currently (1998-2000 phase), the Didactic of Mathematics Department of the University of Granada offers 21 courses covering a total of 63 credits.

Main Contents

There are 15 courses on basic contents covering 43 credits, and two research seminars in Didactic of Mathematics (8 credits).

Those courses constitute the core of the program and one is required to take 15 credits worth for these subjects. While the courses on methodology and similar are shared with the doctoral programs of other disciplines, the main-content courses are specific of each program. These can be divided into two groups: core courses and courses of specific research lines.

Common subjects are compulsory for all students and among them are the following:

Mathematics Education Theory.- This course is focused on the foundations of Didactic of Mathematics, its problems, information sources, theoretical frames, epistemological foundation, research patterns and schools.

Design, Development and Assessment of the Mathematics Curriculum.- This course is devoted to the study of curricular theory in mathematics education and to the problems arising from the complexity of the training plans developed in educational institutions.

Each research line offers optional specific courses and explains their status within specific ambits of the Didactic of Mathematics. Problematic issues and research priorities are dealt with by assignments such as: Numerical Thinking, Didactic of Statistics and Probability, Ethno-Mathematics, Arithmetical problems or problem-solving strategies. These courses are also taught by foreign researchers, which fosters information exchange between the Didactic of Mathematics Department and other research centers of this field.

These courses are complemented by the Didactic of Mathematics Research Seminar, during the first two years of the program. The student here naturally becomes acquainted with on-going research and addresses the practical problems arising at take-off and evolution of the process.

Courses in Methodology

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Research methodology in Social Sciences has gone through deep evolution in recent years, leaving the positivist model and the laboratory-bound work behind. In Education Sciences, Research Methodology became a specific field of knowledge the complexity of which cannot be easily managed (Jaeger, 1988; Wittrock, 1986). Moreover, upon consideration of the Didactic of Mathematicsí specific problems, the relevance of these courses becomes quite clear.

The importance of having a methodological frame in order to write a doctoral dissertation has become clear through the years; an adequate design including the proper tools for data-analysis together with the discussion of results and findings, contributes to the quality of the final output. Currently, the program involves four methodological courses (12 credits).

Mathematics Education Research: Methodological Advances, is a core course, thus, compulsory for all students.

The other three courses are: Mathematics Education Research Design, Data Analysis, and Application of Multivariate Analysis to Educational Research.

The course Mathematics Education Research: Methodological Advances deals with three problematic issues:

* The logical stages in Mathematics Education research, including delimitation of the research problem, bibliographic review and consideration of the nature of empirical data;

* The differential research methods in Mathematics Education, namely: methods centered on mathematical contents, teacher-centered methods, and learning-centered methods and mixed methods;

* The assessment of the Mathematics Education research process.

The general goals of this course are:

* Preparing the postgraduate student to become a consumer of judgments on Mathematics Education research upon his/her critical assessment of communicates and projects.

* Providing the postgraduate student with a wide range of methodological skills and knowledge that may lead him/her to the conduction of research in Mathematics Education, in other words, help him/her become a research producer.

* Understanding the role played by research in the improvement of Mathematics Education practical work and in the foundation and development of this discipline.

Related Courses

We include here the courses considered suitable for the completion of the postgraduate studentís training regarding his/her research work in the field.

In accordance with his/her tutor, the student may select assignments from other programs, covering up to five credits. The assignments taken by the student before joining the program may be validated (to a certain extent) and are also included in this group.

Summary

The teaching offer of the Didactic of Mathematics doctoral program of the

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University of Granada can be seen in the table below:

Research lines and other courses	Total of courses	Total of credits
Numerical Thinking	4	11
Didactic of Statistic and Probability	3	9
Teachers Training	3	9
Curriculum and Assessment	2	6
Theory of Mathematics Education	3	8
Methodological Courses	4	12
Seminars on Research	2	8

Research Work

One of the requirements of the program is to do research work conducted by a researcher. The student usually devotes the second stage of third cycle studies to do it. This work amounts to 12 credits.

This research work is autonomous and is considered a means to start doing research in the corresponding field, under supervision. Nevertheless, focusing attention on a particular research problem from the beginning is highly recommended to the student. This is why the Department considers such work to be an approach to the doctoral dissertation or part of it, where the student has the opportunity to give a first glance to the research problem and do the pertinent correction or reorientation adjustments.

