

people use mathematical ways of thinking for organizing their personal experience and how they organize and develop those ways of thinking and reflecting.

Sfard used a comparison, and her conclusion was that it is necessary for mathematics education to clarify its position with respect to mathematical knowledge:

Our ultimate objective is the enhancement of the learning of mathematics. However, as researchers, we are producing knowledge (about how people create mathematics for themselves), and as educationists, we are inducing certain knowledge in others. Therefore, we are faced with the crucial question: What is knowledge, and, in particular, what is mathematical knowledge for us? Here, we find ourselves caught between two incompatible paradigms: the paradigm of human sciences (to which we belong as mathematics education researchers) and the paradigm of mathematics. These two are completely different: Whereas mathematics is a bastion of objectivity, of clear distinction between *TRUE* and *FALSE* (for practicing mathematicians at least), there is nothing like that for us. For us, mathematics is social, intersubjectively constructed knowledge.... But we feel somewhat schizophrenic between these two paradigms because our commitment to teach *MATHEMATICS* makes us, to some extent, dependent on [the philosophies of mathematics held by mathematicians]. Therefore, we must make the problem explicit and cure the illness by making clear where we stand with respect to the issue of mathematical knowledge.

R. Mura gave a very personal view of the topic of the group and the discussions:

I came to this group because I had written a paper for this conference on mathematics educators' definitions of *mathematics education*. This experience, in a sense, makes it more difficult for me now to give my own definition. In general, I would say that the object of mathematics education is less problematic than the object of mathematics. Maybe what we want to concentrate on is the border cases. Some of us have had our own work challenged as not being research in mathematics education. The first issue (work not really being *research*) is common to all the social sciences and humanities. The second issue (not research in *mathematics education*) is for us to decide. Some of us have been criticized by people saying that our work is in linguistics, in women's studies, in philosophy, and so forth, rather than in mathematics education. Could we behave in a way similar to our colleagues in mathematics and say that mathematics education is what mathematics educators do?

WHAT ARE THE AIMS OF RESEARCH IN MATHEMATICS EDUCATION?

REPORT OF WORKING GROUP 2

Leader: Ole Bjorkqvist

Reporters: Pedro Gómez and Thomas Romberg

This group was asked to consider at length possible answers to the question of aims in order to clarify the notion of research in mathematics education as an academic activity. In particular, the group was asked to examine 'two kinds of aims: pragmatic aims and more fundamental scientific aims'.

The issue was addressed by first considering two papers; then each was discussed. The two papers served to focus the groups' thoughts on the question. Gilah Leder addressed the diversity of research aims in the field of mathematics education.³ She argued that the purposes for doing research have changed during the past half century; that scholars who conduct research have diverse, often pragmatic, and occasionally scientific perspectives about the aims of their research; that the perspectives have been shaped throughout history by the Eurocentric male-dominated majority culture; and that three pragmatic considerations (esteem for research within academic circles, social or cultural priorities, and allocation or availability of resources) often shape the kinds of research carried out.

Julianna Szendrei presented a classification of four different kinds of 'results' produced by research in mathematics education. The paper had been jointly prepared with Paolo Boero.⁴ Furthermore, she related each type of result to pragmatic or fundamental scientific aims and to three intended outcomes: energizers of practice, economizers of thought, and demolishers of illusions. The first type of a result, which she labeled *innovative patterns*, would include teaching materials, reports about projects, and so forth. Obviously, such results have practical consequences and are designed to 'energize practice'. The second type of result is *quantitative information* about the choices concerning the teaching of a peculiar mathematical content, general or specific learning difficulties, possible relationships with factors influencing learning, and so forth. Such results have both practical and scientific aims and are designed to both 'energize practice' and 'demolish illusions' about current practices or beliefs. The third type of result is *qualitative information* about the consequences of some methodological or content innovation, and so on. These results are related primarily to scientific aims and designed to 'demolish illusions'. The final type of result is *theoretical perspectives* regarding reports that reflect on descriptions and classifications or interpretations of phenomena, models, historical or epistemological analyses of content, and so on. Obviously, such results have scientific aims designed to 'economize thought' and perhaps to 'demolish illusions'.

Following these presentations, the members of the working group entered on three occasions into a spirited discussion of the ideas presented in the conference position paper and these papers. The discussion was also fueled by the plenary talks on the balance between theory and practice, the social and cultural conditions under which each of the members of the working group operate, and the other sessions and discussions each member participated in during the conference. The contents of the discussions ranged over several issues related to all five working groups. Working group members submitted written comments to summarize thoughts; a first-draft synthesis was written, points discussed, and format agreed upon, and after two chances to revise the report, this final version was completed.

