Mathematics for Prospective Primary Teachers.
A Pilot Experience for Adapting to the European Higher Education Area.

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Abstract

The future implementation of the European Higher Education Area requires thorough reflection on how to design and develop teacher training courses. In this reflection, it is important to reconsider, among other issues, the role of prospective teachers in their own learning process and the professional competencies that they must develop in the course of their higher education. Since 2004, the University of Granada has undertaken the development of pilot experiences to adapt some degree programs to this new framework, within them the degree Teacher in Primary Education. This degree includes several courses for promoting prospective teachers' development of mathematical and pedagogical knowledge. In this paper we first analyse the general process of adapting these courses. Secondly we describe its theoretical and practical structure, with some examples of practical activities. Finally, some results of the implementation are discussed.

Keywords

Competencies; European Higher Education Area; Mathematics Education; Teachers Training; Theory and Practice.

As a response to the challenge set out in the Bologna Declaration, since 2001 over 175 European Universities had worked intensively to create the European Higher Education Area (EHEA). This project is not only European. In 2004, when the Latin American Tuning Project was created, many latinamerican universities joined this deep reflexion about higher education guided by commom interests such as (a) promoting the compatibility, comparability and competitiveness in higher education, and (b) tailoring programs to the needs of society at both local and global levels (Beneitone, Esquetini, González, Maletá, Siufi and Wagenaar, 2007).

Within this context, here we describes an innovative experience in the effort to adapt mathematics education training for prospective primary school teachers to the new directives proposed, within the current Spanish legal framework. We explain the adaptation process and the results obtained within a pilot experience developed in the Department of Mathematics Education at the University of Granada.

1. Changes towards the EHEA

The ultimate goal of the EHEA is to have universities adopt a system of degree programs that will be comprehensible and comparable in all European Union countries by 2010. Some of the directives established have been given concrete form in Spain in laws governing the development of the future degrees. One of these is the so-called “European credit” or ECTS (European Credit Transfer System) credit, which involves a change in the way of measuring student workload. Another consists of characterizing each degree in terms of the general and professional competencies that define the degree-holder’s professional profile. These two directives require a substantial change in the teaching activity. They focus on the student and conceive the professor’s role as that of a guide and an advisor to ensure that students acquire the established competencies.

In embracing these directives, since 2004 the Andalusian Regional Government has promoted experimentation with current university degree programs to draw conclusions from the experience that will enable the design of new degrees in accordance with the EHEA. Forteen degree programs have been chosen for undertaking this experimentation, among them the degree Teacher in Primary Education.

2. Primary Teachers´ training in Mathematics and Mathematics Education at the University of Granada

In the degree Teacher in Primary Education at the University of Granada, there are two Mathematics Education courses: *Mathematics and its Pedagogy* (a core course of 4.5 theoretical and 4.5 practical credits) and *The Mathematics Curriculum in Primary Education* (a compulsory course of 2 theoretical and 2.5 practical credits). Students can complete their training by choosing between other several mathematics education elective courses.

The core first-year course mentioned above is designed to ground the students’ mathematical knowledge. It aims to deepen the mathematics skills that they developed during their required secondary schooling, before introducing new concepts in Mathematics Education. The term we used to designate this option is: *Mathematics for teachers.* In this course the contents are mathematical, involving the concepts and procedures, their representations, phenomenology, modelling and history (Rico, 1997). The scope of these concepts and their area of formalization, application, signification, representation and study, corresponding to those a Primary Education teacher should have. Mathematics is presented from a cultural, social and epistemological perspective consistent with the current official curricula for this education level.

The compulsory course, indicated for the second year, seeks to professionalize the future teacher in his or her role as math educator and thus requires that he or she masters certain concepts in Mathematics Education. These concepts are always channelled to facilitate the comprehension of the curriculum, as well as the design, implementation and evaluation of the teaching of Mathematics in Primary Education.

