

## Applets in learning fractions: a look from instrumental genesis to emergency remote teaching

**Vania Sara Doneda de Oliveira**

Secretaria de Estado de Educação do Paraná

Campo Mourão, PR — Brasil

✉ [vania.oliveira28@escola.pr.gov.br](mailto:vania.oliveira28@escola.pr.gov.br)

 0000-0001-5229-1880

**Maria Ivete Basniak**


Universidade Estadual do Paraná


Campo Mourão, PR — Brasil

✉ [basniak2000@yahoo.com.br](mailto:basniak2000@yahoo.com.br)

 0000-0001-5172-981X



2238-0345 

10.37001/ripem.v13i3.3539 

Received • 21/02/2023

Approved • 28/03/2023

Published • 10/09/2023

Editor • Gilberto Januario 

**Abstract:** This article investigates contributions by the applets *Cuisenaire environment*, *Geometry* and *Fraction Models* for learning fractions from the measurement perspective in classes based on Exploratory Mathematics Teaching developed during the Emergency Remote Teaching. This qualitative-interpretative research was performed with students of 6<sup>th</sup> grade groups of Elementary School. Instrumental Genesis is the theoretical-methodological lens to analyze the intentional action of the subject, who using mental schemes of use, transforms the artifact into an instrument. *Cuisenaire environment* allowed students mobilize strategies to carry out multiplicative comparisons among the *rods* and establish fractions equivalence understanding symbolic representation and numerical magnitude signaling. The applet *Geometry* allowed students performing quadrilateral measurement (sides, perimeters, and areas), and using the applet *Fraction Models* they validate or not the same magnitude of fractional and decimal representations. Associating the three applets favored students to understand the product property, which when multiplying two fractions different from zero and one, the result may be less than one of the two factors.

**Keywords:** Fractions as Measure. Exploratory Mathematics Teaching. Instrumentalization. Instrumentation. Rational Numbers.

### **Applets en el aprendizaje de fracciones: una mirada a la enseñanza remota de emergencia desde la génesis instrumental**

**Resumen:** Este artículo investiga las contribuciones de los *applets* denominados barras Cuisenaire, cuadriláteros y modelos de fracciones en el aprendizaje de fracciones desde la perspectiva de la medición en las clases a partir de la enseñanza exploratoria de las Matemáticas desarrollada en la enseñanza remota de emergencia. Se trata de una investigación cualitativa interpretativa, realizada con estudiantes de dos secciones del 6<sup>o</sup> grado de la Educación básica. El *applet* barras Cuisenaire permitió a los estudiantes movilizar estrategias para realizar comparaciones multiplicativas entre barras y establecer equivalencias de fracciones, incluyendo representación simbólica y señalización de magnitud numérica; el *applet* cuadriláteros permitió realizar la medición de los cuadriláteros (lados, perímetros y áreas) y; el *applet* modelos de fracciones permitió la validación o no las magnitudes de representaciones fraccionarias y decimales. La asociación de los tres *applets* ayudó a los estudiantes a comprender la propiedad del producto que indica que al multiplicar dos fracciones distintas de cero y uno, el resultado de la multiplicación puede ser menor que uno de los factores.

**Palabras clave:** Fracciones como Medida. Enseñanza Exploratoria de las Matemáticas. Instrumentalización. Instrumentación. Números Racionales.

## ***Applets* na aprendizagem de frações: um olhar para o ensino remoto emergencial a partir da gênese instrumental**

**Resumo:** Este artigo investiga as contribuições dos *applets* Barras Cuisenaire, Quadriláteros e Fraction Models na aprendizagem de frações na perspectiva da medição em aulas assentes no Ensino Exploratório de Matemática desenvolvidas no Ensino Remoto de Emergência. Trata-se de uma pesquisa qualitativa de cunho interpretativo, realizada com alunos de duas turmas do 6º ano do Ensino Fundamental. O *applet* Barras Cuisenaire permitiu, aos alunos, mobilizar estratégias para realizar comparações multiplicativas entre as barras e estabelecer equivalência de frações, compreendendo a representação simbólica e a sinalização de magnitude numérica; o *applet* Quadriláteros realizar a medição dos quadriláteros (lados, perímetros e áreas) e; o *applet* Fraction Models validar ou não a mesma magnitude das representações fracionárias e decimais. A associação dos três *applets* favoreceu que os alunos compreendessem a propriedade produto que, ao multiplicar duas frações diferentes de zero e um, o resultado da multiplicação pode ser menor do que um dos dois fatores.

**Palavras-chave:** Frações como Medida. Ensino Exploratório de Matemática. Instrumentalização. Instrumentação. Números Racionais.

### **1 Introduction**

We identify difficulty to teach in our teaching practice, and consequently students have difficult to understand rational numbers as a diverse field from natural numbers as well. This difficulty is detached when, for example, they add fractions of different numerators summing their numerators and denominators, and it might be associated to teaching fractions focused on the part-whole perspective (Escolano & Gairín, 2005; Lamon, 2012). With a different perspective to teach fractions, Powell (2018; 2019a) uses physical *Cuisenaire rods* to enable students to advance from non-symbolic fractions to symbolic ones, under the measurement perspective.

We understand technology as a human production (Vieira Pinto, 2005) inserted in people's daily lives for activities related to personal communication and/or work situations, and we admit DT as "a set of technologies that mainly allow the transformation of any language or data into numbers, e.g., zero and one (0 and 1)" (Ceale, 2014, n.p.). Hence, we adopted DT in this work in reference to the artifacts that use digital technology in their development process, such as a computer, tablet, smartphone, internet, software, learning objects, applets<sup>1</sup>, among others.

Hereupon, Instrumental Genesis (IG) by Rabardel (1999) as theoretical-methodological lens allows us understanding schemes developed by students when using applets, and consequently to solve tasks, elucidating how the complex mobilizations of thinking happen when the artifact is transformed into an instrument. Bittar (2011, p. 169) affirms that "it is important highlight that this theoretical approach may be, and it is used to investigate learning with instruments, that is, to study how the student learns in the presence of instruments". Thus, IG allowed us understanding and analyzing how the artifacts usage are associated to knowledge used by students to perform determined task.

Especially when the world was dealing with the pandemic caused by COVID-19 and the Emergency Remote Teaching (ERT) has become reality, discussing technology within school space, or permeating this space, is not an option, but a need. Teaching institutions have needed emergency adaptation to the new context. It has led to an obligatory adoption of Digital

<sup>1</sup> We consider un applet an application executed within a website or a larger program that do not need installation in the artifact.

Technologies (DT for ERT practices occurred with no planning) (Moreira, Henriques & Barros, 2020). Therefore, ERT “become protagonist in a crisis moment, placing teacher facing challenges to build new ways of teaching-learning, redefining their pedagogical practices” (Valente, Moraes, Sanchez, Souza & Pacheco, 2020, p. 10).

