USE OF *DRAGGING* AS ORGANIZER FOR CONJECTURE VALIDATION

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In this article, we report on a study centred on the teaching and learning of proof in which there is evidence that dragging becomes a source for significant student participation in the validation of conjectures. The findings highlight the teacher's use of dragging as an organizer of the activity, in cases when there are conjectures that students consider acceptable but for which they do not have the theoretical elements to validate them.

INTRODUCTION

The teaching and learning experience of proof reported here took place in a university plane geometry course, during a problem solving activity, which required a construction carried out with a dynamic geometry program. The problem is one of a set of tasks proposed throughout the academic term to favor student participation in the collective construction of part of an axiomatic system. Students get involved in the exploration of geometric figures, formulation or interpretation of conjectures and their proof. Our general premise is that genuine student participation in the production of ideas with which mathematics knowledge is constructed —thanks to the dynamic geometry context — leads to a significant approach to proof.

We think that this experience contributes to the request formulated by Herbst (2002) which expresses the need to devise class organizations that favor student participation in proof formulating activities, for the different levels of education. Particularly, we want to communicate a novel use of the *dragging* function — specific to dynamic geometry software— employed by the teacher to treat some of the conjectures that the students consider acceptable but cannot validate since the required theoretical elements are not yet part of their axiomatic system, since it is constructed throughout the semester. A review of the literature shows that studies carried out about the *dragging* function in teaching and learning to prove have centered mainly on how students use it to solve problems (e.g. Olivero, 1999; Arzarello et al., 2002; Stylianides and Stylianides, 2005) but its potential use as a class activity organizer has not been explored enough.

THEORETICAL REMARKS

The context in which the activity that we report took place is based on the following ideas about proof and learning to prove. For us, *proving activity* includes two

2009. In Tzekaki, M., Kaldrimidou, M. & Sakonidis, H. (Eds.). *Proceedings of the 33rd Conference of the International Group for the Psychology of Mathematics Education*, Vol. 2, pp. 257-264. Thessaloniki, Greece: PME.

processes, idea that coincides with the proving process described by Olivero (2002). The first process consists of actions that lead to the production of a conjecture; these actions generally begin with the exploration of a situation to seek regularities, followed by the formulation of conjectures and their validation. The actions of the second process are concentrated on the search and organization of ideas that will become a proof; this is considered as an argument of deductive nature based on an axiomatic system in which the proven statement can be included. In this sense, we coincide with other researchers (Hanna, 2000; Mariotti, 2006) who consider proof as the fundamental activity mathematicians carry out to remove any doubts about a statement's truth and to organize ideas in a deductive discourse, with the purpose of validating it within a theoretical system. Since the principles and deduction rules that govern the production of the discourse are established by a specific human group, we recognize the sociocultural character of the proving activity, conditioned by the context and the specific domain within which it takes place (Alibert & Thomas, 1991; Hoyles, 1997; Radford, 1994; Godino & Recio, 2001; Mariotti, 2006).

We view the mathematics class as a community of practice (Wenger, 1998) in which students have the opportunity to learn to prove as they commit themselves with a repertoire of practices suitable for proving activity. With these practices, they gain competency and develop ideas of what it means to prove and how they can participate legitimately in the production of proofs. Undoubtedly, the repertoire of practices is conditioned by the class community resources available to carry out the proposed enterprise and the norms that are negotiated for their use. Particularly, when a dynamic geometry program is available, the use of its functions becomes a characteristic aspect of the community's practices. Sometimes, in midst of proving activity actions in class, an unplanned use of a software function can appear, that is later evaluated as favorable for the practice that is taking place and considered as a useful form of taking advantage of the dynamic geometry program. It is the case of the use of the *dragging* function that we discuss in this article.

RESEARCH CONTEXT

The paper focuses on a series of two 2-hour sessions in which pairs of students taking a university level geometry course were asked to solve the following problem, using dynamic geometry software (no figure was included):

> Given line m and two points P and Q in the same half-plane determined by m, determine point R on m for which the sum of the distances PR and RQ is least. a) Describe the geometric construction used to find R. b) Formulate a conjecture. c) Write the main steps of the proof of the conjecture.

The sessions took place at the end of the semester. Throughout the semester, the students had participated in the collective construction of a portion of the Euclidean geometry axiomatic system including properties of angles and triangles. They were used to solving open problems and could skillfully use dynamic geometry software to explore figures and verify conjectures. The data collected consists of transcriptions of class audio and video recordings complemented by video recordings of private conversations of the teacher with each group of students and field notes of one the members of the research group, who acted as a non-participant observer.

