Lessons from PISA 2012 About Mathematical Literacy: An Illustrated Essay

Ross Turner

Mathematics is central in school curricula around the world, yet outcomes are mixed at best. This is true both in relation to attitudes expressed towards mathematics, and in relation to observed assessment outcomes, showing how poorly individuals are prepared to use mathematics in a world in which the ability to do so is of increasing importance. Why? This essay uses some mathematics outcomes from the Programme for International Student Assessment (PISA) 2012 to seek explanations and possible ways forward. The main message is that negative outcomes of school mathematics arise for a variety of reasons, and they can be reversed through deliberate and concerted action.

Keywords: Anxiety; Competencies; Mathematical literacy; PISA

Lecciones de PISA 2012 sobre competencia matemática: un ensayo ilustrado

Las Matemáticas son fundamentales en los currículos escolares de todo el mundo, todavía los resultados se mezclan en los mejores. Esto es cierto tanto en relación con las actitudes expresadas hacia las matemáticas, como en relación con los resultados observados en la evaluación, mostrando cómo los individuos tienen una preparación pobre para utilizar las matemáticas en un mundo donde la habilidad para usarlas tiene una creciente importancia. ¿Por qué? Este estudio se basa en algunos resultados sobre matemáticas del Programa Internacional para la Evaluación de Estudiantes (PISA) 2012, para buscar explicaciones y posibles formas de abordarlos. El principal mensaje es que los resultados negativos en las matemáticas escolares se producen por una variedad de razones, y pueden revertirse a través de acciones deliberadas y coordinadas.

Términos clave: Alfabetización matemática; Ansiedad; Competencias; PISA

My starting point consists of two statements that provide a background and underpinning to this essay.

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- Maths is hard.
- There is frequent collusion among teachers, parents and students around judgments to the effect that "you're just not a maths person".

At ACER's 2010 research conference[1], my presentation (Turner, 2010) included a reference to what I named the "mathematics terror index" depicted in Figure 1.

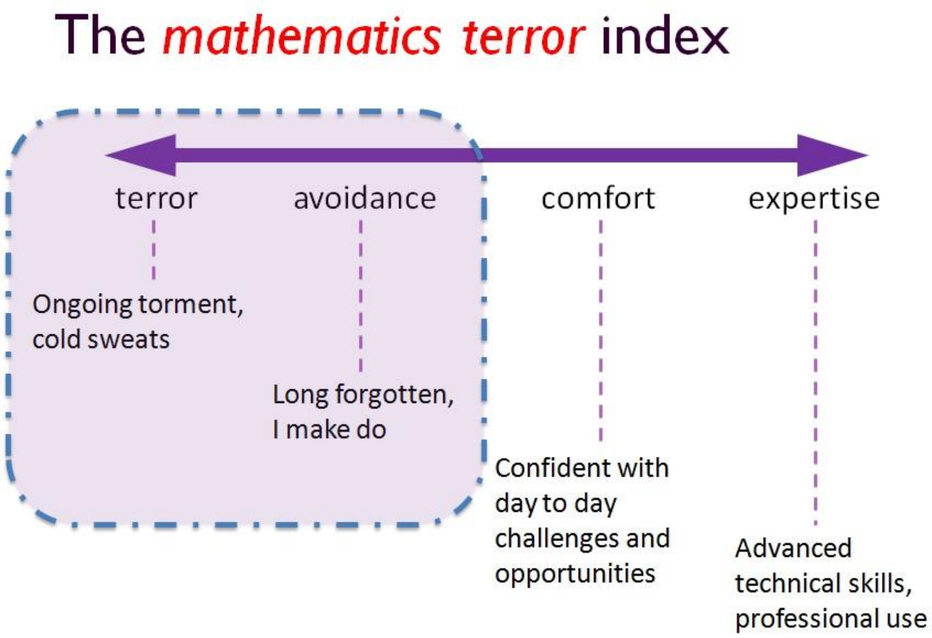


Figure 1. Depiction of the mathematics terror index

Essentially, this idea hypothesises the existence of a continuum of attitudes towards mathematics observed in the adult population at large, with at one end (the negative end) a high degree of "terror" evidenced by ongoing torment about anything that looks like mathematics, and frequently accompanied by cold sweats in the face of any mathematical challenge; and at the opposite extreme, more positive attitudes accompanied by behaviour that could be characterised as "expertise", indicated by the presence of advanced technical skill and knowledge, and frequently by some ongoing professional use of mathematics. In between these extremes, closer to the negative end, we see "avoidance" behaviour whereby individuals just make do without thinking about or using mathematics—whatever they learned was a long time ago and was long forgotten (and blessedly so); and progressing a little further toward the positive end to a point that could be labelled "comfort", indicated by a level of confidence with day to day challenges and opportunities to make use of relevant mathematical knowledge.