We understand that in ERT, DT is used as an access means, and without it there is no ERT. However, teaching methodologies used by numerous teachers with ERT context were chosen to corroborate traditional teaching practices (Moreira *et al.*, 2020; Valente *et al.*, 2020). Although, we consider necessary and possible incorporate, even in ERT, new methodologies to mediate mathematic representations, concepts, and relations. Then, we investigate contributions by the applets Cuisenaire environment, Geometry and Fraction Models in learning fractions under the mediation perspective in classes based on Exploratory Mathematics Teaching developed in ERT. Changing the scenery paper-pencil into simulators, dynamic geometry software, among others, further accelerating the visualization process of mathematic representations, concepts, and relations, enables constructing, deconstructing, and reconstructing such representations in a dynamic way.

Fractions do not have a single concept or definition, they assume different interpretations, as a tangle of ideas with multiple meanings, articulating individually and with each other, among them is possible highlight: part-whole, reason, operator, quotient and measure, and operator (Kieren, 1976; 1980; Behr, Lesh, Post & Silver, 1983; Escolano & Gairín, 2005; Lamon, 2012). We believe that fractions teaching should consider these different interpretations, as we briefly discuss in the next section.

## 2 Fractions Teaching Introduction

According to Kieren (1976; 1980), Behr *et al.* (1983) and Lamon (2012), understanding the fractions concept is paramount for the students’ mathematic performance, further being essential for other mathematical contents, such as Algebraic (Bailey, Hoard, Nugent & Geary, 2012; Torbeyns, Schneider, Xin, & Siegler, 2015; Powell, 2018, 2019a). Moreover, Kieren (1976; 1980), Behr *et al.* (1983) and Lamon (2012) detach the need that students are presented to different interpretations of rational numbers (Kieren, 1980).

Behr *et al.* (1983, p. 91) affirm that “rational number concepts are among the most complex and important mathematical ideas that children meet along elementary school”. Although Behr *et al.* (1983) believe that part-whole interpretation is fundamental for the rational number concept, and consider important for teaching other interpretations, professors and textbook authors seem to use this interpretation as a fraction definition. Lamon (2012) criticizes this approach, highlighting that the word *fraction* does not refer only to part-whole interpretation, which generates obstacles to understand rational numbers.

[...] the restriction of instruction to part-whole interpretation left students with an impoverished notion of rational numbers, and increasingly teachers are becoming aware of alternative interpretations, referring to them as operator, measure, proportion, and quotient. Part-whole comparisons are on an equal footing with other interpretations and no longer deserve the distinction of being synonymous with fractions (Lamon, 2012, p. 33).

Escolano & Gairín (2005) and Lamon (2012) warn that introduction to the rational number concept by the part-whole interpretation leads to epistemological obstacles due to the need for carry out a double count (the whole and the parts taken), using natural number principles. Still according to these authors, didactic purposes justify the introduction to the

fractions study by this interpretation because it allows students to quickly manipulate fractional representations/symbols without understanding their fractional density and numerical magnitude, which are associated with the idea of measure.

Therefore, likewise Powell, we believe that fractions teaching introduction must be performed with interpreting fractions as a measure, since this multiplicative comparison relationship between quantities is in line with the historical emergence of fractions (Powell, 2018, 2019a; Roque, 2012). Then, it favors students realize the need for a new numerical field: rational numbers. As previously mentioned, Powell (2018, 2019a) uses physical Cuisenaire rods in his research on fractions teaching under measurement perspective.

We search for consider these issues in our studies, which are permeated by context nuances in which the research was developed, presented in the following section.

### 3 Research Characterization and Data Collection

This is a qualitative-interpretative research and empirical data were collected in Elementary School 6<sup>th</sup> grade in a public school. The first author was the mathematics teacher in 2020. Data collection was carried out from September to October 2020. In mid-August, a form prepared by the teacher-researcher was sent to students asking information on days and schedules for synchronous classes (lives), type of device available (computer, notebook, smartphone) and type of internet connection (Wi-Fi or mobile data). From seventy-one (71) students, thirty (30) returned the completed form and the terms of consent for the research.

For our investigation, we structured classroom based on EMT to be developed at ERT. For them, we planned and elaborated four exploratory nature tasks for students in the Elementary School 6<sup>th</sup> grade. For each task, one or more applets were proposed to favor the understanding by students on the fractions as measure, and to enable them to identify differences among properties of natural numbers and fraction ones, such as *numerical magnitude signaling* (bigger numbers in the fraction do not determine the fractional magnitude); *symbolic representation* (there are an infinity of fractional representations for the same magnitude); *density* (fractional number has no immediate predecessor or successor); and *product* (multiplying two fractions different from 1 or 0 among them might produce a product less than one of the two factors) (Obersteiner, Dresler, Bieck & Moeller, 2019).

#### 3.1 Students and Available Resources Characterization for ERT

Students who participated in the survey are identified by pseudonyms chosen by them, according to the signed research consent terms. In the Chart 1 first column, groups are identified by the letter G and sequential numbering. The second column, next to the pseudonym, identifies equipment available to ERT. Regarding the type of internet connection, only Boom, Flora and Maluquinha, all of them belonging to the G3 had just mobile data available. All other students had broadband internet access. The third column informs days and schedules for synchronous classes (lives).

**Chart 1:** Weekly meetings organization

Groups	Participants and Available Artifacts	Lives
G1	Docinho (smartphone), Doguinha (notebook and smartphone), Florzinha (smartphone), Fofinha (smartphone), Lindinha (smartphone) and Thor (computer with no sound and smartphone).	Tuesdays at 8am
G3	Boom (smartphone), Flora (smartphone), Maluquinha (smartphone),	Tuesdays at

Groups	Participants and Available Artifacts	Lives
	Mazarect (notebook) and Olívia (computer).	6:30 pm
G4	Anubis (notebook and smartphone), Bob (smartphone), Fifo smartphone), Magrão (computer and smartphone), Spider-man (smartphone) and Ymercurius (computer and smartphone)	Wednesdays at 10am
G5	Luffy (computer and smartphone), Mulher Maravilha (smartphone), Poster (notebook and smartphone), Viúva Negra (smartphone), Zorro (smartphone)	Wednesdays at 2:40 pm
All students in the two sixth-grade classes, research participants and non-participants.		Fridays at 2:40 pm

Source: Data survey (2020)

Each student should participate in two (2) lives per week via Google Meet<sup>2</sup>, and their records of tasks development and systematizations of math learning would be posted on Google Classroom<sup>3</sup>.

### 3.2 EMT and Classes Organization in ERT

EMT is a demanding approach for students and teachers. Some studies (Ponte, 2005; Oliveira, Menezes & Canavarro, 2013; Basniak & Estevam, 2018; 2019) discuss that exploratory tasks instigate students to develop and discuss different strategies for their resolution, encouraging the communication of individual ideas and the construction of collective ideas, providing opportunities for reflection and information analysis in a collaborative work for the construction of mathematical learning and negotiation of meanings. Therefore, EMT is pointed as alternative for traditional or direct teaching (Ponte, 2005), in which the maxim prevails: teacher explains the content, gives examples, students assimilate and reproduce learning with fixation exercises.