3. The Pilot Experience in the Course “Mathematics and its Pedagogy”

Now we focus on the innovative experience developed in the course “Mathematics and its Pedagogy”. We began the experimentation with the European model during the academic year 2004-2005.

This experience became the focus of greatest reflection and attention in the Department’s teaching seminars. In confronting this course, in addition to adapting the program numerically to the ECTS and redefining the objectives to adapt them to competencies, it was found important to establish a precise differentiation between theoretical and practical credits.

Diverse variables utilised managed to distinguish the theory and the practice. Ferrater (1991, p. 2652, 2661) shows us different ways to understand these variables, both in training processes in general and in teacher training. These differences were grouped into two blocks: the first corresponding to the epistemological separation between theoretical and practice knowledge and the second to the competencies and thus the way of teaching.

The division between the *theoretical and practical credits* in the courses is based on the actions performed during teaching and the competencies that the students are to acquire. One way of viewing the practical credits in this context is to focus on tasks in which the theoretical concepts are applied *to solve problems in the professional world* (professional practices, such as class planning, the design and qualification of exams, etc.) *or in the everyday world* (mathematical practices for solving problems, analysing phenomena, interpreting information, etc.). Another perspective is that work for the practical credits must tackle *procedural competencies* tied to know-how, such as those related to the use of technological means for teaching (professional practice) or the use of mathematical procedures (mathematical practice) (Monereo, 1994, Pozo and Monereo, 1999).

Finally, but not unrelated to the foregoing aspects, another differentiation between theory and practice can be made based on the degree to which students acquires a *leading role* in the performance of tasks. This idea enables us to consider as ***practical activities*** those in which students ***act.*** The idea of practice would thus be tied not to the content encountered but to the way this content is related to knowledge. A theoretical, conceptual content can be considered in a practical way if we give students a leading role in performing activities that enable them to interpret it, debate it with classmates, share it with others, present it, and contrast it with texts in which it is defined, characterized, exemplified, etc. Thus, practice is understood like Resnick and Ford (1981), as exercise.

In the organization of the course, we have adopted this last criterion to distinguish the practical credits. The teaching has been organized by differentiating the theoretical classes from the practical classes according to the kind of action undertaken by the teacher and the students, not by the content proposed. This criterion agrees more closely with some of the basic ideas for change under debate, which are directed to emphasizing that students assume a greater role in their learning.

The teaching load for the course involves 3 hours a week throughout the academic year which are divided into one session of two hours and another of one hour, with the following time organization:

- Theoretical credits: Two hours of class a week for 21-weeks of the academic year, orientation seminar of one hour a week for 21 weeks, and individualized tutorials during the professor's office hours throughout the year.
- Practical credits: Practical action seminars for the students for three hours in the course of nine weeks, three weeks at the end of each trimester (in December, March, and June).

3.1 The theoretical credits

In the theoretical class sessions, the teaching responsibility is the professor's. The professor provide material so that students can undertake the study of the course in a significant way, without this involving the presentation of all contents and results that students must learn. To achieve this, the professor performs tasks like those students are expected to perform, making explicit the actions involved; students observe in order to construct a conceptual model of these tasks (*modelling*) or the teacher carries out part of the task and encourages the students to perform other parts with the professor's help (*scaffolding)* (Vizcarro, Liébana, Hernández, Juárez and Izquierdo, 1999).

These teaching processes consider the contents holistically (Moral, 2001), leaving the specific parts for the students to perform with the support of the recommended documents. This method seeks to produce deep learning of the concepts covered by developing them with growing complexity and diversity and global abilities (Vizcarro et al., 1999). During the development of the theoretical contents, the practices of contextualization, application and evaluation are performed (Díaz-Godino, 2005) through the methods of instruction indicated.

The orientation seminars take place every week for an hour. In these sessions, the students must express their needs, questions, etc. The activities requested by the students are also performed; activities related to those developed previously by the professors and inspired by the questions that will later be used in the evaluation tests. To this end, students are encouraged to perform the proposed tasks, from the interpretation of the data to the search for necessary information, while the professor provides the information requested (*training*). In other cases, students choose the tasks to solve and demonstrate their abilities by reasoning and interpreting the concepts employed in solving them (*articulation*).