In the next sections, we analyze the way students and teacher used dragging during the solution of the problem and the process of validating conjectures.

USUAL USE OF DRAGGING

Ten pairs of students were formed. All groups constructed the required elements, measured the segments and found the sum *PR* + *QR* (Figure 1).

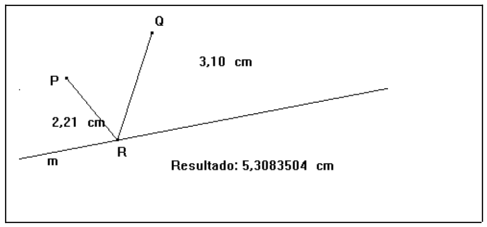


Figure 1: Initial construction

Each group began the exploration process using "linked" *dragging* (Arzarello et al., 2002), that is, moving point *R* on line *m* to determine the position of the solicited point *R*. When they were sure that such a point existed, they searched for the geometric properties that characterize the position, which led them to make auxiliary constructions, find measurements, and move points *P* and *Q,* to determine special configurations or regularities in the figure. Each group wrote their result of the exploration as a conjecture.

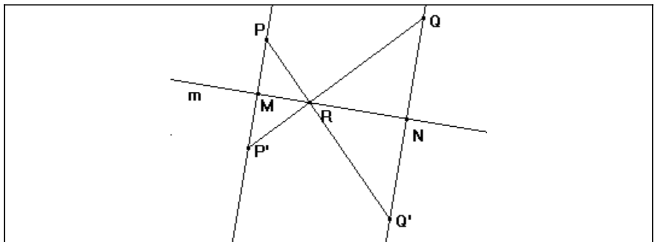
Due to her conversations with each group, the teacher obtained information about the exploration process carried out, the conjecture formulated and the ideas brought up for the corresponding proof. In the explanations that three of the groups gave to the teacher, their use of "linked" *dragging* to verify whether the conjecture was plausible is mentioned.

Having found what the different student conjectures were, the teacher organized them according to their degree of complexity and then moved onto the discussion of results. Some groups presented their conjectures, showing their Cabri representation. Using *dragging*, the class decided whether they were acceptable or not. Seven different conjectures were proposed, one of which was refuted by a student, at the end of the second session, by showing a counterexample by *dragging*. With the presentation of the conjectures, the first session terminated. The teacher asked the students to work on a proof of their conjecture. She suggested using another point on line *m* and comparing the sum of their distances to *P* and *Q*.

In sum, in the course of solving the problem and verifying the formulated conjectures, the students used *dragging* as a means to explore and verify the properties of the figure they constructed. This use of *dragging* has been widely documented (e.g. Olivero, 2002, Arzarello et al., 2002).

NOVEL USE OF DRAGGING IN THE ORGANIZATION OF THE CONJECTURE VALIDATING ACTIVITY

The teacher started the second session asking students for their proofs. Darío offered to prove Leopoldo's and his conjecture (Figure 2). To do his proof, Darío explained that he only needed segment *P'Q* because its intersection with segment *PQ'* is on line *m.* (Figure 3(a)).



P' and *Q'* are points on the lines, perpendicular to *m,* through *P* and *Q* such that *PM* = *MP'* and *QN* = *NQ'*. If *R* is the intersection point of *PQ'* and *P'Q* then *PR* + *RQ* is the minimum sum.

Figure 2: Darío's and Leopoldo's conjecture

Darío used triangle congruency criteria to show that segment *PR* is congruent to segment *P'R* and, therefore, that the sum of *PR* and *RQ* is equal to the sum of *P'R* and *RQ*. (Figure 3(b)):

Darío: [...] since I have a point [*M*] on the line and a perpendicular [to *m* through *M*] and I draw $\overline{PR}$, then [the triangles] *PRM* and *P'RM* are going to be congruent using side-angle-side.

