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Learnings and Reflections by (Future) Teachers on Anticipation in Exploratory Mathematics Teaching

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ABSTRACT

Background: Given its nature guided by inquiry, collaboration, communication and reflection, Exploratory Mathematics Teaching (EMT) practices are considered challenging to the teacher, particularly due to the characteristics and role given to the action of anticipating. **Objectives**: Investigating learnings arising from teachers' reflections based on the anticipation of EMT practices**. Design**: It is a qualitative study, of interpretive nature, about collective reflections of Mathematics teachers who developed EMT practices raised in a focus group. **Setting and Participants**: Five teacher participated in the study after accepting a voluntary invitation, sent by email, for a focus group conducted remotely through the Google Meet platform. When conducting EMT practices, two of them were already working on Basic Education and three others performed them in actions of supervised internship, research projects, or extension ones during their undergraduate course, which justifies referring to them as future teachers in this study. **Data collection and analysis**: The data were produced from a focus group, guided by a script that focused on teachers' reflections about EMT practices, whose video recording was transcribed; and, in this study, the aspects related to the action of anticipating were focused. The analyses problematize, in an interpretative way, actions, reflections, dilemmas, possibilities of referrals and considerations about the EMT, in reference to a framework that guides practices of this nature. **Results**: The results point out some demanding actions for the teacher anticipating and learnings raised by reflection on these aspects highlight changes in understanding planning and the mathematical task, as well as about the roles played by the students and teachers. **Conclusions**: While the study highlights the potential investigations located in practices to emphasize specific ways of appropriating demanding teaching perspectives – such as Exploratory Teaching – it highlights the

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effectiveness of said practices and reflects on them as potentially enrichening for professional learning and for expanding theoretical frameworks for guidance.

Keywords: Teaching Professional Learning; Exploratory Mathematics Teaching; Professional Practice; Teachers' Reflections; Focus Group.

Aprendizagens e reflexões de (futuros) professores sobre o antecipar no Ensino Exploratório de Matemática

RESUMO

Contexto: Em virtude de sua natureza orientada pelo *inquiry*, colaboração, comunicação e reflexão, práticas de Ensino Exploratório de Matemática (EEM) são consideradas desafiadoras ao professor, particularmente pelas características e pelo papel conferido à ação de antecipar**. Objetivos**: Investigar aprendizagens suscitadas em reflexões de professores, esteadas na antecipação de práticas de EEM. **Design**: Trata-se de uma pesquisa qualitativa, de natureza interpretativa, acerca de reflexões coletivas de professoras, que desenvolveram práticas de EEM, suscitadas em um grupo focal. **Ambiente e participantes**: Participaram do estudo cinco professoras que aceitaram convite voluntário, enviado por correio eletrônico, para uma seção de grupo focal realizada remotamente, por meio da plataforma Google Meet. Ao realizar as práticas de EEM, duas delas já atuavam na Educação Básica e três as realizaram em ações de estágio supervisionado, projeto de pesquisa ou extensão, durante o curso de licenciatura, o que justifica referi-las, no estudo, como futuras professoras. **Coleta e análise de dados**: Os dados foram produzidos a partir de um grupo focal, orientado por um roteiro que focalizou aspectos reflexivos das professoras acerca de práticas de EEM, cuja gravação em vídeo foi transcrita e, neste estudo, focalizam-se os aspectos relacionados à ação de antecipar. As análises problematizam, de forma interpretativa, ações, reflexões, dilemas, possibilidades de encaminhamentos e considerações sobre o EEM, em referência a um *framework* que orienta práticas dessa natureza. **Resultados**: Os resultados apontam algumas ações exigentes para o professor no antecipar e aprendizagens suscitadas nas reflexões sobre esses aspectos salientam mudanças de entendimento a respeito do planejamento e da tarefa matemática, assim como acerca dos papéis de aluno e professor. **Conclusões**: Ao mesmo tempo em que o estudo destaca o potencial de investigações situadas nas práticas para evidenciar formas próprias de apropriação de perspectivas de ensino exigente – como o EEM – evidencia a efetivação de práticas e a reflexão sobre elas como potenciais para a aprendizagem profissional e para a ampliação de quadros teóricos de orientação.

Palavras-chave: Aprendizagem profissional docente; Ensino Exploratório de Matemática; Prática profissional; Reflexão de professores; Grupo focal.

INTRODUCTION

Exploratory Mathematics Teaching (EMT) can be understood as an expanded understanding of inquiry-based teaching (Oliveira & Cyrino, 2013). This perspective differs from traditional teaching due to the roles the teacher and the students play, the mathematical tasks proposed and class dynamics (Ponte, 2005). This approach brings the student to the center of the pedagogical process and confers the teacher a mediating role, instead of allowing them to monopolize knowledge and merely transmit it to their students.

Different works dealing with EMT present potentialities and contributions in relation to learning, to the students' mathematical communication and the improvement of the teacher's practice (Cyrino, 2016; Ponte, 2014). However, they also show that dynamics carried out in this perspective are really demanding, both for the students, in regard to the nature of the tasks and activities performed, and for the teacher, due to the actions necessary for the preparation and effectiveness of the class dynamics (Estevam, Cyrino, & Oliveira, 2017). Thus, this research focuses on this second component, allowed as a problem related teacher's experiential knowledge, to identify and problematize reflections on actions related to teacher practices guided by the EMT perspective, with a particular focus on the preparation stage, which precedes classes.

In particular, this article seeks to investigate, based on a focus group carried out with teachers and future teachers who performed EMT practices, learnings raised in reflections of these teachers based on the anticipation of EMT practices. In this sense, teachers' reflections are understood as aspects that identify and recognize important situations and actions, as well as reveal reasoning and interpretations that highlight understandings, potentialities, difficulties, dilemmas, and possibilities related to this type of pedagogical practice. These reflections are considered to suggest changes in the patterns of teacher participation in professional teaching practice, which are manifested in what they do or do not do (or say), as well as in the meanings that sustain these sayings and doings, thus affecting the way they see themselves. Therefore, they denote professional learning in a social perspective (Vilas Boas & Barbosa, 2016).