The Didactic of Mathematics Department of the University of Granada regards this work as the means for learning how to do research so students are encouraged to submit a third-cycle report showing the results previously obtained. This record is not compulsory since it can be replaced by writing an article or even by previous research works. However, it has become evident that those students who have decided to write this report have soon finished their dissertation or made great progress thereafter.

Several sessions of the Research Seminar are devoted to lengthy discussion about the design, progress and ulterior development of the research work. Once this piece of work has been finished it must be submitted to the Seminar before it goes through assessment session.

By doing the supervised research work, the postgraduate student tests the processes, techniques, strategies, and tools that he/she will be using when writing his/her doctoral dissertation. Regardless of the curricular contents, there are methodological issues leading to systematic kind of work that the student should become acquainted with. One of the usual strategies is doing regular work together with the research tutor, one of his/her techniques being the writing of a notebook

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about his/her research process.

The studentís dissertation activities must be jot down on a research notebook so there is available information about the activities that were scheduled and performed, the analysis and valuations that were done, the tests that were applied, the decisions that were made and the progress that has been achieved. Also, continuous monitoring and assessment of such activities must be carried out through periodic sessions together with the supervisor. Recording of such sessions and ulterior transcription to include the corresponding summary in the research notebook.

We know through experience that regular work sessions with the tutor foster research activity. A fortnight meeting is good enough to receive proper monitoring, although this periodicity may be adjusted to studentís needs. The matters discussed in these meetings are: precise delimitation of research problem, bibliographic background scrutiny and delimitation of the theoretical frame that best suits the problem, studyís methodological design, selection and setting up of information-gathering tools, data treatment and analysis, and output assessment.

Writing the Doctoral Dissertation

Before completing the program, students must submit a doctoral dissertation project to the Department, supported by its director or directors. At the University of Granada, the dissertation project is submitted to the Didactic of Mathematics Department in one of the sessions of the Research Seminar. The project is discussed here and suggestions made for its final writing; after this session, the project is approved.

In order to be awarded the Doctorís Degree in Mathematics within the Didactic of Mathematics doctoral program of the University of Granada, apart from the courses and works mentioned above, the student must write a doctoral dissertation.

The Doctoral Dissertation

A Doctoral dissertation is an original research work on a subject related to the Didactic of Mathematics conducted by the postgraduate student. Doing research is carrying out an inquiry activity based on the scientific method supported by a theoretical frame, leading to the transformation of the environment. Today there is no legal deadline for the completion of a Doctoral Dissertation.

Science intends to provide answers to real issues and problems, as well as to transform the world, not only the physical world but also the social one. Such aim implies the improvement of our knowledge about the issues that are of concern to us. Investigating is equitable to producing science and because of this, research can be considered a problem-solving activity. Since science is considered to be transforming action, research is thought to be a problem-solving task (Laudan, 1986). Such conceptualization applies to research on mathematics education as well (Shumway, 1980; Grouws, 1992; Kilpatrick, 1994; Malara, 1998; Ponte et al., 1998; Rico and Sierra, 1999).

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From this point of view, a dissertation work can be seen as the solution of a problem, an ambitious and complex problem.

This problem-solving task we call research involves systematic inquiry through the scientific method; every piece of research must be founded on a method, expressed by a paradigm, several approaches, techniques, and tools. Thus, the Didactic of Mathematics research must be founded on a theoretical frame as well. Moreover, due to the fact that research in Didactic of Mathematics aims at having an impact on the social environment, it represents transforming action and that is why its work ambit must be considered.

Therefore, a doctoral dissertation on Didactic of Mathematics is defined by three components: its theoretical frame, its methodological frame and its activity scope within the mathematics education field.

Stages in Writing a Dissertation

In the problem-solving process experts usually identify several key stages (which they identify under similar names; for example, Polya, 1979; Bransford and Stein, 1986). In the same general way, we distinguish five stages within the process of writing a doctoral dissertation. These stages are as follows:

1. Detection and identification of a problem
2. Problem representation within a conceptual frame
3. Technical handling of the problem through different strategies
4. Explaining results or arriving at a solution
5. Result assessment and interpretation

We will discuss the relationship between the three general components of a dissertation and the stages of the problem-solving process that have just been pointed out.