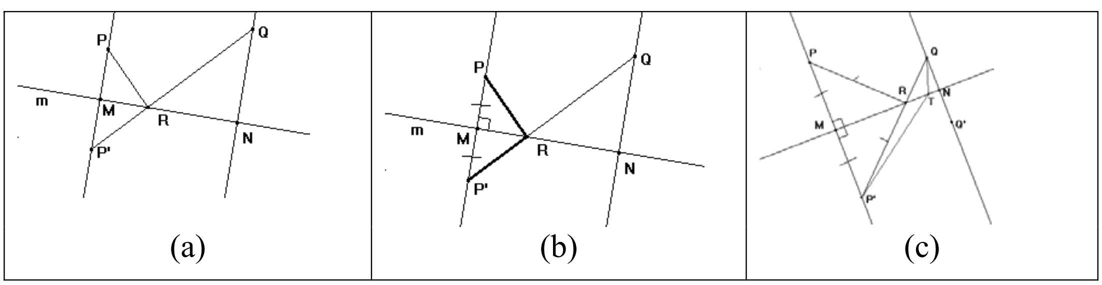


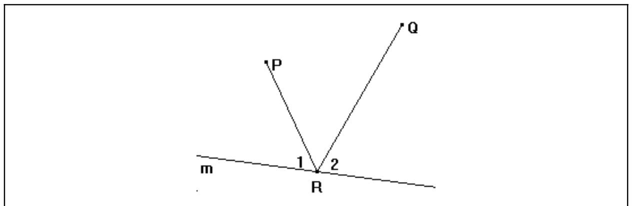
Figure 3: Figures that support Darío's proof

He then used the Triangle Inequality Theorem to show that for any other point *T* on line *m*, the sum of *P'T* and *TQ* is greater than *P'R* plus *RQ* (Figure 3(c)):

Darío: [Draws segments *P'T* and *TQ*, Figure 3(c).] Yes… then we do not have *T* between *Q* and *P';* then we have a triangle. By Triangle Inequality, I have that *P'T* plus *TQ* is greater than *P'Q*; that is, *P'T* plus *TQ* is greater than *PR* plus *RQ*. And this happens with any point that I use.

The student uses in his proof theoretical statements of the axiomatic system that the students have at their disposition, linking statements starting from the properties they fixed for point R in their construction. *Dragging* does not play any role in the validation of his conjecture.

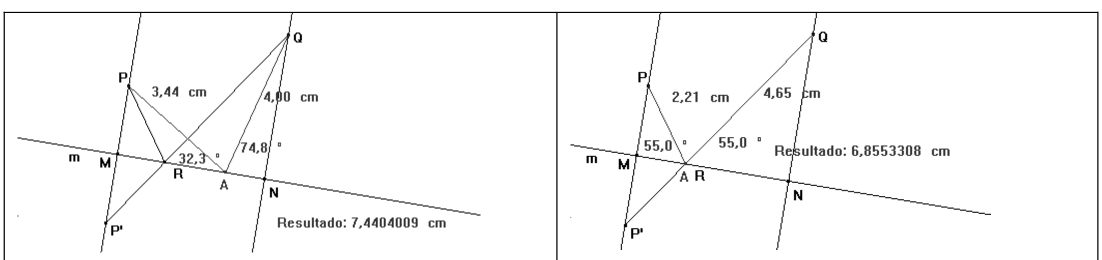
Afterwards, the teacher invites the students to look at Henry's and Antonio's conjecture (Figure 4) and makes them notice that it refers to congruent angles, a geometric property that can be checked, but that they did not give a geometrical construction proposal for point R, a marked difference with Darío's and Leopoldo's conjectures.



If angles 1 and 2 are congruent then $PR + RQ$ is the minimum sum.

Figure 4: Henry's and Antonio's conjecture

At this point, the teacher could have opted for explaining to the students that there was no way to validate Henry's and Antonio's conjecture, using the available axiomatic system. According to the norms established in the class, since the conjecture could not be validated, it had to be discarded and could not become an element of the axiomatic system being constructed. The student's effort would not be valued as relevant mathematical production. Instead, the teacher decided to take advantage of *dragging* to find a way to validate the conjecture. She projects Darío's Cabri construction on the wall, locates point A, different from R, on line m, constructs $\overline{PA}$ and $\overline{AQ}$, finds the sum of the distances and, additionally, measures $\angle PAM$ and $\angle QAN$ (Figure 5 (a)). Then, she *drags* point A, until the sum of the distances became a minimum; at that moment the angle measurements were equal and R and A coincided; this meant that the constructions proposed by both groups produced the same point (Figure 5 (b)).



