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Study of graphs mediated by learning object: a discussion based on initial teacher training

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Abstract: This article presents a study that analyzes a Learning Object (LO) of graph interpretation, conduced with teachers in initial training in the Institutional Program of Scholarships for Teaching Initiation — PIBID. The qualitative empirical study aimed to analyze and discuss pedagogical and exploratory conditions of the LO built in the GeoGebra software. Data were collected from the participants through activities created and shared in the LO, field notebook annotations, observations, records and description of representations. In the organization and analysis of data, the categorization was used. The theoretical framework encompasses reflections on policies for Initial Teacher Training, in interaction with digital alternatives, study of graphs and mathematical visualization. The results suggest that LO, mediated by digital technologies in Mathematics Degree, promotes discussions and reflections regarding future pedagogical possibilities of representation, creativity and construction of mathematical thinking.

Keywords: Initial Teacher Training. Educational Policies. Basic Education. Mathematics Education. Learning Objects in GeoGebra.

Estudio de gráficos mediados por objeto de aprendizaje: una discusión a partir de la formación inicial docente

Resumen: Este artículo presenta un estudio que analiza un Objeto de Aprendizaje-OA de interpretación de grafos, realizado con docentes en formación inicial del Programa Institucional de Becas de Iniciación Docente — PIBID. El estudio empírico cualitativo tuvo como objetivo analizar y discutir las condiciones pedagógicas y exploratorio del OA construido en el software GeoGebra. Los datos fueron obtenidos con los participantes a partir de actividades creadas y compartidas en la LO, de anotaciones en el cuaderno de campo, observaciones, registros y descripción de representaciones. En la organización y análisis de los datos se utilizó la categorización. El marco teórico incluye reflexiones sobre políticas para la Formación Inicial Docente, en interacción con alternativas digitales, estudio de gráficos y visualización matemática. Los resultados sugieren que la LO, mediada por las tecnologías digitales en la Licenciatura en Matemáticas, promueve discusiones y reflexiones sobre las futuras posibilidades pedagógicas de representación, creatividad y construcción del pensamiento matemático.

Palabras clave: Formación Inicial del Profesorado. Políticas Educativas. Educación Básica. Educación Matemática. Objetos de Aprendizaje en GeoGebra.

Estudo de gráficos mediado por objeto de aprendizagem: uma discussão com base na formação inicial de professores

Resumo: Este artigo apresenta um estudo que analisa um Objeto de Aprendizagem-OA de interpretação de gráficos, realizado com professores em formação inicial no Programa

Institucional de Bolsas de Iniciação à Docência — PIBID. O Estudo empírico qualitativo teve por objetivo analisar e discutir condições pedagógicas e exploratórias do OA construído no *software* GeoGebra. Os dados foram obtidos com os participantes a partir de atividades criadas e compartilhadas no OA, anotações em caderno de campo, observações, registros e descrição de representações. Na organização e análise de dados, utilizou-se a categorização. O quadro teórico contempla reflexões sobre políticas para Formação Inicial de professores, em interação com alternativas digitais, estudo de gráficos e visualização matemática. Os resultados sugerem que o OA, mediado pelas tecnologias digitais na Licenciatura em Matemática, promove discussões e reflexões quanto a futuras possibilidades pedagógicas de representação, criatividade e construção do pensamento matemático**.**

Palavras-chave: Formação Inicial de Professores. Políticas Educacionais. Educação Básica. Educação Matemática. Objetos de Aprendizagem no GeoGebra.

1 Introduction

The concern for education in the present day has taken on new dimensions with the Covid-19 pandemic. We are faced with the necessity of restructuring, primarily regarding the technological evolution in schools, with an experience that values digital technologies in pedagogical practice, implying mainly the students' interest and/or their distancing from school. These concerns extend across the different educational levels, including institutions that train teachers for Basic Education, with a focus on the potential of school education that follows the transformations of reality.

The study presented here concerns the possibilities for pedagogical practice, interpretation, and analysis of graphs with teachers in initial training. It was developed in collaboration with academics from the PIBID Program, Mathematics Nucleus of Campus Chapecó, SC, from 2020 to 2022. The established discussion is part of an empirical research project in Mathematics Education[1] that aimed to analyze and discuss LO of mathematics regarding concepts of the final years of Elementary School, focusing specifically on the study of graphs.