Firstly, in order to detect and identify a problem, a researcher-to-be resorts to the work ambits of mathematics education, there he/she delimits his research problem, the individuals he/she is going to work with, and the institution where he/she will conduct his/her investigation. This choice is initially very broad and arises from a problem experienced in his/her professional work. He/she then tries to specify the issue he/she is interested in jotting down his/her first intuitions clearly and precisely. This is how the general components characterizing the problem are outlined and the need arises to contemplate it within the scope of a concrete framework.

This stage is mainly linked to the selection of an activity field that concludes with researcherís setting up of a general, clearly stated and unambiguous purpose for his/her study. The choice of such field of activity is a key decision the student must make since it will condition his/her future work and will place him/her in the activity scope of a research team.

When the beginner joins a work team that has been following its own research agenda and has already defined its work scope, detection of problems becomes easier and its previous expertise helps identifying difficulties that may become research problems in the future.

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Secondly, problem representation is based on a theoretical and epistemological frame. This stage includes precise description of the research problem the enunciation of which derives from a conceptual framework of reference. In order to accomplish this, several techniques are used such as: making an initial list of problem-related terms and key words, reviewing the related bibliography and consulting databases, selecting and reading background literature. Finally, writing a report on the issueís current status and results is very convenient.

A good strategy to define a research problem is to break the general goal into partial aims. Thus, the components included in the theoretical frame of the problem can be determined in their different dimensions, that is, cognitive, conceptual, pedagogical and sociological.

At this stage, it is also interesting to make a general description of the methodological design that must include the type of research, the sample, the tools used to collect information and its timing. Finally, the conjectures that need to be checked can result in several research hypotheses.

The kind of study that is going to be done is linked to its rationality, and both notions -study nature and rationality- show the set of ideas the researcher uses to represent the problem under consideration.

In this case, the work of the postgraduate student requires an important intellectual effort, which involves critical reading and review on his/her own, as well as participating in the Seminar where he/she must present his/her continuous attempts to express the problem. At this stage of the process, taking a general course on Research Methodology is essential. Discussing work with his/her Director and other researchers is also very useful at this point. This stage includes the period devoted to setting-up and writing of the doctoral dissertation project.

Thirdly, the researcher-to-be approaches the research problem empirically or through review work that demands technical treatment. At this point, research is supported by a particular methodological frame and demands the application of the previously outlined techniques. Dealing with individuals on an empirical basis is frequent.

Some key steps at this stage include the delimitation of variables, the definition of tasks and the creation of information-collecting tools. Experimental testing as well as tool and task revision are also important aspects in this stage.

During the development of the experimental work some control criteria must be followed and the student is required to have a detailed record of activities so that a performance analysis can be done later on. This stage is also devoted to the treatment and analysis of information, which involves tasks such as data collection, the use of criteria and categories to analyze them, as well as the characterization, codification, organization and treatment of such data.

This stage is considered to be the core of the research process, which makes the previous two being considered less important; nevertheless, success of this executive stage highly depends on the elucidation and specification work previously developed.

Fourth, the result achievement stage is reached, which has no continuity with the

previous stages. The findings of a piece of research are the explanation about events that were observed within a particular theoretical frame. At this point overall as well as specific interpretation of the obtained data on the basis of the criteria and categories previously established becomes rather relevant. Here again we come back to the theoretical framework, since data and observations must be explained in the light of theory in order for them to become research results. It is here that that the students must show full explanatory capacity regarding the research problem.

Some of the tasks performed at this stage are the review and methodical exposure of results in relation to the problem statement; elucidation of their relationship with the designed work-scheme, exposure of research findings and achievements and of confusing results (of difficult interpretation), and finally, elucidation of the links between the results and the initial hypotheses, and their valuation.

Lastly, an evaluation and final interpretation of results must be exposed, an explanatory framework accounting for the obtained information must be proposed and new research trends must be opened. This is a critical moment for it makes the overall interpretation of the problem and the results possible. At this point, the three basic research components - activity environment, theoretical frame and methodological frame - come together as a whole, gaining full dimension in the frame of a general scheme, where global reflection opens up new queries.