[1] The proceedings of the 2010 research conference of the Australian Council for Educational Research (ACER), with the theme *Teaching mathematics? Make it count: What research tells us about effective teaching and learning of mathematics*, can be found at this link:
http://research.acer.edu.au/cgi/viewcontent.cgi?article=1094&context=research_conference

My hypothesis was that a very large portion of the adult population in my country, and I suspect in many other places, lies in the terror and avoidance parts of the spectrum.

There is a large literature on mathematics anxiety. Buckley (2013) provides a brief introduction to key issues, highlighting US research estimating that approximately 20 percent of the US population are highly maths anxious. The PISA 2012 survey included a student questionnaire, containing questions seeking information on a wide range of background variables, to which every sampled student was expected to respond. The student questionnaire included questions on a range of self-beliefs: mathematics self-efficacy (the extent to which students believe in their own ability to handle mathematical tasks effectively and overcome difficulties), mathematics self-concept (students' beliefs in their own mathematics abilities), mathematics anxiety (thoughts and feelings about the self in relation to mathematics, such as feelings of helplessness and stress when dealing with mathematics), and student engagement in mathematics activities at and outside school. The Organisation for Economic Co-operation and Development (OECD) report of the main results from the part of the student questionnaire relating to mathematics attitudes stated that some 30 percent of students reported feeling helpless when doing mathematics problems, and that greater anxiety is associated with a 34 point drop in PISA performance—the equivalent of almost one year of school (OECD, 2013, p. 88).

Of course much of the interest in PISA data centres on student responses to the cognitive questions. Each sampled student is presented with a test booklet that contains about 50 questions on the test domains of mathematics, reading and science; with the particular selection of test questions presented to each individual sampled student being a random assignment from a rotated booklet design. For PISA 2012, mathematics was the major test domain, and all sampled students were presented with at least some mathematics test items (between one-quarter and three-quarters of each booklet comprised mathematics questions). What do the PISA results show that 15-year-old students are able to do in response to those mathematics questions?

Almost 80 percent of 15-year-old across the more than 60 PISA-participating countries can follow instructions that would enable them to identify a particular data value from a simple bar chart. For example, they might be responding to a question such as "what was the total value of sales for a particular year?" by reference to a given sales chart. Less than half are likely to be able to interpret, extract and combine related data from two different graphs and perform a calculation involving percentage ("what was the value of a particular export item in a given year?"—given both a graph of total export value and a graph of the percentage distribution across a handful of export commodities for that year). Less than one-fifth of students internationally are likely to be able to effectively use geometric reasoning and insight about the comparative perimeters of different—but familiar—shapes. These observations are based on percent correct data for three PISA mathematics items (Exports Q1 and Q2, and Carpenter Q1) released following the PISA 2003 survey, the items being reproduced in

Chapter 2 of OECD (2004) and the percent correct figures in OECD (2005, pp. 412-413).

One of the salutary lessons I have learned over the 15 years during which I have been closely associated with the international implementation of the PISA survey is the existence of a substantial gap between the hopes and expectations of the mathematics education community, one of PISA's key interest groups, and the reality of observed student performance on the PISA mathematics survey items. The test developers, the mathematics experts forming the reference group for this part of the PISA work, the key decision-makers within the OECD's PISA Secretariat, and many of the item reviewers from participating countries hope and expect that 15-year-old students will be able to make intelligent use of the mathematical knowledge they have accumulated over their time at school to solve a range of meaningful and worthwhile problems. Indeed, there is an expectation among many in those groups that the mathematical knowledge students acquired recently (material that might be regarded as Year 9 mathematics or Year 10 mathematics) would be legitimately and satisfactorily incorporated into the PISA test.

But what do we actually observe when we ask 15-year-olds to complete PISA tasks? These tasks tend not necessarily to be presented in the form to which students are accustomed from their mathematics classes; and typically students have to first recognise that some mathematical knowledge might be relevant, then to identify exactly which elements of their knowledge that could be, and to activate it effectively to deal with the challenge before them. We see again and again when problems are expressed in a context, that many students are immediately faced with a severe and even debilitating challenge and find it very difficult indeed to make the relevant connections between their knowledge and the demands of the problem, and to activate the relevant knowledge.

Why is this? My answer lies in a few generalisations that I admit are certainly not true in every case, but are true enough in enough cases to be plausibly regarded as major contributors to the situations I mentioned in the paragraphs above.

