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Conceptualizing and Describing Teachers' Learning of Pedagogical Concepts

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Abstract: In this paper, we propose a model to explore how teachers learn pedagogical concepts in teacher education programs that expect them to become competent in lesson planning. In this context, we view pedagogical concepts as conceptual and methodological tools that help teachers to design a lesson plan on a topic, implement this lesson plan and assess its results. Concepts such as the notions of learning goals, errors, conceptual structure, representation systems, resources, grouping, interaction or assessment strategies are examples of such pedagogical concepts. We propose a model that involves three types of knowledge of a pedagogical concept—theoretical, technical and practical—for describing teachers' learning of it. The knowledge classification proposed by this model assumes that a teacher should know the theory about the pedagogical concept (theoretical knowledge), be able to use it for analysing the topic and producing information about it (technical knowledge), and be able to use this information for making decisions in the planning process (practical knowledge). We present examples of the development and enactment of those types of knowledge by a group of mathematics future teachers in a teacher education program we have worked on.

Introduction

There is an ever-growing literature about teachers' learning (Adler, Ball, Krainer, Lin, & Novotna, 2005; Buitink, 2009; Gess-Newsome & Lederman, 2001; Sánchez, 2011). An important proportion of this research is concerned with teachers' knowledge: the knowledge necessary for an effective teaching (Beijaard, Korthagen, & Verloop, 2007; Hill, 2011; Hill, Ball, & Schilling, 2008; Hurrell, 2013). However, there is less literature on how teachers learn when they participate in teacher education programs (Borko, 2004; Carter, 1990; Cavanagh & Garvey, 2012). This might be a consequence of the fact that what and how teachers learn in those circumstances depend upon what they are expected to learn and upon the opportunities they are given for learning it (Putnam & Borko, 2000).

In teacher education programs that expect teachers to become competent in lesson planning that promotes active learning (that is, in designing a lesson plan on a topic, that includes how the lesson is expected to be implemented and assessed), teachers have to adopt an active role as curriculum designers, holding a leading role in curriculum decision-making (Xu, 2009), as opposite, for instance, to book-knowledge transmitters. In order to become effective planners under this approach, teachers need to get to know the topic from multiple perspectives—the school content behind the topic, the cognitive aspects involved, and the teaching and assessment strategies that might be specific to it (Cooney, 2004, p. 511; Shulman, 1986)—and analyse the topic from these perspectives in order to produce information that can support their planning choices (Charalambous, 2008; Clarke & Roche, 2010). Teachers need analytic and methodological tools for performing such analyses.. Those tools are the pedagogical concepts we refer to in this paper and that we will specify hereinafter. They are concepts "of mathematics teaching and learning and are distinct from mathematical concepts" (Simon, 2008, p. 20) and pertain to teachers' pedagogical content knowledge base. The following are examples of the kind of pedagogical concepts we refer to: the notions of learning goals, errors, conceptual structure, representation systems, resources, grouping, interaction or assessment strategies. These pedagogical concepts can be used in many disciplines. In this article, we use school mathematics as an example and adopt the meaning that mathematics education research assigns to them. Several authors have shown that they are useful for mathematics lesson planning (i.e. Rico et al., 1997).

Mathematics teacher education programs expect teachers to learn pedagogical concepts. However, as Simon (2008) has argued, literature on the articulation of pedagogical concepts that should be developed in mathematics teacher education is scarce and there is a need for studies of learning and teaching of specific concepts. We contribute to this line of research by describing a model for conceptualizing teachers' learning of pedagogical concepts. This model is inspired by the Aristotelian categories of episteme, techne and phronesis (Aristotle, 350 BC/1994). It involves three types of knowledge: theoretical, technical and practical. We will show that this model can be useful for understanding how learning takes place in mathematics teacher education programs that expect teachers to learn and use pedagogical concepts in order to become competent in lesson planning.

Our work fits within the research literature on teachers' professional learning and knowledge (Eraut, 1994). This literature addresses those topics from multiple viewpoints. We highlight two perspectives. The first perspective focuses on the knowledge that the teacher is expected to have. This is the case of Shulman's (1987, p. 8) categories of teachers' knowledge base: content knowledge, general pedagogical knowledge, curriculum knowledge, pedagogical content knowledge, knowledge of learners, knowledge of educational contexts, and knowledge of educational ends. Some of these categories have been characterized in detail in specific disciplines. For instance, in the case of mathematics, Hill et al. (2008) have proposed and characterized concrete categories such as common and specialized content knowledge, knowledge of content and students, and knowledge of content and teaching. The second perspective focuses on kinds or forms of teacher's knowledge. In this perspective, Shulman (1986, pp. 10-12) proposed three forms of knowledge (propositional, case and strategic), Rowland, Huckstep, and Thwaites (2005) proposed the knowledge quartet (foundation, transformation, connection and contingency), and one finds distinctions such as technical – practical, knowing that – knowing how, propositional – procedural, conscious – tacit, or cultural – personal. Most of these categories refer to how the teacher develops and enact the knowledge he is expected to have.

Our work is related to these two perspectives as follows. On the one hand, we see pedagogical concepts as the building blocks of the categories that configure the teachers' knowledge base. For instance, pedagogical concepts such as learning goals and students' difficulties and errors refer to Shulman's category of knowledge of learners and Hill et al.'s category of knowledge of content and students. Similarly, the representation systems pedagogical concept fits into Hill at al.'s specialized content knowledge category. On the other hand, the model we propose focuses on how teachers develop and enact their knowledge of pedagogical concepts. The theoretical, technical and practical knowledge are forms of knowledge (in the sense of the second perspective mentioned above). These forms of knowledge are specific to the learning and using of pedagogical concepts in lesson planning.