The LO, according to Scheffer et al. (2018), can be programmed and planned by the teacher to involve and use pedagogical materials from the teaching lab integrated with digital technologies, incorporating simple elements such as a video or a song. It can also be presented practically through a PowerPoint presentation or through more elaborate programs that require an understanding of programming languages, using possibilities with digital technologies that are within the reach of teachers in the school context.

According to Kalinke e Motta (2019), these initiatives constitute the use of digital technologies in teaching and learning processes, whether in the classroom or not, they are interactive resources evaluated as digital entities, to be utilized, reused, or referenced as technological support for educational activities. The LO presented in this article focuses on the study of graphs and describes interactive activities developed in the GeoGebra software. These activities aim to interpret graphs, which are topics typically covered in the 9th grade, highlighting applications of content intended to be taught by mathematics teachers in the later years of Elementary School.

The theoretical reflections encompassed the analysis of pedagogical conditions involving digital technologies in the initial formation, study of graphs and the role of

[1] Research approved by the Research Ethics Committee, CAAE: 50898721.80000.5564, Opinion Number: 4,990,198, on September 9, 2021.

mathematical visualization when interacting with dynamic software. The theoretical framework is based on authors who discuss initial teacher training and conceptions of teachers, LO, as well as the interactions between initial teacher education and digital alternatives.

Thus, in the investigation of the study we address the following research problem: "What pedagogical conditions does a Learning Object assume for the study of mathematical graphs in the initial training of mathematics teachers?". Considering the reflection on practice, we initially sought, in the literature, contributions regarding the initial training of teachers and pedagogical conditions related to the use of digital technologies, with the study of graphs, mathematical visualization and with the use of LO in the teaching of mathematics.

The data analyzed refer to teachers in Initial Teacher Training in the analysis of activities that were created and shared through the LO, which integrated the interpretation of mathematical graphs. From this, we illustrate representations that have produced meaning by combining digital media, fields of concepts and conceptions that identify and relate to the professional knowledge of teachers in initial training.

The study, in its methodology, includes the possibility of describing the interpretations and conceptions of future teachers in order to present the mathematical thinking mediated by the digital alternatives of the LO. Thus, in this interaction, through the identification and analysis of representations, properties and didactic possibilities, we aim to understand the conceptions that future teachers have in the Mathematics Degree.

The structure of the article includes: the Introduction, which provides the contextualization of the theme and the study; the Methodological Choices of empirical research and data treatment; the Review and Theoretical Perspectives regarding Initial Teacher Training and educational policies for this training, Learning Objects and Mathematical Visualization; the LO for the study of Graphs develop in the GeoGebra software, including a selection of activities where we report actions and data. Lastly, we present a categorization that indicates the analysis of data and results with the understandings built about the problem investigated and the Concluding Remarks.

2 Methodological Choices — The study

The research of qualitative approach considers the complexity of the process of learning to be a teacher, in which “researchers make an interpretation of what they see, hear, and understand” (Creswell, 2010, p. 209).

The study was conducted with a sample of nine participants, academics who entered the initial training course, a Mathematics Degree offered in the face-to-face modality, during the period from 2019 to 2023, and who were part of the PIBID Program from 2020 to 2022.

The data collection occurred through the analysis of the LO during its application with the participants of the sample, considering a practical approach, which makes it possible to see and review the activities proposed within the object, as well as the intended theme and objectives, thereby enhancing the interpretation process at any given moment. Among the alternatives considered in the data collection are the field notes, observations, records, and descriptions of representations constructed within the graphical analysis object, as well as the responses obtained from the evaluation questions of the LO, at the end of the activities that involved its development, discussion, and application with the participants.

The organization and analysis of data were conducted through categories that emerged from the data, for Bardin's (2016), involves a set of techniques and procedures for analyzing content and its meanings across three phases: pre-analysis, exploration of the material, and

treatment of the results, contemplating inference and interpretation.

The method of investigation, content analysis, as described by Moraes (1999), comprises special procedures for processing scientific data. According to the author, the raw material for content analysis can originate from any form of verbal or non-verbal communication. Hence, data from various sources are received by the researcher in their raw state and require processing to facilitate the work of comprehension, interpretation, and inference aimed at by content analysis.

3 Initial Teacher Training

The discussion regarding learning to be a teacher begins during initial teacher training and continues throughout the teaching Journey in the relationship between learning about how to teach and how to be a teacher. This learning is related to the responsibilities, functions, roles, actions, thoughts, and beliefs in the professional life inside and outside the school context.