Here, together with the conclusions there appear new problems to be approached, issues that were not contemplated in the problem statement and the connection of the study with fields that had not been initially contemplated.

At the same time, there may appear connections between the results obtained here and data of previous studies approaching the same query. That is why we stress the relevance of convergent efforts towards common projects and towards insertion in research groups or research lines, thus forcing the candidate researcher to share scientific assumptions, foundations and methods and to interpret his/her results in the frame of other researcher's results.

Guideline for Doctoral Students

During the process of writing a doctoral dissertation there are several moments where critical and complex decisions have to be made. One of the tasks of a Director is to provide the postgraduate student with the appropriate guidance to help him/her overcome difficulties. On the basis of the above mentioned research scheme, we highlight several problems that might be encountered during the research:

Problem Identification

The first difficulty the Didactic of Mathematics student has to deal with is the selection and identification of the research problem. In this sense, some reflections come at hand. On the one hand, the student usually has personal concerns regarding the teaching and learning of Mathematics and he/she tries to make them into a research problem. His initial intuition might be useful, but in most cases it is inadequate and not well defined and cannot be readily included within the research

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lines of the research group the student intends to join, and to which his/her Director belongs. On the other hand, the postgraduate student thinks that having a general problem is enough to start doing research and he/she is not aware of the need of having an appropriate theoretical frame that may allow his/her concern to be delimited and approached adequately.

At this point, the postgraduate student must follow the discipline of the research group he has joined and assume its general theoretical frame. In some cases, this choice is not simple and it should be assumed as an active and critical involvement with a group having more expertise than him/her. The student must not take a passive attitude but rather be prepared to learn and also to discuss and to reject those aspects that appear unintelligible or confusing.

Problem Representation

Representation and approach of the research problem involves delimitating and enunciating it. In our opinion, the main difficulty of this stage is stating the research hypothesis or enunciating the issue with fair precision.

In order for the student to delimitate his/her research problem adequately and enunciate it with precision, he must have comprehensive understanding of the study field and a theoretical framework that may lead him/her to meaningful questioning of issues that had been traditionally raised in relation to the problem.

> "Research goals result in queries that lead to the clear statement of a research problem. Such queries constitute the pathway for the enunciation of the research hypotheses -possible or provisional explanations that bring to mind the factors, events or conditions the researcher intends to elucidate-. These hypotheses include facts that go beyond the known and pretend to provide feasible explanations to unknown conditions. When relating known facts to conjectures about unknown conditions, hypotheses increase knowledge, whether they are proved or rejected by research". (Segovia, 1995)

The research hypotheses shape the research problem as regards the theoretical framework and the methodological framework from where it shall be approached. Hypotheses express conjectures that intend to explain the phenomena under study. Their acceptation or invalidation -or its joint consideration- brings light to the study field. Strictly speaking, hypotheses express the totality of the problem under study. Therefore, they must bring about interpretations about unsolved questions that are of intrinsic interest, whatever the reason to go deeper into such matter. Finally, hypotheses must express the research problem with rigor and clarity.

Stating the research problem in terms of one or several hypotheses is formal condition to conduct an investigation successfully; it also helps one to calibrate the depth and interest of the proposed study. In the case of a doctoral dissertation it represents one of the studyís cores. The effectiveness of an hypothesis (or the research results) is one of the criteria to ponder the dissertationís quality. One of the important moments the dissertationís Director must intervene is at the initial delimitation and precise enunciation of the research problem.

Creation of Tools

A variety of problematic issues emerge when writing a dissertation. We want to stress the importance of the issue regarding the information-collecting tools. Except

for refutation studies, the research work involves delimitation of various tasks to be proposed to the sample subjects.

Characterization and selection of tasks is a complex procedure that faces the researcher with a challenge: that of gathering the maximum amount of information for the research purposes, with a minimum of procedures. Tools are an essential part of the design for they condition the sample and pace the studyís timing. Variables must be delimited and criteria for the information analysis and treatment should be established in accordance to the different tasks. Tasks reveal the core of the research in detail and, though they do not guarantee the work's quality, they do bring its essential characteristics to the surface.