In the first part of the paper, we describe the role of pedagogical concepts in lesson planning within teacher education programs, specify the focus of the paper, introduce the model as a knowledge classification of pedagogical concepts, and define the theoretical, technical and practical knowledge of a pedagogical concept in terms of what teachers are expected to learn in a teacher education program. In the second part of the paper, we present examples of the development and enactment of those types of knowledge by a group of mathematics future teachers during their education. In the last section of the paper, we discuss about the usefulness of the model and point to lines of further research.

Mathematics Education Pedagogical Concepts for Lesson Planning

The concept of error is an example of a pedagogical concept. Mathematics education research has characterized errors as wrong generalizations from previous learning; they are neither lapses nor oversights; instead, they inevitably arise in the learning process and show significant ways of thinking about the students' school mathematics learning. Knowing this theoretical information on the error concept helps teachers to analyse a mathematical topic with a practical purpose: they can identify the usual errors in which students incur when learning that topic and they can use this information for selecting tasks and justifying such selection (Borasi, 1996).

Pedagogical concepts, as the concept of error, are conceptual and methodological tools that guide the ways in which mathematics teachers enact their knowledge. With a given pedagogical concept, the teacher can analyse a topic and produce specific information about it (as in the example concerning students' errors) that he can use for his lesson planning. Thus, as we argued above, pedagogical concepts are notions that support and give structure to teachers' professional knowledge base.

In this paper, we focus on those pedagogical concepts that can play a role in teachers' lesson planning. Lesson planning is a key teacher's competence (e.g. Carlgren, 1999; Zazkis, Liljedahl, & Sinclair, 2009). Following the ideas of the dialogical model of lesson planning proposed by John (2006), teachers do not follow fixed-order planning processes and the "main core is fixed by the aims, objectives, and goals of the plan" (p. 491). What is important is to put the focus on (a) the design and implementation of tasks that can enable students to achieve a given set of learning expectations for a topic and (b) the systematic justification of the realized choices (Liljedahl, Chernoff, & Zazkis, 2007; Sherin & Drake, 2009). In this context, teachers are expected to have a deep enough knowledge of the topic so that they can support the choices and decisions they make for their lesson plan (Charalambous, 2008). This is a topic-specific knowledge that involves, for instance, selecting concepts and procedures, representation systems and phenomena related to the topic, establishing the learning goals that teacher expects students to achieve, foreseeing the students' learning, including the errors that they might encounter when learning the topic, and establishing the teaching (e.g. tasks, resources, grouping, interaction) and assessment strategies that are more appropriate for developing the learning expectations identified (e.g. instruments and procedures for collecting and analysing the students' performance).

Focus of the Paper

In this paper, we refer to teachers learning of pedagogical concepts as a basis to develop the lesson planning teacher's competence, as described in the previous section. In this context, we have two purposes:

1. to propose a model, configured around three types of knowledge, for conceptualizing mathematics teachers' learning and enacting of mathematics pedagogical concepts, and
2. to show some examples of the development or enactment of those types of knowledge of specific pedagogical concepts by a group of mathematics future teachers during their education.

In the next two sections, we develop these two purposes.

Theoretical, Technical and Practical Knowledge of a Pedagogical Concept

In the context of teacher education we are interested in, and in relation to each pedagogical concept involved and a specific school topic, teachers are expected:

1. to know the pedagogical concept so that, for example, they can distinguish instances pertaining to it;
2. to be able to use the pedagogical concept for analysing the topic and producing information about it that can be used in the planning process; and
3. to be able to use the information produced with the pedagogical concept for making decisions in the planning process.

These three learning expectations involve three types of knowledge that we call theoretical, technical and practical knowledge of the pedagogical concept. These types of knowledge are the basis of the model we propose.

The three types of knowledge proposed fit into the Aristotelian categories—episteme, techne and phronesis (Aristotle, 350 BC/1994)—which have been adopted and adapted by several authors for referring to teachers' knowledge and performance and for exploring the duality between theory and practice in teacher education (Back, 2002; Orton, 1997; Saugstad, 2005). Episteme refers to the theory—the universal—, while techne and phronesis refer to pragmatic knowledge —the particular— (Kinsella & Pitman, 2012). Flyvbjerg (2006, p. 361), proposes the following characterization of these types of knowledge.

Episteme. It is the scientific knowledge. It is universal, invariable and context-independent.

Techne. It is a pragmatic, variable and context-dependent knowledge oriented toward production and governed by a conscious goal.

Phronesis. It is a pragmatic, variable and context-dependent knowledge, oriented toward action and concerned with rational thinking, wisdom and deliberation about values and interests.

We note that this knowledge classification resembles Habermas' classification of learning domains (technical, practical and emancipatory) of his critical theory of knowledge (Mezirow, 1981). Similarly, the triadic nature of this scheme evokes Sternberg's classification (analytical, creative and contextual) of his triarchic theory of intelligence (Sternberg, 1985). Whereas Habermas' areas of cognitive interest are related to aspects of social existence (work, interaction and power) and Sternberg's proposal centers on the individual's effectiveness for fitting to his or her environment and contends with daily situations, our interest focus on the types of knowledge of a pedagogical concept that teachers can develop.