In the following sections, we present the theoretical frameworks focusing on the stage of anticipation of EMT practices, with emphasis on teacher's actions. Then, aspects related to learning and teaching professional practice are discussed. In the methodological section, the process of constitution and implementation of the focus group is presented, as well as the

profile of its participants. Finally, in the results section, the teachers' reflections are presented alongside the analyses, based on theoretical aspects that guide this study. The last section explains the conclusions of the work and some considerations.

ANTICIPATING EXPLORATORY MATHEMATICS TEACHING

In EMT practices, students are encouraged to communicate their conjectures, strategies, and difficulties, as well as to question their ideas and those of their colleagues, to reflect on needs, potentialities, and resolution strategies, involving a process of collaboration and negotiation in the classroom. In this sense, four fundamental aspects of this kind of practices stand out, namely: collaboration, inquiry, reflection, and communication (Chapman & Heater, 2010).

For the mobilization of these aspects, a class in this perspective is normally organized in phases, which admit different structures and forms of organization. Among these different structures, we consider the one discussed by Cyrino and Teixeira (2016), which relates the teacher's actions to five phases, in the following way: proposing the task; monitoring task resolution; selecting and sequencing resolutions for discussion; discussing resolutions; and systematizing learnings.

When proposing the task, the teacher must explain the dynamics to be carried out, as well as enable its resolution, the organization of the students, their role and that of the students, and the time allotted and the resources used. Likewise, they should encourage the emergence of different resolution and communication strategies, considering the planned actions and situations that may occur, anticipated in the planning stage.

When monitoring the resolution of the task, which aims to identify the reasoning and emerging strategies, in addition to the potential of the resolutions for the objectives of the class (Stein *et al.*, 2008), the teacher needs to observe, question, and guide the work carried out by students, considering the anticipation of possible resolution strategies that may emerge, as well as doubts or errors.

The action of selecting is performed by the teacher when identifying resolutions with the potential for discussions appropriate to the purpose of the class, according to criteria previously established in the anticipation stage

regarding different strategies, reasoning, and representations. In turn, the action of sequencing aims to order the students' resolutions, also according to pre-established criteria, and it may consider recurring errors, particular or well-founded resolutions (Canavarro, 2011).

Discussing the resolutions, in turn, consists of the moment students are invited to share their resolutions, according to the teacher's guidelines, who must request justifications and encourage communication between students, using what was planned. The aim is to oppose ideas, establish relationships between the resolutions, and analyze their specificities and mathematical potential according to the objectives of the class (Canavarro, 2011).

Finally, the action of systematizing learnings involves, with the collaboration of the students, the formalization of the results of previous discussions. Thus, it is possible to insert and/or discuss, for example, representations, concepts, and properties, articulating mathematical knowledge with emerging resolutions and those foreseen in the planning stage (Cyrino & Teixeira, 2016).

From the EMT perspective, it is possible to perceive that before, during and even after the students' work, the teacher plays complex roles and performs some interferences with different intentions. Thus, Cyrino and Teixeira (2016) summarize these actions in Table 1, explaining elements that make up the action of anticipating.

Table 1

Teacher actions before class (Cyrino & Teixeira, 2016, p. 86)

Stage	Action	Elements that compose actions

Before class	Anticipating	▪ Establish the specific objectives of the class.
		▪ Choose/adapt/elaborate the task(s), considering:
		  ▪ the objectives of the class.
		  ▪ the nature of the task, prioritizing those with a high level of cognitive demand.
		  ▪ the students' prior knowledge.
		  ▪ the resources available at the school.
		▪ Solve the task(s).
		▪ Predict possible student resolutions, doubts, and errors.
		▪ Think of possible questions, guidelines or other resources that can be suggested to students,

Stage	Action	Elements that compose actions
		making sure to maintain the level of cognitive demand. ▪ Establish connections between: ▪ the planned resolutions. ▪ the planned resolutions and the mathematical knowledge to be developed in the classroom.

Thus, in the anticipation phase, carried out prior to the implementation of the practice, the teacher initially seeks to establish learning objectives enabled by the dynamics to be carried out, chooses the set of tasks to be explored in the class(es), and takes into account aspects and possibilities regarding how students can think and learn under the proposed conditions (Simon, 1995 apud Serrazina, 2012).

In this sense, the task is understood as a proposition made by the teacher to focus students' attention on a mathematical idea (Jesus, 2011), in which EMT practices privilege ideas with high level of cognitive demand (Stein & Smith, 1998). Kraemer (2008 apud Serrazina, 2012) also emphasizes that these tasks must also be able to meet the established objectives. However, it is not enough to choose good tasks, it is necessary to be careful with how their accomplishment in the classroom is proposed and conducted (Ponte, 2005), since the teacher can change them, even unintentionally (Jesus, 2011). This occurs, for example, when they reduce the cognitive demand of the task by indicating (re)solutions, manage time and the classroom inappropriately, and do not hold students accountable for the results found (Stein & Smith, 1998).

In addition to the task, the teacher needs to organize structural situations related to the delimitation of time, space, resources used and student organization. These anticipation actions allow flexibility while conducting the class (Oliveira, Menezes, & Canavarro, 2013). Likewise, the importance of considering the characteristics of the students, the way in which they engage with the tasks, and the previous knowledge they have is highlighted, so that the teacher is able to foresee situations that may emerge and interventions to be carried out (Cyrino & Teixeira, 2016). For this, it is important for teachers to know their students so that they can, to some extent, predict the ways in which students can deal with class proposals from the perspective of EMT (Estevam, 2016).