In order to observe the subjects' performances carefully, the researcher should build adequate tasks that bring beliefs, obstacles and errors to light. To put to test an assumption about scholars' learning or teachers' professional skills, thought-revealing activities eliciting the complexity and richness of underlying thinking should be applied. For this reason, information-collecting tools (tests, questionnaires, surveys and interviews) and their corresponding tasks (items, problems, open questions and activities) are essential parts of the research work and they form an assessment criterion of the work in progress.

The methodological design guarantees adequate control and treatment of the gathered information; it leads one to optimal use of obtained data which is utmost important for the research. Through the applied tasks, the researcher may get a precise picture of how students and teachers interpret and provide meaning to their actions within the Mathematics teaching-learning process.

One of the important tasks of the adviser is to discuss the tasks the research is going to be focused on.

Results

Interpretations and connections arrived at throughout the research process become a delicate issue when writing the study's conclusions.

Frequently, the results arrived at do not answer all of the questions posed at the beginning of the research process, which may frustrate the researcherís expectations. Oriented by precise questions and with the aid of adequate tools and tasks, the researcher has gathered substantial data producing information of all kinds, which apparently surpasses the initial forecasts, or else, they may point in unexpected directions.

It is a crucial moment within the research process. The researcher now seems to count with less information than he/she thought and, on the other hand, he/she has in hands finding he/she had not thought of. The puzzle pieces seem not to fit the scheme. The avalanche of resulting information may exceed the researcher and hinder his/her comprehension. At this moment, the researcher should have in mind that data must be organized in a way such that he/she obtains the most powerful explanations, even if he/she should take distance from some of his/her initial expectations. If data are not erroneous, the researcher should review his/her interpretive frame and go deeper into the diverse options connecting the results to be in a better position to offer a well-structured and coherent interpretation of them.

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Interpretation and Final Conclusions

Finally, results ought to be connected to a wider framework and open questions must be brought up. The researcher now has gone through the inquiring process and has faced the studyís complexity and the richness and variety of the data. He/she has also verified the connections between the studied matter being studied and other important issues that were not direct object of the study. He/she knows that the information that has been gathered is just part of the studied phenomena where the process is under control and refutations can be put forward. Now he/she must make the most sense out of the obtained data and information, highlight findings and substantiate achievements within the structure and coherence of the research framework.

This is the most difficult -and interesting- stage of the research process. It is difficult to place the results of isolated or single research works into a more general framework; they tend to open up new queries and conjectures. Research works conducted within the scope of given research lines are provided with available resources to have their results organized, since their data do not have a unique, absolute value but must be interpreted in relation to and in contrast with a wider field of knowledge to which they contribute.

Research Results

Research Assessment

The evaluation of studies and pieces of research is mandatory as far as work quality, improvement, revision or refutation is concerned. It is essential to the progress of a scientific community and to that of specific fields of knowledge. The assessment of a research work is the core of its critical revision:

> "What is essential to science is assuming a critical attitude. First we create theory, then we criticize it. Because we tend to assume a rather humane attitude in relation to our theories and to defend them in stead of criticizing them, scientists appear to relate to each other in some kind of friendly-hostile way. And we must feel grateful for it". (Popper, 1997)

Before the presentation and defense of a doctoral dissertation, or any other kind of research work, each member of the jury who is going to participate in its assessment must issue a written document validating the work as suitable for scientific assessment. In this report, the jury members may put forward pertinent remarks aiming at the improvement of the study. The researcher may not defend his/her work until he/she has made the required modifications. Members of the jury might also ponder the convenience of postponing the defense of the work until revision has been completed. Research seminars are offered in most departments, where research works are submitted to periodic discussion and evaluation for their improvement.

The Research Seminar of the Didactic of Mathematics Department of the University of Granada looks up to the public exposure, discussion and evaluation of research production and results of students of the doctoral program, at different stages of their work.

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Quality Criteria concerning Research

There has been continuous concern for research in the Didactic of Mathematics field ever since its take-off, and parallel to the advances achieved in the methodology of Education research. (Rico and Sierra, 1999). The developments in education research and permanent interest for research quality have lead to the establishment of assessment criteria and assessment standards (Wittrock, 1986). Usual criteria are: Empirical character, Feasibility, Clarity, Relevance to the community, Relevance to practice, Relevance to theory, Originality, Ethics and Precision (Fern·ndez, 1995). Each of these issues expresses a basic concern of researchers on education.