The terms theoretical, technical and practical are used with multiples meanings in the teacher education research literature. In the rest of this section, we establish the meaning we give to these types of knowledge in the specific context of the learning and enacting of pedagogical concepts for lesson planning in teacher education programs. We illustrate those meanings with an example in the following section. In the context of a teacher education program, the *theoretical knowledge* of a pedagogical concept refers to the meaning that the educators choose among the multiple meanings that exist in the pedagogical research knowledge base. They usually make this choice on the basis of its usefulness for planning purposes. Hence, the meaning of a pedagogical concept is program specific. In its intensional definition, it is usually presented in terms of its properties and its relationship with other concepts. Those are the key ideas that characterize the meaning of a pedagogical concept as a theoretical knowledge and set it apart from the meanings that the term has outside the education literature. It can also be exhibited in extensional terms by means of examples that describe the collection of instances that compose the concept.

The *technical knowledge* of pedagogical concept is a type of pragmatic knowledge that is oriented toward production and is program and concept-specific. Analysing a topic with a pedagogical concept requires putting it's meaning into play in order to produce information that can be used in the planning process. That is, it is necessary to operationalize the key ideas that characterize it's meaning into techniques. Those techniques configure the technical knowledge of the pedagogical concept and should satisfy two conditions: (a) to be grounded on the meaning of the pedagogical concept and (b) to make possible to produce information about the topic that can be used for planning purposes. Among all the techniques that satisfy these two conditions, educators propose and make explicit those that they consider most effective for planning purposes.

The information that emerges from enacting the techniques that configure the technical knowledge of a pedagogical concept can be used for planning purposes. The *practical knowledge* of a pedagogical concept refers to the set of techniques involved in such process. These techniques should satisfy two conditions: (a) to use the information that emerges from enacting the technical knowledge of the pedagogical concept and (b) to involve decisions about the planning process. These decisions refer to two scopes: the technical knowledge of another pedagogical concept and the selection of tasks for lesson planning. The techniques are oriented towards a concrete action and they are associated to a prediction of its effects in practice. Among all the possible techniques, educators propose and make explicit those that they consider most effective and better suited to each scope and purpose. Other techniques might emerge in practice. As the technical knowledge, the practical knowledge of a pedagogical concept is a kind of pragmatic knowledge.

The three types of knowledge that we have proposed are related. Figure 1 presents a summary of the main characteristics of each type of knowledge and their relationships, and portrays our model. The theoretical knowledge of a pedagogical concept is the option that educators chose for a given program. The key ideas that characterize the theoretical knowledge of the pedagogical concept give rise to the techniques—with which a given topic can be analysed—that configure its technical knowledge. This information can then be used for planning purposes—in other analysis or in lesson planning—. The techniques that guide this use ground the practical knowledge of the pedagogical concept.

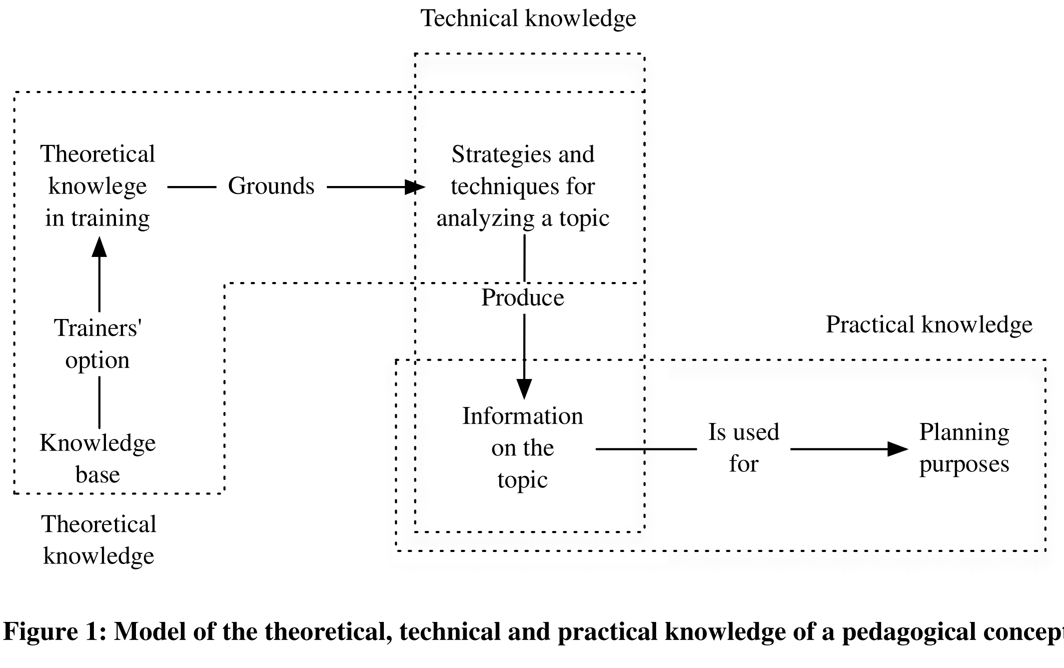

Figure 1: Model of the theoretical, technical and practical knowledge of a pedagogical concept

Example of the Three Types of Knowledge: the Learning Goal Pedagogical Concept

In this section, we illustrate the three types of knowledge proposed in the context of a mathematics teacher education methods course we have implemented. The example concerns the learning goal pedagogical concept. We first describe the course and the method we used in order to identify the three types of knowledge in the teachers' interactions.