Taking into account the action of anticipation and the influence of the task in the EMT perspective, it is interesting for teachers to solve the task they

intend to propose so that, in this way, it is possible to reflect on possible strategies, doubts or errors of the students, as well as about actions that will be carried out to conduct the class and maintain the cognitive demand (Stein & Smith, 1998; Stein, Engle, Smith, & Hughes, 2008). In this sense, the teacher acquires confidence to help students explore the task properly and maintains focus on the objectives outlined for the class (Jesus, 2016). Likewise, this action helps in the decision-making process regarding the structure of presentations and the management of discussions, establishing criteria related to mathematical learning (Canavarro, 2011).

Anticipating also involves estimating the time needed for each phase of the lesson to provide students with sufficient time to perform the conjectures, tests, discussions and records they deem necessary. Allowing students a long time to develop the task may not be ideal, as everyone will have opportunities to think about it in the moment of collective discussion. Idle time should also be avoided for those students who perform the task more easily (Jesus, 2016). Likewise, in the discussion phase, the way in which the presenters and groups will be divided to expose their ideas should be considered. In the systematization phase, there must be enough time for the teacher to be able to link previous phases with the purpose of the class, still leaving time for questions and misunderstandings to be clarified or expanded (Stein & Smith, 1998; Canavarro, 2011; Oliveira, Menezes, & Canavarro, 2013).

These highlighted aspects show, therefore, the complexity of the action of anticipating, which is a challenge, even for experienced teachers (Oliveira & Carvalho, 2014), mainly because it is an action that requires rethinking teaching practices in a different perspective from that usually carried out in Mathematics classes (Estevam, 2016) and the unpredictability and diversity of paths that students can follow (Pires, 2011). Planning classes from the perspective of EMT involves choosing tasks with the potential to investigate mathematical ideas, as well as requiring teachers to put themselves in the student's shoes, thinking about possible resolutions and errors that allow anticipating teachers' actions in face of what they can see and hear, supporting students' work according to the established objectives (Baldini, 2016). In this way, the action of anticipating demonstrates centrality in making sustained decisions during the class, reducing the need for improvisation, which can lead to the posing of questions and direct guidelines that can condition students' productions and reduce the level of reflection on their work (Silva & Rodrigues, 2017).

PROFESSIONAL LEARNING AND PRACTICE

When dealing with professional practice, Tardif (2002) highlights teaching knowledge as plural, temporal and heterogeneous, formed by knowledge arising from professional training, understood as the set of knowledge transmitted by teacher training institutions; disciplinary knowledge, selected by the university institution and incorporated into teaching practice; curricular knowledge, which involves contents, objectives and methods with which the school institution categorizes and presents social knowledge; and by the experiential knowledge, which consists of those that emerge and are validated in the experience. The latter – experiential knowledge – is highlighted by Tardif (2002), because it keeps a deep connection with the functions of teachers, since it is in this context of practice that the teacher develops, mobilizes, and models this knowledge, constituting a practical knowledge in the sense that its use depends on the problems and situations of the work. In the same sense, it is an interactive knowledge, which is also modeled in the interactions between the teacher and other educational actors.

When discussing a knowledge base for teaching, Shulman (1986) also highlights learning from experience as a fundamental component of knowledge that makes up a profession since learning from practice makes it possible to deal with unexpected and complex situations. Although, while several professions have the possibility of learning a set of experiences, mistakes, successes, and strategies (considered a framework of experiences) arising from the practice of other colleagues in the profession, the teaching profession still faces challenges. According to Almeida and Biajone (2007), education professionals are still working on these aspects, whose notes emphasize the need to abandon individuality to look at the practice of their peers and be seen.

Considering the influence that practice and the experience arising from it have on professional knowledge, as well as the need to involve professionals who carry out similar practices, it makes sense to consider professional learning in social terms, directly related to what teachers do and the meanings it produces in the course of these experiences. Thus, the assertions of Lave (1996) and Lave and Wenger (1991) regarding social learning are accepted, using the re-reading made by Vilas Boas and Barbosa (2016) to conceptualize professional teaching learning.

In addition to training courses, teacher learning can take place in other contexts, even in those not oriented towards this purpose (Vilas Boas & Barbosa, 2016). By recognizing that learning can be related to forms of participation in social practices, it is admitted as "a change in the patterns of the subject's

participation in a given practice" (Vilas Boas & Barbosa, 2016, p 1104). It should be noted that the practice involves actions that have a meaning shared by a certain social group, and that participation does not refer to any type of involvement in an activity, but to when there is mutual recognition of this involvement among the members who share the practice, in a way to awaken these individuals' feeling of belonging to those practices.

Considering the conceptualization of learning with a focus on participation processes presented by Vilas Boas and Barbosa (2016), it should be noted that the negotiation of meanings that sustain learning in social terms is characterized by the combination of participation and reification processes (Wenger, 1998). According to Estevam and Cyrino (2019), while, in the process of participation, we recognize each other reciprocally because of the relationship with other individuals and experiences of meaning; in the process of reification, we project our meanings onto the world, so that this projection takes on an independent existence. In this sense, both the changes in practice carried out in the classroom and the meanings attributed to support them are indicative of learning. They can be manifested, beyond what they do, in what they say (and what they do not say), and especially in the meanings and interpretations that support their sayings and doings, which make use of knowledge, beliefs and conceptions, including the way they see themselves. Therefore, this is the conceptualization that we practice regarding teacher learning and that guided the analysis of emerging reflections in the conducted focus group, whose foundations and directions are explained below.

METHODOLOGICAL PROCEDURES

This research has a qualitative character and uses the focus group as a means of producing data. The work by Gatti (2005) defines a focus group, presented by Powell and Single (1996), as the selection and gathering of a group of people to discuss and comment on a topic based on their personal experiences. Other definitions are presented in the work by Trad (2009), in which focus groups are referred to as a qualitative research technique, derived from group interviews, which seeks information through interactions (Morgan, 1997), and a form of interviews with groups based on communication and interaction (Kitzinger, 2000). In this way, the three definitions are related by highlighting aspects of communication and interaction, as well as the knowledge derived from the experiences of the focus group participants.