- Very few people are able or willing to view their world through mathematical lenses. Their mind-set does not predispose them to seeing the mathematical possibilities around them and to activating their mathematical knowledge when it might be useful to do so.
- Too little school time, particularly mathematics class time, is devoted to grappling with real-world situations and problems and exploring the ways mathematics might be relevant to them. Students simply do not have enough experience thinking about, discussing, exploring, and experimenting with situations in their environment that might have mathematical content, teasing out that mathematical content, and bringing to the surface of awareness those mathematical features of the situation that might otherwise remain hidden. This lack of experience can be wrongly interpreted as a lack of ability.

- ♦ For a variety of unfortunate historical and structural reasons, such as the identification of important elements of knowledge and its packaging and codification in school curricula and presentation in defined year-related chunks, usually in text books, much of students' experience with mathematics involves dealing with pre-packaged, pre-defined routines and very controlled applications, typically very artificially constructed to facilitate the teaching of a particular concept, skill or procedure that is part of the syllabus for a particular school year. Teaching and learning practices too often assume that the task at hand is for everyone in a class to learn more or less the same material in more or less the same order and at more or less the same speed.

The student experiences in this typical environment build lack of awareness about mathematical opportunities around us, lack of practice in exploring mathematics around us, and frustration with school mathematics experiences. To the extent that those generalisations apply to students during their school years, why would we expect this to change when schooling has finished? Why would we expect our adult population and workforce to adopt different attitudes towards mathematics and about their mathematical ability? These and related issues are taken up further by Masters (2013) in a call for a rethinking of approaches to educational assessment.

Is the situation changing? The release of the PISA 2012 results in December 2013 featured a focus on the countries whose students did comparatively well and this was accompanied by the trumpeting of a number of countries whose PISA scores had substantively improved. The international report (OECD 2014, Chapter 2) revealed that of the 64 PISA participants for which some trend data are available for the period from 2003 to 2012 (the years of the PISA surveys in which mathematics has so far been the major focus of the assessment), 25 showed an average annual improvement in mathematics performance, 14 showed deterioration, and for the remaining 25 countries there was no change.

But let's look at this from the point of view of the keen mathematics educationalist. What do the PISA 2012 results tell us about how 15-year-olds respond to particular mathematics questions? A number of PISA test questions have been released into the public domain since the 2012 survey administration. In fact a large body of released items is now available. These can be found at the following website http://www.oecd.org/pisa/pisaproducts/pisa-test-questions.htm.

An example of a released item from PISA 2012 is in Figure 2. It is based on a kind of activity, and demands a kind of thinking, that is common in day-to-day life —not only in a food preparation context as presented here, but in many real-life settings. Ingredients are to be mixed together, with specific volumes of three ingredients needed to make a particular volume of the final product, but we want a different volume of the final product. What is required?

You are making your own dressing for a salad.

Here is a recipe for 100 millilitres (mL) of dressing.

Salad oil:	60 mL
Vinegar:	30 mL
Soy sauce:	10 mL

How many millilitres (mL) of salad oil do you need to make 150 mL of this dressing?

Answer: ……………….. mL

Figure 2. Sauce (a released mathematics item from PISA 2012)

The question calls for a form of mathematical thinking referred to as proportional reasoning. It is presented in a way that corresponds to the kind of challenge that could commonly be met in several contexts. The question requires some thinking in order to formulate it as a mathematical problem (*formulating situations mathematically* being one of the mathematical processes the PISA 2012 mathematics framework emphasises). No explicit guidance is given as to what kind of mathematical knowledge is required—the problem solver needs to work this out. The level of mathematics required is not high—I would say it should be within the grasp of many 12-year-old students. How did PISA students respond?

Table 1 shows the percent correct data for Australia, Spain, Poland, Korea, Indonesia, and the OECD average on the Sauce item shown above. These countries have been selected as a convenience sample that reflects some geographical and cultural diversity and a range of levels of performance.

Table 1
Percent Correct Data for Selected Countries on Sauce

Country	Percent correct
Australia	56%
Indonesia	30%
Korea	73%
Poland	73%
Spain	62%
OECD average	63%

The results vary considerably by country, but on average, less than two-thirds of the surveyed 15-year-olds answered this question correctly. Perhaps a salad dressing that doesn't taste as it is meant to taste would not be a disaster, but what if the situation demanded the mixing of chemicals or medicinal ingredients in a particular proportion?

A second released item is located squarely in a workplace context, in a situation where mathematical skill is clearly relevant (as it would seem increasingly to be in a wide variety of workplaces as we progress into the twenty-first century). Part of the PISA mathematics item Drip Rate is shown in Figure 3.

Infusions (or intravenous drips) are used to deliver fluids and drugs to patients.

Nurses need to calculate the drip rate, D, in drops per minute for infusions.

They use the formula $D = \frac{dV}{60n}$ where d is the drop factor measured in drops per millilitre (mL), v is the volume in mL of the infusion, and n is the number of hour the infusion is required to run.