Finally, the tutorials provide individualized attention in the professor's office, resolving difficulties that students encounter with the topics covered in the course and the tasks required of them. During these tutorials, the instruction provided is based on training, scaffolding, modelling and reflection.

In order to direct the action, we have developed some work guides and activity sheets that orient the student and guide the professor's work in class (see example in First Topic of the program in the Appendix). The students perform the assigned tasks, working in groups or individually, participate in presentations, and develop their work and present the work developed using the appropriate technological media.

3.2 The practical credits

In the practical class sessions, students are the ones who perform the tasks that the professor has planned. In this classess the tasks consist of problems of exercitation, application and evaluation (Díaz-Godino, 2005); or experiences, illustrative experiments, practical exercises and research (Caamaño, 2003). All of these include activities of observation, prediction, critique, generation and analysis (Llinares, 1998). The work model proposed covers reading the guide document, carrying out group work with the material provided, presenting results and completing a team workbook. In the course of the class, we perform the following models of instruction:

- *Training:* Students perform tasks, while the professor observes this performance and gives advice and help.
- *Articulation:* Students resolve tasks, demonstrating their reasoning and abilities.
- *Reflection:* Students compare their processes for solving problems with those of their classmates, first through group work and then by means of performing presentations and discussions.

- *Exploration:* Students develop new situations to which to apply the concepts learned. In designing the tasks they propose, they are encouraged to find new situations affected by the concepts learned.

Practical activities, in which the students act on concrete material under the professor's supervision, are also stimulated.

To achieve this, students are divided into three subgroups for their practical classes. These groups rotate through three different weekly scenarios: the classroom (mathematics workshop), the math laboratory (manipulative materials) and the computer lab. Three professors supervise the practical classes, each one in a specific scenario.

In each subgroup, the students work in teams of four, remaining in the same small group for the whole academic year. Likewise, the same professor who directs the practice activity in a particular scenario supervises the three blocks of content for which they perform the practical classes in that scenario. We plan tutorials directed to meet with the teams several times throughout the course, such that each team has at least one tutorial with one of the professors for the practical tutorials.

The work in practical credits requires students to act, first as individuals and then as a team. The professor presents the activities, responds to questions, encourages the students in their work, and coordinates the presentations. This requires the use of "practical workbook guides" (Flores and Segovia, 2004) with the relevant instructions and activities. Now we present and illustrate with examples these guides.

3.3 Practical workbook guides

The workbooks for the practical sessions are designed to foster autonomous work in our students (prospective teachers), while also constituting one of the indicators for evaluation of their performance.

There are individual and team workbooks, classified into three blocks of content (Arithmetic, Geometry and Measurement-Statistics) and into three practical scenarios (Mathematics Workshop, Manipulative Materials, and Computers lab) for a total of 18 workbooks. Each student has an individual workbook, which includes the explanations, activities, resources and documented sources to enable the student to tackle each of the practical activities. The team workbook unifies the work performed individually and includes additional reflection activities on the earlier individual work.

To illustrate the contents and structure of the practical workbooks, we now present some examples. Each example corresponds to one of the scenarios and describes just one of the activities included in a workbook (the text is directed to the students).

3.3.1 Practical workbook for working in the Computers lab. Activity: The Factor Game.

> In this activity, you will work with natural numbers and the main arithmetical operations. Specifically, we will study the divisors of a number, which are all the divisors of this number except that number itself. The study of problems of divisibility is of great importance, since they constitute the solution to a wide number of problems in everyday life, such as calculating the dimensions of a piece of rectangular cloth to make a curtain, distributing a class of students in rows, or distributing the prizes in a contest according to the position in which the participants have placed.

The main objective of this exercise is to work on the following notions through an interactive computer program: divisibility, divisors of a number, decomposition into its factors, and prime and composite numbers.