The Teacher Education Course

A Spanish university has implemented for several years a half-year methods course in which secondary mathematics future teachers are expected to become competent in mathematics lesson planning through the learning of pedagogical concepts. In the class of the course we are going to refer to, the educator introduced the pedagogical concepts of representation system, conceptual structure, phenomenology, capacity, learning goal, error, learning path, resource, types of mathematical tasks and assessment criteria. The method used in the course was the same for each pedagogical concept: the educator started with the presentation of the theoretical option he has chosen on the pedagogical concept; then, he described with examples how a given secondary school mathematics topic could be analyzed with that pedagogical concept; next, each group of future teachers was asked to analyze its topic with that pedagogical concept. Once the information from the different pedagogical concepts of the course was produced, the groups were asked to propose and support a lesson plan for their topic. Lesson planning in the course involves delimiting the content to be worked on, establishing the learning goals to be achieved, proposing the learning tasks with which the students are expected to develop such goals, and designing instruments and procedures for assessing students' learning.

Method

We recorded 210 minutes of interactions among three future mathematics teachers participating in the course. They were third year mathematics students. They chose the topic "area of plane figures". We audiotaped their interactions while they (a) studied the theoretical information provided in the course on the pedagogical concepts of capacity, learning goal, competence, error and learning path; (b) worked on the analysis of the topic with those pedagogical concepts; and (c) designed a lesson plan for the topic for 15-years old students.

Each researcher identified, in the recordings, those sentences or groups of sentences—that we refer to as episodes—in which the group mentioned some of the pedagogical concepts provided in the course. We compared and discussed on the results of this process, and agreed on 225 episodes that we transcribed. Then, each researcher interpreted each episode by means of a qualitative-interpretative approach in order to decide if it showed some kind of evidence related to the three types of knowledge. We compared our interpretation of each episode and discussed on those on which we had differences, until we reached an agreement. This process allowed us to refine the criteria we used to interpret the episodes. We describe in detail those criteria below.

We interpret that, in an episode, there are evidences of meaning development and enactment when some of the future teachers mention the key ideas, properties and relationships that characterize the meaning of the pedagogical concept proposed in the course. This is the case when future teachers: (a) rephrase, declare, discuss or reflect on the pedagogical concept properties; (b) establish its relationships with other pedagogical concepts; or (c) distinguish instances that correspond to the pedagogical concept.

We interpret that, in an episode, there are evidences of technical or practical knowledge development and enactment when some of the future teachers mention or use the corresponding techniques introduced by the educator. We also consider the case in which future teachers propose their own techniques. Interpreting the episodes in terms of technical and practical knowledge involves specific difficulties. When teachers enact techniques, those techniques do not get necessarily registered explicitly in their oral performance. The episodes might include only the results of the use of techniques by the future teachers. In these cases, technical and practical knowledge are recognized through the product of the techniques enactment: information about the topic—technical knowledge—or planning decisions based on that information—practical knowledge—.

Based on these criteria, and after reviewing and agreeing on our interpretations, we found that 53 episodes showed evidence of the development and/or enactment of at least one of the three types of knowledge. Figure 2 shows the time distribution of the episodes (over the 210 minutes of recordings), grouped by type of knowledge (theoretical knowledge at the bottom, technical knowledge in the middle, and practical knowledge on top).

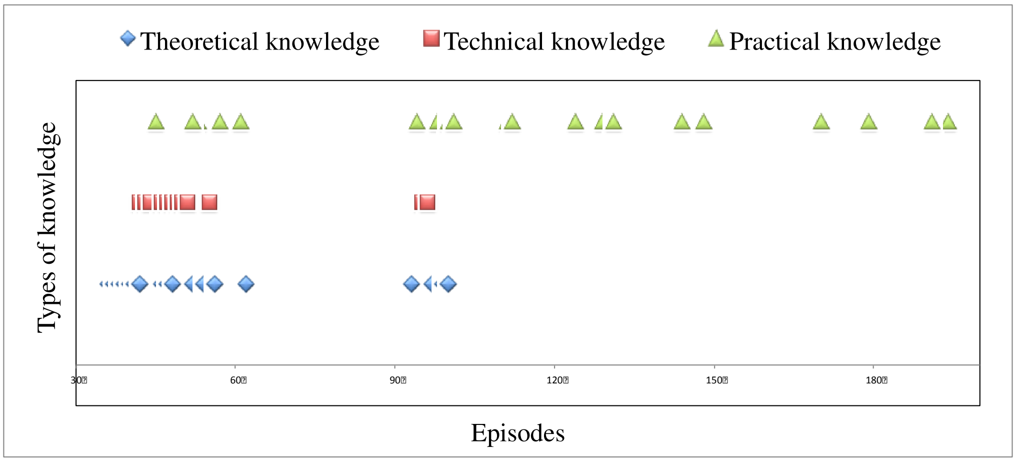

Fig. 2: Time distribution of episodes

In what follows, we show, from the 53 episodes identified, a few concrete examples of the three types of knowledge. In each example, we give details on how we interpreted the episodes.

Theoretical Knowledge of the Pedagogical Concept

In the methods course, we use the term learning goal to capture a student learning expectation related to a particular mathematical topic in a specific course. The main theoretical option, specific to mathematics education, that we, as educators, have chosen on this concept is that it has to involve "connected knowledge", as described by Mousley (2006, p. 361). For us, a learning goal expresses learning expectations in terms of (a) the connections that students make among the concepts and procedures involved; (b) the relationships that students develop between the multiple representations of those concepts; and (c) the links that students make between school concepts and the mathematical aspects of everyday contexts related to them. For instance, a learning goal for 16-year-old students concerning the topic of surface areas of two and three-dimensional shapes—that takes into account the representation systems and the links with everyday contexts— is the following.