Assessment criteria for research in Didactic of Mathematics is not specific but coincide, in general terms, with those to assess Education research. Freudenthal (1982) proposed three quality criteria: Reliability, Validity and Relevance, and discussed their relevance within our discipline. In general terms, we may say that reliability evaluates quality and suitability of the method; validity evaluates quality and coherence of the theoretical framework and relevance evaluates quality and practical results.

Kilpatrick and Sierpinska (1993) establish more sophisticated criteria to evaluate research quality in Didactic of Mathematics which are similar to those of education research. Criteria proposed by them are: Relevance, Validity, Objectivity, Originality, Rigor and Precision, Predictability, Reproducibility and Relatedness to Mathematics and Mathematics Education. These are valid up to date.

Each of these issues focuses on one of the components of research above mentioned. Thus, rigor and precision, and reproducibility are criteria focused on the methodological aspects of a research. Thus, rigor, precision and reproducibility are centered on methodological aspects; they set up quality standards according to the selected method.

Validity, objectivity and relatedness are oriented towards the theoretical framework. Whereas pertinence and predictability evaluate research coherence and meaningfulness as to the ambit it pertains.

Originality is basically linked to the overall character of the study and is a comparative valuation criteria regarding other research focused on the same problem.

Dissemination of Research Results

The usual means of disseminating research results is a document containing: the research problem, the selected method and the conclusions arrived at.

Those pieces of research conducted in the university fit a conventional format which applies to minor theses, third-cycle doctoral theses and doctoral dissertations. The final output is a generally lengthy research report including the introduction, the delimitated research problem, the conceptual and methodological framework, the designed instruments, the collection, treatment and analysis of information and, finally, the findings, results and conclusions. Research journals are technical studies focused on ever more specialized and concrete fields of study where it may be

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difficult for the studious to orient him/herself.

There are conventional strategies to disseminate the results of research and they involve different levels of communication. At the first level, is the publication of research journals themselves or research monographs. These are extensive and specialized works of limited edition which constitute the bulk of the so-called *gray* literature and are exchanged among specialists and have no commercial version. In Spain, these works are of limited edition, carried out by universities and other research institutions and can be found in their research centersí libraries, or else, in the web pages of university departments.

Also, the *Mathematics Education Department of the Advanced Studies and Research Center of Mexico* publishes limited-edition result monographs of research that was carried out in the Department. Some of these works are of commercial edition including some materials that reach a somehow wider public. Such is the case of *Grupo Editorial Iberoamericano* in Mexico, those of *una empresa docente* in Colombia and the *Mathema Collection* by Comares Publishers in Spain.

At a second level, research results are briefly communicated through articles in specialized journals, seminar debates and at specific research congresses. These are concise and synthetic documents that present only the core of the research work.

This literature serves various complementary purposes: it presents the work to the specialists critical point of view transforming the study into public knowledge; it extends the knowledge of the research community and forces the author to reduce his/her work to its very essential parts. It is addressed to specialists although it is open to a broader public.

In Spain, this second level is being constituted and consolidated and we do not have specialized journals devoted to Didactic of Mathematics exclusively. *Enseñanza de las Ciencias* is the publication that best meets the above mentioned criteria but must still become independent from experimental sciences, expand its production and foster its critical apparatus. Publications from different countries such as *Educación Matemática* in Mexico, *EMA* in Colombia, *Quadrante* in Portugal and the transamerican journal *RELME*, have been doing a high service on behalf of research dissemination in Didactic of Mathematics published in Spanish or Portuguese. On the other hand, articles in this field are occasionally published in Education Research Journals. The material is public in this case and may be found in specialized data basis.

Research Production in Spain

Background and recent developments in Didactic of Mathematics research in Spain are proportionally similar to those of countries with equivalent social and cultural development. (Rico, Sierra, and Castro, 1999). We briefly present some of the recent production in Spain as follows.

Curricular research has been somewhat developed in the University of Granada, one of its research lines being: Design, Development and Evaluation of the Mathematics Curriculum; there is substantial theoretical discussion about the

curriculum concept published in various works, and eight doctoral dissertations have been devoted to such matter.

