Generalization in Chess Thinking

Giuliano D'Eredità and Mario Ferro

In this work we deal with generalization in chess thinking. Generalization is a complex process based on information people acquired during previous experiences. In the field of chess, chess books, chess education and personal game practice supply the information for generalization to occur. The way in which generalization is performed in chess is still a topic that deserves more research. In this article we dwell on early theories about chess thinking. We underline the role played by what we call configural concepts, in which geometrical patterns and logical expected developments coexist. We suggest that the idea of configural concepts, along with generalization and abduction constitute the basis of chess thinking.

Keywords: Abduction; Chess thinking; Configural concepts; Generalization; Strategy; Tactics

Generalización en el pensamiento al jugar ajedrez

En este trabajo abordamos la generalización en el pensamiento en el juego del ajedrez. La generalización es un complejo proceso basado en información adquirida durante experiencias previas. En el campo del ajedrez, los libros de ajedrez, la educación en el ajedrez y la práctica personal con el juego aportan la información que posibilita la generalización. La forma en que la generalización se produce en el ajedrez es todavía un tema que merece más investigación. En este artículo consideramos teorías tempranas sobre el pensamiento en el ajedrez. Subrayamos el papel que juegan lo que llamamos conceptos configurales, en los cuales coexisten los patrones geométricos y desarrollos lógicos esperados. Sugerimos que la idea de conceptos configurales, junto con la generalización y la abducción constituyen la base del pensamiento en el ajedrez.

Términos clave: Abducción; Conceptos configurales; Estrategia; Generalización; Pensamiento en el ajedrez; Táctica

Psychologists have very often stressed the role of perception in chess. Perception, which we discuss in the first section of this article, plays a fundamental role in strategy and tactics, both considered the main types of chess thinking. In both, strategy and tactics, generalization is fundamental because the next moves the player has to play are frequently chosen from previous knowledge. The player has to generalize previous knowledge and adapt it to the context at hand. Such endeavour implies a complete or partial pattern recognition. The pattern has not to be considered merely something static. It is rather a dynamic aspect linked to expected developments in a game. It is this dynamic aspect of chess thinking that we try to capture through the idea of configural concepts in the second section of this article. The dynamic aspects involve specific, technical chess concepts the player acquires in the course of time, but of course subjective beliefs and personal experiences are relevant too.

THEORETICAL ASPECTS OF CHESS THINKING AND THE ROLE OF PERCEPTION

Chess was called the *drosophila* of the Psychology, because of its specific characteristics, especially the rules worldwide accepted and a sharp universal rating of expertise, called the ELO system. A fundamental study was carried out by Adrian De Groot in 1965. He was the first scholar to carry out an experimentally based psychological analysis of chess thinking (De Groot, 1965). An important topic in his study dealt with the ability to recall a position. Masters performed significantly better only if positions made sense, that is, if they represented a chess position that could occur in a standard game. When pieces were set randomly there was no difference between masters and beginners. There is a semantic or meaning field that organizes recollection. De Groot understood the role of perceiving complex of pieces, a key item that was studied by important scholars in the subsequent decades. Another aspect investigated by De Groot was the structure of chess thought by searching for the features underlying skills and talent. Somewhat surprisingly, chess masters did not show a clear superiority in depth analysis, or in the number of variations analyzed. More recent studies show that experts analyze slightly deeper than non experts (Gobet, 1998, p. 28), indicating that depth analysis is not the characteristic of skilled chess players.

Chess skill reveals in early finding/troubleshooting and in selecting the right variations to analyze. It recalls the way minimax and alfabeta algorithms proceed. This finding and selection are enhanced by players' chess knowledge.

De Groot identified the critical role of perception, which allows quick access to information stored in long-term memory (Gobet & Campitelli, 2002, p. 106). Perception in chess seems driven by expertise. We know anyway that similar results about perception were also found in different disciplines.

More in general, protocols in De Groot's study show some typical features in chess thinking, summarized as follows (Di Sario, 2002, p. 28).

- Identification of the problem(s).
- Various levels of depth, according to a classical tree-structure.

- ♦ Players always look for a subjective reason of choice.
- ♦ Players apply a continuous feedback mechanism (goal feedback).
- ♦ The feedback mechanism may cause a radical change of the problem itself, and of the ways to tackle it.

It is at the third bullet that forms of generalization appear, as we will develop in the following.