A nurse wants to double the time an infusion runs for.

Describe precisely how D changes if n is **doubled** but d and v do not change.

Figure 3. Drip rate (a released item from PISA 2012)

What happened when 15-year-old were confronted with this problem in the PISA 2012 administration? It contains an algebraic formula, which definitely looks like mathematics. That may immediately have a proportion of students on their guard. To answer the question, some reasoning is needed to understand the relationships defined in the formula. As with Sauce but with an added level of complexity, a form of proportional reasoning is needed. If the denominator of the right-hand-side of the equation were doubled (and its numerator remained unchanged), then D would be halved. The doubling is a simple ratio, but the thinking and reasoning required is critical, and this question had an added challenge, namely it required students to write a very brief explanation of their conclusion. How did PISA students response to this item? Table 2 shows the percent correct results for mentioned countries, and the OECD average on the Drip rate item.

Table 2
Percent Correct Data for Selected Countries on Drip Rate

Country	Percent correct
Australia	21%
Indonesia	3%
Korea	43%
Poland	23%
Spain	19%
OECD average	22%

Once again, as with every PISA mathematics item, the results vary considerably by country, but on average, less than one-quarter of the surveyed 15-year-olds answered this question correctly. Unlike the Sauce example, the ability to manage the reasoning involved here could easily be a matter of life and death. Three further released items related to a single stimulus (shown in Figure 4), are the Revolving door questions, shown in Figure 5, Figure 6 and Figure 7.

Here the student is presented with a somewhat abstract representation of a revolving door—a stylised view from above, showing the door itself having three wings housed in a circular frame from which two pieces are removed for the entrance and exit, and text that describes the key features of the door.

A revolving door includes three wings which rotate within a circular-shaped space. The inside diameter of this space is 2 metres (200 centimetres). The three door wings divide the space into three equal sectors. The plan below shows the door wings in three different positions viewed from the top.

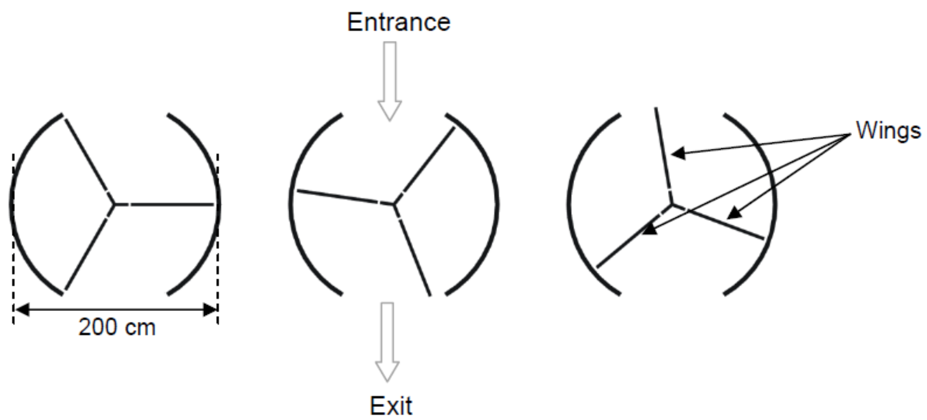


Figure 4. Stimulus for PISA 2012 released items Revolving door

The three questions attached to this stimulus vary quite a bit in the nature and depth of the challenge presented to students. The first question, shown in Figure 5, requires the recall of some basic factual knowledge, but it is situated in a context that demands a small reasoning step, not simply asking for direct recall. Students need to recall a basic fact about circle geometry (360° at the centre), recognise the relevance of the information given that “the three door wings divide the space into three equal sectors”, and divide 360 by 3 to obtain the required result.

Question 1: REVOLVING DOOR

What is the size in degrees of the angle formed by two door wings?

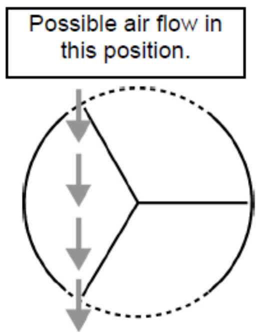
Size of the angle: ....°

Figure 5. Revolving door question 1 (a released item from PISA 2012)

The second question, shown in Figure 6, is completely different. This requires substantial reasoning about the design features of a revolving door that enable it to perform its function as a doorway while maintaining a sealed space that prevents air flowing between the entrance and exit. For many adults, and certainly for many 15-year-olds, this would be a novel problem that would require some creative thought, and not the application of any algorithm or fixed piece of curricula knowledge they would have learned. It is something that an engineer might need to know, or the people involved in manufacturing revolving doors. But it might also be a matter of curiosity that a person who is tuned in to mathematical features of their environment might notice and wonder about. One way of reasoning here would be to focus on the closed parts of the door frame. When two of the wings are near the two ends of the closed part on one side, as shown in the diagram, the closed part must be at least the length that matches the opening between the two wings in order to prevent air flow, hence must span at least one-third of the circular frame. The door is symmetrical; hence the two closed parts must together take up at least two-thirds of the frame. This leaves a maximum of one third, divided into two equal parts (the entrance and exit should be equal); hence the maximum opening of each is one-sixth of the circumference. The data given in the stimulus can then be used to calculate the circumference using a formula that most 15-year-olds would have been taught, and the required arc length is one-sixth of that circumference.