By performing the different activities that we propose, you will become familiar with the main properties of divisibility among natural numbers; you will be able to express one number as a product of factors, and we will approach the study of prime numbers. We will also work on the notions of perfect numbers and see how some of these ideas were studied at a certain point in ancient history.

General description of the exercise

In this activity, we introduce an interactive game for two people. The game is available on http://illuminations.nctm.org/ActivityDetail.aspx?ID=12 (November, 2007). The program puts the main ideas of the exercise into practice, while also developing capacities related to problem-solving, such as exploring winning strategies. The exercise begins with some games for the future teachers, to be played with each other or against the computer, so that they can evaluate the merit of starting the game or being second, the best options for beginning the game, and how the first natural numbers can be classified according to their divisors.

3.3.2 Practical workbook for working in the Math laboratory (manipulative materials). Activity: Operating with the Abacus

The abacus is a tool that is used for centuries to perform calculations. Until the fifteenth century numbers operations were performed with the abacus, and even now this instrument is still used in some countries as a pocket calculator. The spread of positional numbering system, making use of zero, along with the invention of paper, were decisive in replacing the use of the abacus by the use of the pencil and paper algorithms that we use today.

This activity suggests calculations to perform with the abacus directed to identify rules that govern the use of this instrument. Our main intention with this activity is to help you to consolidate your knowledge of the properties of the numbering system and the computational algorithms. The specific objectivies of this activity are:

a) To know about the types of abaci and their use.

b) To recognize the properties of positional numbering system from the use of abacus: recognition of the principle of grouping, notion of value position of the figures, decomposition of a number in powers of 10.

c) To understand the mechanism of algorithms through the manipulation of calculations on the abacus

d) To transfer the manipulative representation through the abacus to the written representation used in the pen and paper algorithms and vice versa.

e) To explore relationships and properties of natural numbers through representations using the abacus

General description of the exercise

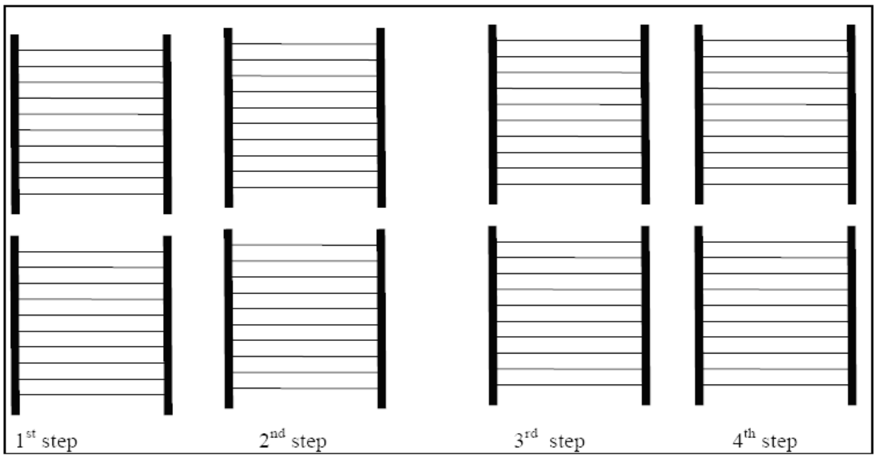
In this activity a summary of different types of abaci are provided to the students (Roman, Chinese, Japanese, Russian) and the difference between the horizontal and vertical abacus is highlighted. Students are asked to use these two types of abaci to perform representations of numbers and arithmetical operations, indicating the appropriate steps to solve such operations.

By way of example we show two of these activities concerning to the sum of natural numbers using the horizontal abacus.

Use the horizontal abacus to carry out the following additions:

65 + 28 = ; 81 + 46 = ; 387 + 575 = ; 3572 + 5849 =

Make a representation of the suitable steps for the addition 387 + 575 = . Since the numbers are greater than hundreds, two abaci (one for units and the other for hundreds) are needed in order to represent these operations.