Canavarro, Oliveira and Menezes (2012) suggest that EMT classes must be planned and developed in phases, which Cyrino and Teixeira (2015) admit being four: *Task introduction, task carrying out, Task Collective discussion, and Learning systematization*.

To ensure the negotiation of meanings, preparation and reworking ideas, concepts and mathematical relationships, classes based on EMT in ERT context need DT so that it is possible to solve group tasks synchronously, and posteriorly discussions and systematization of mathematical learning are carried out with all the groups synchronously as well. Therefore, classes organization has occurred in a different way in ERT. We followed guidelines by Oliveira and Basniak (2021) that bring tables with the teacher's intentional actions in EMT practice for ERT in each of its phases. We organized a live for each group to introduce and develop the class, and a second live for each collective task discussion and systematization of mathematical learning with all the students (Table 2).

For each class, we structured tasks to carry out using specific applets, as explained in the following section.

<sup>2</sup> Google Meet is a Google company solution which offers a free videoconference communication service for up to 250 people for educational institutions.

<sup>3</sup> Google Content Management Resource for Educational Institutions which allows the creation of virtual classrooms.

### 3.3 Suggested Tasks and Applets

In this paper, we discuss the use of applets by students: *Cuisenaire environment*, *Fraction Models* and *Geometry*. The groups solved four (4) tasks; Task 2 and Task 4<sup>4</sup> were subdivided into part 1 and part 2. Each task was developed based on EMT perspective, and they were planned to meet all the phases suggested by Cyrino and Teixeira (2016) within the same week. Chart 2 identifies each task, its objectives and the applets used.

**Chart 2:** Exploratory nature tasks, objectives and applets suggested

Task	Objective(s)	Applet(s) Suggested
Task 1: What is the length?	- Understanding fraction as a measure	- <i>Cuisenaire environment</i>
Task 2: measuring with <i>Cuisenaire environment</i> (Part 1 and 2)	- Understanding equivalence relationships and representing them algebraically, - Understanding fraction equivalence, - Understanding fractional representation, - Comparing fractions; and - Understand adding fractions of the same measure unit.	- <i>Cuisenaire environment</i>
Task 3: Train game	- Comparing fractions, and - Understanding fractions addition and subtraction of different measure units.	- <i>Cuisenaire environment</i>
Task 4: Quadrilateral Area and Perimeter (Part 1 and 2)	- Understanding fractions multiplication, - Associating decimal representation with fractional one, - Relating the multiplication of fractions and decimals to the calculation of quadrilateral area; and - Understanding rational numbers as a field different from natural numbers.	- <i>Cuisenaire environment</i> - <i>Geometry</i> - <i>Fraction Models</i>

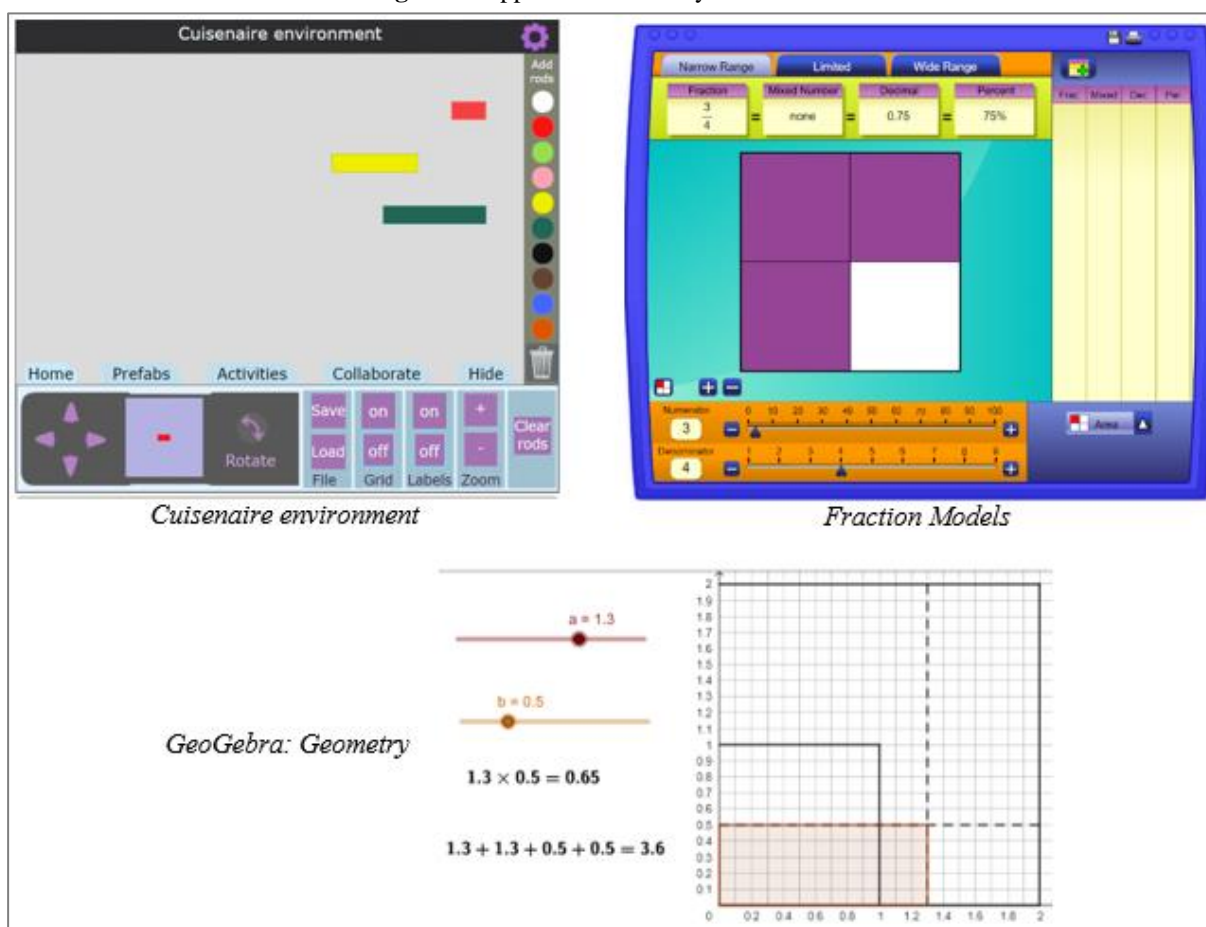
**Source:** Own elaboration

*Cuisenaire environment* were used in all tasks, that despite not having a standardized measurement unit, like the physical rods (centimeter), the white rod may be taken as a reference for all other ones: red rod measures two white rods; light green rod measures three white ones; and so on. However, as the rods do not have a preestablished value, the same rod might be taken for different values, depending on the comparison performed.