Question 2: REVOLVING DOOR

The two door **openings** (the dotted arcs in the diagram) are the same size. If these openings are too wide the revolving wings cannot provide a sealed space and air could then flow freely between the entrance and the exit, causing unwanted heat loss or gain. This is shown in the diagram opposite.



What is the maximum arc length in centimetres (cm) that each door opening can have, so that air never flows freely between the entrance and the exit?

Maximum arc length: .... cm

Figure 6. Revolving door question 2 (a released item from PISA 2012)

The third question, shown in Figure 7, requires different thinking again: yet another example of proportional reasoning. Not a question of life and death this time, but this kind of mathematical modelling is critical to design processes and to safety and other practical decisions that relate to the movement of people in public spaces. One approach would be to start with the door's four rotations per minute, scale this up to 120 rotations in 30 minutes, then to recognise this would provide 360 entry opportunities (scaling up again according to the three door openings for each rotation), and doubling this to take account of the two persons accommodated at each entry opportunity giving a final answer of 720.

Question 3: REVOLVING DOOR

The door makes 4 complete rotations in a minute. There is room for a maximum of two people in each of the three door sectors.

What is the maximum number of people that can enter the building through the door in 30 minutes?

A 60
B 180
C 240
D 720

Figure 7. Revolving door question 3 (a released item from PISA 2012)

So how did our set of comparison countries do with these three items? The PISA 2012 outcomes for the three Revolving door questions are shown in Table 3. Only 58% of tested 15-year-olds could answer the easiest of the three questions, involving making sense of the diagram, a small reasoning step within a fairly straight-forward context and dividing 360 by 3. Question 2 was one of the most difficult questions in the PISA 2012 test, and only 3% of students tested were able to complete this correctly. Less than half of all students tested were able to answer the third question.

Table 3
Percent Correct Data for Selected Countries on Revolving Door Questions 1, 2 and 3

Country	Percent correct Q1	Percent correct Q2	Percent correct Q3
Australia	58%	4%	50%
Indonesia	22%	1%	11%
Korea	79%	6%	53%
Poland	71%	3%	51%
Spain	52%	2%	45%
OECD average	58%	3%	46%

Attention on PISA results is usually focussed on the overall scores and the changes from survey to survey that are observed. My conclusion about the results presented here is that while there has been change at that macro level in PISA results for different countries, when you look at the item level in all PISA surveys it is clear that observed mathematical proficiency leaves much to be desired. Many mathematics educators looking at these findings will continue to be disappointed at the gap between what might be expected and desired, and what is observed. Such a conclusion raises important issues for schools, teachers, employers, parents, students, indeed for the whole community.

However, news in this realm is not all bad—we can certainly find evidence of positive stories arising from the PISA 2012 data. Taking into account the nationality of the author, and the location in which this journal is published, I take Australia and Spain as the focus of a slightly wider analysis of student performance on the mathematics items in the 2012 PISA survey. But a similar analysis could easily be carried out in relation to any country participating in PISA, using publicly available data. Specifically, the source has been the *Compendium for the Cognitive Item Responses* (available at http://pisa2012.acer.edu.au/downloads.php), which provides the percentage of students in each participating country responding in each response category defined for each test question.

One could look at these data in a number of different ways. My approach will be to look for items on which Australian and Spanish students respectively performed relatively well, and items on which they performed relatively poorly. Performance can be looked at in absolute terms (for example, those items on which students in a particular country had the highest and lowest percent correct figures). It can also be viewed in relation to the OECD average percent correct, which might point out particular items that may have generated atypical results. A third way of looking at performance might be to see whether students from a particular country had an unusual profile of results on any of the most and least difficult items compared to other countries seen to be of special comparative interest (for example, in countries with similar curriculum, or countries for which strong mutual economic or cultural ties exist). That kind of analysis is outside the scope of this discussion.