To compute the area of figures given in real situations for which the data required in the formula are not directly known.

The learning goal pedagogical concept expresses a level of learning expectations of a mathematical lesson. In the method's course, we also use the term capacity to express the learning expectations corresponding to lower level cognitive demands. Capacities refer to knowledge of terminology, specific facts, simple procedures, algorithms and techniques and constitute the basis for the student to progress in the development of learning goals. For example, the capacities of "drawing a plane polygon from a real three dimensional situation" and “decomposing polygons in triangles” or “memorizing polygons area formulae” are necessary (but not sufficient) prerequisites to develop the previously stated learning goal.

The following episode took place while the future teachers studied the theoretical information provided in the course on the learning goal concept. In the episode, they are making an effort to reinterpret the key ideas proposed by the educator for the learning goal concept[1]. The future teachers are developing theoretical knowledge on the learning goal pedagogical concept when trying to clarify whom— students—or what—the task—is affected by this notion. Besides, they establish relationships with the pedagogical concept of capacity, and, later on, they propose an example in order to understand and distinguish some of the relationships that characterize the learning goal and capacity concepts.

Future teacher 2: OK, the learning goal is general and the capacity is individual to the student, isn’t it? I think that this is where the difference is.

Future teacher 3: The learning goal is about the task… it is not about the student.

Future teacher 2: The learning goal is what you [the teacher] want to deal with… and the capacity is your expectation about what the student can achieve.

Future teacher 1: But you have that… you have a concrete capacity and with that capacity you solve a task. This capacity corresponds to that task. And the learning goal is more general. In order to achieve the learning goal you have to connect all the capacities.

…

Future teacher 1: For our topic, a learning goal would be to be able to subdivide a surface; and a capacity would be the triangulation, the completion… wouldn’t be?

Future teacher 2: It would be like dividing the context that embrace everything and then…

Technical Knowledge of the Pedagogical Concept

The technical knowledge of the learning goal pedagogical concept consists of techniques for formulating learning goals appropriate for a given mathematical topic in a given planning situation. These techniques imply paying attention to a variety of aspects. For instance, the concepts and procedures involved in the topic have to be partitioned in content foci to which different learning goals will refer to; and for each focus, a technique is needed for deciding which representations and contexts might be included in a given learning goal. From the cognitive viewpoint, there are techniques that help distinguishing learning goals from capacities. For example, tasks used to determine if a student has developed a learning goal are complex tasks, while tasks related to capacities can be solved by routine procedures. Other techniques to formulate learning goals use the errors that students might make when solving tasks, so that learning goals centered in correcting those errors can be formulated. All these techniques can include references to grammar rules concerning, for example, the kind of verbs that better describe the learning expectations to be achieved or the appropriate length

[1] All transcriptions have been translated into English by the authors.

of a learning goal final wording. From the assessment point of view, there are techniques to determine if a learning goal corresponds to the assessment criteria that the official curriculum establishes for a topic. Those criteria are also used in techniques for verifying the coherence and completeness of a set of learning goals for a topic. All these techniques configure the kind of knowledge that we call technical knowledge of the learning goal pedagogical concept.

The following episodes correspond to the moment in which the group of future teachers wrote down the learning goals they expected students to develop on their topic. Future teachers enacted a sequence of techniques for this purpose.

The future teachers start using the technique of linking together several capacities related to the topic (making surface developments and obtaining measures) to produce a first wording of a learning goal.

Future teacher 1: Here, for instance, we can say, "Learning goal: make surface developments and obtain measures that we do not have", OK?

Next, the future teachers read an assessment criterion, showing that they used a second technique (to adapt the learning goals to the assessment criteria found in the official curriculum).

Future teacher 2: "… apply suitable formulae and develop suitable techniques and abilities for performing the proposed measuring."

And, taking into account this information, they enacted two techniques that refer to ways of writing the beginning of the phrase and to the length of the phrase itself.

Future teacher 1: The learning goal would be, for instance, let us see if we can say something like: "to develop strategies", we can start a learning goal like that, can't we? OK, "to develop strategies for calculating unknown magnitudes", this is the learning goal we want to establish [she wanted to emphasize the link between this idea and the assessment criterion].

Future teacher 2: So, we can finish the writing of the learning goal, can't we? Or we can extend it—that is what I was telling you. We can include more things in the learning goal.

Finally, they produced the final wording of the learning goal by relating it to a specific school mathematics topic.

Future teacher 1: Well this [she points to the wording they have produced for the learning goal] is perhaps too broad; it refers to many topics. OK, so we can make it more concrete. What I mean is that this learning goal can belong to many topics. Then, we can set it specifically for the area topic: "To compute the area of figures given in real situations for which the data required in the formula are not directly known."

These episodes show that the future teachers related several pieces of information that they had produced previously in order to establish a final statement of the learning goal. Several techniques appear intermingled in a sequence that portrays the development and enactment of their technical knowledge.