Represent in a table as shown on the right the transformations that you made in each operation of the previous task. In order to do it, write the necessary changes made to the first addend to obtain the result. Carry out the operations and note them in the table more than once, starting with a different order of units each time. Example: 264 + 186

First addend:	2	6	4
Second addend:	1	8	6
1st Transformation:	3	6	4
2nd Transformation:			
…			
Solution:			

I add the hundreds

I add the tens

3.3.3 Practical Workbook for working in the classroom (Mathematics Workshop). Activity: Estimating measurements

Make the following estimates individually (do not modify them, even if they do not agree with or differ greatly from those of your classmates):

L) Estimate the length of the professor's desk

S) Estimate the surface area of the blackboard

C) Estimate the interior volume or capacity of the waste basket

P) Estimate the weight/mass of the chair on which you are sitting

1. Present your results and describe how you performed the estimation (strategies and references used) to the classmates in your (small) group.

2. Now you are going to study of the estimations made by the whole class. Record the different results from the whole class: **L)** Desk (length): **S)** Blackboard (surface area): **V)** Waste basket (volume): **W)** Chair (weight):

The group is going to perform the statistical study of all of the class results from the estimation of one of the four magnitudes studied: **L**ength, **S**urface area, **V**olume or **W**eight. To select the estimation, you must do the following:

- Think of the first letter of your name.
- Determine the magnitude whose initial (**L, S, V, W**) is closest to the letter preceding the initial of your name in alphabetical order (for example, if your initial is **P**, the magnitude you should work on is **L**, since **P** is between **L** and **S**; if your initial is **B**, you should work on magnitude **W**, since **B** is between **W** and **L**, if we assume that **A** follows **Z**).

A, B, C, D, E, F, G, H, I, J, K, **L**, M, N, Ñ, O, P, Q, R, **S**, T, U, **V**, **W**, Y, Z, A, B, C, ...

- Copy the magnitudes that belong to the members of your group. You must study the magnitude that corresponds to the largest number of members (in case of a tie, draw lots).

3. What magnitude do you think has the greatest probability of being studied by the largest number of groups in the class? Explain the reasons.

4. Record the data of the magnitude chosen using the preceding procedure (Length, Surface area, Volume, Weight) and, as a group, develop a statistical study that includes a Table of Frequencies, a Graphic Representation and the calculation of measures of central position and dispersion.

5. Represent the data graphically. From these results, draw some conclusions about the results obtained. What measurement represents the estimations? What degree of dispersion does the data show? Was the class's estimation homogeneous? Was the team's?

<u>General description of the exercise</u>

This exercise aims to estimate measurements of quantities of magnitudes present in the classroom. To contrast them, students must perform a statistical study of the estimations of the entire group. At the same time, students should perform a probabilistic study, in which we hope that they will examine the expected sample space and contrast it with that obtained empirically.

4. Evaluation of the adaptation and conclusions

After the planning and experimentation in the course *Mathematics and its pedagogy*, we not only fulfilled the guidelines for avancing in the adaption of the course to the future EHEA but recognized several issues that we consider very valuable, both in our professional development as trainers involved in the project and in the development of the area of knowledge in the Faculty of Education and at the University of Granada.

First, the experience promoted interesting debates and discussions among the professors related to the planning of teacher training in the area of mathematics. These activities led us to generate classroom materials agreed upon by the professors and tested with the students. Second, the described actions generated significant changes in the students' attitude toward the course *Mathematics and its pedagogy* and to school math. Their participation in the activities, the stimulus to work autonomously, and the fostering of communication among students and between students and professors constitutes one of the great advances in this experience. Further, we have established a work model for classrooms with a large number of students (over 100) where it is difficult a priori to abandon a methodology based on the lecture class.

