

GEOMETRY IN DESCARTES' SYSTEM

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Introduction

What struck the young student of mathematics at the college of La *Fleche* was the absolute certainty of their demonstrations. More specifically, he was impressed by the contrast between these proofs and the literary tradition, for “/ had compared the moral writings of the ancients pagans to the most proud and magnificent palaces built on nothing but sand and mud”. Still, while he grants the are unquestionable, Descartes discovered both in geometry and arithmetic, an essential uselessness and futility, an appreciation he will maintain all throughout his life.

These first impressions might account for two emotions “joy and unease”² frequently to be found at once in Descartes’ mathematical works. In his work we find a combination of joy of discovery³, clear ideas and of mastership over methods, and uneasiness caused by the puerile, futile character of the very mathematical object itself. This ambivalence is present clearly and explicitly throughout *Geometry*.

For Descartes to achieve peace of mind he will have to “discover their *true use*” and recognise that “*the certainty and evidence of their arguments*” which he enjoys so much, does not give assistance “*the mechanical arts only*”. He therefore had to build something “*more essential on such firm and substantial foundations*”⁴.

1. The Truth of The Traditional Corpus

We shall have to re-examine the Cartesian critique of contemporary mathematics, but first of all there is an important point to be made: even in their present disorganised condition, mathematics do produce -generally speaking- true results. The Euclidian corpus, is accepted wholesale- so are the works of Apollinius and Archimedes. In Descartes' work we do not find any discussions of the truth of the first principles of geometry. Definitions, axioms and postulates of elementary geometry are accepted with all the guarantees of the 'natural light of understanding'. As we are told in *Meditations*, apart from the *ego*, the thinking subject, and God, mathematical truths are the last truths I can doubt, and are also the first truths to escape from hyperbolic doubt.

2. The Possible Errors of Judgment

If elementary geometry and its classical results are not open to doubt, this was far from the case for contemporary mathematics (for instance Cavalieri's mathematics and some of Fermat's methods). Generally for one and the same reason: mathematical demonstrations implying infinitesimal methods slip out of the grip of certain knowledge.

Thus we see Descartes triumphing over Debeaune's difficult problem involving an infinite regression, but refusing to validate his own solution: one cannot conceive the limit the relevant series tend to, thus the human mind cannot understand the result⁶.

Some of the methods of the Ancients were shown to involve the same fault. The one the most often referred to by Descartes himself was Hyppias' quadratric.

But there is more to this, since a correct result or construction can nevertheless include an error. This is a very characteristic insight of the Cartesian critique. A logically conclusive demonstrative process can still be questionable in geometrical terms, or to put it more precisely, wrong in terms of method. An additional criteria, conforming to the precepts of the method,

must be respected: that is the principle of simplicity. In the beginning of the third book of *Geometry* Descartes states this explicitly, giving an example. If a construction (in this case we are dealing with a construction of mean proportions) can be produced using different curves, it is imperative to use only the simplest one. To have recourse to a more complex curve, however impeccable the demonstration may be, is a mistake. It might be that the use of a more composite curve offers an easier construction; whatever the case, it must be rejected in favour of a less composite curve, even if the construction is thereby rendered more difficult. This approach conformed neither to the classical tradition, nor to modern mathematical procedures which put a premium on demonstrative elegance and deductive exactness. But the principle is entirely in keeping with the Cartesian method according to which it is necessary to respect this principle in order to maintain a proper procedure in science, unfolding it step by step without omitting a single stage, avoiding any jumps which would break the links of the chain of reasoning.

As we have seen, Descartes was looking for *the true use* of mathematics, and he does find it. It lies at the very heart of the method, and this is granted from the very birth of it. The *Regulae* and *Discours de la Méthode* bring new evidences of the same point. Intuition, the rule evidence, the decomposition of complex questions into absolute elements, the formation of more complex questions according to a specific order, the simultaneous consideration of all data, in a word, the precepts of the method are more obviously employed in geometry and algebra than in any other field of knowledge.

The privileged role of mathematics is stressed right from the start of *Regulae* since, as Descartes tells us, geometry and arithmetic are entirely adequate for the *certain touch of divinity* which is, in the human mind, the place where *the first seeds of useful thought already lie*.