Such aspects are guided by a theoretical purpose present in the proposal of Initial Teacher Training programs, strengthened by academic research, referred by Tardif (2007), as the knowledge of professional education, covered in formative actions and are related to conceptions of knowledge construction, pedagogical concepts, teaching, learning, and pedagogical practice, as well as methods, techniques, and evaluative processes. Thus, the process of learning how to teach, from theories that encompass beliefs, values, ideologies, attitudes, ideas, professional and personal experiences, meaning constituting a field of knowledge that encompasses both education and teaching.

In this sense, according to Imbernón (2011), "education should propose a process that provides teachers with knowledge, skills, and attitudes to develop reflective or investigative professionals" (p. 58). And the profession is built upon initial training with the purpose of providing theoretical, technical, methodological, and practical foundations in knowledge construction and teacher training, permeating conceptions and beliefs that consider such knowledges, ranging from the pedagogical, the students' context and educational policies that have favored this teacher training and professionalization.

We also emphasize that teaching training goes through educational policies, that Mainardes, Ferreira and Tello (2019, p. 156) highlight the perspective of critical theories of discourse and the formulation of policies, as an arena of dispute for meanings, aiming that "the emphasis is placed on the process of policy formulation, and policy is understood as a contest among competitors to define objectives where language and, more specifically, discourse, are tacitly used". Thus, the process of policy formulation is considered as a continuous cycle, in which policies are formulated and recreated all the time in order to define training perspectives for teaching in a constant way.

Consequently, by thinking of teacher training as a commitment of the State and not as a particular responsibility of the individual, educational policy comes to be considered as a right to guarantee the education of all. This aspect is highlighted in the National Curricular Guidelines for Basic Education, by dealing with the pattern of training paths ensured in structured courses to ensure the specificity of teacher training and a pedagogical project that promotes the articulation between theory and practice in the process of teacher education, which is established in scientific and didactic knowledge, in addition to the recognition of Basic Education institutions as (co)formative spaces, necessary for the training of teaching professionals (Brazil, 2015).

Resolutions guarantee Initial Teacher Training, such as: 1 — Resolution CNE (National

Board of Education)/CP (Penal Code) No. 02/2015, which refers exclusively to Initial Teacher Training. The Annex of the Resolution corresponds to the BNC (Nacional Common Core) for Formation (Article 2), which includes the general teaching competencies provided in the BNCC (National Common Core Curriculum) — Basic Education (Brazil, 2015), as well as specific competencies distributed across three dimensions: knowledge, practice, and professional engagement (Article 4). This Resolution emphasizes the commitment to a training model that reinforces the importance of partnership between IES (Higher Education Institutions) and Basic Education institutions, using the term "mentoring" to refer to the school teacher responsible for guiding the pedagogical practices of trainee teachers (Article 7). It also highlights the importance of practice, which should be addressed not only during internships but also from the beginning of teacher education programs, recognizing the partnership between universities and educational networks, particularly public networks; 2 — Resolution CNE/CP No. 2/2019 (Brazil, 2019), which establishes three dimensions as integral parts of teaching action: “I — professional knowledge; II — professional practice; and III — professional engagement”. These dimensions can be observed in internal programs of universities, such as the PIBID and the PRP (Pedagogical Residency Program), promoted by CAPES (Coordination for the Improvement of Higher Education Personnel). These programs aim to foster institutional projects that contribute to the improvement of initial teacher training in undergraduate teacher education programs. Other programs that contribute to these dimensions in universities include PET (Tutorial Education Program), the Monitorship Program, and Study and Research Groups, which are part of teacher education, particularly in public institutions, complementing the formative process of the curriculum in teacher education programs in the present day.

From this last Resolution, it can be inferred, in general, the importance of investing in the scientific, technical, and cultural updating of the graduate student in an inseparable manner throughout their formative journey during the teaching degree program, involving them with the programs of initiation to teaching, with research and extension activities, navigating both the university and school settings to experience the context that will be part of their professional life.

Thus, the future field of professional activity has its alternatives and specialties, not being able to remain on the sidelines of the discussion on the Digital Technologies of Information and Communication — DTIC, very present in education. The conception of education that is part of the pedagogical project of the undergraduate courses and is reflected in the curriculum matrix is related to the formation for the critical and creative domain of the DTIC in the licentiate degree.