Based on these aspects, the objective of the focus group is to gather detailed information on a specific topic, from a group of participants selected through pre-established criteria, according to the problem under study. It is considered that the participants have common characteristics that qualify them to discuss the topic in question and that they have some relationship with the topic to be discussed, with a view to offering conditions to bring considerations based on everyday experiences (Gatti, 2005; Trad, 2009).

In this way, the focus group that supports this study aimed to focus on the reflections of (future) teachers on the actions of anticipating in EMT practices, considering that, with this focus, it is possible to advance in a broader understanding of the contexts and ways in which teachers plan and guide EMT practices in Basic Education and the meanings that support such guidance. For this purpose, to select participants with affinity to the group purpose (Gatti, 2005), teachers who performed practices of this nature were invited, via e-mail (from a survey of information carried out by GEPTEMatE, a research group of which the authors are part), which constituted a common criterion among the participants. In order to encompass a certain diversity of perceptions, participants with different periods of teaching experience and who performed practices in different contexts were invited. We specifically sought Basic Education teachers experienced in teaching mathematics, identified in this research by the acronym P#; and recently graduated teachers who carried out their practices in supervised internship actions, teaching, research, and extension projects while still in Graduation, who were considered future teachers and identified by the acronym FP#.

As a research technique, the focus group still features a certain theorization, since it is constituted and developed from the research problem, guiding the elaboration of relevant and contextualized scripts or questions (Gatti, 2005). Thus, the script created to guide the focus group was based on the actions of teachers before and during the class in EMT practices mentioned by Cyrino and Teixeira (2016), with questions focused on the reflections of teachers in relation to their actions at those times. Changes in relation to traditional teaching and learning arising from studies and practices were also considered, seeking notes that go beyond description and that can be discussed/evaluated with other participants.

Regarding the group's qualification, five recently graduated teachers and three teachers who work in Basic Education were invited. Only five of these invitations were accepted, three of which came from newly graduated teachers and two from teachers who work in Basic Education. In the invitation, in

addition to the guidelines for the focus group and the necessary authorizations, a form was also sent that requested information from the participants [1], involving aspects of training and performance, which are summarized in Figure 1.

Figure 1

Profile of research participants

Identification					
Participant	P1	P2	FP1	FP2	FP3
Professional training					
Higher education area	Degree in Mathematics	Degree in Mathematics	Degree in Mathematics	Degree in Mathematics	Degree in Mathematics and Architecture
Completion of the Degree in Mathematics	2003	2005	2018	2021	2021
Post-graduation[i]	Yes	Yes	Yes	No	No
Conclusion of Post-graduation	2000 and 2021	2007, 2009, 2016 and 2021	2021	-	-
Professional performance					
Year of EMT practice(s)	2020 and 2021	2021	2018	2019	2019
Context for carrying out the practice(s)	Professor (Master's)	Professor (Master's)	Future teacher	Future teacher	Future teacher
Classroom performance time	More than 10 years	More than 10 years	No performance time	No performance time	No performance time
Context for carrying out the practice(s)[ii]	Classes in which they worked as a teacher	Classes in which they worked as a teacher	Mandatory supervised internship	Mandatory supervised internship and extension project	Mandatory supervised internship and extension project

[i] During the focus group period, P1 and P2 had recently completed their master's degree in Mathematics Education, while FP2 was a student of the same master's course.
[ii] The ETM practices were carried out by P1 and P2 linked to their master's research.

Regarding the location of the focus group and the records, Gatti (2005) and Trad (2009) point out that the space should favor interaction between participants and emphasize the importance of face-to-face meetings around a table, for example, seeking verbal and non-verbal interaction. However, due to the pandemic situation during the period of research development (2020-2021), the location of the focus group was adapted to virtual environments, using the Google Meet platform. This platform features sharing audio and videos of each participant synchronously. Considering this, but also the limitation of the

[1] All participants signed a Consent Term in accordance with the guidelines of the Research Ethics Committee. n. 3.951.250. Authors assume and exempt Acta Scientiae from any consequences arising, including full assistance and possible compensation for any damage to any research participants, per Resolution No. 510, of April 7, 2016, of the National Health Council of Brazil.

platform and the unforeseen in relation to technological resources (internet failure, difficulty in accessing the platform, etc.), the participants were instructed to speak one at a time (avoiding noise and external interruptions), and to leave their cameras on, in order to preserve, to some extent, the expressions that involve non-verbal communication. On the other hand, the virtual meeting via Google Meet platform made it possible to record in audio and video the interactions that emerged during the focus group, helping in the transcription and analysis process. Due to the availability of schedules for holding the focus group and the characteristics of the virtual meeting modality, the meeting took just over two hours.

In addition to the selected participants, the focus group needs a moderator for its effectiveness, whose attributions are to introduce and maintain the discussion, emphasize that there are no right or wrong answers, observe and encourage participants, seek opportunities in the discussion itself, build relationships with informants to deepen responses and observe non-verbal communications. The moderator should also make it clear that all opinions matter and that different points of view are expected, leaving the participants free and the discussion open around the proposed question. It should be noted that the dynamic is not reduced to questions and answers between moderator/participant, but also to dialogues and complementation between the participants (Gatti, 2005). In addition to the moderator, there must be support, acting whenever necessary as a second moderator, and there may still be the presence of external observers in order to capture the reactions of the participants (Trad, 2009). In this sense, the focus group carried out in this research had as moderator the first author of this article and two assistants invited by the moderator to perform the functions of second moderator and external observer.

After the implementation of the focus group, we carried out the transcription of the speeches by participating teachers and moderator in their entirety, using the recording and notes made by the second moderator and external observer. The analyzes were carried out according to the theoretical framework adopted (Table 1) and the items of the script prepared to conduct the focus group, focusing on reflections on the action of anticipating.