Looking at the items that Australian and Spanish students found most difficult, for Australia we observe that there were eleven items on which less than 25 percent of students were correct, and these were the most difficult eleven items internationally (though the order of difficulty differed slightly); while for Spain, of the seventeen items on which less than 25 percent of students were correct, fourteen of them were amongst the most difficult items internationally. At the other end of the spectrum, over three-quarters of Australian students correctly answered 15 questions, and all but one of these were among the least difficult 15 questions internationally; while for Spain the seventy-five percent correct hurdle was exceeded on the nine items that were also the least difficult internationally. Performance of Australian and Spanish students in the PISA 2012 sample, therefore, was well aligned with the international average percent correct data when considering the items on which students from those countries performed best and worst on average.

Divergence from the international average percent correct, however, does reveal some interesting observations. Australia's average of item percent correct figures across all items was 1.7 points above the international average. Nine items were found to be relatively much easier for Australian students than the overall average would predict because the percent correct difference was greater than 6 (approximately $1.7+4$). Eleven items were relatively more difficult for Australian students with percent correct difference less than -2 (approximately $1.7 - 4$). Spain's average of item percent correct figures across all items was 2 points below the international average. Items diverging markedly from this include the eight items for which the percent correct difference was greater than 2 (i.e., $2 - 4$)—these were found to be relatively easy for Spanish students based on the overall average percent correct figures, and the seven items for which the percent correct difference was less than -6 (i.e., $2+4$)—these were relatively difficult for Spanish students.

Using knowledge of the items in the PISA 2012 survey, including some that remain secure, and with permission of the OECD's PISA Secretariat to use that knowledge in this paper, these instances of divergence from the international average percent correct lead to some generalisations about the kinds of tasks and task demands that Australian and Spanish students respectively appear to have found unusually difficult or easy. These are summarised in Table 4.

Table 4
Mathematical Demands of Items Found Unusually Easy or Unusually Difficult by PISA Students in Spain and Australia

Mathematical demands of items PISA students found relatively difficult	Mathematical demands of items PISA students found relatively easy
Spain	
Interpret and use a given relationship expressed algebraically	Interpret probabilistic statements (e.g., about outcomes in a game of chance, or about a medical diagnosis)
Interpret and use a relationship expressed in words	Apply a multi-step calculation algorithm
Interpret a geometric structure, apply reasoning about angles	Transpose a given graph to a different scale
Apply scale information to estimate a distance on a map	Substitute numbers into an equation
Link functional changes with their graphical representation	Identify and extract data from a graph or table
Interpret data and verify conclusions about the situation represented	Carry out arithmetic calculations

Table 4
Mathematical Demands of Items Found Unusually Easy or Unusually Difficult by PISA Students in Spain and Australia

Australia	
Items for which the *interpret* and *formulate* processes are central were highly represented in this set	Use basic algebra, such as substituting into a formula, applying a formula, or formulating an equation in a geometric context (e.g., using circumference)
Items involving a strong reasoning demand, particularly proportional reasoning and reasoning about probabilities, appear to present the biggest challenge	Interpret graphs, locate specified data points in a graph or table

Even on the basis of this cursory analysis, it seems clear that 15-year-olds in different countries have relative strengths and weaknesses and that these vary, presumably as a reflection of different curriculum emphases.

Returning to the lessons I have learned through my PISA experience, these frequently take the form of more questions—about the possible ways in which some of the shortcomings observed might be addressed. How do we better encourage our students to see their world through mathematical lenses? A broadening consensus suggests that it is of increasing importance in workplaces of the 21st century, indeed in many life situations people occupy these days, that mathematical knowledge is at the least useful, and sometimes even essential, to deal with many challenges that people confront. Certainly there is a body of evidence (e.g., de Baldini, Rocha, & Ponczek, 2011; Green & Riddell, 2012; Leigh, 2008; Shomos, 2010; Vignoles, De Coulon, & Marcenaro-Gutiérrez, 2011) that links higher levels of literacy and numeracy with improved outcomes in a wide range of later life contexts (such as education, health, employment, income levels). Where do we start to better prepare people to use mathematics when it may be productive and beneficial to do so? Is on-the-job training sufficient? Would more effective school education in mathematics provide students with less negative attitudes towards mathematics and their mathematical ability, a better preparedness for changing life and work environments, greater flexibility in confronting an ever-changing world and therefore better access to informed choices about their place in that world?

What will it take to increase our students' experience in grappling with real-world situations and problems and exploring the use of their accumulated mathematical knowledge in the course of such grappling; and using that grappling to further develop and extend their knowledge? In particular, how can we better fit students to grapple with unusual problems that may not look like the kinds of problems they are accustomed to working with in their mathematics classes, problems that may require the problem solver to first transform the problem as presented so it is amenable to mathematical treatment with the tools they possess?