Thus we are convinced that in these sciences **we** will find the sparks of certain truths. The proof is that, even without a good method, the fruits of geometry are already spontaneously remarkable.

While distinctions are made in the fields of knowledge and there are many different sciences, there is but a single and unique way for progressing from true intuitions to secure deductions and from the most simple statements to the most complex theorems. Now, this approach is best conceived and

understood in matters which are particularly simple and easy, such as geometry, arithmetic and algebra. The *Discours de la Méthode* is based on this argument.

No other science can offer so obvious and certain examples. But this should not be misinterpreted; if facility, adequation and exemplarity save mathematics as a discipline, it remains nevertheless no more than a trifle, concerned with useless problems which arithmeticians and geometers play to while away their leisure time.

Descartes' claim is not therefore to have provided mathematics with a larger body of fragmentary knowledge, it is of producing a unified picture. This project, announced in the *Discours de la Méthode*, consists specifically in replacing the different fields of mathematics by a unified theory.

Ease and certainty, the two characteristics of ordinary mathematics, are a sign of something more fundamental: a field open to human reason, but being so poorly developed and still producing such impressive fruits, is necessarily exceptionally fertile. What more could be expected by organising it and developing it according to the method! A mass of new results, original theorems, and above all the recognition of the power of the acting method, and thus, a new, reorganised, unified and well-structured mathematics. Naturally this second, dual result is far more important than an accumulation of new propositions, even if they are true.

On the other hand, the tools of the method were in any case going to be forged and sharpened in that very process of the reorganisation of mathematical knowledge. According to Descartes, it was through a consideration of the faults and advantages of the various fields of mathematics that he “came to think some other method had to be looked for”⁹. The most relevant examples relating to intuition were taken from that science (especially from geometry) and the process and validity of deductive chains were apparent in it.

“These long chains of reasonings, all of them simple and easy to understand, which geometers are accustomed to using to build up their most difficult demonstrations, had given me cause to imagine that everything which can be accessible to the human understanding may be logically ordered in the same way...”¹⁰. This confirms what the *Regulae* had already taught us, occasionally in greater detail.

A counterpoint lowering the elevated object is however ever-present, “considering that, among all those people that looked for true knowledge in the sciences, *only* the mathematicians were able to find out demonstrations, that is to say, certain and evident reasons, I had no doubt that it was by the same things which they had examined that I should begin, although I did not expect any other usefulness from this but to accustom my mind to feed on truths and not to be content with false reasons”.

Such statements should be kept in mind to explain why mathematics does not appear in the tree of philosophical knowledge represented in the Introduction of the *Rincipes de la Philosophie*.

And so mathematics was to remain modestly as a separate science, despite its grandiose role in the overall System. In its reorganised and properly elaborated form, the role of mathematics was to help form the real tools for attaining certain knowledge. *This is Professor F. Alquié's view when he writes* “we understand clearly the meaning of the last four precepts. They are drawn from his reflection of mathematical method, and Descartes hoped to extend the mathematical method to all the sciences”.

3. Criticisms of The Traditional Fields Of Mathematics

The Discours de la Méthode presents an inventory in which the three arts or sciences involved in mathematics, namely formal logic, geometrical analysis and modern algebra, are all found to be faulty and insufficient.

“This was why I thought I must look for some other method which would cumulate the advantages of these three branches and be free from their mistakes”³.

3.1. A Critique of Formal Logic

Logic claims to be an autonomous, distinct subject, a foundation beyond all knowledge. However, it has two particularly serious faults. Firstly, it replaces living thought with automatic thinking, and in that sense “*Syllogistical figures are of no help to discover truth*”

This is to say that this single, autonomous science is hollow, that it is dangerous since it has no object. A relatively simple solution is proposed: to dedicate the confident methods of the syllogism to living intellectual activity, one which produces knowledge.