For task 4 we used the applet *Geometry*, in which is possible construct squares and rectangles by changing their length and width by moving slide controls a and b, making possible view the calculated area and perimeter on the screen. We also suggested the applet *Fraction Models*, in which is possible writing the fractional representation at the bottom of the applet by typing at Numerator and Denominator fields; or sliding the indicator at the wished value; or even clicking the minus or plus buttons, then returning to the pictorial representation, the mixed form of the fraction (if there is one), decimal and percentual. Figure 1 shows the applets used.

<sup>4</sup> Task 4 was planned with no subdivision, but when performing the phases *Introduction* and *Task carrying out* with G1, students could not solve the whole task in the planned time. Then the Task 4 was subdivided as follows: part 1 with items a and b; and part 2 with items c and d.

**Figure 1:** Applets used to carry out the tasks



Source: Data survey (2020)

Instrumental Genesis (IG) provided subsidies to understand mental schemas elaboration in the use of applets and the way how the applets interfere in students' thinking and vice versa to understand fractions as measure, according to detailed in the following section.

### 3.4 Instrumental Genesis as Theoretical and Methodological lens

IG, based on Cognitive Ergonomics theory and on ideas by Vygotsky was developed by Rabardel (1995), when studying the subject's action mediated by an instrument.

This perspective is based on the transformation process the artifact into an instrument, as an intentional construction of the subject who appropriates the artifact and by developing usage mental schemes becoming it useful: constitutes an instrument, according to the usage schemes mental assigned to the artifact.

Drijvers *et al.* (2010, p. 108) detach that "Instrument = Artifact + Schemes and Techniques for determined type of task". Thus, the instrument is more than an artifact, it is an individual construction for each person, through their own usage schemes (Rabardel, 1995; Bittar, 2011). Therefore, "[...] instrument is not something done and finished; it may be elaborated and re-elaborated by the subject throughout activities performed with the artifact, now an instrument, since it has already suffered the subject's action" (Bittar, 2011, p. 162). Then, usage schemes associated to the artifact may result from "a subject's own construction, autonomously or through the appropriation of pre-existing social schemes" (Almeida & Oliveira, 2009, p. 88), so it is different for each person because "the instrument as an artifact is

constituted in the usage that the subject makes of it”.

In addition to the distinction between artifact and instrument, the dimensions of instrumentation and instrumentalization approach the double relation between subject and artifact. While in instrumentation the subject appropriates the artifact and adapts it to his/her work; in instrumentalization, artifact potentials and limitations collaborate to structure the subject’s activity (Rabardel, 1995; Almeida & Oliveira, 2009; Basniak & Estevam, 2019; Bueno & Basniak, 2020). Thinking on DT which permeate teaching and learning process, we consider that, while instrumentation “refers to how the artifact affects student behavior and thinking”, instrumentalization “concerns how the student’s thinking affects the artifact” (Basniak & Estevam, 2019, p. 741). Ergo,

instrumental genesis is a complex process that seeks integration between the artifact characteristics (potentials and limitations) and the subject’s activities. This process occurs in two different directions: in the internal direction of the subject itself, called instrumentation, and in the external one, of the artifact, called instrumentalization (Padilha & Bittar, 2013, p. 7).

For data analysis from IG perspective, we consider:

- *Subject*: the student who, by employing individual schemes in the artifact, transforms this artifact into a unique instrument. Particularly given the context of research development, the artifacts/instruments which are the focus of this article are those directly related to teaching and learning fractions as a measure: the applets.
- *Schemes*: refer to the “structured set of generalizable characters of activities of instruments usage” (Rabardel, 1999, p. 210).
- *Instrument*: comes from the relationship between the subject and the object, in which the artifact added to the usage scheme(s) that the subject employs in this artifact transforms it into an instrument.

Chart 3 makes explicit describers of instrumentation and instrumentalization associated to the applets related to our planning for fractions teaching under measure perspective.

**Chart 2:** IG elements, subject and applets

	<b>Instrumentation</b>	<b>Instrumentalization</b>
<b>Subject</b>	<ul style="list-style-type: none"> <li>- Develops ideas and/or generalize them from what he/she reads in the applet.</li> <li>- Uses applet tools to develop ideas and/or generalize them.</li> <li>- Appropriates the characteristics and capabilities of the applet to perform measurements.</li> <li>- Uses the applet to determine and/or verify equivalent fractions.</li> </ul>	<ul style="list-style-type: none"> <li>- Understands how the applet works and manipulates it.</li> <li>- He/she is aware of the possibilities and limits of the applet.</li> <li>- May create/adapt applet to solve task.</li> <li>- Investigates the applet components and functionality.</li> <li>- Explores the features available by applet.</li> <li>- Appropriates the characteristics and capabilities of the applet to perform the task.</li> </ul>
<b>Cuisenaire envi-</b>	<ul style="list-style-type: none"> <li>- Takes measurements.</li> <li>- Establishes comparative</li> </ul>	<ul style="list-style-type: none"> <li>- Perceives overlapping rods.</li> <li>- Performs multiplicative comparisons.</li> </ul>



	<b>Instrumentation</b>	<b>Instrumentalization</b>
<i>ronment</i>	relationships. - Establishes equivalence relationships.	- Uses the rods to elaborate ideas and/or generalize them. - Makes comparisons between same color or different color rods aligned side-by-side, and same color or different color rods aligned below.
<b>Geometry</b>	- Represents the requested quadrilaterals. - Associates length and width to slide controls. - Associates decimal representation to fractional ones. - Identifies when the quadrilaterals do not meet the requested conditions.	- Moves the slides controls and observe changes related to the length, width, area, perimeter, and values shown by the applet. - Use the applet to find/check equivalent fractions.
<b>Fraction Models</b>	- Associates decimal representation to fractional ones. - Uses equivalent fractions.	- Changes numerator and denominator and observe the decimal, pictorial, and percentage representations that the applet returns. - Comprises the chosen measure unit.

**Source:** Own elaboration (2020)

We reiterate that these describers were anticipated by us from our intention with the applets. From data analysis, we identified those that were outstanding during the classes as shown by the next section.

#### 4 Data Analysis

To situate the reader regarding the use of applets by students, and the IG process throughout the classes, we analyzed the resolutions and discussions of the following tasks: *Task 1* because it was the first one that Cuisenaire environment was used; *Task 3* due to the task objective, which required that students understand the fractions magnitude; and *Task 4* because we used other applets further the Cuisenaire environment. To analyze students' evolution during the tasks, we chose presenting excerpts from the same group, G1, because it was the one that more clearly explained doubts and (mis)understandings during tasks development. It should be noted that when the group was developing the task, the teacher often closed the camera and the audio so that the students could develop autonomous work, and they called the teacher through a WhatsApp group when necessary.

For Task 1 carrying out, students needed to determine the total horizontal length of the applet region using the entire rods, which did not complete the length.