The searches of Bairral e Henrique (2021), Assis (2021), Rocha e Palha (2021), Scheffer, Finn e Zeiser (2021), Oliveira, Canavarro e Menezes (2021), Borba, Silva e Gadanidis, (2014), contemplate a discussion about the conception of education underlying the formative process, with emphasis on preparation for teaching with digital technologies, as a possibility to understand the alternatives offered by mobile devices, such as computers, tablets and smartphones, and its role in the classrooms of Basic Education, with a focus on performance, providing training for working with the media. In view of overcoming what Kenski (2018) highlights by stating that it is not about the presence of digital technologies in the classroom, but of “difficulty in mastering the competencies for the use of Information and Communication Technologies (ICT) by teachers” (p. 105).

On the other hand, when it comes to a new professionality, it is worth considering Nóvoa (2002), that at that time already referred to the need for teacher construction, aiming to the problem of the crisis in teaching activity and professional identity. For the author, it is more

evident that teachers are not only consumers, but also producers of teaching materials, being, in addition to executors, also creators and inventors of pedagogical instruments:

> [...] there is no teaching without a permanent renewal of the pedagogical means, without a daily conception of new materials: whether it is the contents or the didactic situations, whether it is the tasks to be proposed to the students or the curricular organization, whether it is the planning or the evaluation system, teachers are faced with a constant activity of production and invention (Nóvoa, 2002, p. 36-37).

In this perspective, we can highlight in this study the importance of critical and creative training for the DICT on teaching degree, that goes beyond technologies as a didactic resource, which will allow the future teacher to understand and establish relationships between society, technologies and education, rescuing, thus, the potential of digital technologies for education at all levels and being prepared to create, plan and transform pedagogical practice supported by new possibilities for work in mathematics.

A creative possibility to produce didactic resources by mathematics teachers and future teachers is using digital technologies, as in the case of the study presented here with GeoGebra, is the production of LO that can be built in an integrated way. Such objects present themselves with different representations for mathematics classes, acting as main or complementary resource, or even as an extra-class activity. These interactive resources allow students to work at their own pace, establishing connections, making conjectures, and creating representations, in order to deepen their knowledge that encompass digital technologies in the field of mathematics for the final years of elementary school.

4 Learning Objects

The LO, according to the researchers, Nunes (2004), Hay e Knaack (2007), González e Ruggiero (2009), Scheffer, Comachio, Cenci e Heineck, (2018), Kalinke e Motta (2019), whether they constitute initiatives that use digital technologies for teaching and learning processes in the classroom or not, considered interactive resources evaluated as digital entities, to be used, reused, or referenced as technological support for educational activities.

A LO came to be considered as a discrete piece of educational content, which can be built in environments such as the Microsoft PowerPoint, Canva, Google slides, as in the GeoGebra software (Scheffer e Zeiser, 2022) Like a text, a graphic, an animation, an audio file, a video, an interactive activity, with specific educational purpose to support situations of reflection and learning of concepts, in order to expand or guide the processes of teaching and learning. Consequently, they are activities that employ different media and educational resources, including books, videos, slides, activity scripts or digital animations with the aid of digital technologies, which can be created in any media, such as applet, Flash animation, video or audio files, photo, PowerPoint presentation or website, constituting part of the scenario of investigations and relationships experienced in the classroom.

The LO are considered resources with determined functions and objectives, which can be combined with other objects and reused, that is, they are technologies to support the classes, which can guide the reflection, interpretation and contextualization of the concepts, including in an interactive way. They can constitute a self-instructive pedagogical material to be viewed in a practical way, from a PowerPoint presentation, or involve more elaborate programs that require an understanding of programming languages, making visualization possible, an aspect that strengthens learning and attribution of mathematical meanings, a resource that can be used to support mathematical learning and interactive discussion.

In this sense, the knowledge that teachers produce and execute in the LO begin to assume their value because they are related to the reality, experiences and cultures in which the school is inserted. For Souza Junior e Lopes (2007), when the teacher is conceived as a reflective professional and investigator of their practice, occurs the confrontation of the problem of distancing and strangeness between the scientific knowledge, practiced/produced by the academy, and those practiced/produced by the teachers in the reflected teaching practice.

Regarding the applicability of LO, Kalinke and Motta (2019), highlight that

> the use of LO has the potential to change the dynamics of classes, incorporating new possibilities for pedagogical work that involve exploration, simulation, and interactivity, with activities in which the pursuit of knowledge is a constant factor (p. 11).