This experience is important to adapt instruction to the new European directives in the material of higher education as evidenced by the interest in this experience shown in diverse forums (Flores, Segovia and Lupiáñez, 2006; Flores, Segovia, Lupiáñez and Molina, 2007; Lupiáñez, Segovia and Flores, 2006; Segovia, Flores, Lupiáñez and Molina, 2007). We have also been invited to help other Spanish universities (Flores and Segovia, 2006), and our work has become the subject of a chapter in a book on the European convergence (Segovia, Lupiáñez and Flores, 2006).

Recently we have initiated an evaluation of the practice part of the program of this course which aims to study its potential as element of development of mathematical professional competencies for prospective teachers (Cecilia, 2007). This analysis is based on a unified list of professional mathematical competencies for teachers elaborated after discussions about the following four list of competencies: (1) list of competences for primary teachers from the ANECA (Spanish Nacional Agency of Quality and Qualitication Evaluation); (2) list of competences for primary teachers agreed on Andalusia for the ECTS experimentation, (3) list of competentes from the Tuning project; and (4) list of competentes for primary teachers by Pollard (1997).

Cecilia has examined the quantity of activities which promote the development of each competence and the weight that each competence plays in the prospective teachers´ evaluation processes. These processes are explicited in the team workbook that students must complete after the practical classess.

Till the moment just the part of the practice program related to the content of arithmetic and to the laboratory scenario has been analysed. This analysis has shown that this part of the program is coherent and mainly promotes the development of competences related to knowledge and understanding of mathematical contents and to teaching materials and resources. The competences related to actitude towards mathematics are less present. We are still working on the evaluation of the rest of the program.

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Appendix. Work guide for the professor and students in the first topic of the course *Mathematics and its Pedagogy*

MATHEMATICS EDUCATION DEPARTMENT. UNIVERSITY OF GRANADA

Course: Mathematics and its pedagogy. Degree: Teacher in Primary Education. Academic year 2004-05

Guide to Topic 1: NATURAL NUMBER. NUMBER SYSTEMS.

1. Uses of natural numbers (1, pp. 123ff.).
2. Concept of natural number (1, pp. 128ff.).
3. Ordering. The sequence of counting numbers (1, pp. 131-132). Quantifying. Strategies. The zero (1, pp. 133ff) & (2, pp. 31).
4. Representation of the number. Number systems: antecedents and their evolution (1, pp. 138ff) & (2, pp. 31).
5. Place value numeric systems. Decimal Number System (1, pp. 140) & (2, pp. 55).
6. Materials and resources (1, pp. 141) & (2, pp. 163).

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Orientations for the students´ work

There are three basic ideas associated with this chapter: the idea of the natural number, which is extracted from reflection on the mathematical concept, its use and its forms of representation. Relative to this concept, we hope that students will develop an idea of the significance of the concept of number from its ordinal and cardinal conceptions. As to its use, students should grasp the importance of understanding the sequence of counting numbers as well as know the basic principles of the activity of counting and the strategies that are employed in the fundamental uses of the number: ordering and quantifying. Concerning the forms of representing numbers, students should master the principles of functioning of the decimal number system and other forms of representation that enable reflection and analysis of it (e.g. about the presence of the zero, place value, etc.). Finally, students should know some of the most common materials and resources in the teaching and learning of numbers and the number system, such as Cuisenaire rods, multibase blocks and the abacus. We propose activities that involve the questions that students should be able to answer. We also present some examples that can serve for reflection while also illustrating whether students understand the theoretical questions and knows how to apply them.

Activities for reflection and evaluation

- What is a number? When is it employed (used)?
- What is counting / pairing? What kind of number results from counting/pairing? How is the number that results from counting characterized? How does one count?
- What is ordering? What kind of number results from ordering? How is the number that results from ordering characterized?
- What are the ways to represent the number? What are the characteristics of the written decimal number system? And of the oral system?
- How is the Roman numeral system different to the decimal number system?
- What other systems share any characteristics with the decimal number system?
- What are the particular qualities of the zero? What function does the sign of zero have in the decimal number system?
- What educative materials can be used to work on the learning of the decimal number system? How is each of these materials used?