**Chart 4:** Task 1: *What is the length?*

Access the link <https://nrich.maths.org/cuisenaire/responsive.html>

- 1) When clicking on the colored circles on the tools, right side, different sizes and color rods appear. To select the rod to be used, click on the color you wish.
  - a) Using the same color rods, determine the horizontal length of the applet region. Do it for all

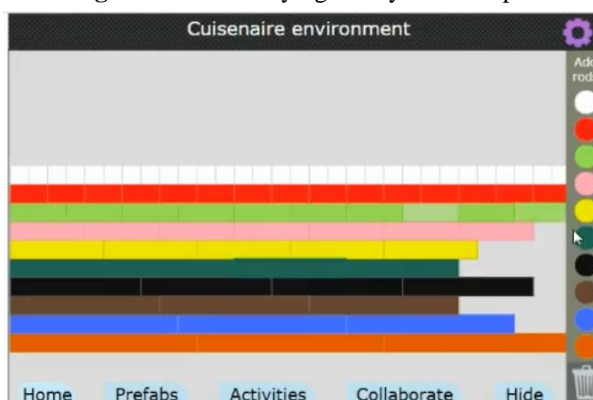
the colors.

- b) You have already noticed that some rod colors do not complete the full horizontal length of the applet region. How can you explain to colleagues the total horizontal length of the applet region using this rod (which does not complete the length) in the explanation?

Source: Data survey (2020)

After reading the assignment with students, the teacher asked who could share the screen to solve the task, and the group nominated Thor. Thor's computer microphone had a problem during this task, so he interacted via Google Meet chat. He aligned all the bars on the applet as shown in Figure 2.

Figure 2: Task carrying out by the Group 1



Source: Data survey (2020)

Any Cuisenaire rod may be used as a measure unit reference since they do not have a fixed unit value. Hence, looking at the arranged bars (instrumentation), students needed to decide (instrumentalizing) how they would answer the question, establishing as the measure unit the color used to measure the applet horizontal length. However, initially, they started using approximations/estimates for the rod parts that were missing to complete the applet horizontal length, without naming the bars using different colors. As example, regarding the blue rod, they concluded that the length measurement was 3.30 blue rods. then a discussion was started on how to name the missing piece of blue rod to complete the applet horizontal length.

Docinho: Teacher! So, may we put the little white blocks there and see how much it gives? [instrumentalization]

Teacher: You know there are nine white [rods] to one blue, right? You have already told me this.

Lindinha: Right.

Teacher: [You know that] one blue has nine white ones. But to complete that piece, how many white ones do it [...]?

Docinho: Thee. Three of them do it, teacher.

Teacher: Put the little white ones there [instrumentation]. You may see that there are three left. But the task does not affirm that you can complete with three rods, you must speak that is blue. What is this piece, what measure is this in relation to the blue one?

Docinho: Teacher, so, if I were to measure the blue counting with the white ones there, I would give three little rods.

Teacher: So, it's three what?

Docinho: Three and thirty.

Teacher: Three what?

Thor: [types in meeting chat] ninths.

Teacher: Thor typed there.

Docinho: Three ninths.

Task 1 – G1. Task carrying out, 09/15/2020.

From the teacher's questions, students used the white rod as a measure unit and started making multiplicative comparisons between the rods using the applet to validate the initial ideas.

They chose to align the brown rod above the white ones, and when comparing what the applet returned (instrumentation), concluded that the brown rod measured eight (8) white ones (Figure 3).

**Figure 3:** Task carrying out by the Group 1



**Source:** Data survey (2020)

However, Doguinha made a mistake when named the missing part to complete the horizontal length as  $\frac{3}{8}$ . Thor noticed it out and began to arrange six (6) white rods in the space that was left to complete the applet horizontal length (instrumentalization), aligned with the brown rod (between the blue and black ones), according to the Figure 4.

**Figure 4:** Task carrying out by the Group 1



**Source:** Data survey (2020)

Doguinha: The brown one is three-eighths.

Thor: [writes in the chat] 6 eighths.

[...]

Docinho: To complete there, six white rods are missing, not three. As Thor put there, so it's six-eighths.

Lindinha: Ok, so, there will be three [entire] rods and six eighths.

Task 1 – G1. Task carrying out, 09/15/2020.

Unable to justify his ideas by speaking, Thor used the applet, mobilizing a resolution strategy (completing the missing piece of the brown rods using the white ones as the measure unit), and then he wrote 6-eighths in the chat. The artifact (applet) was turned into an instrument through the comparison strategy, and instrumentalization was identified when Thor found a strategy using the artifact to prove/demonstrate his idea to his colleagues. For the other rods which did not complete the horizontal length, Thor's colleagues asked him to do so, then he completed with white rods the lengths of the pink, black and blue ones, being able to measure these pieces.

Task 3 started with the Train game (Chart 5).

**Chart 5:** Task 3: *Train game*

Access the link <https://nrich.maths.org/cuisenaire/responsive.html>

3) Let us play the Train game. We will call *wagon* each rod and *train* the horizontal alignment of one or more *wagons*.



To do so, each group must be divided into pairs, or two play and other waits to play the next turn. Each pair or player chooses a wagon. This is how the game works:

- Each pair or player only will use the wagon chosen to play, i.e., always the same color.
- The chosen wagon for each player (or pair) will be vertically aligned on the screen.
- Starts to play who has chosen the shorter train.
- Always plays who has the shorter train, until his/her train became bigger than the opponent's.
- The game finishes when all the trains became the same size.
- The player or pair who put the last wagon wins the game.

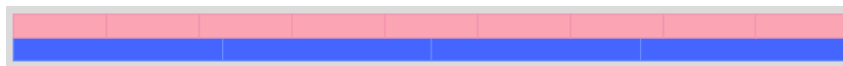
After playing some games, answer:

- a) What is the best strategy to win the game? After completing a game round, considering each train separately, write down the fraction that each wagon represents regarding the entire train.
- b) By observing the fractions written down to the item a, build trains with the same size of each wagon of each player. There is one condition: the trains must have the same color. Write down equivalent fractions regarding the trains and wagons.
- c) Considering the fractions that each wagon of each train represents regarding the entire train (item a), which is the smallest of these fractions?
- d) Considering the fractions that each wagon of each train represents regarding the entire train (item a), what is the result of the sum of these fractions? Explain it representing with Cuisenaire environment.
- e) Considering the fractions that each wagon of each train represents regarding the entire train (item a), what is the result of subtracting the largest fraction minus the smallest fraction? Explain it representing with Cuisenaire environment.

Source: Data survey (2020)

This game uses *Cuisenaire environment*, and it may be played individually or by pair, so that each player or pair chooses only one-color rod to play, according to the rules mentioned in Task 3 (Chart 5). After a few game rounds, students should choose one of them and report, on their notebook or file, the fraction that each wagon represented regarding the entire train. G1 chose the round below (Figure 5).