(a) (b)

Figure 5: Comparing conjectures

Due to the comparison carried out by *dragging* of the conjectures, the teacher's idea is to use syllogisms to prove that point *A,* as proposed in Henry's and Antonio's conjecture, has the same geometric properties of point *R*, as suggested in Darío's and Leopoldo's conjecture. Since Darío had already proven that *R* satisfied the condition established in the problem, they could conclude that *A* also satisfied the condition. The teacher suggests using as the only "given" condition that $\angle 1$ and $\angle 2$ are congruent (Figure 6 (a)) – as the conjecture states – and to prove that *A* is collinear with *Q* and a point on the perpendicular line *PM* the same distance from *m* as *P*, as indicated in Darío's conjecture (*P'* in Figure 2).

María, another student, suggests drawing the ray opposite to $\overrightarrow{AQ}$ and finding point *S,* the intersection of that ray with $\overleftrightarrow{PM}$. As María explains, this guarantees that *S, A* and *Q* are collinear, due to the definition of "opposite rays" (Figure 6 (b)). Showing that the distance from *S* to *M* is equal to the distance from *M* to *P* remains. Melisa shows this is true because triangles *PMA* and *SMA* are congruent since $\angle 1 \cong \angle 3$ ($\angle 3$ and $\angle 2$ are vertical angles and the latter is congruent to $\angle 1$, Figure 6 (c)), angles *PMA* and *SMA* are right angles and the triangles share $\overline{MA}$. Therefore, *PM* is equal to *MS* and point *S* corresponds to point *P'* of Darío's and Leopoldo's conjecture.

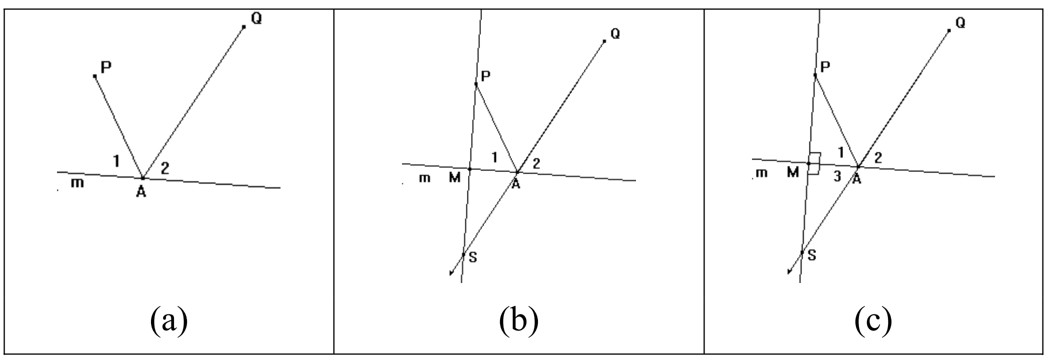


Figure 6: Figures that support theoretical validation of Henry's and Antonio's conjecture

This way, they prove that if angles 1 and 2 are congruent, then *A* is collinear with points *S* and *Q* and *PM* is equal to *MS*. Therefore, point *A* is R. They have already proven (Darío's and Leopoldo's conjecture) that if *R*, *S* and *Q* are collinear points, and *PM* is equal to *MS* then *PR* + *RQ* is the minimum sum. Therefore, they concluded that if $\angle 1 \cong \angle 2$ then *PR* + *QR* is the minimum sum.

To summarize, since Henry's and Antonio's conjecture did not provide geometric properties that were useful for a proof that is within the available axiomatic system, the teacher suggests using *dragging* to verify the coincidence of point *A* and point *R*. The latter point was obtained through a geometric construction that does provide the necessary elements to construct a proof. This allowed validating the conjecture in a way in which, instead of trying to show directly that the sum *PA* + *AQ* is a minimum, the teacher, together with the students, constructs a deductive argument to show the coincidence of points *A* and *R*. Through this ingenious resource, the teacher organizes the validating activity of Henry's and Antonio's conjecture.

CONCLUSIONS

The analysis of the events that we carried out permits us to state, as other researchers have mentioned (Olivero 1999, Olivero 2002, Arzarello et al, 2002; Stylianides and Stylianides, 2005), that the *dragging* function has an important role in the generation of a favorable environment for learning proof, not only during the exploring, discovering and conjecture verifying moments, but also as a means to generate ideas that are a source for the construction of a proof.

In this article, we emphasize the teacher's creative use of *dragging* to organize student activity during the proving activity. Undoubtedly, since the validation done does not ascribe to the type of proofs constructed in class, it is quite improbable that a student would have thought it up. This is why we think that *dragging* becomes a teacher resource with which student proving activity can be fostered.

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