The rules of syllogism can never surpass or obscure intuition and deduction, the only two true means to knowledge. Now, intuition can never be separated from its object in any way, and deduction can never be purely formal. The stages of reasoning which are so clearly identified in mathematics invalidate formal logic as a separate science. The logic of syllogism must be replaced by a living logic, an analysis which “*shows the real path by which something is invented methodically, and reveals how effects depend on causes; such that, if the reader wishes to follow it, carefully examining everything it contains, he will no less conceive the conclusion thus demonstrated or make it his own than if he himself had discovered it*”

3.2. A Critique of Classical Geometry

The geometry of the Ancients, the science of figures and construction of lines, offers both the best attributes and some of the worst faults to impede knowledge.

Geometry’s strength is that it is related to extension, which is essential to all material bodies. Its object is produced by an examination of dimensions and continuous quantity, and it is therefore a privileged path to true mathematics. In that way, Cartesian mathematics are above all geometry. I will return to this issue, but let us observe now that according to Descartes other mathematical fields were not to separate themselves from lines, nor from the continuous magnitudes of geometry which alone provide the content for symbolic processes.

Still, the geometry of the Ancients appeared to be doubly in fault. Firstly, because it was fragmentary. As M. Marie reminds us “*There was no link between the curves studied by the Ancients, nor was there any means of establishing one, so that the study of one was of no benefit to any of the others; finally, their very identity was not even established, for one and the same curve of some complexity endowed with an infinite number of different properties, will therefore have an infinite number of definitions which were often very difficult to bring into concordant relationships*”.

Secondly, traditional geometry was *too restricted to the consideration of figures* and depended too heavily on a strictly reproductive imagination which considered only such images resembling the objects causing them. The figures of the Ancient’s geometry were accepted because of the spatial reality they reflected, which was in fact a source of obscurity rather than clarity; consequently Descartes proposed to abandon this spatial realism¹⁴. The reform was intended to keep geometry in the field of continuous magnitudes or extension, while involving its lines in a general, orderly algorithm.

3.3. A Critique of the Algebra of the Modems

The algebra Descartes criticised is then that science which works on letters those operations which are supposed to be valid for numbers. The advantages Descartes recognised in the algebra of the Modems are well-known. This “*sort of arithmetic relieves the imagination*” and helps to make clear difficulties hidden by the confusion of numbers. However he also had considerable reproaches to make on this new field.

1. The notation was still confusing and heavy-handed; Descartes proposed a thorough going reform of it.
2. Many results concerning the solution of equations were still missing. The theory of equations and polynomial roots were developed arbitrarily and in a disorderly manner.
3. Above all, ordinary algebra was based on a particularly poor understanding of dimension. This restrictive notion was why algebraists could on-

ly deal with roots, squares and cubes by associating them with length, surface and volume, treated as three categories of magnitude.

The representation of all dimensions and powers by lines unquestionably outgrows the algebra of the Moderns. This form of representation was possible because dimension is nothing other than the mode in which a subject can be measured and thus all quantities can be reduced to length according to the method demonstrated in the first pages of *Geometry*.

It was necessary to envisage and bring about this *rapprochement* of geometry and algebra in order to reveal the real potential of mathematics.

The goal of the essential method presented at the beginning of *Geometry*, book one, should allow to reduce the study of figures to that of lines, indeed as straight lines, in order, at a later stage, to define those figures through the inter connexions of those lines. This method reduces disorder, the major fault of the geometry of the Ancients. The examination and classification of ratios between lines provides a criteria for the ordering and unity of geometry: from the most simple to the most complex ratios, this method covered the entire area of geometry.

It was only after this that the contribution of the Ancients could be incorporated, when needed. Their discoveries were worthy of consideration and were true. Descartes needed to give form to a geometry in which Euclidean propositions, Apollonius' theories of the properties of conicals and the theory of proportions, would all be demonstrated again. They would fit into position like puzzle pieces which have already been pieced together and which finally fit into the finished picture. J.L Marion writes "*In this sense the Method undertakes a recuperation of arbitrary inventions in terms of certainty*".

The idea of relating mathematical objects to lines naturally requires an examination of the rules for using these lines.