Another milestone in understanding chess thinking was set by Chase and Simon (1973a, p. 80), proposing their famous chunking theory to explain De Groot's results, also using Jongman's results on chess players' ocular movements (Jongman, 1968, p. 187). The chunking theory is a model of chess players' perception. The chunk is a typical piece ensemble, immediately recognized by the skilled player, and perceived as a whole. In the Figure 1 an example of chunk is shown: the typical King's *fianchetto* (white pieces in the bottom right corner).

Figure 1. Example of chunk

For the skilled chess player, the white pieces' ensemble occupies an elementary unit of memory. Then the player skimps on his or her own memory space. It does not happen to beginner chess players; they use more bits of memory to recall the chunk. Chunking is a cognitive resource for chess skill, in particular in tasks of recalling a position.

This is the first concept for understanding the processes of pattern recognition in chess. Pattern recognition is considered the most important cognitive resource, as confirmed in almost all relevant chess studies.

It is clear again that chunking is strictly connected with knowledge and practice, otherwise test results about recalling a position would be hard to explain. Chunks are supposed to be stored in long-term memory.

> *Through years of practice and study, masters have learnt several hundred thousands of perceptual patterns, which, once recognized in a particular position, give rapid access to information such as potential moves or move sequences, tactics, strategies, and so on. Simon and his colleagues proposed that pattern recognition explains a number of important phenomena, such as highly selective search (even chess grandmasters rarely search through more than one hundred moves before selecting a move), automatic and "intuitive" discovery of good moves, and extraordinary memory for game-like chess positions. Chase and Simon (1973a, p. 56) suggested that at least ten years of practice and study were necessary to acquire the minimum knowledge required to become a grandmaster.* (Gobet & Campitelli, 2002, p. 106)

Chunks are surely a form of generalization, a visual generalization.

Another, very interesting, theoretical item proposed by Chase and Simon (1973b) is the so-called mind's eye. In a nutshell, mind's eye is a model of chess problem solving. The player solves the problems making in mind visuo-spatial operations. It's an active mental imagery. In fact players often use the typical sentence "I saw it", meaning the discovery of a move. However, they do not refer to a real visual operation, but to a kind of perception similar to the one of the mathematician who, walking after hours of work on a problem, suddenly "sees" the solution. By the mind's eye theory, chess players solve problems in a visual-perceptive way. Is it a rational, deductive approach? We believe that it is, but certainly not a classical one. The mind's eye is a very interesting theoretical structure with potential links with other disciplines, including mathematics, and is useful to better understand chess thinking.

Coming back to the basic structure of chess thinking as summarized by De Groot, skilled players use to select the more plausible options, and then analyze them trough and trough. It is a deep visit of the tree, performed in a classical, deductive manner. This way of operating, which we believe is reasonable, is supported by players' reports after tournament games and by tests from the most important and classical chess books. Moreover, chess players are able to verbally reconstruct their thoughts, even if their reports are sometimes not so clear as their moves on the board. This aspect is highlighted also by Montero and Evans (2011, p. 187), in arguing versus Herbert Dreyfus' theory of expertise, that expert action generally occurs automatically and unreflectively.

Dreyfus (2005, p. 25) argues that experts' actions are based on intuitions and, although proceeding in a very effective way, are taken without full consciousness of deliberation, as a pilot driving a race car or someone`s climbing his/her home's stairs. The argument is subtle because the great players' moves seem often part of an automatic, spontaneous flow, especially during rapid games, albeit this is true only in some phases of the game.

On the other hand, in different phases of the game, a logic, deliberate, and sometimes very deep analysis occurs in tournament practice, as highlighted by

Montero and Evans quoting Larry Evans' report of one of his own games. The Evans report is logic, verbal and, indeed, reconstructs apparently correctly his thoughts during the game, showing a full awareness and not any kind of "automatic pilot". We concur with Montero and Evans' opinion. It does not seem appropriate to reduce chess reasoning to a sort of automatic retrieval and application of information. The player recognizes configurations and considers his/her experiences, but the analysis proceeds in an analytical way, and only eventually heuristic and synthetic considerations are made (D'Eredità, 2012, p. 54). Just in specific standardized positions, playing is somewhat automatic. Several generalized concepts concur in the player's mind.

Within the psychological theoretical framework of Gestalt, humans' perception of reality is considered to be not a sensorial mosaic, but a synthesis, a structural unit. This form, the Gestalt, may be the main human model of reality. Gestalt is depicted by perception, and learning consists in the perception of a situation, recognition of its troubling aspects, and its solution obtained by perceptive insight (D'Eredità, 2012, p. 55).