REFLECTIONS OF (FUTURE) TEACHERS ABOUT THE ACTION OF ANTICIPATING

Regarding the notes of (future) teachers in the anticipation stage, the first aspect highlighted involves teachers' ability to put themselves in the student's shoes, according to the following dialogues.

> **P1**: My process [of anticipation] was *to put myself in the place of the student*, mainly because they are [learning] remotely, *to solve the task*, how could I think, what common mistakes happen, what are the successes and all this context they were in.
>
> **P2**: On this topic, then, since P1 is talking about this task, it is so important *to put yourself in the student's shoes*. I'll tell you from the experience I've had in doing this: I couldn't see all the possibilities on my own.
>
> **FP3**: I also think that there is one thing I always do, which is *to solve the task several times*, because, each time I solve the task, *I end up realizing other ways* [of solving it] or I end up making a mistake that maybe I would miss.
>
> **FP1**: [...] So, how will they [the students] *feel* while they are doing this task? How will they *think*? What are they going *to use* to understand this concept? [...] So, I think it's about you thinking both there in your classroom and about what that task will add up *in the future* for them.
>
> **FP2**: One thing I always look for [...] is *to plan all the wrong actions* [...] the mistakes that are going to be, in some way, logical, because another thing about planning that for me is complicated is this question that you don't want students to come up with *the same resolution*.

In order to anticipate student solutions, doubts, and errors, the (future) teachers highlighted that putting themselves in the student's shoes involves solving the task, identifying resolution strategies that may emerge, identifying the context of use of resources by students, especially when considering remote teaching, and enabling the preservation and encouragement of students' individuality with regard to their resolutions and representations (Stein & Smith, 1998). This anticipation is highlighted by Oliveira, Menezes and Canavarro (2013) as fundamental for supporting the students' work to be productive,

taking care not to standardize resolutions based on the path that the teacher has in mind. In terms of the nature of resolutions, the concern with providing incomplete or incorrect resolutions stands out in the reports, which suggests that this type of resolution has the potential for exploration and discussion.

Directly related to the previous actions, incorporating the task and assimilating the anticipation framework are actions, in relation to the preparation of the teacher, considered important for guiding the class, in order to have clarity of what is asked and what is expected from each task item (Stein *et al*., 2008). In this sense, teachers emphasize the search for elements to encourage the participation of students, supporting those with more difficulties and challenging those who show more ease in solving them, in order to be able to develop the tasks and achieve the goals of the class (Ponte, 2005; Jesus, 2011). On the other hand, these actions demand a lot of study and preparation from the teacher, as P1 refers, after having also developed practices based on the tasks and plans prepared by P2.

> **P1**: For P2's task [...] it was really about *assimilating* that *anticipation framework* that she had prepared, solving the task [...] in the sense of looking for elements so that they [the students] could really develop the task […], so that I wouldn't be looking at the framework. Because having a paper in my hand to check what I will answer if they say this or that is very bad [...] so I read it for two or three weeks, every day I checked it out a little bit.
>
> **P2**: Another thing about what P1 said, about *reviewing this framework before class*, because really, during class time, it seems that you have a guide there, which will follow that sequence, and really it will not, because during class time there is that student who anticipates and does everything already in letter a). Then you have to be flexible [...] because there are those who don't understand well, who need more time. So it is, in fact, something challenging for the teacher, it is not easy, even with all this planning that precedes the implementation of the task [...].

However, the teachers understand that no matter how much previous preparation the teacher has, different unexpected situations can arise during the class, which, even if not directly dependent on the teacher, must be circumvented by them. Similarly, the teachers investigated by Oliveira and Carvalho (2014) anticipated students' mistakes that could arise from the

incorrect use of procedures or misunderstandings of the concept, and even so, unexpected situations emerged that deserved teacher interventions. In this sense, Oliveira, Menezes and Canavarro (2013) note the need to prepare the teacher for what is expected in each stage of the class, as well as the relationships between the different stages, in order to contribute to students' mathematical learning. With this preparation, it is possible to make the conduction of the class more flexible, making decisions and changing the planning based on the students' activity (Oliveira, Menezes, & Canavarro, 2013).

Another aspect identified in anticipation involves the task, its design, articulation with objectives, with the curriculum, adaptation of other materials and its influence on student engagement.

> **P2**: I had a lot of problems with this [establishing the objective of the task], including for the understanding of my task [...] but to see, well, how much *an objective can influence the development of the task*. It practically transformed into another after I added one of the objectives, [...] it [the task] advances, takes on another direction [...] because it would suddenly become a good task with very little use. I was going to address just one concept, when I could address so many other things.
>
> **FP2**: I agree with what P2 said and I also find this issue of the objective complex, because I also *prepared* [the task] *with the objective in mind*. And then *the objective permeated the entire task*, so it's not like I added one more objective, I'm going to add one more question here so that I reach the objective. No, because I have the view that the task has be connected in its entirety. So, if you have a goal there, all the *items* have to be *interconnected* to achieve these goals that are proposed, and that ends up making it difficult, because it's not a work that is separated, individually.
>
> **FP3**: Besides, [...] when you take a task from someone else, who already has a proposed objective and [...] have *to adapt an objective*, I find it much more difficult, because in addition to the objective, you end up changing the assignment. [...] I think it is *different* when you are doing [designing] the task from when you are just *using someone else's material*. I had both experiences. So, [...] even if you read the plan [...], they are different situations. I think you're much more appropriate for the task when you develop [elaborate] it, and that's different.

> [...] But I think it varies from adapting the task to the students you have.
>
> **P1**: But creating the task is extremely difficult. In my case, I still had to elaborate it using technology and think in a way that they [the students] would work together on the same application. But *to make yourself understandable in the tasks, the language for the student*, so it is in a way that they can understand and *that is not the answer to the next question* [...], we have to leave this cognitive demand of the task open, but in a way they can resolve it. So, there are many elements, there, to think about this task elaboration, it is not easy.
>
> **FP3**: [After reflecting on the practice carried out in the internships, I concluded that] the task was not good enough in our view [interns] for them [students] to succeed, there was something missing that we were unable to do. So, this [cognitive] demand part is very important.