Consequently, LO come in various forms from a technological point of view, such as simulators, animated infographics, games, audiovisuals, videos and other possibilities. Therefore, such resources can be constructed and reformulated according to the concept to be worked, its use, role and pedagogical dynamics expected.

5 Mathematical visualization

Mathematical visualization, especially in this study, assumes its function in the learning environment, because it involves students and teachers in the interaction with digital technologies, when assigning mathematical meanings, constructing mathematical arguments, symbolizing and analyzing representation.

In this sense, Palais (1999), when referring to visualization in mathematics, emphasizes that

> the advantage of supplementing these and other such classic representations of mathematical objects by computer-generated images is not only that a computer allows one to produce such static displays quickly and easily, but in addition it then becomes straightforward to create rotation and morphing animations that can bring the known mathematical landscape to life in unprecedented ways. Even more exciting for the research mathematician are the possibilities that now exist to use mathematical visualization software to obtain fresh insights concerning complex and poorly understood mathematical objects. (p. 647).

This author describes visualization as an important aspect in the representation of images that occur on the computer screen with dynamics of movement, rotation and animations that present themselves to the understanding of mathematical objects, mainly with software.

Presmeg (2006), on the other hand, when referring to visualization with digital technologies in the processes of teaching and learning mathematics, points out that "the visual depiction clearly manifests the difference between various types of algebraic word problems, and this visual process is encouraged and enhanced by the dynamic software" (p. 226). Therefore, according to this author, visualization retrieves algebraic relationships among various types of problems in mathematics teaching and learning, and the visual process is also encouraged and enhanced by dynamic software.

Oliveira and Barbosa (2022), when referring to visualization in teacher education, emphasize:

> The theories that contribute to what we advocate as visualization consider language as an important element in the production of meaning. Mathematical language is imbued with a symbolism that often makes it inaccessible to many students at different stages of schooling, and therefore, it is often regarded as abstract in nature. In schools, the pedagogical practices in mathematics classes largely focus on presenting mathematical content through definitions, examples, and direct applications. This approach leaves out the opportunity to broaden the perspective of mathematical knowledge, with an emphasis on the production of mathematical meanings by students (p. 53).

For the authors, one way to understand what they understand as visualization is to admit the existence of a close relationship between language and thought. For instance, it is only possible to know what the other thinks if they express themselves through language. Therefore, in a context of teaching and learning, it is through dialogue, gestures, textual or pictorial production that the teacher can have access to the student's thought. When these authors refer to the relationship between language and thought, considering the perspective advocated by Castro e Frant (2011), clarify that this relationship is not ordered, that is, thought does not precede language, nor the other way around. They explain that thought and language are constituted in constant dialogue and the ways of thinking change to the extent that language is produced and the same occurs with language, which gains new contours as thought is constituted.

In this perspective, the authors consider that a schema is not limited to an image but is a visual representation, coupled with questioning that seeks to understand a thought process. Therefore, in the interaction with a schema, the individual produces knowledge. That is, the use of schemas in the visualization process contributes to the production of mathematical knowledge. Thus, the concept of visualization, advocated by the authors, is a process that involves the dialogue between "seeing" the schema, reflecting on the proposed questions, and providing a response.

It is through this path that we consider visualization, in this study, which promoted a reflection on the analysis of graphic representations.