**Figure 5:** Task 3 carrying out by the Group 1



Source: Data survey (2020)

G1 correctly identified the fractions as representing  $\frac{1}{4}$  for the blue wagon and  $\frac{1}{9}$  for the pink one. When comparing these fractions to determine which of the fractions was the bigger one, Docinho, after having announced that  $\frac{1}{4}$  was bigger than  $\frac{1}{9}$ , has changed her mind and said she did not think this was right, that she believed to be the opposite. Facing the teacher's question about her doubt, Lindinha explained, as showed below, that Docinho was comparing only the fraction denominators.

**Teacher:** *But what are you seeing, Docinho? You can see that the blue one is bigger. Why aren't you understanding that the blue fraction is bigger?*

**Lindinha:** *Teacher, she is saying that  $\frac{1}{9}$  is bigger than  $\frac{1}{4}$  because 9 is bigger than 4.*

**Task 3 – G1. Task carrying out, 09/15/2020.**

Despite seeing that the blue rod  $\frac{1}{4}$  was bigger than the pink one  $\frac{1}{9}$ , the student had not understood the property of signaling the fraction numerical magnitude, and then she could not understand the fraction  $\frac{1}{4}$  as bigger than  $\frac{1}{9}$ , because number 9 is bigger than number 4 in the set of natural numbers. In view of Lindinha's explanation, Docinho stated that one should not compare fractions by numbers (she was referring to the denominator), but by the *Cuisenaire rods*. The teacher had to intervene, explaining that *Cuisenaire rods* and respective fractions represent the same magnitude. Then the teacher asked G1 to compare the fractions  $\frac{1}{2}$  and  $\frac{1}{10}$ . Thereunto, they aligned vertically white and yellow rods on the applet, as if they were going to play the train game, being able to compare the rods. Using the applet enable the students visualize and understand fractions magnitude from the measurement perspective, revealing the instrumentation process.

Other applets further than *Cuisenaire environment* were used to the Task 4.

**Chart 6:** Task 4: *Quadrilateral Area and Perimeter*

4) Open the GeoGebra Geometry file available at <https://www.geogebra.org/geometry/gvd5vbsq>. (If you use the applet on your mobile, tick the options desktop version on the browser). Move *a* and *b* slide controls, observe what happens in the represented figure and answer:

- What does change in the figure represented by GeoGebra as you move *a* and *b* slide controls?
- Since the area of squares and rectangles is the result of multiplying the length by the width and the perimeter is the sum of all sides, use the GeoGebra Geometry file available at <https://www.geogebra.org/geometry/gvd5vbsq> (if you use the applet on your mobile, tick the options desktop version on the browser) and the applet Fraction Models available at <https://www.nctm.org/Classroom-Resources/Illuminations/Interactives/Fraction-Models> to complete the table referring to the sides, perimeters and areas of the quadrilaterals in their decimal and fractional representations.

	Rectangle 1		Square 2		Rectangle 3	
	Decimal representation	Fractional representation	Decimal representation	Fractional representation	Decimal representation	Fractional representation
<b>Length</b>	0,2				0,7	
<b>Width</b>	0,6			$\frac{9}{10}$		
<b>Perimeter</b>						
<b>Area</b>					0,35	

	Square 4		Rectangle 5		Square 6	
	Decimal representation	Fractional representation	Decimal representation	Fractional representation	Decimal representation	Fractional representation
<b>Length</b>			4,5			
<b>Width</b>						
<b>Perimeter</b>			13			$\frac{24}{5}$
<b>Area</b>	1					

c) Return to the previous task. Observe the tables and answer: What difference(s) is (are) there in the multiplication of Rational Numbers when compared to Natural Numbers? Is it possible to say that the results always increase in the multiplication of Rational Numbers? Explain your reasoning and, if necessary, explore the GeoGebra file further, testing different values.

**Source:** Data survey (2020)

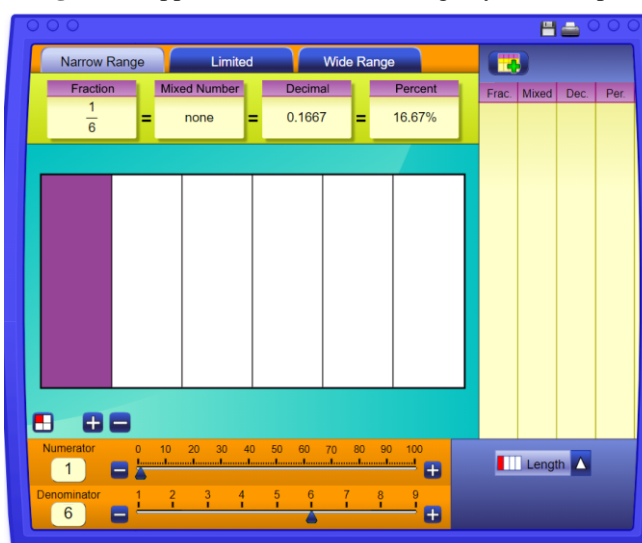
In the resolution phase of task 4 part 1, Lindinha was chosen by her group (G1) to share the smartphone screen with the Geometry applet so that the group could discuss its manipulation (artifact usage schemes). After manipulating the applet Geometry and choose computer version to improve viewing, she started dragging left and right a and b slide controls (instrumentation). After some time of group discussion on the applet usage (instrument), Docinho described the relationship between the slide controls and the quadrilateral sides:

Docinho: This is how I've done the first: when we move the control a, it [length] goes from one side to another, and when we move the control b [slide control b], it [width] goes up and down, but also when we move a and b, we form quadrilaterals which might be tall, short, wide, and long. A [slide control a] forms the length and b one [slide control b] makes the height.

Task 4 – G1. Task carrying out, 10/20/2020.

Then, for the item *b* in this task, the group's first concern was that they would have to use two applets to complete the table. Considering that ERT for group discussions is not possible share the screen with all the applets simultaneously, the teacher urged them to observe the values and calculations that appeared below the slide controls in the applet Geometry. With these questions, the students realized that such values were related to the perimeter and area of quadrilaterals, and therefore the values requested by Task 4 table, referring to the decimal representation of Rectangles 1, 3 and 5, and of Squares 2, 4 and 6, were explicit by the applet.

Thus, to perform conversions from fractional to decimal representation or vice versa, students used another applet *Fraction Models*. The student indicates the numerator and denominator (instrumentalization) at the applet, and it returns a figure of area, fractional representation, mixed number (if any), decimal and percentage, which the students needed to interpret (instrumentation). G1 students used this applet to check if decimal representation of Rectangle 1 (Task 4 table), two tenths (0.2), would be equal to the fractional representation  $\frac{2}{10}$ . Thereunto, they enter 2 at numerator and 10 at denominator, and the applet returned 0.2 decimal notation. However, when checking the perimeter of Rectangle 1, whose decimal representation is 1.6, they enter the applet 1 as numerator and 6 as denominator (instrumentalization). As a result, the applet returned decimal 0.1667, as in the Figure 6.