Now, knowledge and culture do not come after perception, intervening a posteriori, but in our opinion they interact at the same time, in a complex and dynamic occurrence. Indeed, "rather than being a purely biological act, human perception is a social process through and through. It is a cultural artifact shaped by our own historically changing practices" (Wartofsky, 1984, p. 865).

As mentioned in the introduction, usually, chess thinking is roughly divided in strategic and tactical thinking. One strategy is the formulation, even in an implicit form, of a plan of action which, even on a long term basis, is taken as a reference for the coordination of the activities addressed to the reaching of a predetermined goal. The word strategy is derived from ancient Greek and meant general (*στρατηγός*). The first necessary option is precisely the determination of the goals, that is, the explicit identification of the objectives on the basis of an evaluation of the situation. Strategy in chess is based on experiences, knowledge and beliefs, and strongly influences the evaluation, i.e. the choice of the next move(s).

Tactics comprise the methods used for achieving established objectives. Tactics are the means, real or logical, used to obtain a goal, be it partial or total (D'Eredità & Spagnolo, 2009, p. 265). A tactical operation has the goal of realising a single action within the strategy, or also for gathering possibilities offered by an adversary or from the physical or logical environment in which it is found. A famous Tartakower[1] motto is "Tactics is knowing what to do when you have something to do; strategy is knowing what to do when you don't have anything to do" (Schoenberg, 1975, p. 127). Tactics in chess means concrete sequences of moves.

An advantageous tactical operation, which stands out can convince one to decisively change his formulated strategic plans if necessary.

[1] Savielly Tartakower (1887-1956) was one of the strongest chess player of all time. Although he never earned the title of World Champion, he is remembered for his chess studies and publications.

The evaluation phase is perhaps the most delicate and depends on the data at hand, on the decision-making ability of the subject in terms of aptitude, experience, and knowledge. Here, abduction and generalization are present. The player recognizes in the position he is facing some basic details that allow him to link or insert the position itself in a sort of known class. This is abduction. The appearance of known patterns, both visual and logical, helps lead the player in this kind of process. As a consequence of this abduction, the choice of the next move(s) depends greatly on the behaviour of that class, i.e. on its known outcomes. This is generalization. In fact, the player relies on these behaviours, and on how much effective is the generalization he is concerned with.

In fact, frequently in chess, beliefs change, often as a consequence of a complete or partial failure of a generalization. We must walk a thin line between flexibility and consistency (Kasparov, 2007, p. 38). Furthermore, it is not trivial to emphasize that the abduction processes mentioned above may be multiple, i.e. a position could be considered as belonging to various of the so called classes..., therefore the consequent behavior it is not so easy to forecast. Chess is not a simple game. These mechanisms will be discussed in the following paragraphs.

Generalization and Configural Concepts

From an epistemological point of view, the cognitive functions involved in chess visual processes are somewhat different from the geometrical ones. The concepts are not the same in those fields. Although in a given chess position all the visual information is directly accessible, the visual information conceals conceptual relationships that have been culturally created and refined by generations of individuals over years of chess practice.

Among these conceptual relationships, we are interested in investigating those characterized by spatial and conceptual relationships between particular pieces, with other pieces on the chessboard, and the chessboard itself. It is this idea that the concept of configural concept tries to capture.

A configural concept is made up of chess objects and their conceptual relationships. Its meaning comes from the hierarchical linkage of the conceptual relationships between the involved chess objects and from its position in the whole theoretical structure of the pieces in the chessboard (Ferro, 2012, p. 15).

The use of a configural concept depends on the goal that an individual is pursuing in a chess game. It involves the identification of general structure of the game at a certain moment and the role of the configural concept therein. In terms of learning, configural concepts become noticed and valued through a process in which the student becomes conscious of the chess objects and their conceptual mutual relations (Ferro, 2013, p. 5). This process is what Radford calls objectification (Radford, 2010a, p. 3).

In this regard, one of the fundamental skills developed by chess players is to configure and reconfigure chess objects in order to invoke different configural concepts and then produce different evaluations of a position. It is a process of continuous construction and deconstruction (Di Sario, 2002, p. 35).

So chess players improve the ability of disclosing the correct and useful configural concepts and manipulating them with the purpose of achieving a goal.

Due to the fact that the meaning of a configural concept depends strongly on figural and conceptual relationships between chess object involved in it and from its position in the whole theoretical structure of the pieces in the chessboard, the generalization of a configural concept occurs through two different processes.