6 The Learning Object for the study of graphs: some highlighted colors

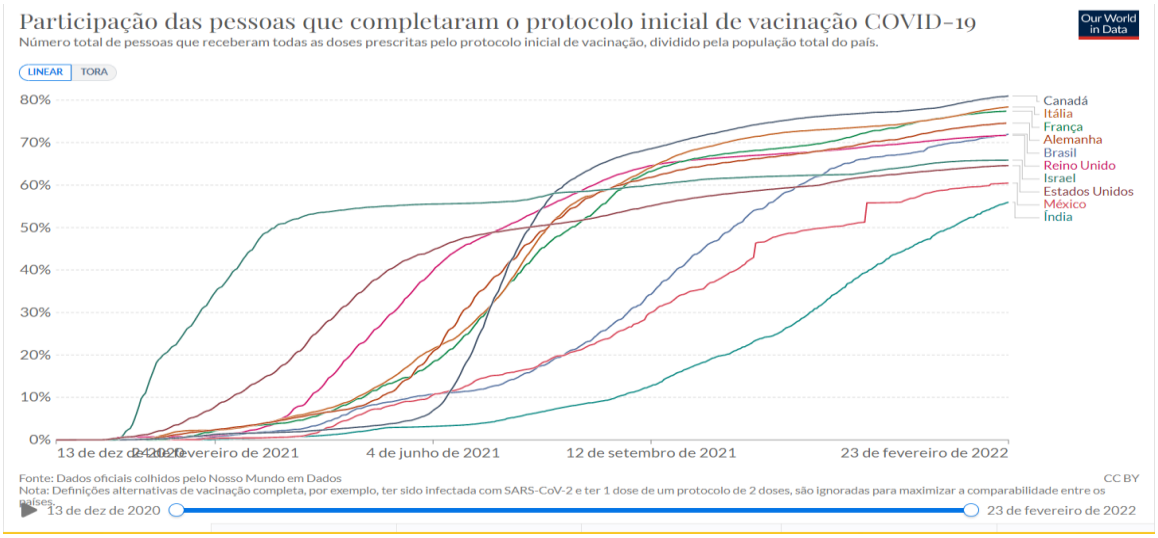
The LO of graphical analysis was a proposal of activity elaborated to work Probability and Statistics in the 9th grade of Elementary School, developed in the software GeoGebra and in the platform Google Slides, both of free access. LO provides an interactive opportunity for studying graphs, aiming to promote the ability to interpret graphs and statistical data. In its first part, it works with the ability to interpret from hypothetical situations, exploring graphs through questions about characteristics and variations observed in graphic constructions. In the second part of the OA, the work focuses on the application and discussion of the concepts reflected in the first part, and a sequence of exploratory questions of graphs related to confirmed cases and vaccination rates of Covid 19 in the world, during the Pandemic. The interaction provided by the object allows the activities to be answered directly in the book created in GeoGebra software, enabling students to respond and automatically save their answers.

Initially, the LO provides a brief Introduction to graphical analysis by discussing hypothetical graphs, specifically focusing on bar graphs and pie charts. Through these types of graphs are highlighted exploratory questions that promote visualization and data presentation, while exploring the main characteristics of both types of graphs. Thus, the LO also addresses the visualization of mathematical representations, the characteristics and differentiation between pie charts and bar graphs, as well as variables on the Cartesian plane to construct other

types of graphs, such as line graphs, and the interpretation of data.

The example provided in the presented LO includes data on *Covid-19 vaccinations during the pandemic*. It highlights the graphical exploration of a real-life situation, using data on the vaccination status in several countries and worldwide. In Figure 1 there is the graph considered for this activity. The countries considered in the activity are the United States, India, United Kingdom, Germany, France, Brazil, Italy, Mexico, Canada, and Israel, as indicated by the graph used in the activity.

Figure 1: Line Graphs — *Participation in Covid-19 Vaccination during the Pandemic*.



Source: Website: Our World in Data (Available at: https://ourworldindata.org/coronavirus; accessed on 12 jun. 2022)

With the exploration of the graph, we propose to discuss the data related to vaccines, considering financial and social issues, the ranking presented, classification of Brazil in the period from February 2021 to February 2022, positive or negative positions in relation to the period in our country, the vaccination of other countries in the world, and comparative data. In addition to exploring the data using the "slider" command in GeoGebra, it is possible to observe the time periods of each country and their respective percentages.

Another aspect contemplated in the LO was the exploration of the confirmed cases accumulated in the period of the Covid-19 Pandemic in the world, other questions sought to explore the cases per million people, which means to say that the numbers presented refer to the number of people infected per million.

From this study, it was possible to extract information about the data presented in order to critically analyze differences between the contamination that occurred in the countries represented in the graph under study, especially about the restriction measures practiced in each country.

7 Categorization, a brief discussion

In the categorization process, we are considering the second phase of analysis, foreseen by Bardin (2016), which is *the exploration of the material*, which consists of the coding, decomposition and enumeration of the data, in view of the rules already formulated, and the third phase of analysis foreseen by the author, which is *the treatment of the results, inference and interpretation*. In this phase, the results are treated so that they become significant and valid, allowing to build tables, figures, diagrams that represent the information obtained by the

analysis. We present a brief discussion on the subject in three categories: about pedagogical conditions, digital technologies, and practical approach to the LO; the conceptions and professional knowledge of future teachers; and the potentialities of discussion and mathematical analysis.