Practical Knowledge of the Pedagogical Concept

The practical knowledge of the learning goal pedagogical concept implies a set of techniques for using the list of learning goals that a specific group of students is expected to achieve. This list can be used in different scopes, with different purposes and in conjunction with the information produced with other pedagogical concepts. For instance, it can be used for analysing and selecting the mathematical tasks that configure the lesson plan; establishing the assessment criteria of the lesson; and establishing levels of achievement of those learning goals and identifying tasks that students are expected to solve for each level. The analysis and selection of the tasks that configure the lesson plan requires a technique for establishing to what extent the tasks contribute to the learning goals' development. This technique implies establishing a list of capacities associated to each learning goal and analysing whether a task that is supposed to contribute to a learning goal activates the corresponding capacities.

Once the future teachers established a final wording for the learning goals, they enacted some techniques for selecting tasks that could promote their development. Throughout the course, the future teachers collected several mathematical tasks on the area topic from different sources. For instance, they found tasks that promoted the connection between several representations of the topic or collected various modeling tasks. They assigned a label to each task—e.g. "the goat problem", "the perimeter problem" or "the cone problem"—. For producing the final selection of tasks, they took this collection as reference and used several techniques to arrive to their final decision.

For example, in the following episode, future teachers are taking into account two aspects of the learning goal: whether the data given is the data required for using the area formula ("the one that we develop and the center is given", "the goat problem, in which we were giving the circumference radius"), and whether the task relates to a real situation, which is the implicit reason they are using for rejecting the perimeter problem.

Future teacher 1: This [learning goal] corresponds exactly to the cone problem, the one that we develop and the center is given.

Future teacher 2: Ah, and also the goat problem, in which we were giving the circumference radius...

Future teacher 3: Do we have to choose only one problem per learning goal? In this case, I prefer the perimeter problem.

Future teacher 2: The perimeter problem is great. But it does not refer to this learning goal.

Next, they solve the goat problem as if they were students, and determine whether the capacities linked to the learning goal are triggered (identify the surface, draw it, locate the data in the drawing, use the properties of the hexagon). During this process, they realize that the area could be calculated using the properties of the hexagon, without making a surface partition. But they seemed very interested in promoting the learning of surface partitions as part of their interpretation of the learning goal. Thus, they found a new wording of the problem that is better adapted to this purpose.

Future teacher 1: Ah! Look, and they have to use the properties of the hexagon, because we have said that the circumference has radius r. That is, before locating the data [in the drawing], they have to know the properties of the hexagon.

<u>Future teacher 2:</u> But, imagine that instead of a hexagon, you have a heptagon. Then, you would have to make a partition… If we want them [the students] to use this property [the surface partition], then we do not ask about the hexagon. We ask about the heptagon.

In these episodes, the future teachers foresee the ways in which the tasks can be solved, and select and modify one of them, so that it suits their interpretation of the learning goal. The modification of a task in order to adapt it to a learning goal is one of the clearest indications of the development and/or enactment of the practical knowledge of this pedagogical concept.

Discussion

In this paper, we have described three types of knowledge of a pedagogical concept—theoretical, technical and practical—that can be developed and enacted in education programs that expect teachers to become competent in lesson planning. We have illustrated them with episodes from a specific mathematics teacher education methods course. In this final section, we reflect on and identify some questions concerning the complexity of teachers' learning processes from this perspective. Finally, we suggest that the model and the results that emerge from it can be useful for the design and development of teacher education programs.

We have shown that our knowledge model is useful for interpreting concrete teachers' performances when they analyse a mathematical topic with a specific pedagogical concept. These interpretations provide "snapshots" of how teachers develop and/or enact their knowledge of the pedagogical concept at a given moment in time. The time-ordered sequence of these snapshots can be used for describing how their knowledge evolves over a given period of time. Therefore, one can characterize the teachers' learning processes and answer the "how" question that we mentioned at the beginning of this paper. We have explored elsewhere this type of research question and found some unexpected results (Gómez, González, Rico, & Lupiáñez, 2008). Since it is natural to think that in order to use a concept, one has to know it, one could postulate a standard learning process of a pedagogical concept in which developing the theoretical knowledge of the concept should precede the technical knowledge development and this knowledge should come before the practical one. But, in the preliminary analyses that we have performed, we have found that this standard process seldom appears in practice. The development of theoretical, technical and practical knowledge appears as interconnected events in the teachers' learning process of a pedagogical concept. For example, in the learning goal case, we have found that most of the episodes in which the future teachers enacted theoretical knowledge were interconnected with episodes of technical knowledge enactment or development. For instance, when teachers were trying to formulate learning goals on the surface area topic, they were not sure whether a statement was a learning goal or not. Hence, they returned to clarify the meaning of the learning goal concept given in the course. Sometimes, they proposed two versions of a learning goal for the topic at hand and returned to determine which of them was aligned to the meaning given by instruction. Thus, the information that emerged from enacting their technical knowledge led them to review and improve their theoretical knowledge. Similarly, there were moments in their learning process when the information that emerged from enacting their practical knowledge led them to modify their theoretical knowledge of the pedagogical concept. For example, they enacted the practical knowledge of the learning goal concept with the purpose of selecting a task by assessing whether the sequence of capacities that are put into play by the task corresponded to the learning goal. But they asked themselves how many tasks were necessary in order to develop a learning goal. Reflecting on the number of tasks associated to a learning goal, future teachers were questioning their interpretation on the significance and effect of the learning goal in a planning sequence. Therefore, they were returning to develop theoretical knowledge on the learning goal pedagogical concept. These examples indicate that learning a pedagogical concept over a period of time is a dynamic process in which teachers develop, in a simultaneous and interconnected manner, its theoretical, technical and practical knowledge. Our initial explorations suggests that each pedagogical concept may impose particular conditions that shape how it is learned, and those conditions may imply certain learning patterns—in terms of the three types of knowledge—that most teachers could follow.