- The first one—we will call it the *productive process*—consists of processes of induction in which the individual observes and describes the communal and distinctive characters of particular positions, extending them to a representative pattern.
- The second one—we will call it the *decisional process*—consists of processes of deduction in which the player makes decision in the game and evaluate the positions, in this way he/she gain awareness about the configural concept in the whole theoretical structure of the game.

To explain these concepts we will analyze the example in the position in Figure 2 (the *Réti endgame*), in which white seems to have no chance to draw the match because its king is too "far away" to help promoting its pawn or to stop the opponent one. On the contrary, by passing the squares g7, f6, e5, the king could draw the game promoting its pawn or taking the opponent pawn.

To solve correctly this position, the players must recognize and handle the configural concept that synthesizes the idea of distance in this kind of chess endgames: the square rule. Its use, supported by a strongly figural sense (the square shape), gives to the square rule a hierarchical position on the visualization processes in this kind of endgames. In this case, the decisional process consists of the deductive processes in which the player associate square rule in a precise moment of the game, to a result: win, lose, draw, good, bad, uncertain. These processes are mostly linguistic, and during the processes of generalization the statements move from personal ones like "I didn't like to play these positions" or "I didn't understand these positions", to the categorial ones like "this game is lost/won".

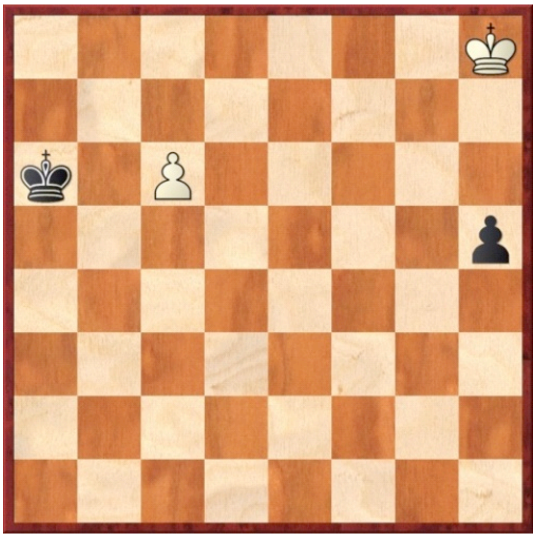

Figure 2. The Réti endgame show the diagonal idea and the square rule in the endgame of king and pawns

Let's study the examples in Figure 3 to better understand the idea of productive processes in generalization. In those endgames (white to move) the black king cannot prevent the promotion of the white pawn because the king is too far away from the promotion square. However, this aspect could be hidden by other figural aspects.

In the first position, the black king is behind the white pawn and in the second position the white King *obstructs* the passage of black King. Thus, the chess player, playing, studying or simply observing these kinds of endgames, will compare them generalizing the configural concept (correctly or not).

Let's try to show how a chess player could become aware of the configural concept of square rule. By definition, the square rule is based on the idea of distance —an idea that in chess is different from the idea of distance of Euclidean geometry. The endgame in Figure 2 is an example in which the figural aspect of distance in chess generates a problematic situation.

As we shown in Figures 3a and 3b the chess player can initially think that the king can't reach the enemy pawn because it is behind the pawn, or obstructed by the other king, referring thereby to relationships between pieces.

As we have had the opportunity to observe again and again in our work as chess teachers, in the course of chess practice the novice chess player may become progressively aware of the fact that there are also other aspects to be noticed other than distances. This awareness occurs in what Radford (2002, p. 14) calls a process of objectification. As a result of it, the chess player may change what he/she is looking for: from relationships between pieces the player may also include in his/her analysis relationships involving pieces and the chessboard. In this way he/she will compare distance (chess distance) between pieces and zones of the chessboard and he/she will endows these zones with conceptual considerations (in this case a square with the enemy pawn as vertex).