**Figure 6:** Applet Fraction Models usage by the Group 1

**Source:** Data survey (2020)

Therewith, they concluded that fractional representation was not expected (instrumentation). So, the teacher asked the group how to read 1.6, and Docinho answered sixteen (16) tenths. Whence, the teacher encouraged them to enter this information into the applet. Lindinha, who was sharing the screen, inserted 16 at numerator and 10 at denominator, and they could verify that decimal representation in the applet resulted in 1.6. Next, to verify if they were correct regarding fractional representation of the area measurement, 0.12, they realized that the Fraction Models applet is limited to the denominator 25, so they called the teacher to help.

Docinho: And twelve hundredths... Is there 100 here? [looking at the applet how to enter 100 at denominator].

Lindinha: No, just until 25... Teacher, are you online?

Teacher: Hi, guys, what happened?

Docinho: There is not 100! [referring to the applet Fraction Models].

Teacher: So?

Docinho: It is because I think that in the area the value is  $\frac{12}{100}$  (twelve hundredths) but there is not 100 there.

Task 4 – G1. Task carrying out, 10/20/2020.

The applet *Fraction Models* is limited to the measurement unit 100 (denominator), its maximum is 25. The teacher asked them to continue the task by completing the table referring to the other quadrilaterals, and in the meantime, they would think about how to check it.

They then proceeded to complete the table referring to Square 2, whose only information was the width, in the fractional representation  $\frac{9}{10}$ . To complete the length, area, and perimeter, both in decimal and fractional representation, they had no difficulties. Firstly, they used the applet *Fraction Models*, entered 9 as numerator and 10 as denominator to convert fractional representation  $\frac{9}{10}$  into decimal 0.9. Then they opened the applet *Geometry*, slid the slide controls  $a$  and  $b$  until the measure 0.9 and found the perimeter and area to complete the table referring to decimal representations (instrumentalization). Next, they returned to the Fraction Models

applet to check if the perimeter they wrote as fractional representation was correct: they entered 36 at numerator and 10 at denominator and verified it was 3.6. They were also unsure about the fractional representation of the 0.81 area measure, as they had already found that the Fraction Models applet is limited to the denominator 25. So, even in doubt, they wrote  $\frac{81}{100}$  and proceeded with the task resolution regarding Rectangle 3. For this rectangle, the length 0.7 and area 0.35 were informed.

Docinho: [...] So, enter there, Lindinha, 0.7 length [she slid the slide control a to the 0.7 measurement].

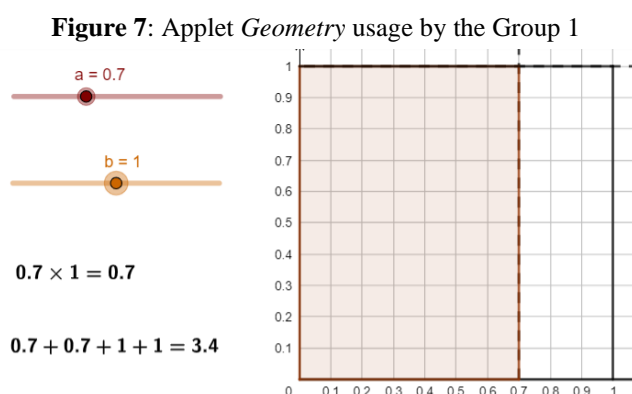
Lindinha: Done.

Docinho: Now go ahead with width [She slid the slide control b] until 0.35 there, in times [at the value of the area].

Lindinha: Look there, 3.4 [Lindinha slid the slide control b until the measure 1].

Task 4 – G1. Task carrying out, 10/20/2020.

This discussion resulted in the Figure 7.



**Source:** Data survey (2020)

Docinho: No, over there 0.7 times something, the result must be 35 [referring to 0.35 at the area measurement].

Lindinha: It is on plus, isn't? [referring to the perimeter calculation].

Docinho: No, it is on plus [referring to the area].

Lindinha: There! [Lindinha slid the slide control b until 0.5].

Docinho: Ah, then 0.5 is the width!

Lindinha: Why did you choose 35? [referring to 0.35].

Docinho: Because over there, in the area that the teacher sent to the group [referring to the task table], there is 0.35, so I've talked to you enter the result 35 to check what the width result was.

Lindinha: Ok, I got it!

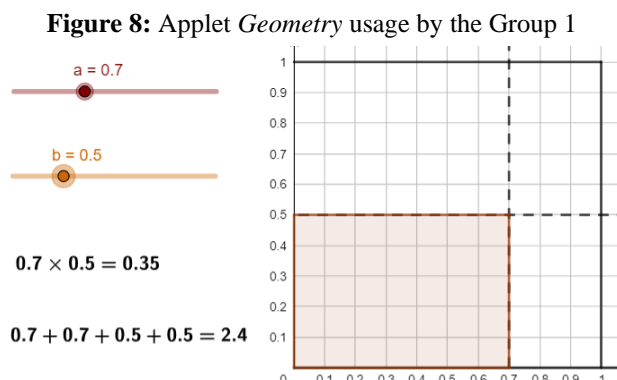
Docinho: Now the perimeter is 2.4. So, the with will be 5/10, the perimeter will be 24/10, and the area 35/100.

Task 4 – G1. Task carrying out, 10/20/2020.

When students represent the requested quadrilateral, they associate length and width to the quadrilateral slide controls, and link the decimal representation to the fractional one, so the instrumentation is revealed. The action of moving the slide controls while observing changes



related to length, width, area, perimeter, and values shown in the software is related to instrumentalization.



**Source:** Data survey (2020)

Our intention was for students to conclude that, when multiplying fractions whose factors are less than 1, the product is smaller than the factors; when the product equals 1, the factors are also 1; and when the factors are bigger than 1, the product is bigger than the factors. In other words, it is different with natural numbers, that the product is always bigger than the factors, in fractions, this is not a rule. Hence, to complete the Task 4 table, referring to the representations of Square 4, the only information given was the area measure, 1, as decimal representation. There were no difficulties in finding the length and width measurement, but the students were in doubt about the fractional representation of natural numbers. At the time of the dialogue, which can be read in the excerpt below, Lindinha shared the applet *Geometry*.

Teacher: [...] But what fraction could you write to represent one?

Docinho: One one? [referring to the fraction 1/1].

Teacher: What do you think? Is there another?

Docinho: I do not know, teacher... It is one entire.

Lindinha: Is it  $\frac{1}{2}$  teacher?

[...]

Docinho: Ok, I will check the other applet [referring to the Fraction Models, and she starts to share the screen]. It is 1 by 1, teacher, 1 at numerator and another 1 at denominator [she enters the Fraction Models 1 as numerator and 1 as denominator, which returns decimal representation equals 1].

Lindinha: And on the bottom [referring to the perimeter] it will be 4, the result for the perimeter.

Docinho: Then it will be 4 by 4... let me see... [she inserts in the Fraction Models 4 as numerator and 4 as denominator, which returns the decimal equals 1]... [Then, after some seconds in silence, the student enters 16 at numerator and 4 at denominator and verifies that this fraction  $\frac{16}{4}$ , returns the decimal 4]. It is  $\frac{16}{4}$ , teacher.