Besides the issues involved in learning a specific pedagogical concept in relation to its three types of knowledge, developing lesson-planning competencies involves processes of integrating and coordinating several pedagogical concepts. This coordination has two sources. The first one involves the descriptions of the three types of knowledge that we have presented. Those descriptions comprise links among the pedagogical concepts. We have seen that the meaning of a pedagogical concept (theoretical knowledge) usually involves the meaning of other concepts, as we have shown for the learning goal case and its relations with the capacity concept. We have also seen that the information that emerges from enacting the technical knowledge of a pedagogical concept can be used to produce information with another pedagogical concept; that is, the technical knowledge of pedagogical concept B involves the practical knowledge of pedagogical concept A (see Fig. 3). This is, for instance, the case when teachers use students' frequent errors on a topic for establishing the topic's learning goals. The second source is related to the task analysis and selection that teachers perform towards the end of their education, after having analysed the topic with the pedagogical concepts. This process requires teachers to put together and relate the information they have gathered using all the pedagogical concepts. How to conceptualize the coordination and integration of several pedagogical concepts is an open question that we are beginning to explore.

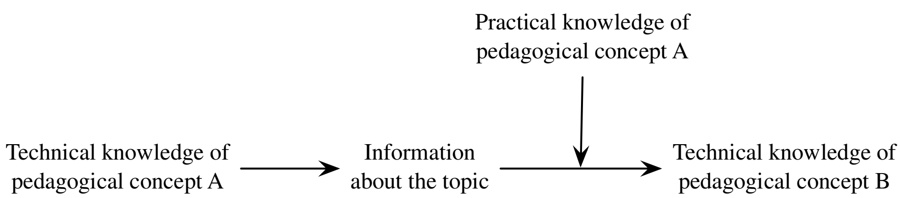

Fig. 3: Relation between practical and technical knowledge

The model is also useful for designing and developing teacher education programs that expect teachers to learn and use pedagogical concepts. The model organizes the teaching and learning of pedagogical concepts on three features of the programs: (a) it focus the teaching and learning of theory on the key ideas that characterize the theoretical knowledge of a pedagogical concept; (b) it highlights the importance of teaching and learning the techniques—emerging from those key ideas—that provide relevant information about the school topics; and (c) it calls attention to the use of such information in lesson planning. Focusing on the key ideas that characterize the theoretical knowledge and the techniques that configure the technical and practical knowledge of pedagogical concepts, without the need to make explicit these three types of knowledge to future teachers, teacher educators can improve the design and implementation of their programs and of the teaching and learning activities that configure them. Furthermore, the model provides teacher educators with a scheme for assessing teachers' performance and determining the effectiveness of instruction.

We have put into play these features in mathematics teacher education programs in which we are involved in Colombia and Spain (Gómez & González, 2013). Even though these programs involve different profiles of teachers, are set up in different educational systems and cultures, and focus on different types of education (pre-service and in-service), the model has been useful in both cases. It has leaded us to change several aspects of our approach to the design and implementation of the programs. For instance, when planning our teacher education lessons, we analyse the pedagogical concept in order to identify those key ideas that establish the main techniques that characterize its technical knowledge. We present those ideas and techniques in class and define precisely the work that we expect the groups of teachers to produce. We give importance to the practical knowledge of each concept. When interacting with the groups of teachers while they analyse their topic, we induce them to develop their theoretical, technical and practical knowledge in an interconnected way, asking them to review systematically their previous work every time they finish the analysis of their topic with a pedagogical concept. Informal comparison of the future teachers' work along the years suggests that these changes have been fruitful. They indicate that the model and the research results that emerge from it can be useful in the design and development of teacher education programs of this type.

References

Adler, J., Ball, Deborah, Krainer, Konrad, Lin, Fou Lai, & Novotna, Jarmila. (2005). Reflections on an emerging field: researching mathematics teacher education. *Educational Studies in Mathematics, 60*(3), 359-381.

Aristotle. (350 BC/1994). *Nicomachean Ethics* (W. D. Ross, Trans.). Cambridge, MA: Harvard University Press.

Back, S. (2002). The Aristotelian challenge to teacher education. *History of Intellectual Culture, 2*(1), 2-4.

Beijaard, Douwe, Korthagen, Fred, & Verloop, Nico. (2007). Understanding how teachers learn as a prerequisite for promoting teacher learning. *Teachers and Teaching, 13*(2), 105 - 108.

Borasi, Raffaella. (1996). *Reconceiving mathematics instruction: A focus on errors*. Norwood, NJ: Ablex Publishing Corporation.

Borko, H. (2004). Professional development and teacher learning: Mapping the terrain. *Educational Researcher, 33*(8), 3-15.

Buitink, Jaap. (2009). What and how do student teachers learn during school-based teacher education. *Teaching and Teacher Education, 25*(1), 118-127.

Carlgren, I. (1999). Professionalism and teachers as designers. *Journal of Curriculum Studies, 31*(1), 43-56.

Carter, K. (1990). Teachers' knowledge and learning to teach. In W. R. Houston (Ed.), *Handbook of research on teacher education* (pp. 291-310). New York: MacMillan.

Cavanagh, M. S., & Garvey, T. (2012). A professional experience learning community for pre-service secondary mathematics teachers. *Australian Journal of Teacher Education, 37*(12), 57-65.