As a set of a general arithmetic, algebra could not be of any use in this project. It was therefore necessary to separate this algorithm and its connected symbols from the concept of number. The letters to be used in calculations and transformations would no longer represent numbers nor measurements, instead they were to indicate the length of lines.***

Although the ordinary objects of algebra were replaced, the rules of calculations were retained intact. The certainty of algebraic procedures, as received from arithmetic, had to remain unharmed and unaltered even as the nature of the objects designated by the letters of the algebraic script were altered. The procedures themselves were not at all in doubt and as Descartes emphasised from the time of the writing of *Regulae*, algebraic procedures respected methodical requirements: “*And by this means we not only save the space of many words, but what is more important, we shall make the elements of the difficulty so clear and simple, that, even while nothing useful is omitted, still nothing superfluous will ever be found in it which could encumber the mind when it is necessary to consider several things at once*”

Having posited that in geometry figures should be reduced to lines, Descartes named them with letters. The letters of algebraic geometry are not numbers, they are magnitudes. The property of these magnitudes is that they can be constructed and are therefore under the jurisdiction of the geometry of curves and figures and may be manipulated according to the rules of a specious algebra. This is why algebra, in Descartes' System, does not aim at an autonomous status, and it is also why the geometry of curves and figures is controlled by calculation. Lines are accepted in geometry because they can be expressed in terms of ratios which are regulated by algebra. Descartes' dream, or rather his conviction, was that the two criteria for acceptance as certain knowledge, namely the property of being exactly constructible and of being expressed algebraically, would turn out to be one and the same.

4. The Theory of Proportions

One of the central and most important ideas in Cartesian mathematics is the theory of proportions, which is the heart of his algebraic geometry. The innovations of Descartes' procedure were the new notation and the power of generalisation, however, the rules of transformation, simplification and the authorised procedures are founded in Euclidean theory. From *Regulae* to *Geometry* Descartes repeatedly stated that to replace a geometrical problem by its symbolic writing and the solution of that equation are but applications of that ancient theory: “*...those particular sciences which come together under the name of mathematics, even though their objects are different, are*

all concordant in so far as they consider only the different relations [ratios] or proportions to be found among these objects"²⁰.

Rule VI also demonstrates *the principal secret of the method*² : by means of an examination of a proportion which starts off simple and then, through an initial result, leads on to a problem of greater complexity, and from there to an even higher level of complexity, etc.. Rule XIV categorically states: "*Let us conclude safely and firmly that perfectly determined questions are devoid of difficulties, or nearly so, except the one of reducing proportions into equalities or equations, and any time such a difficulty is met, it can easily, and must, be let apart from all other matters and then be translated into extension and figures*"²².

Jules Vuillemin expresses his view categorically in this context when he writes: "*More than being just a theory of the parallelism between functions and curves, Geometry is first and foremost a theory of proportions*"

Which is no punctual conception of the method since, to quote Vuillemin again "*to the philosopher, the invention of analytic geometry was secondary to the invention of the universal method of thought contained... in the general theory of proportions*".

5. From Regulae To Geometry

The question of the relation of *Regulae* to *Geometry* can be best understood by recognising the role of the theory of proportions in Descartes' thought. The grand theory of ratios is the center of his statements of project. As stipulated in *Regulae*, the objects of knowledge, in their more general aspect, are orderly examined in terms of progressive proportions. Thus the theory of proportions is the key to the *Mathesis Universalis*. On the other hand, the theory of proportions is the means by which geometry becomes algebraic, thus unifying the mathematical sciences.

If a common, identical kernel (noyau) is acknowledged as being a part of the scientific Project presented in *Regulae* and in the 1637 treaty, the question remains, how much was the treaty a realisation of the *Regulae*? Was *Regulae*'s objective of establishing a universal science part of the same project to unify the fields in mathematics which were previously considered to be separate?

Not at all, and some commentators have considered the 1637 *Essay* as the abandon of the past program aiming at founding a science of everything available to human understanding.

6. The Mathesis, An Inaugural Project

Hence it is necessary to assess what Descartes' project was in the *Mathesis Universalis*. In a letter he wrote on the 26 March 1619 to his friend Beeckman, he confides his idea of “*a science with a new foundation, which would allow us to resolve in general all the questions one might ask oneself in whatever quantity, continuous or discontinuous, but each one according to its nature [...] An incredibly ambitious project*”.