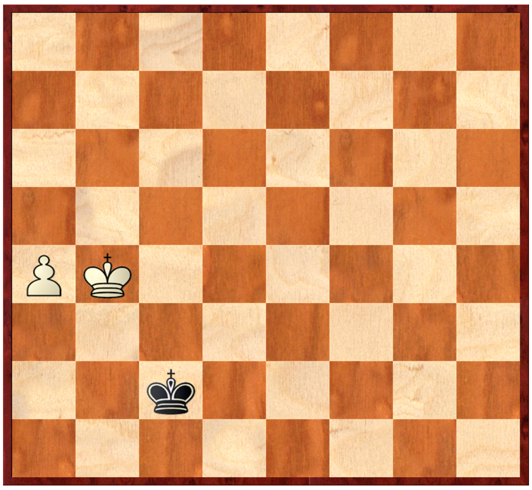

Figure 3a

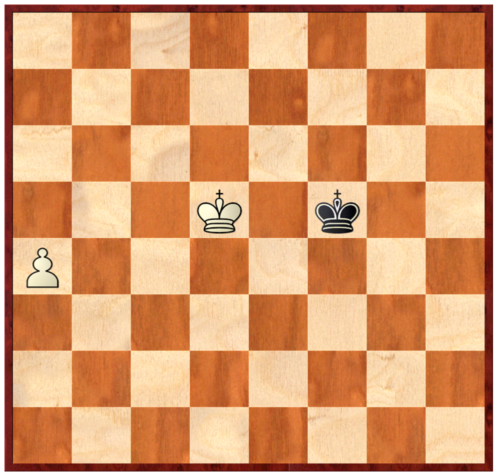

Figure 3b

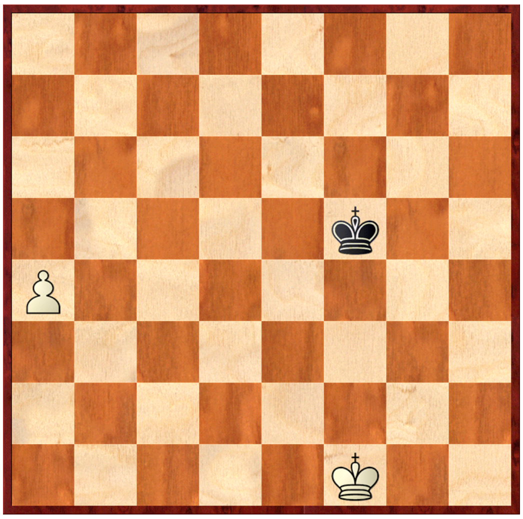

Figure 3c

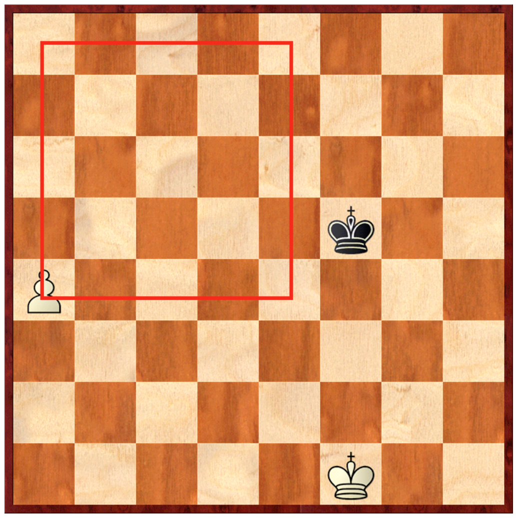

Figure 3d

Figure 3. Four endgames with king and pawn vs king and the square rule

To investigate this phenomenon in chess teaching and learning, we study the evolution of particular semiotic means of objectification (e.g., words, gestures, perception). We observe that in gaining awareness about a chess object, the students change or refine the use of the semiotic means of objectification.

> *[…] in lower level of awareness the student use the gesture to point squares, to keep in hand pieces or to tap them over the chessboard. These gestures (in particular the pointing gestures) were modified (or simply contracted) into his eyes action. When he achieved highest level of awareness he moved his eyes and his head to individuate the squares on the chessboard without using gestures. […] By the way these eyes' motions were not "alone", they were*

coordinated with the language that the student improved in "calling" the columns or the squares. (Ferro, 2013, pp. 104-105)

Furthermore, the chess player can be aware of the square rule if he/she knows the promotion rule (the pawn in last row can become rook, knight, bishop or queen) and knowing the elementary endgames (endgames involving king vs king and piece). Through these processes, the chess player can recognize and evaluate the square rule in the position, and so become aware of it.

Notice that the idea of promotion has not been always the same and, as a result of its historical transformation, it has changed the idea of the square rule too. For example in the 18th century the Pawn could only be promoted into the piece of the column on which it is promoted or on which it started. So in the 18th century, if the Pawn was in b-file, the promotion of the pawn did not ensure the victory of the game because the endgame king and knight vs king is a draw.