To reach the categorization stage, it was necessary to make an organization with the pre-analysis, the exploration of the material and the coding to determine the categories. Some categories, according to the author, can be established a *priori*, and in other cases, a *posteriori*, in the present study, the categories emerged by having contact with the data and the answers entered by the research participants in the LO itself.

The elements extracted from the text as units of analysis are categorized and grouped in order to continue the research. Categorization can group several of these elements that are classified according to common characteristics related to a theme. Bardin (2016) defines that "classifying elements into categories requires investigating what each of them has in common with others. What allows their grouping is the common part that exists among them" (p. 146).

The first category relates to pedagogical conditions, digital technologies, and the practical approach of the LO. In this stage, some verbs were selected to represent the possible meaning attributed to graphical representation in Mathematics Degree. Similarly, some expressions were also selected, considering the definition of pedagogical conditions and digital technologies.

The second category, which focuses on the conceptions and professional knowledge of future teachers, presents the data collected from the evaluation of the LO by academics and teachers who are in the initial stages of teacher training. In this stage, verbs and expressions were selected from the responses with the intention of representing possible conceptions and professional knowledge of future mathematics teachers.

The third category, which relates to the potential for mathematical discussion and analysis of the LO, is based on the analysis of the LO in its application, considering the practical approach, the involvement of participants, visualization, and mathematical interpretation. Thus, we present in Table 1 a selection of data to be considered in the analysis of this article.

Table 1: Graphical Analysis LO Data Analysis — PIBID

Events	Manifestations	Data Units	Categories	Observations
Pedagogical conditions	Overcoming difficulties	1	1	Academic-machine interaction, student-teacher interactions, concept.
Digital technologies	Preparation for working with DICT	1	1	Interactions between students and teachers and DICT
Conceptions	Preparation to teach, functions of the teacher	2	2	Teaching, human relationship, learning, mathematical language.
Professional Knowledge	Functions, roles and actions for teaching, planning, argumentation, visualization and symbolic language	1 2	2	Natural and symbolic language
Potentials	Graphical exploration and mathematics.	3 2	3	Communication in mathematical language.
Discussion	Discussion and mathematical	2	3	Visualization,

	analysis	3		mathematical representation.
Mathematical Analysis	Visual representation, language and thought	3	3	Structures in technical language. Mathematical visualization.

Source: Research Data

8 The pedagogical conditions, digital technologies, and practical approach of the LO

The pedagogical conditions, when working with LO with teachers in Initial Teacher Training in mathematics through the PIBID program, are inherent to the formative process. Participants point out "*the need to overcome difficulties*" and "*preparation to work and teach with digital technologies*". These aspects highlight possibilities for understanding the alternatives offered by mobile devices such as computers, tablets, and smartphones, emphasizing their role in Basic Education classrooms. Thus, this practical approach to LO in the Initial Teacher Training program, specifically in the mathematics education field, involves the participation of teachers in activities using ICT in mathematics teaching, as evidenced by Bairral e Henrique (2021).

The contributions of the participants, from the answers given to the questions related to the evaluation of the LO for the graphic exploration, also pointed to positive aspects of the pedagogical conditions and its applicability in the "*future practice in the classroom*". In the sense designated by Kalinke and Motta (2019), when referring to the applicability of the LO, when they highlighted that "the use of LO has the potential to change the dynamics of classes, incorporating new possibilities of pedagogical work" (p. 11). This attitude also corroborates the overcoming of difficulties with the digital technologies highlighted by Kenski (2018), when she affirms that it is not about their presence in the classroom, but about the "difficulty in mastering the competencies for the use of ICT by teachers" (p. 105).

8.1 The conceptions and professional knowledge of future teachers

The conceptions and professional knowledge of future teachers was another aspect analyzed during the development of the study. Participants expressed: "*I am worried, I need to learn the main functions, roles and actions needed to teach*", that occur in professional life, especially in relation to pedagogical practice. Another aspect, pointed out by the participants when they expressed concern with professional knowledge was: "*How should I be aware to make a good planning of the activities?*", which was taken up in the data, when the participants responded to the assessment instrument. The knowledge of professional formation included in the training actions, referred to as Tardif (2007), is related to conceptions of knowledge construction, pedagogical conceptions, teaching and learning conceptions, and pedagogical practice. In other words, they are part of the field of knowledge that encompasses teacher education. Furthermore, they are anchored in the Resolution CNE/CP nº 02/2015, which, by valuing principles of popular education, proposes the construction of an Initial Teacher Training that considers the conception of an inclusive curriculum, ensuring spaces for the participation of different social subjects, aware of the commitment to sustainable development.