The (future) teachers consider that both elaborating and adapting class proposals of an exploratory nature constitute quite demanding actions for the teacher, particularly when considering the articulation of tasks with the objectives of the class and the different possibilities they offer for exploration. Likewise, the use of technologies and preparation for the dynamics, in order to involve the work of students in groups, are complex actions, and P1 emphasizes that this aspect is even more demanding in the context of remote classes. In this sense, (future) teachers emphasize that, when elaborating a proposal considering the particularities of their classes, there is greater appropriation by the teacher, and it facilitates the guidance during the class.

Possibly, the difficulty in adapting a proposal is related to the particularities of the class, since, when anticipating, teachers take into consideration their teaching objectives, the characteristics of the students, their involvement in the tasks and their previous knowledge (Cyrino & Teixeira, 2016; Garcia & Oliveira, 2016).

These perceptions are in line with what is presented in Oliveira and Carvalho (2014), whose investigated teachers point out the selection and adaptation of tasks as important but also demanding actions, because they require large investments from the teacher, being one of the most complex and time-consuming moments in the preparation of the conducted plan.

As characteristics of the task that allow the involvement of students, (future) teachers highlighted the use of resources and environments that favor group work, while, for the development of the task in the expected way, an accessible and understandable language for the student is essential, but that does not give evidence of answers to the questions, seeking to provide conditions for students to solve the task. However, it must preserve its open nature in terms of strategies to be employed (Ponte, 2005) and the level of cognitive demand (Stein & Smith, 1998). If students are faced with obstacles that prevent their progress in relation to the development of the task, either because they do not understand what must be done or because they do not have compatible prior knowledge, the proposal may be compromised and should be reassessed.

Another aspect highlighted in the anticipation action is the selection and sequencing of the students' resolutions for the discussions, according to the following notes:

> **FP2**: My colleague and I had a lot of *difficulty selecting* who would go to the discussion and *then systematizing it* [...]. What we thought to help with this was [to identify] classes [...] [with] a break in the middle. So, we tried, already in the planning, to do the development process, and then, at break time, look at their [students'] resolutions and compare whether those resolutions that we had proposed to select for discussion emerged or not, so that we could *select with more time* and not there, inside the classroom.
>
> **FP3**: Just to complement that thought, my [internship colleague] and I ended up doing both options. [...] *It's hard to think at the time*, and there comes the importance of, at the beginning, *having established what I* want to be discussed. When you already have that in your head, it's easier, because you only look for it in the groups. Now, when you don't, this is very difficult. And it wasn't perfect, our choice, but we ended up following what we had proposed in the lesson plan, that's *the importance of planning*.
>
> **P2**: *In practice*, it's much more like what you said, FP3, having to choose than happening what you had planned, because you can think about a lot of things, we make a plan, but the name already says it, it may happen, but it is likely it does not happen or happen in unplanned ways. [...] So, the chance of you having

to make a decision, choosing teams on the spot is much greater [the challenge for the teacher].

P1: I did [...] P2's task [...]. I had to choose right there, right away. So, I gave them [the students] the deadline very precisely, [...] and I organized who would speak first. But I know that if I had brought it home, looked at the answers, perhaps I would have changed the order, but it's not easy to fix it right away. And when I read those Portuguese people solving, doing everything on the spot, who they choose to present before and such, but then we'll see they have few students in the classroom, there aren't 35 [students] there [...], and in my task with the sixth year I was all lost in knowing who presented first, who presented later, it was a struggle. Then it was kind of like 'whoever goes first...'

FP3: [This] is something that happened to me. I adapted [a task] [...], but I don't think I did it as I should have [...]. When it came to choosing [selecting students for discussion], I got desperate: I didn't know who to choose, and then I chose like this [arbitrarily]. And I had time, because it wasn't in the same class, but I hadn't planned [prepared] before. I ended up choosing very badly and ended up with that kind class that doesn't go anywhere.

These notes are related to what Oliveira and Carvalho (2014) highlight when they identify from the reports of the surveyed teachers' difficulties in predicting how students would react to the task, as well as possible strategies and difficulties that could emerge, due to lack of knowledge, familiarity with this type of action and the habit of approaching concepts directly, based on student training, through typical exercises.

Teacher's actions for the phases of selection and sequencing of resolutions, discussion of resolutions and systematization of learning were identified as difficult to perform during the same set of classes, especially with regard to the articulation with the students' resolutions. These difficulties involve the large number of students in each class, different situations that may (or may not) arise during class, and above all the limited time available for teacher and proposal preparation.

When highlighting the challenges for teachers who carry out EMT practices, Canavarro (2011) suggests that the teacher be prepared to avoid

postponing the discussion of resolutions and/or the systematization of learning using students' productions to the next class. This is because they consider that the consequences may involve the loss of student involvement and the distancing of the mathematical productions carried out, requiring large investments in effort and time to recover them to some extent. However, another challenge presented suggests favoring effective discussion by students in order to enable the learning of mathematical concepts and procedures, as well as the development of skills, with a focus on mathematical communication (Canavarro, 2011).

In this sense, even knowing the possible implications of postponing the stages of the class, (future) teachers seem to consider that the losses are smaller when resuming the planning to privilege more effective discussions, since, according to FP3, the teacher needs to know what they want with that class, guided by the established objectives, and if you can't perform the actions properly, you can end up compromising your class. Thus, they highlight as possibilities which helped in the stages of resolution selection and sequencing the establishment of well-defined criteria for the choice of groups of students for the discussion, the realization of the practice of EMT together (doubles) during the initial training and carrying out the practice on different days or at times that have breaks between classes. This allows some time to analyze the students' resolutions more carefully, identify whether the anticipated resolutions have emerged or not, with a view to selecting and sequencing the resolutions for the collective discussion stage.