Later on *Mathesis Universalis* was presented in *Regulae*. In Rule IV we are told “*this discipline ought to contain the first rudiments of human reason, and be broad enough to bring out the truths of any subject whatsoever*” and this is defined more precisely later on: “*there must be a certain general science which explains everything which can be asked about order and measure, and which is concerned with no particular subject matter, and that this very thing is called “universal mathematics“, not by an arbitrary appellation, but by a usage which is already accepted and of long standing, because in it is contained everything on account of which other sciences are called parts of mathematics. How much this science surpasses both in usefulness and in facility the others which are subordinate to it is apparent from the fact that it is concerned with everything with which the latter are concerned, and many other things as well*”.

Descartes adds that he has already pushed the study of this science far enough and that he will give the main outlines of his results; there follow the rules of the method.

7. The Perfectly Formed Questions of Geometry

As we have already seen, mathematics is both the proof that a universal science exists, but also acts as a mask to conceal it. Geometry, arithmetic and even logic are *the dressing rather than the constitutive parts* of the only science which is of real value. The Cartesian project does not aim at exhibiting that science in its absolute purity but at *dressing and adorning (orning) it, so that it could be more acceptable to the human mind. Since Mathesis Universalis* was purely intellectual, “*it is dressed with figures and numbers as a kind of tribute to the imagination, conceived as an auxiliary to the mind*”.

In this formulation there is a connection between *Mathesis Universalis* and a reorganised form of mathematics. *Mathesis Universalis* presents itself in masked form since it is, by essence, purely intellectual and abstract and cannot be brought before us for mathematical consideration, rather we consider its masks and clothing as found in those perfectly posed questions which are geometry and algebra. Then these sciences which have been dressed in their adequate veil are in a position to be examined by reason, aided by the imagination.

In order therefore to save the decorations (figures and numbers) from mere vanity it must be stressed that they are but the outfitting of another knowledge, far grander than the one to be found in their own proceedings.

One will not attempt to do away with the mask or the clothing, but it is extremely important to identify it as such, or else (sinon) the only object of study would be a sheet of ectoplasm, and not the appearance behind which the most sublime method of using the human understanding is accessible.

An examination of the structure of the *Regulae* will help us define the relationship of algebraic geometry to *Mathesis Universalis*^{***}.

After the presentation of precepts related to simple *propositions* in the first twelve rules there is an examination of *perfectly understood questions* in the next twelve rules. Following. Finally the last twelve rules were to deal with *imperfectly understood questions*. This section was never written, but it can

still be compared to the part which was written, even though this too was incomplete. Descartes outlines a study project and methodology completely adapted to mathematics in the general sciences section in which constants and variables, premises and conclusions, known and unknown terms are so clearly identified that the problems can be posed in the form of an equation.

The important role of figures and algebra in the nine rules drawn up in this section was inevitable since perfect questions are only found in arithmetic and geometry. But Descartes also indicates that it is necessary to pass *through* this part of the method to arrive at the next stage. This following stage was to be concerned with precisely those questions which were not perfectly formed, for instance in the experimental sciences, the natural sciences and parts of physics such as magnetism and sounds.

Does the third Essay achieve the project envisaged in *Regulae* On the basis of my preceding observations, the answer is no, in as much as *Geometry* is obviously not the *Mathesis Universalis* treatise. And yet we can view this in a far more positive light if we refer to the writings in the *Regulae* concerned with the examination of *perfectly formed questions*. The 1637 *Geometry* does achieve this part of the general project even with all its inadequacies and errors.

It is then possible to claim that the 1637 treatise does not reflect a change in the project, to say that it is faithful to the hopes of the 1620s, even if it only completes the first part of the project. It is therefore a partial but decisive success since what can be known is understood by applying the precepts of the method, and the world and all the sciences which can be used to understand it make use of the essential attribute of matter which is extension, and extension is understood by means of the science of continuous magnitudes laid out in *Geometry*. This is why Descartes wrote to Mersenne, "I claim to have demonstrated [the superiority of the Method] through my *Geometry*".