Throughout the sessions some issues related to aims emerged again and again. These can be summarized under three headings:

Research as a Human Process

The term *research* refers to a process – something people do, not objects one can touch or see. Furthermore, research cannot be viewed as a set of mechanical procedures to be followed. Rather it is a craft practiced by scholarly groups whose members have agreed in a broad sense on what procedures are to be followed and on the criteria for acceptable work. These facts led us to the following assertions:

- An important aim of all research should be ‘to satisfy the curiosity of the researcher about some situation’. [Note, first, that *situation* is used here to refer to all the objects of study being specified by Working Group 1, and second, that the researcher’s ‘interest’ is often influenced by policymakers, school boards, and so forth.]
- That curiosity should lead to an understanding of situations. Many situations involve the teaching and learning of mathematics in classrooms with the expectation that understanding such situations could lead to improved practice. Other situations may be outside schools and may lead to improvements in the workplace. In this regard, we recognize that there are several levels of understanding such as describing or explaining.
- The actual situations a researcher might investigate are embedded in the institutional, social, political, and cultural conditions in which the researcher operates. The personal aims of different researchers will differ because of different beliefs and their membership in particular scholarly groups with differing notions about disciplined inquiry. [Note also that these groups may have differing aims.] And there may be a difference in the aims for a particular study and a set of studies or a research program.

One member of the group proposed that Figure 1 be used to illustrate the variety of things a scholar may be influenced by when deciding on the aim of a particular study.

Diversity of Aims

The teaching and learning of mathematics in schools at any level in any country is complex. When one also considers mathematics outside schools and in adult education, the complexity is compounded. These facts, when added to individual curiosity, make it clear that there has been and will be a diversity of aims. Individual studies and even research programs conducted by different persons or groups will inevitably have different aims. The concern of the group was that such diversity might make impossible any coherent compilation of findings.

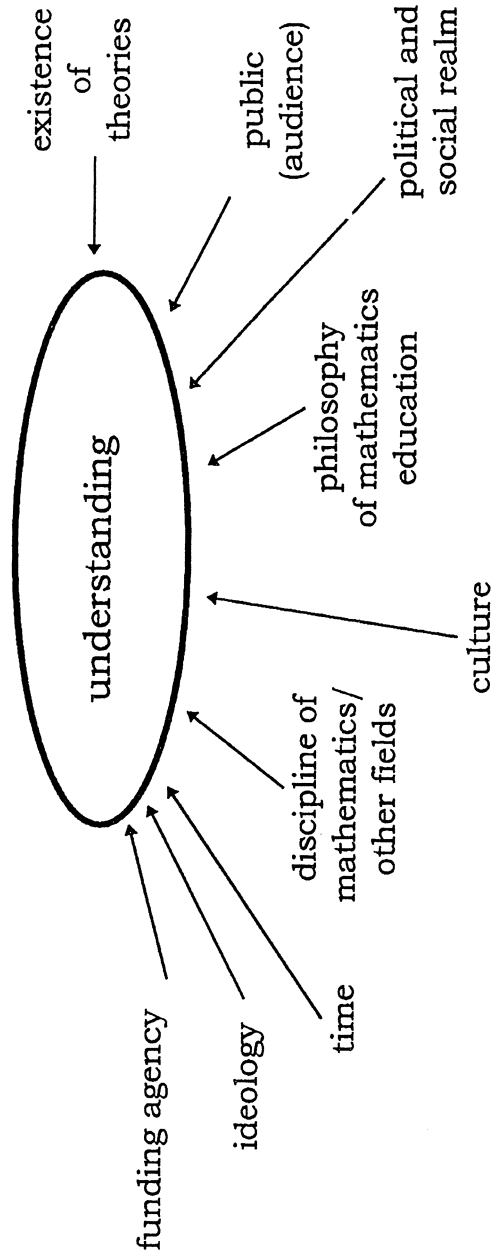


Figure 1 Things to consider about the aims of a study

Nevertheless, some factors were considered by the group as helpful in making the specific understandings useful:

- The situations we aim to investigate must include mathematics.
- We need to differentiate specific aims between short-term and long-term aims.
- We need to consider the possible alignment of personal aims with the external aims of professional groups, policy reports, or funding agencies.

It became clear in the discussions that the community of mathematical sciences education needs to become politically active in order to shape external expectations for research.

Practical Aims or Theoretical Aims

In the group's view, given the diversity of aims and the fact that the results of attempting to understand a situation can have a variety of implications, both the differences between theoretical knowledge, professional knowledge, practical knowledge, and their interrelatedness should be appreciated. The group also understood that such knowledge is provisional. Nevertheless, pragmatically it should be obvious that some research studies will have been designed to have practical implications (i.e., energize practice), and others to contribute to theory. In fact, the group agreed that in either case all research should eventually have a positive impact on practice. John Dewey's dictum 'that there is nothing as practical as a good theory' should be remembered. In addition, one should recognize that:

- There are differences between theory-driven research and theory-generating research.
- Some studies should aim to establish the limits of a theory.
- Some studies should identify and contribute to the elimination of obstacles to the growth of research and to the acceptance of research results.