I would suggest that teaching and learning practices should focus on conceptual understanding before rules and algorithms. An example of evidence in support of this is Professor Jo Boaler's research showing the importance of number sense and the benefits to students of learning to use numbers flexibly (Boaler, 2014; and see various resources at Boaler's Stanford website youcubed.stanford.edu). Mathematical concepts and knowledge should be embedded in the contexts in which they might be useful, since the connection between the experienced world and the mathematical concepts and skills that might be used to understand that world must be constantly emphasised and given prominence in mathematics classes. Research undertaken over several years using data from the OECD's PISA survey (Turner, Blum, & Niss, 2015) shows the importance of a number of mathematical competencies and behaviours as significant drivers of mathematical literacy, suggesting the importance of seeking opportunities to promote them in the classroom. These include mathematical communication (explaining, describing, arguing, listening to the explanations and arguments of others, verbalising reasoning steps), which is also emphasised in discussion by Boaler (2014) where she refers to the use of *Number Talks* to promote mathematical communication as a concrete means of building conceptual understanding and number sense. Other competencies ought to also be given greater prominence in mathematics classrooms to build the links between mathematical knowledge and the contexts in which it might be used. Mathematical modelling, in particular the formulating part of the modelling cycle that sees problems arising in the real world transformed into mathematical form; exploring and using a variety of representations of mathematical ideas and phenomena; and strategic thinking, whereby solution processes and pathways are thought about and articulated in advance as a way of connecting mathematical thought and action with the objects of that action.

The volume of good and useful research on effective teaching and learning practices continues to grow, yet we continue to observe the assessment outcomes referred to earlier in this essay, and in particular we continue to observe evidence of widespread mathematics terror among students and in the community at large. In Figure 8, I present a model that captures some of the steps and salient processes that contribute to this situation. The five connected shapes at the left see children beginning life as creative and exploratory individuals. Evidence of mathematics anxiety is observed early in the primary school experience of many children. By the time students reach secondary school, we see many dropping out of mathematics classes as soon as it ceases to be compulsory, clear evidence of growing mathematics anxiety and assessment data showing evidence of widespread failure against teaching and learning objectives exemplified by the kinds of results achieved by 15-year-old students in the PISA survey mentioned earlier. At the stage of entry to the workforce, evidence of gaps in work readiness appears, and workplace training comes into play to ensure labour force skills are developed to meet workplace needs. And out of this we see a community whose members, if ever challenged to use mathematics, frequently exhibit anxiety and a lack of confidence in their mathematical ability, with consequent poor actions and decision making.

On the right-hand side of Figure 8 two sets of factors are shown that bear on this developmental sequence. One set of very influential factors operates in schools. These factors relate to the quality of teaching, and to school culture. In Australia, too many teachers are required to teach out of their field of expertise. The level of mathematical expertise held by teachers of mathematics (including elementary school teachers, most of whom are required to teach mathematics) varies enormously. The mathematical content and pedagogical content knowledge in pre-service teacher training programs is also variable. And the in-service training of mathematics teachers varies in quality, effectiveness, and in its reach. Finally, school cultures vary in the extent to which they are able to engage and excite students about the whole learning enterprise, and to create high expectations of success among all students. Is it any wonder that teachers not adequately equipped to excite their students about mathematics and the possibilities it provides to better understand, predict and manipulate the world around, so often fall back on methods and resources that provide little challenge, excitement or impetus to be inventive? Should we be surprised that high levels of engagement with and excitement about mathematics are so uncommon?

The factors in the second set are broader societal factors that play on the developmental sequence presented in Figure 8. Historically, mathematics has often been used in schools, universities, and by employers to sort and select a few students as worthy recipients of special benefits. This has had serious distorting effects on the design and delivery of school mathematics curriculum, and has fed the growth of anxiety around mathematics achievement and a sense of failure among all who were not selected. It has infected students themselves, but has also affected schools, teachers and parents. How many parents feel they are not up to the task of guiding their children through their mathematics schoolwork? Many parents express inadequacy in their ability to meet the expectations on them and to support their children in meeting their expectations—and this comes from a combination of the legacy of their own schooling experience, and from a sense that the world is rapidly changing, not the least in the very areas for which mathematical competence is said to be so crucial. How many teachers feel that the routines they are expected to follow are not adequate to effectively deal with the range of student interests, backgrounds and needs in the class before them? Even teachers with strong mathematical backgrounds themselves face severe challenges in tailoring their teaching practice to the real needs of their diverse students. They struggle to work effectively with the students who work slowly, or who are not interested in mathematics; they struggle to effectively extend and challenge the faster students and those who are especially talented and enthusiastic; and the great bulk of students in between also deserve their utmost energy and attention. How many schools have developed a culture that effectively counteracts the negative messages about mathematics, and that stretches, challenges and caters properly for all students no matter their current level of achievement? Schools are expected to cover a particular curriculum sequence within a particular time frame, and this expectation is held over everyone's head. But does this approach recognise the real needs of all students?