INDUCTION AND DEDUCTION

As we said before, generalization in chess involves induction and deduction. Chess players apply deduction when using a specific rule to make a choice in a given position (e.g., in endgames with king and pawn vs king, if the pawn reaches the promotion square, the player can use the specific rule "king and queen or king and rook do win against the king alone", to decide to promote the pawn into a queen or a Rook). Chess player uses induction when starting from a position (e.g., black in Figure 3a) and observing an outcome (the king cannot stop the promotion of the pawn, losing the game), he/she produces and generalizes a rule, in this case "when the king is behind the enemy pawn, the pawn cannot be blocked". So we can say that *decisional generalization* involves deductive inferences, and productive generalization involves inductive inferences.

A chess player uses abduction when starting from an unexpected enemy move (e.g., a piece sacrifice) he/she supposes some rules that could justify that move, trying to see the same position through the opponent's eyes. In fact, the move in a chess game could be seen as a hypothesis "I believe that this is the best move in this position and against this opponent", but to prove it, he/she has to wait until the opponent's move to confirm or invalidate the hypothesis.

Chess players need to generalize and to evoke configural concepts to understand a position and make a choice. It forces chess players to make continuous deductions, inductions and abductions.

The concepts he/she evokes are strongly depending on culture in general, in chess culture in particular. In fact chess knowledge is continuously changing and, as a consequence of it, the assessment of a given position or opening may change in the course of time. Obviously, in each step of the game, the chess player does not describe all the conceptual relationships that may arise. However, these relationships are embodied in figural patterns, some of which become more salient than others in the processes of chess knowledge objectification. These processes may generate some paradoxes: an expert chess player often knows the correct move in some position, but he/she doesn't know why it is the best move. In a specific example, an endgame with king and pawn vs king, the expert chess player motivates a move talking about spatial advantage that is totally wrong without a pawn structure (Ferro, 2012, p. 23). Just analyzing the position more in detail she/he very probably will change the move.

EVALUATION AND GENERALIZATION IN CHESS: STUDY OF AN EXAMPLE

In this paragraph we want to discuss the relationships among configural concepts, generalization, tactics and strategy. We will analyze the example shown in Figure 4 that represents a position played in the game Stein-Bronstein (Dvoretsky, 1991, p. 82).

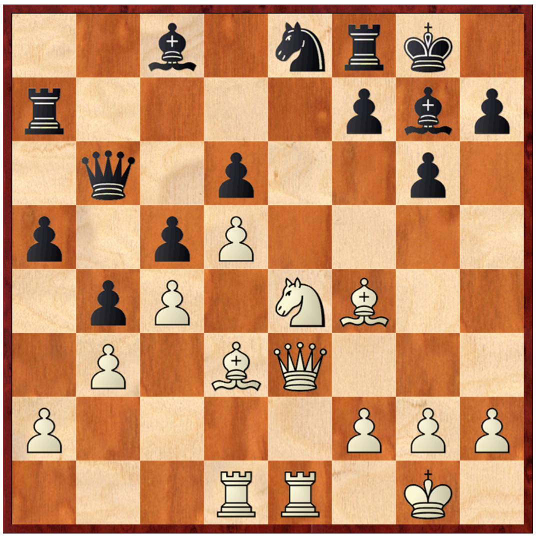

Figure 4. Stein-Bronstein, Moscow 1971

How does a chess player evaluate this position? How does he/she find the moves and/or the best variation?

To find these answers we will use separately the tactical and strategically approach. To use a tactical approach means to look at some chess objects (pieces and squares), match them to one or more configural concepts and then calculate if they could be useful in this concrete case. Referring to the position in Figure 4 we consider the pattern in Figure 5.

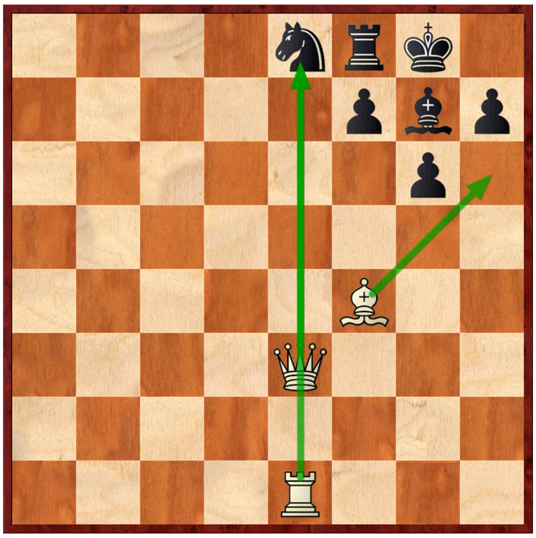

Figure 5. A pattern extracted from Figure 4

This pattern involves some fundamental elements:

1. Black has castled with fianchetto (pawns f7, g6 and h7, and bishop in g7)
2. White has some major pieces (queen or rook) in an opened line (in the example, the e line)
3. White has a bishop that can go in the h6 square, and
4. Black has the 8th rank weak, i.e. black has not control enough of the 8th rank.