Teachers also highlight the support through collective discussions, which helps to deal with unexpected situations (Oliveira & Carvalho, 2014). These signs of joint involvement constitute another aspect highlighted in the reflections of (future) teachers, referring to the contributions of collaboration to their practice.

> **P2**: So, it's the fact of *discussing collectively*, [...] planning in a collaborative way is very important and *greatly enriches the task*. So, speaking of my experience specifically, it has been improving and developing in the *community*, and even so I'm sure that, if we review it, it still has things to add. So, the more we think about possibilities that they [the students] can get [different strategies for solving], that they will bring during class, it will enrich their learning and ours as well.

> **FP3**: [...] With our (internship) advisor, [...] the best option was to get there, do a systematization beforehand and then move on to the next part [...].
>
> **FP3**: I think what contributed [in the stage of selecting and sequencing of students] was that we were two [interns], because if it was just one, just me, I think I would have chosen a bit randomly [...].

(Future) teachers emphasize the centrality of collaborative work as a means of sharing their ideas and receiving suggestions from colleagues. The different perceptions and opinions that emerge in the research group are evaluated in order to allow reflections about the task, aiming at expanding the possibilities of solving and conducting the task, (re)constructing the planning and enriching the anticipation framework. On the other hand, considering the action in the classroom, in the initial formation, aspects of collectivity are highlighted in the support found in carrying out the internship in pairs, as well as in external orientations, based on the perceptions of experienced teachers and advisors.

When questioned about possible changes in planning and pedagogical practice and perceptions that differ from the traditional practice of EMT, (future) teachers highlighted:

> **P1**: [...] I think it's a class with an idealized plan, but that might not work out. So, there is not this fixed thing of a traditional class that you know [how] it will start, if the students didn't [do it], just copy its resolution and that's it. But it is very different [...].
>
> **P2**: Yes, and [...] we must start understanding that it's okay if a class didn't work out. It's not because it didn't work out in this class [that] it won't work in the next [...]
>
> **P1**: It is a [learning] *process for them and for us*.
>
> **P2**: Exactly, [...] it doesn't mean that everything went down the drain. Maybe for this class you have to follow another style of task, but the students have to correspond too.
>
> **FP3**: We go around blaming ourselves, but reflecting later, I see that it contributed a lot, because [...] now I know where I went wrong and what I can't do to not mess up my class. So, [in] each experience you learn a little more.

> **FP2**: It looks like *there's a lot more to be done* [in EMT practices].
>
> **P1**: It's *more laborious*.
>
> **FP3**: [...] You have to be much more prepared than in traditional teaching, which, despite always having that student who will ask you things, like, that you don't know, in Exploratory Teaching you make more room for this [...]
>
> **FP2**: And this *superficial knowledge* will not be enough [...]. With Exploratory Teaching, it seems that the teacher's actions will always be aimed at deepening, and then they need much more.
>
> **P2**: [...] In the groups of teachers that I have worked with [...] we did not plan, like, for a specific content, we planned either for the bimester or entire quarter, if not the year. [...] So, we don't review that planning again, we prepare that class following the textbook. This is something that really caught my attention [...]: we focus on content and so many ideas emerge that you can, in a task [...], take advantage of so many opportunities [...], it's better than simply doing a lot of exercises to calculate, to solve, which are useless.

The reflections of (future) teachers in relation to teacher's actions in the anticipation stage, with reference to the EMT practices they performed, reveal a certain change regarding the planning itself, compared to traditional teaching. This change derives particularly from the objective development of a set of tasks, focusing on specific situations and contents, which support the objective(s) of the classes, instead of the planning commonly carried out by teachers, organized bimonthly, quarterly or even annually, essentially guided by the textbook and hardly revised. This observation corroborates Oliveira and Carvalho (2014), who mention that planning is not a usual practice, especially with such level of detail, being perfected from the interaction with qualified teachers. This change of understanding regarding the importance of planning for the teacher's practice, whether from external involvements (studies of the anticipatory action) or internal involvements (anticipation and effectiveness of practice) provides evidence of changes in the practice of (future) teachers.

Another highlighted aspect involves the uncertainties related to the control that the teacher has over the class in the EMT perspective, when compared to the more fixed aspect of traditional teaching, in which the teacher

leads the class and often limits the role of students. This dynamism in the sense of the different situations that may occur implies a more consistent preparation of teachers, with regard to the actions they will perform in the classroom and what they expect their students to perform. Hence the need for more detailed planning (Oliveira, Menezes, & Canavarro, 2013), which differs from the planning that is normally carried out in traditional classes, even in the textbook. On the other hand, there is no script to be followed that will imply in the (un)success of a practice carried out from the perspective of the EMT, so that more experienced teachers indicated, in their reports, the understanding of planning as an instrument of flexible guidance for practice, which adjusts according to emerging activities. However, future teachers, despite considering the lesson planning and teacher preparation, seem to have a more rigid view regarding the conduction of the classes they carried out.

Other considerations point out that the practices in the EMT perspective are more laborious for teachers, whose mentioned preparation emphasizes the insufficiency of superficial knowledge, since, in EMT practices, teachers' actions are aimed at deepening mathematical ideas, raising and encouraging reasoning from the students.

Carrying out EMT practices is considered a process of change, both for students and for teachers, so that if the class has come to an end and the teacher feels unmotivated for believing that they have not reached the expected goals, it does not mean that other practices will also "go wrong". These moments constitute learning opportunities for the subjects involved in this practice. Such reflections, raised in the focus group, show that the EMT should not be seen as something to be experienced sporadically, but as a movement that needs time and continuity for the improvement of teachers' practice, as well as the development of students in relation to the learning of mathematical content and the ways of producing this knowledge, from a community in which they are members (Canavarro, 2011).