Conclusion

Let us therefore reconsider the four Cartesian fields involved in this discussion.

- Traditional mathematics. In this are found both the seeds (graines) of divine certainties and the criteria for generalisation. It is a disorderly, poorly developed discipline which runs the risk of being fruitless.
- *Mathesis Universalis* is the general science of ordering and measurement. It is concerned with universal knowledge. This is the only science with which *Regulae* are concerned.
- The Method is a strategy for the acquisition of all certain knowledge. This is also presented in *Regulae*, as well as in the *Discourse* of course.
- Algebraic geometry. This is the unified image, the scaffolding (échafaudage) or architecture of things understood in terms of extension and perfect definition. It is served by algebra.

And here are their respective positions as I understand them.

Geometry is obviously not *Mathesis Universalis* since it is not concerned with imperfectly known questions. But *Geometry* is nevertheless adequate, in conformity with the method as stated in *Regulae* and in *Discourse*.

Geometry is close to *Discourse* not only in terms of publishing chronology, but also because a more precise wording, like “*I will take the best one of them...*” reflects the maturation of the project.

Geometry is however much more readable and more structured by statements from *Regulae*, particularly those concerning the theory of proportions, the absolute and relative characteristics of the elements of a question and the formation of equations from problems, even though important aspects of the doctrine had evolved.

Thus ordinary mathematics fed and inspired the method like a muse. The method was brilliantly illustrated and supported by *Geometry* without being accused of being a ‘deduction’ or ‘application’ of the *Discourse*.

Mathesis Universalis can also claim *Geometry* as one of its elements. Thus the *Regulae's* project was realised, within limits indeed, but in this realisation it was remarkably complete. The coupling of 'traditional mathematics-reorganised mathematics' is a particular expression of the coupling 'individual sciences-Mathesis *Universalis*'. In this sense to succeed in the passage of the first coupling is a partial realisation of a corresponding passage in the second coupling. The success of this passage of course depends upon the method.

It has been said that for Descartes to know is to construct; this claim informs the radical constructivist thesis. It is true that the way in which we understand exterior objects is through extension (as also for geometrical objects). It is also perfectly correct that the first certainties, the first knowledge, that we acquire in geometry are concerned with extension. Likewise, the first certain knowledge we have is generally intuition.

It goes without saying that a science restricted to the founding intuitions would be more or less meaningless. Scientific statements are composed from the results of long deductive chains. To know, for Descartes, is thus above all to be able to guarantee the validity of these chains of reasoning. While the first two precepts of the method are related to intuitions, those following them are concerned precisely with the processes of creating and mastering deductive chains. Without them there would be no method, and therefore no science, neither universal, nor particular.

Now, if extension, or lines themselves, are the constituents of geometrical intuitions, the rules of algebra are the means of controlling the deductive chains of geometry. This is the subject matter of the beginning of Book I. One could thus say that algebra is to geometry what the third precept of the method is to the first, and what the procedures for the creation of deductive chains of reasoning are to intuitions.

This seems to be an important point to make since it reduces what all too often appears to be a conflict between knowledge by geometrical construction and knowledge by algebraic generalisation. This is to contrast a constructivist Descartes to an algebrist Descartes even within his own single mathematical treatise. Admittedly this conflict is present from the point of view of the history of mathematics, but it makes little sense in the context of Descartes

the philosopher-mathematician. As the particular appearances of a science demonstrated according to the precepts of the method, algebra and geometry, the two complementary aspects of an orderly and guaranteed area of knowledge, could not be opposed to each other. Philosophically this is understood, and *Geometry* provides some brilliant mathematical arguments in favour of an adequation of knowledge attained by the construction of curves and solutions and the knowledge of algebraic expression. Of course in mathematics argument does not replace demonstration, and this is lacking. What is -mathematically speaking- missing in Descartes' work is precisely the demonstration of this main contention: what we may know through construction, we may also know through algebraic symbolism.

I would even be tempted to add 'and reciprocally'. To my mind this absence is in principle unimportant or harmless: the certainty of the general methodical and philosophical reasons, secured by mastery of a series of argument-examples in which truth can be seen as dressed geometrically and then algebraically, is sufficient⁵. The reception of the third Essay in *Discourse on Method* was rather cool. It was certainly a failure in the eyes of the public, and a stubborn reticence was most of the specialists' responses to the treatise. There were weighty polemical reasons for this response, but all the same they are not enough to explain this reaction.