The idea in this pattern is that white can capture the Knight on e8, sacrifying the Queen, and so after 1.QxKe8 RxQe8, 2.RxRe8 Bf8, 3.Bh6 with the threat RxBh8 checkmate.

But in the position in Figure 4 there is a knight in e4, so white has to analyze how to move it along the e line to reach the discussed pattern. Obviously 1.Nxd6 or 1.Nf6+ are not good because the knight in e8 can take that piece and escapes from e line. So, the white need to find a good place to move the knight by e4. After some trial considerations we can affirm that the best move is 1.NxPc5 and then after 1…QxNc5 or 1…PxNc5 the white can play 2.QxNe8 gaining a pawn in this trade. However, continuing with the variation, it seems that 1.NxPc5 is not really good because after 2…Bg4, 3.Qb5 Qxb5, 4.PxQb5 BxRd1, 5.RxBd1 Rb7 is not clear (Dvoretsky, 1991, p.82).

If the chess player uses a strategically approach he/she finds out some configural concepts from the position and then configures the pieces in order to achieve one or more goals. Referring to the Figure 4, white can evaluate the position as shown in Figure 6. In this operation we can underline the weakness of pawn in d6, the bad position of rook in a7 and the bishop in c8. Also white has space advantage and his/her pieces are well placed on the center of the board so he/she can manoeuvre the pieces easily. In this case a proper move for white is 1.Qg3 attacking the pawn d6 and forcing the black to move the rook in d7 and then block the movement of bishop c8, so after 1…Rd7 a good idea is 2.h4 and improve the space advantage and pressing on black castle.

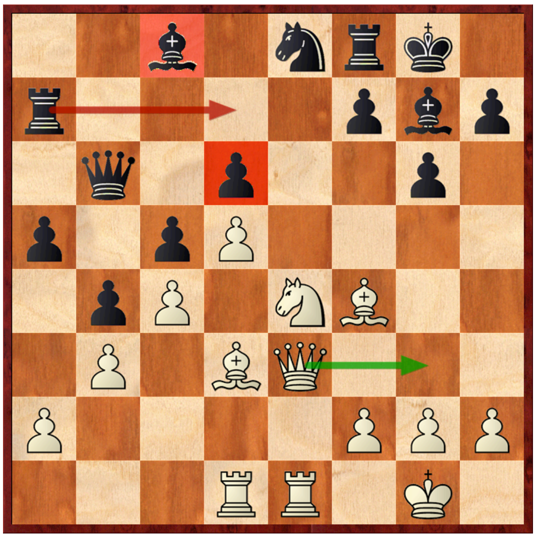

Figure 6. A strategic evaluation of position in Figure 4

These two examples show that there is not a unique way to deal with a position. There is not a unique evaluation of the position and every evaluation produce different plans, ideas and variations. In the position in Figure 4 stein played 1.NxPc5 and he preferred this variation to the other ones. He thought that this position belonged to those positions in which white achieves concrete results based on his/her positional superiority by a tactical approach. What we want to emphasize is that, according to these considerations, the evaluation of a chess position could be considered as a process of abduction and generalization. First, the chess player finds out some configural concepts and decides that the position belongs to a class of positions (abduction), and so he/she can produce a move or a complete variation, based on an expected common behavior (generalization).

This is what we called above decisional generalization. The relationships among patterns and ideas, considerations, moves, etc. are built in the process of productive generalization, in which the players playing or studying chess compares and describes similar patterns and produce the ideas and moves used in the decisional generalization.

CONCLUSIONS

We think that chess is a *drosophila* more and more, with respect to various field of human thinking, mathematics included. It is so because chess thinking is purely abstract. Therefore, the features of mental processes that we tried to highlight in chess may be probably usefully connected with thinking in other disciplines.

The mechanisms we deal in this work, in particular the construction and deconstruction of configural concepts leading to their objectification, are sensuous mental operations realized by continuous use of generalization. As we have shown in the last paragraph, it occurs through various levels of generalization, and it may be interesting to compare them with the layers of generalization suggested by Radford (2010b), where the layers are defined according to the generalization afforded by the semiotic systems in which the generalization is expressed.