In this sense, the LO, of this study, was considered a resource with determined functions and objectives, as well as a technology to support the classes, in order to guide the reflection, interpretation and contextualization of the concepts, in different types of graphs in an interactive way. The LO was a pedagogical material that made possible the visualization, involving constructions and exploration in GeoGebra. The proposal of the LO made possible the learning

in the relationship with digital technologies, strengthening the attribution of meanings and the interactive discussion. As Imbernón (2011) points out, "a training program should propose a process that provides teachers with knowledge, skills, and attitudes to create reflective or inquiry-based professionals" (p. 58). As well as the visualization in the teachers training, was another aspect found in the reports of the participants of the study, which indicates the valorization of the argumentation and mathematical symbology, which, according to Oliveira e Barbosa (2022), involves language as an element in the production of meanings, especially because mathematical language is imbued with unique symbolism.

8.2 The potentialities of mathematical discussion and analysis offered by the LO

From the analysis of the LO in its application, considering the practical approach, we observed that the participants highlighted the LO as "*an instrument that made it possible to see and resume activities*" that "*the proposals highlighted in the object considered the theme and objectives foreseen for the exploration of the content, Graphs in mathematics*". From these observations, we can say that the LO, in the case of this study, potentiated the respective process of mathematical interpretation, supported by visualization, as Oliveira e Barbosa (2022) highlight, for whom, understanding visualization is to acknowledge the existence of a close relationship between language and thought. Thus, we can recognize that the teachers in initial training of the PIBID Mathematics Program of this university campus were so involved with the LO that they were able to identify potentialities to promote the discussion and mathematical analysis of graphs, from the argumentation, the registration of impressions and the answers obtained. Aspects highlighted by Presmeg (2006), when it comes to visualization with digital technologies in mathematical teaching and learning processes, which allows us to conclude that visual representation clearly manifests differences among several types of problems enhanced by dynamic software when the representation occurs on the screen.

9 Some final words

When revisiting the research question regarding the search for pedagogical conditions assumed by an LO for the study of mathematical graphs in the Initial Teacher Training of mathematics teachers, we established three categories for the analysis and discussion of the data obtained in the study: 1) The pedagogical conditions, digital technologies, and practical approach of the LO; 2) The conceptions and professional knowledge of future teachers; and 3) The potentialities of mathematical discussion and analysis offered by the LO.

Based on these categories, the analyses conducted, considering the units of record, as well as the manifestations and observations obtained from the data, allowed us to highlight that: in order to learn about how to become a teacher, it is necessary to understand aspects related to the processes of teaching and learning, considering the prior knowledge to be addressed, the articulation between theory and practice, and the dialogue established with students and the knowledge that emerges from it. Another aspect to highlight is the orientation established by the teachers of the Degree in Mathematics, and regent teachers who worked in the school in which PIBID had a partnership, from the planning of the class, the relationship with digital technologies, the production of creative activities such as LO and the relationships established between teacher and academic, that culminated in the evaluation of the activities.

In a movement of diverse and distinct actions, we verified that most of the study participants had the desire to be a critical, creative, and reflective teacher, which influenced the ability to engage with practical activities, aiming at the test of conjectures that allowed the reflections on the graphs. Thus, the activities developed promoted the understanding that a LO can value visualization, representation, creation, reflection, and knowledge. In this study, the

reflections established in the LO involved the analysis of conditions necessary for pedagogical practice digital technologies in initial teacher training, learning, and discussion of different possibilities of graphs in the interaction with a study of representation analysis for the final years of Elementary School.

Therefore, we consider that the research question has been addressed. In order for Initial Teacher Training through the use of LO to provoke actions related to the discussion and learning of concepts, from the planning of actions that result in a critical-reflective pedagogical practice, the study made it possible to have moments of investigation, analysis, and work in the search for understandings, educational conceptions, pedagogical conditions, digital technologies, and potentialities of LO, all of which imply the construction of professional knowledge of being a teacher. To conclude, it is worth considering the continuous desire to review and rethink teacher education, considering the theoretical-practical reflection and educational policies for this purpose. Specifically, in mathematics teacher education programs, their pedagogical projects have shown a concern for the professional development of these teachers.

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