The greatest misunderstanding was a result of the fact that Descartes thought that he had, and indeed claimed to have, 'exhausted geometry', which was obviously untrue, as further developments of the science soon proceeded to demonstrate. The Cartesian claim must however be treated seriously for it is not unfounded. The switch to algebra, a decisive move of Descartes', will soon explode the traditional boundaries of mathematics: the character of numbers and lines, the possible new algorithmic expressions, the new writing were soon to free themselves of their geometrical roots. New equations and new loci which found algebraic expression expanded the limits of the mathematical science. This is suggestive of Leibniz's thought, which showed an exceptionally clear awareness of the expansion of mathematics. Analytic and infinitesimal geometry, the theory of equations, transcendental curves... these were the vast new chapters to be opened just after the composition of *Geometry*.

Now, what is the matter for Descartes in the third Essay? It was certainly not an attempt to transform the nature of the objects of this science. The

stakes (enjeu) involved in the great algebraic translation did not appear immediately. Initially it was surreptitious (subreptice), *and* appeared to be a commodity whose essential consequences the authors (Descartes, as well as Fermat) did not immediately recognise: the powerful overthrow (renversement) of the contents, objects and even the methods of mathematics.

The establishment of a new way of writing mathematics only seemed to be a memory aid, a way of extending the traditional geometry of figures and curves more clearly, which was no meaner achievement.

Born as a memory aid, as an abbreviated notation, switch to algebra will find its accomplishment by exploding the very status of mathematical objects and their relations.***

All the rigour of Cartesian geometry, and even its metaphysics were needed to restrain and withstand the impetuous stream he had freed when he blew up the flood gates of symbolic writing.

It was indeed a question of memory. The overloaded memory and the accumulation of information did not only lead to forgetting, it also made their presence confusing to understanding. The intellectual economy also aims at clarifying contents so as allow an actual and faithful memorisation.

This is the idea which Leon Brunschvicg considers as being common to both Descartes and Spinoza: *“The characteristic of Cartesian geometry is that it applies an original method to problems which already were or could have been resolved through the System of synthetic reasoning of the Ancients. Without properly speaking, modifying the reality to which the mathematics were applied, this approach transformed the way in which the mind applied itself to this reality; it restricted the imagination and put into play the activity of intelligence”*^{3^}.

Cartesian memory is the method at work, and in this case algebra organises knowledge of extension.

The enterprise was thus not to transform the nature of the mathematical sciences but to transform the study of it. Then, an illusion arose, a chimera: the switch to algebra appeared to be a definitive solution, an end to confusion and blindness, and ultimately it seemed to announce, or even to realise, the

closure of mathematics. This illusion is understandable: without changing the nature of mathematical science itself, in the context of what the reasonable and rational exercise of understanding authorised to be examined and understood, in a word, by not expanding the field of legitimate knowledge and at the same time multiplying the means of investigation of the human mind adventuring in the realm of geometry, by considerably increasing its ability to synthesise and make deductions, it was very logical to consider that it would be possible to explore the entirety of mathematical knowledge.

In order to understand this illusion more completely it would be necessary to examine how Descartes treated those mathematical questions which were obviously not contained in the framework he had established. He both resolves them and rejects them. Such is the case when he achieves, through the method of limits his solution of the fall of bodies in the early exchanges with Beeckman, an other example would be his quadrature of the cycloid which he managed by a method of indivisibles⁷ but whose importance he underestimated; a third example comes up with Debeaune's remarkable problem for which he gave the transcendental solution while at the same time claiming that this was the result of two movements which were "*so incommensurable that they cannot be adjusted exactly to each other [...] this line is of the kind that I rejected out of my Geometry as being only mechanical ones*"; such again are those imaginary roots which he allowed to appear in his third book but to which he only ever accorded some sort of "spectral existence"⁹.

Thus are preserved the boundaries of the country which he pretended to have entirely explored.