Charalambous, C. Y. (2008). Mathematical knowledge for teaching and the unfolding of tasks in mathematics lessons: Integrating two lines of research. In O. Figueras, J. L. Cortina, S. Alatorre, T. Rojano & A. Sepúlveda (Eds.), *International Group for the Psychology of Mathematics Education* (pp. 281-288). Morelia, México: PME.

Flyvbjerg, B. (2006). Making organization research matter: power, values, and phronesis. In S. R. Clegg, C. Hardy, T. B. Lawrence & W. R. Nord (Eds.), *The Sage handbook of organization studies* (2nd edition ed., pp. 370- 387). Thousand Oaks, CA: Sage.

Gess-Newsome, J, & Lederman, N. G. (Eds.). (2001). *Examining Pedagogical Content Knowledge. The Construct and its Implications for Science Education*. Dordrecht: Kluwer.

Gómez, P., & González, M. J. (2013). Diseño de planes de formación de profesores de matemáticas basados en el análisis didáctico. In L. Rico, J. L. Lupiañez & M. Molina (Eds.), *Análisis didáctico en Educación Matemática. Formación de profesores, innovación curricular y metodología de investigación* (pp. 121-139). Granada: Comares.

Gómez, P., González, M. J., Rico, L., & Lupiáñez, J. L. (2008). Learning the notion of learning goal in an initial functional training program. In O. Figueras, J. L. Cortina, S. Alatorre, T. Rojano & A. Sepúlveda (Eds.), *Joint Meeting of the International Group for the Psychology of Mathematics Education (IGPME 32) and North American Chapter (PME-NA XXX)* (Vol. 3, pp. 81-88). Morelia: Cinvestav-UMSNH.

Hill, H. C. (2011). The nature and effects of middle school mathematics teacher learning experiences. *Teachers College Record, 113*(1), 205-234.

Hill, H. C., Ball, D. L., & Schilling, S. G. (2008). Unpacking pedagogical content knowledge: Conceptualizing and measuring teachers' topic-specific knowledge of students. *Journal For Research in Mathematics Education, 39*(4), 372-400.

Hurrell, D. P. (2013). What teachers need to know to teach mathematics: an argument for a reconceptualised model. *Australian Journal of Teacher Education, 38*(11), 54-64. doi: 10.14221/ajte.2013v38n11.3

John, Peter D. (2006). Lesson planning and the student teacher: re-thinking the dominant model. *Journal of Curriculum Studies, 38*(4), 483 - 498.

Kinsella, E. A., & Pitman, A. (Eds.). (2012). *Phronesis as professional knowledge. Practical wisdom in the professions*. Rotterdam, Holanda: Sense Publishers.

Liljedahl, P., Chernoff, E., & Zazkis, R. (2007). Interweaving mathematics and pedagogy in task design: a tale of one task. *Journal of Mathematics Teacher Education, 10*(4-6), 239-249.

Mezirow, Jack. (1981). A critical theory of adult learning and education. *Adult Education Quarterly, 32*(1), 3-24. doi: 10.1177/074171368103200101

Orton, Robert E. (1997). Toward an Aristotelian model of teacher reasoning. *Journal of Curriculum Studies, 29*(5), 569-584.

Putnam, R. P., & Borko, H. (2000). What do new views of knowledge and thinking have to say about research on teacher learning? *Educational Researcher, 29*(1), 4-15.

Rico, L., Castro, E., Castro, E., Coriat, M., Marín, A., Puig, L., . . . Socas, M. (1997). *La Educación Matemática en la enseñanza secundaria*. Barcelona: ice - Horsori.

Rowland, T., Huckstep, P., & Thwaites, A. (2005). Elementary teachers' mathematics subject knowledge: The knowledge quartet and the case of Naomi. *Journal of Mathematics Teacher Education, 8*(3), 255-281.

Sánchez, Mario. (2011). A review of research trends in mathematics teacher education. *PNA, 5*(4), 129-145.

Saugstad, T. (2005). Aristotle's contribution to scholastic and non-scholastic learning theories. *Pedagogy, Culture & Society, 13*(3), 347-366.

Sherin, Miriam Gamoran, & Drake, Corey. (2009). Curriculum strategy framework: investigating patterns in teachers' use of a reform-based elementary mathematics curriculum. *Journal of Curriculum Studies, 41*(4), 467-500.

Shulman, L.S. (1986). Those who understand: knowledge growth in teaching. *Educational Researcher, 15*(2), 4-14.

Shulman, L.S. (1987). Knowledge and teaching: foundations of the new reform. *Harvard Educational Review, 57*(1), 1-22.

Simon, M. (2008). The challenge of mathematics teacher education in an era of mathematics education reform. In B. Jaworski & T. Wood (Eds.), *The International Handbook of Mathematics Teacher Education* (Vol. 4, pp. 17-29). Rotterdam: Sense Publisers.

Sternberg, Robert J. (1985). Implicit theories of intelligence, creativity, and wisdom. *Journal of personality and social psychology, 49*(3), 607-627. doi: doi:10.1037/0022-3514.49.3.607

Xu, Yuzhen. (2009). School-based teacher development through a school-university collaborative project: a case study of a recent initiative in China. *Journal of Curriculum Studies, 41*(1), 49-66.

Zazkis, R., Liljedahl, P., & Sinclair, N. (2009). Lesson plays: Planning teaching versus teaching planning. *For the Learning of Mathematics, 29*(1), 40-47.

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