Finally, the teachers consider that the time available for planning EMT practices is another aspect that differs from traditional teaching.

> **FP3**: Every time I developed [the practice became effective], I ended up *taking longer*, or else I ended up having to do it for another class. [...] we did the task, and the students were not able to achieve what we wanted, we took more time in the development [and] they did not get there.

> **FP2**: And on this issue there, of *time*, which FP3 commented, I think firstly that *the time to do all the planning*, which will be to anticipate the students' actions and so on. Our plan for 12 lessons with 4 tasks and a diagnosis was 57 pages long. So, we took a long time to do, to anticipate all these actions.
>
> **FP3**: I adapted [an assignment] on permutation and arrangement, the assignment was excellent. [...] but it takes a lot to plan ahead, not only there, in the minutes before class, because I had time from one class to another, from one week to another, but I didn't have time because the amount of *activity time,* I think, it's *little* for you to *prepare a class* in Exploratory Teaching.

In this sense, the issue of time in EMT practices is treated with different approaches, so that the anticipation generates great demands, as it involves the preparation of tasks and anticipation of the actions of students and teacher, which is considered incompatible with the workload outside the classroom made available to the teaching professional. Still, the implementation of practices of an exploratory nature seems to require more time, by making the planned conduct more flexible considering the students' actions. However, this may be related to the need for continuity and maturation time for teachers and students to become familiar with this type of Mathematics teaching practice.

CONCLUSION AND CONSIDERATIONS

From the analysis of the emerging reports in the focus group, it is possible to conclude that (future) teachers consider that putting themselves in the students' shoes, solving and anticipating aspects related to the task; assimilating the anticipation framework, aiming at preparing the teacher to conduct the class; articulating the task with the objectives of the class and with the particularities of the students; as well as defining criteria and anticipating issues related to the selection of resolutions are challenging aspects. However, these aspects must be considered in the anticipation stage, and are privileged by the possibility and availability of time and involvement in collaborative environments.

These considerations reveal their own meanings, based on the studies, discussions and practices carried out. They elucidate the teachers' intentions and justifications according to their conceptions and different contexts for carrying out the practices, denoting changes in relation to the different roles

that involve the anticipation of a class, especially an EMT one and, therefore, learning about these aspects (Wenger, 1998). On the other hand, even though they highlight difficulties, the reports do not directly explain understandings about the anticipation of the action of sequencing the resolutions, since the teachers' notes regarding the intentions and ways of planning/effective this action were poorly elaborated. However, there are indications that suggest that meanings referring to this action are in the process of being constructed.

The teachers refer to the dilemma involved in developing or adapting exploratory tasks and consider that proposals of this nature are not replicable, in the sense that a planning elaborated in this teaching perspective carries with it the particularities of the teacher and the students. These particularities are present in the tasks, in the anticipation of resolutions, and in the guidelines that will be given. Thus, it is considered that there must be adjustments in the use of proposals prepared by third parties, considering the new context in which the practice will be carried out. Likewise, teacher preparation is necessary, demanding time and study, since the task alone does not guarantee student learning and the teacher's actions influence the class to a great extent (Ponte, 2005).

As potentialities, the ability to put oneself in the student's place is highlighted in the reports, as presented in the literature (Canavarro, 2011; Oliveira & Carvalho, 2014), involving demanding actions, which require different teacher movements, which then seek to predict actions of students and establish guidelines for conducting the different stages during the class. The work in pairs (in the case of future teachers) when carrying out EMT practices and participation in study and research groups stands out as possibilities to give and receive support in terms of improving the task and the anticipation framework, as well as for the preparation of the teacher aiming at the actions during the class.

As a possibility of referral, carrying out EMT practices in different classes, in order to separate the phases of proposing the task and monitoring the resolutions of the others, despite being pointed out in the literature as something to be avoided because it distances students from their productions, seems to be an alternative in assisting the teacher in organizing the discussion of resolutions and in systematizing learning in a more effective way. This is because it makes it possible to analyze, under conditions of lower tension, the results and records of the students, articulating them with the contents and objectives proposed for the class.

The emerging reflections point to different understandings provided by the anticipation of the EMT practice regarding planning, the characteristics of the task and its development, the anticipation of the actions of the students and the teacher, many of which emerge in the practice itself. In this sense, evidence of teachers and future teachers' learning is identified as they engage in practices that assume other ways of acting (and thinking) inside and outside the classroom (Vilas Boas & Barbosa, 2016), as well as how to relate with students and with the mathematical content, providing new understandings raised in the interaction with fellow professors of the research group. Reflections were also identified during and after the practice, provoked in the execution of the planning with the students.

Thus, this study advances theoretically by highlighting, through reflective practice, ways in which the elements that make up Table 1 were referred and conferred meaning by the teachers, complementing and giving meaning to these elements situated in the practice itself. In the same direction, the teachers refer to aspects that complement the guidelines for EMT practices, namely: assimilating the framework of anticipation of the students' and teacher's actions; putting yourself in the students' shoes and solving the task to identify different forms of resolution and representation; discussing the planning elements collectively; valuing wrong strategies for exploration and discussion; and establishing clear and objective criteria for selection and sequencing of resolutions for discussion.

However, there is room for future investigations and studies on the reflections and lessons learned in carrying out EMT practices, including considering the contrast between planning and what is effectively implemented in the classroom. Likewise, studies involving other groups, with other experiences, can complement and expand the notes presented here.

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AUTHORS' CONTRIBUTIONS STATEMENTS

EPOR was responsible for the production of the data, analysis and interpretations presented, as well as for the elaboration of the initial proposal of the manuscript. EJGE was responsible for the theoretical and methodological design of the study, as well as complementing the analyzes and the final articulation of the manuscript. Both authors actively participated in the discussion of the results, reviewed, and approved the final version of the work.

DATA AVAILABILITY STATEMENT

The data that support the results of this study will be made available by corresponding, EPOR, upon reasonable request via e-mail.

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