Generalization is fundamental in chess, because it allows the player an useful employment of his/her experiences. Generalization is not, however, the only thought mechanism involved in chess thinking. The second basic mechanism is abduction. Abduction allows the chess player to improve his/her experiences and knowledge during the game. Generalization and abduction, we tried to show, play an important and complementary role in chess thinking.

To end with a didactical note, let us note that various studies show the positive effect of practicing chess on mathematics learning. According to our experience, the benefits of chess practice related to mathematics learning are particularly observable in scholastic environment, dealing with grade 1 to 5, working with teachers and chess instructors for a minimum of 30 hours of chess practice. Nevertheless, there are a many possibilities for new inquires and research on chess thinking and the role of generalization as well as other thinking mechanisms like induction and abduction.

REFERENCES

Chase W. G., & Simon H. A. (1973a). Perception in chess. *Cognitive Psychology, 4*, 55-81.

Chase W. G., & Simon H. A. (1973b). The mind's eye in chess. In W. G. Chase (Ed.), *Visual information processing* (pp. 215-281). New York, NY: Academic Press.

De Groot, A. D. (1965). *Thought and choice in chess*. The Hauge, The Netherlands: Mouton Publishers.

D'Eredità, G. (2012). *Chess and mathematical thinking-cognitive, epistemological and historical issues* (Unpublished PhD Thesis). Palermo University, Italy. Available at http://math.unipa.it/~grim/PhD_Tesi_Deredita_2012.pdf

D'Eredità, G., & Spagnolo, F. (2009). Le diversità culturali nelle concezioni di Strategia e Tattica tra Oriente ed Occidente osservate attraverso gli scacchi ed il Wei-ch'i e le connessioni con la Didattica [The cultural diversity in the strategic and tactic conception between Orient and Occident observed accross the Wei-ch and its conection to the didactic]. *Quaderni di Ricerca in Didattica, 19*, 263-283.

Di Sario, P. (2002). *Apprendere ed applicare strategie seguendo un modello cognitivo per il gioco degli scacchi* [Learn and apply following a cognitive model for the game of the chess] (Unpublished degree dissertation). Università di Bologna, Italy.

Dreyfus, H. L. (2005). Overcoming the myth of the mental: How philosophers can profit from the phenomenology of everyday expertise. *Proceedings and Addresses of the American Philosophical Association, 79*, 47-63.
Dvoretsky, M. (1991). *Corso avanzato di'analisi scacchistica* [Advanced course on statistical analysis]. Rome, Italy: Prisma Editore.
Ferro, M. (2012). Chess thinking and configural concepts. *Acta Didactica Universitatis Comenianae,* 15-30.
Ferro, M. (2013). *A multimodal semiotic approach to investigate on synergies between Geometry and Chess* (Unpublished PhD Thesis). Palermo University, Italy.
Gobet, F. (1998). Chess players' thinking revisited. *Swiss Journal of Psychology, 57*, 18-32.
Gobet, F., & Campitelli, G. (2002). Intelligence and chess. In J. Retschitzki & R. Haddad-Zubel (Eds.), *Step by step. Proceedings of the 4th Colloquium Board Games in Academia* (pp. 103-112). Fribourg, France: Editions Universitaires.
Jongman R. W. (1968). *Het oog van de meester* [The eye of the master]. Amsterdam, The Netherlands: Van Gorcum.
Kasparov, G. (2007). *Gli scacchi e la vita* [Chess and life]. Milan, Italy: Mondadori.
Montero B., & Evans, C. D. A. (2011). Intuitions without concepts lose the game: Mindedness in the art of chess. *Phenomenology and the Cognitive Sciences, 10,* 175-194.
Radford, L. (2002). The seen, the spoken and the written: A semiotic approach to the problem of objectification of mathematical knowledge. *For the Learning of Mathematics, 22*(2), 14-23.
Radford, L. (2010a). The eye as a theoretician: Seeing structures in generalizing activities. *For the Learning of Mathematics, 30*(2), 2-7.
Radford, L. (2010b). Layers of generality and types of generalization in pattern activities. *PNA, 4*(2), 37-62.
Schoenberg, H. C. (1975). *I Grandi maestri degli scacchi* [The Great masters of chess]. Milan, Italy: Garzanti.
Wartofsky, M. (1984). The paradox of painting: Pictorial representation and the dimensionality of visual space. *Social Research, 5*(4), 863-883.

Mario Ferro
Palermo University
ferro@math.unipa.it

Giuliano D'Eredità
Palermo University
giuliano.deredita@gmail.com