Finally, one member of the group expressed the belief that there are, in the present meeting, different uses of the words *practice* and *theory* corresponding to different points of view that can be summarized as follows:

<i>Point of view</i>	<i>Practice</i>	<i>Theory</i>
level (role)	teaching	researching
person (status)	teacher	researcher
place	classroom	laboratory
product	technique	knowledge
methodology	collect data	analysis
situation	natural	experiment
time	short-term	long-term
generality	projects	fundamental knowledge
research vs. development	development	research

To paraphrase D. Lacombe in the article '*Didactique des disciplines*' in the *Encyclopaedia Universalis*: In the end, if the researcher wants 'recognition' for improving practice, it is enough to become a salesperson for a brand of instruments, or even better, several brands of instruments put together. On the other hand, if researchers want to leave a 'trace' of their intellectual contribution, they must be prepared to see their work the object of criticism or even derision and to undermine the comfort of the establishment.

Sharing the Findings of Research

One aspect of conducting a research study is that a study is not complete until a report is written explaining the results and anticipating actions of others. Thus, one important aim must be with respect to sharing the results with others. This aim must involve:

- deciding on an audience (or audiences), and
- considering the potential consequences of the results for that audience.

In conclusion, the following two questions need to be considered by all when talking about the aims of research in mathematics education:

1. To whom are we (mathematics education researchers) talking when we list the aims of our research?
2. Are we trying to determine what the aims of mathematics education research have been and are, or are we trying to make proposals about what the aims could be in the future?

The first question is relevant since there are 'outside factors' (funding agencies, government bodies) that have in the past shaped at least partially what the aims have been. On the other hand, the research community could and should influence the way these external agencies have an impact on those aims.

Researchers can have a programmatic or a descriptive approach concerning the aims of research in mathematics education. The former could amount to the proposal of some kind of research agenda, whereas a descriptive approach could require an analysis and description of how the aims of research in mathematics education have evolved and how we can use this evolution as an indication of their future status.

The group saw research in mathematics education as the activities and results of a community of scholars. Therefore, from a descriptive point of view, the aims of research in mathematics education are the aims of this community as far as its activities and results. In this respect, a programmatic approach is not very helpful if one sees the research community as a body that evolves according to multiple interests and perspectives.

This multiplicity of research perspectives tends to characterize the way researchers see the aims of their work. One can see the aims as to explain, predict, or control (empirical-analytic); or to understand (interpretive); or to improve or

revise practice (critical). However, these perspectives are not specific to mathematics education research. What makes the aims of mathematics education research specific are the phenomena and the practices we are trying to explain, predict, control, understand, and improve.

WHAT ARE THE SPECIFIC RESEARCH QUESTIONS OR *PROBLÉMATIQUES* OF
RESEARCH IN MATHEMATICS EDUCATION?

REPORT OF WORKING GROUP 3

Leader: Mariolina Bartolini-Bussi

Reporters: Bernard Hodgson and Iman Osta

A general problem of the group was the ambiguity of the task: Is it possible or even meaningful to discuss 'specific research questions' without considering at the same time the problems of objects of research in mathematics education, aims of research in mathematics education, results of research in mathematics education, and criteria for research in mathematics education? However, the group tried to accomplish the task.

Two presentations introduced the discussion: Nicolas Balacheff spoke on the case of research on mathematical proof, and Ed Silver spoke on research questions in the international community of researchers. In his presentation, Balacheff elicited three different components of a didactical situation that establish relationships with different research fields: the content (defined by a pragmatic epistemology of proof, where researchers in mathematics education can question mathematicians' practices); the learning process (where social interaction can act as a catalyst for developing proofs or counterexamples, and hence where psychology and sociology can provide elements of frameworks); and the classroom situation as an object of study (entity) (where questions specific to didactics occur, e.g. related to the didactical contract and to the *milieu*; the latter concept may lead to the idea of mathematical phenomenology, which in turn leads one to question mathematicians).

In his presentation, Silver discussed two sources of research questions: (a) theory and prior research; and (b) educational practice and problems. The relationship between theory and practice can make international communication difficult (the example of the QUASAR project was presented as context-bound research, difficult to discuss and evaluate in an international forum when compared with the author's research on problem posing). In mathematics education research, in addition to disciplinary issues, it is necessary to consider political, financial, and other societal issues.

After the two presentations the group started the discussion. The main points of the discussion were the following (different streams went on with continuous intersection; what follows is not a chronicle but a kind of reconstruction of the outcome of the discussion).