Research on the teaching and learning of concrete topics has been more widely developed in various universities. In the Autonomous University of Barcelona, and the universities of Cantabria, La Rioja and Valencia research on the learning of geometry are conducted. In the case of Barcelona, there has developed a common research project involving the use of new technologies and analysis of interactions in the classroom is being developed. In Valencia, research is focused on the Van Hiele model for the understanding of geometrical concepts as well as for the analysis of geometry learning processes.

In the universities of Granada and Jaén there is a research team working in Didactic of Statistics, Probability and Combinatory; seven doctoral dissertations have been written on the matter. This team has good international status. The La Laguna and the Murcia universities are also working in Didactic of Statistics, while the Autonomous University of Madrid and the university of Cádiz work on Didactic of Probability.

In the universities of Alicante, Castilla-La Mancha, La Laguna, Salamanca, Santiago de Compostela, Valladolid, the Autonomous University of Barcelona and the Public University of Navarra there are teams working on Didactic of Calculus and Analysis. These universities have produced seven doctoral dissertations and keep good interchange, which has lead to joint projects.

In the universities of Granada, Málaga, La Laguna and Valencia there are research teams working on Algebraic and Numerical Thinking, focusing on the processes involved in the teaching and learning of school Mathematics, numerical structures, representation systems, the transit from arithmetic to algebra and problem-solving. This field is also studied in the universities of Alicante, Almería, Cantabria, Córdoba, Barcelona, Zaragoza, Valladolid and the Public University of Navarra whose production amounts to sixteen doctoral dissertations so far.

As far as procedural aspects of Mathematics are concerned -notably problem-solving-, the Autonomous University of Barcelona and the universities of Extremadura, Huelva, Granada, La Laguna and Valencia, as well as at the Instituto de Estudios Pedagógicos (Institute for Pedagogical Studies) of the Pontifical University of Somosaguas, are doing research in the field. At the same time, the universities of Cádiz, Granada, Málaga, Oviedo and the Public University of Navarra have intense work on children education.

The universities of Barcelone, Cádiz, Extremadura, Granada, Huelva, León and Sevilla also conduct research on teachers' professional knowledge and the training and development of Mathematics teachers. The production of dissertations in this area is high too, amounting to ten doctoral dissertations written so far and the development of several research projects and international contributions.

Specialists have paid less attention to the mathematics assessment field; nevertheless, there is stable activity in the universities of Almería, Barcelona, Granada, León, PaÌs Vasco and the Autonomous University of Barcelona, where research projects are supported by the Spanish General Directorate for Scientific and Technological Research (DGICYT) and by the Research, Documentation and

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Assessment Center (CIDE).

Research on History of Mathematics is confined to the universities of Barcelona, Murcia and Salamanca, where three doctoral dissertations have been defended and various others are in progress. Research on the learning of Mathematics by handicapped children are mainly based in the University of Barcelona, although the University of Málaga is working in this field as well.

In the Autonomous University of Barcelona, and the University of Granada, there are research teams focused on Ethno-Mathematics maintaining intense and internationally recognized activity. Also, there is a group of researchers working in Didactic of Mathematics as a scientific discipline, who intend to encourage theoretical reflection. These researchers area based in the universities of Granada, Zaragoza, the Complutens University of Madrid and the Autonomous University of Barcelona.

Conclusions

These reflections aim at providing guidance for those beginning researchers in Didactic of Mathematics, mainly those in the process of writing a doctoral dissertation. Within the frame of this international report: Supervision of Higher Degrees in Mathematics Education - An International Perspective, we have highlighted the activity carried out in Spain over the last years. It contemplates the academic standards for third-cycle studies and the processes involved in the completion of a doctoral dissertation in order to become a doctor in Didactic of Mathematics.

Another important aim of the present contribution is to provide a well-structured system of ideas that may help writing a doctoral dissertation, from the studentís perspective as well as from the Directorís perspective. For this reason we have distinguished work contexts, components of the theoretical framework, design, techniques, variables and instruments, process stages, and assessment criteria. We wish to offer this system of ideas as a practical reflection and guidance tool for the new researcher, for it is the result of our three decade experience as researchers in Didactic of Mathematics and of the critical observation of the Spanish community of researchers in Mathematics Education.

We hope this may serve as a useful tool contributing to the progress and development of many.

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