The easiest solution would appear to be to adopt a stance that says some students are capable, deserving, and can appropriately aspire to greater levels of achievement; while others are not. That way we can excuse unacceptable outcomes by externalising responsibility. In particular, we can explain and accept the incidence of fear and anxiety about mathematics as a reflection on the individual concerned rather than looking at deeper contributing factors.

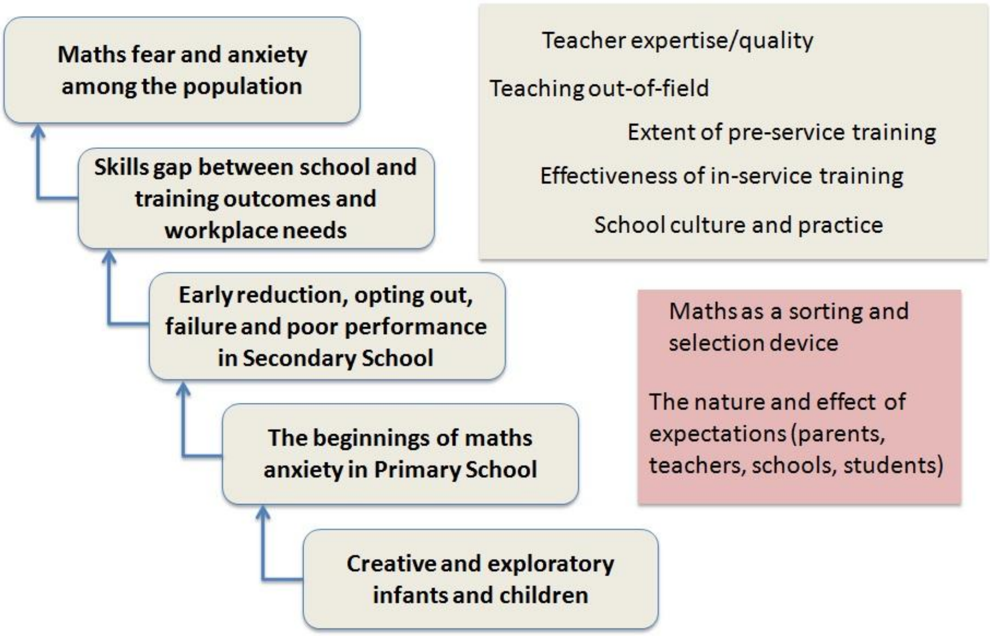


Figure 8. A model for the production of mathematics fear and anxiety

To conclude this essay, my remarks about societal expectations bring me back to the statements with which I opened. "Maths is hard" seems to be a statement that most people would regard as true, and which is a common way of justifying an inability or unwillingness to explore and use mathematical knowledge. In my view the truth of the statement is something to celebrate. It creates opportunities, since humans thrive on challenge. Indeed some go even further to say that hard work is its own reward. The notion of succeeding through effort and perseverance, which is one contributor to the high levels of achievement observed in students from East Asian countries influenced by Confucian cultural values (Leung, Graf, & Lopez-Real, 2006), takes us immediately to the second statement, alluding to the fallacious belief that the mathematical ability required to do well at school is a genetic endowment that some people have and some do not. The second statement at the opening of this essay was made by a Victorian Ministry of Education official in his remarks to ACER's 2014 National Adult Literacy, Language and Numeracy Assessment Conference. Related statements are made by three prominent US mathematics education academics in a forthcoming documentary on *The American Math Crisis*, in a trailer that can be found on the YouCubed website (http://youcubed.stanford.edu/the-american-math-crisis-forthcoming-documentary/). Those comments are displayed in Figure 9.

Professor Alan Schoenfeld (Uni of California Berkeley)
"In the United States, people believe there is a math gene – either you are born with it or you are not. If you don't have it, then you just don't have it."

Professor Jo Boaler (Stanford University)
"Teachers believe that, parents believe it, students believe it, and it's one of the reasons that we have such widespread math failure and math trauma in the States."

Dr Robert Moses (Algebra Project Founder)
"… the idea that there are math people and other people who are not math people, it's a cultural concept that really needs to be destroyed."

Figure 9. Three quotes on mathematical ability in the USA

We must all make our contribution to destroying this idea wherever we see it has taken hold. This will be a critical step in meeting the urgent need to look more deeply at the causes of the unsatisfactory levels of mathematics achievement and the unacceptably high levels of anxiety about mathematics we so frequently observe.

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Ross Turner
Australian Council for Educational Research
Ross.Turner@acer.edu.au