Lindinha: So, enter  $\frac{16}{4}$  [referring to the filling in the table referring to the square 4 in the perimeter fractional representation].

Docinho: And the area...

Lindinha: It will be 1/1.

Task 4 – G1. Task carrying out, 10/20/2020.

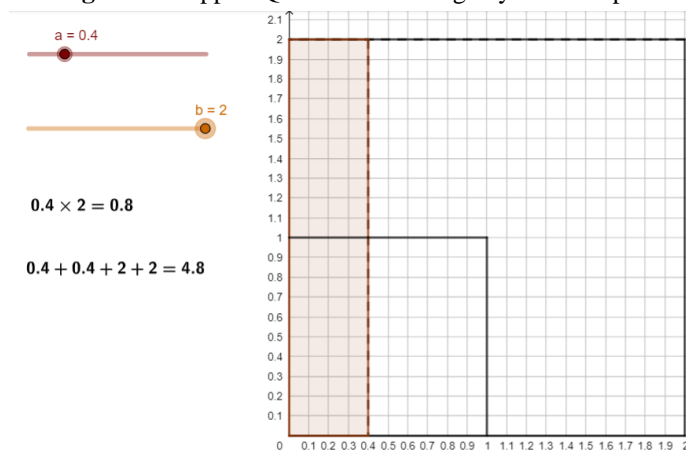
The intention of putting length and width measuring 1 was for the students to understand that, in the multiplication of rational numbers, multiplying two different fractions of 1 together

may produce a result (product) smaller than one of the two factors. Length and width 1 would be parameters to understand this.

As provided by the Task 4 table, regarding the Rectangle 5, information that students had to fill in decimal and fractional representations was length 4.5 and perimeter 13. To find the width, they would need to subtract twice the length, 9, from 13, and then divide the result by 2. However, they found the following problem: the applet Geometry only accepts length and width up to measurement 2. When trying to slide the slide control  $a$  to the measure 4.5, they realized this limitation (instrumentalization). After some minutes discussing this issue without concluding how to do, they decided leaving to answer this question about Rectangle 5 later, and work with the next quadrilateral, Square 6.

In the Task 4 table regarding the Square 6, only the perimeter fractional representation was informed,  $\frac{24}{5}$ . Then, Docinho, using the applet *Fraction Models*, inserted 24 at numerator and 5 at denominator, and concluded that the decimal representation was 4.8 (instrumentation and instrumentalization). Lindinha was sharing the applet *Geometry*, and she moved the slide controls  $a$  and  $b$  for the perimeter to measure 4.8. However, the quadrilateral should be a square.

**Figure 92:** Applet Quadrilaterals usage by the Group 1



**Source:** Data survey (2020)

Docinho: Teacher, the perimeter in fractional representation is  $\frac{24}{5}$ , but decimal representation is 4.8 [observing that Lindinha found a rectangle with 4.8 perimeter and not a square].

Teacher: Ok, so, enter 4.8 there on the table perimeter [referring to the fraction conversion into decimal one].

Docinho: But it will be that rectangle, Teacher [referring to the Figure 9].

Teacher: But it should be a square, shouldn't it?

Docinho: Yes.

Teacher: So, you need to find out what measure is for all the sides. These measurements are equal, and added together they should measure 4.8.

Docinho: Put everything on the maximum, Lindinha [referring to the slide controls  $a$  and  $b$ ].

Teacher: Do you remember what a perimeter is?

Docinho: Yep, now go ahead decreasing [referring to the slide controls  $a$  and  $b$ ] to be 4.8 there [referring to the result the perimeter sum, below the slide controls].

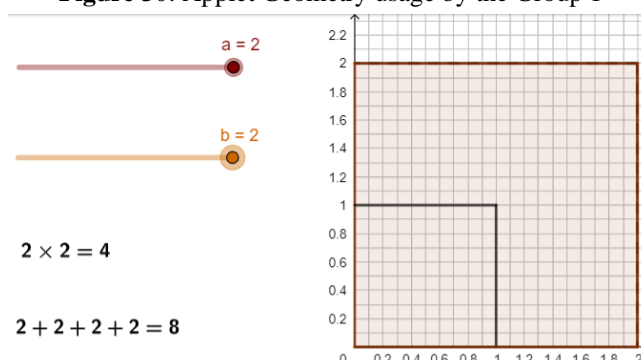
Lindinha: For both? [referring to the slide controls  $a$  and  $b$ ].

Docinho: Yes.

Lindinha: [Set the slide controls a and b to a maximum of 2, as shown in Figure 10].

Task 4 – G1. Task carrying out, 10/20/2020.

**Figure 30:** Applet Geometry usage by the Group 1



**Source:** Data survey (2020)

In the excerpt above, Docinho realized that the formed quadrilateral is wrong (instrumentation), despite the perimeter measurement is correct. Then, the teacher intervened so that the students would remember the perimeter concept of the square. At this moment, Lindinha found a strategy: set slide controls  $a$  and  $b$  to maximum, that is, with measure 2, and reduce them until reaching the desired perimeter, 4.8, whose formed quadrilateral should be a square. We note that instrumentation is highlighted when students appropriate the features and capabilities of the applet to perform measurements, and they observe changes in the perimeter and formed figure, related to changes in the slide controls. They continued the discussion:

Docinho: Now go on decreasing [referring to the slide control a] to reach 4.8 [referring to the perimeter].

Lindinha: [She slides the slide control a to the left, decreasing the measure, and stop at 1.4].

Docinho: Go ahead, further up. Further... [meaning to Lindinha moving the slide control a to the left].

Lindinha: Wait... [move the slide control a until 1.2].

Docinho: Further, Lindinha. Further up, not down [meaning to Lindinha moving the slide control a to the left side].

Lindinha: [She moves the slide control a until the measure 2 once again and start moving the slide control b until 1.7].

Docinho: That's it! No! Return...

Lindinha: [She moves the slide control b until 2].

Docinho: There! No... Wait. A little bit down [referring to move the slide control b to the left].

Lindinha: [She decreases the slide control b until the measure 0.4].

Docinho: There, now decrease the other [referring to the slide control a].

Lindinha: [She decreases the slide control a until the measure 0.4 as well] ... But then the other will also decrease [referring to the perimeter].

Teacher: So, length and width must be bigger, mustn't it?

Lindinha: [She starts increasing measures equally with slide controls a and b: she slides the slide control b until 0.7, then the slide control a until 0.7. She observes the perimeter, 2.8, and moves the slide control a, and then the b one until 1].

Docinho: Leave the a at 8 (referring to 0.8).

Lindinha: [She slides the slide control a until 2 and starts decreasing, until reaching 1.4].

Docinho: There! Now increase the other [referring to the slide control b].

Lindinha: [She moves the slide control b until reaching 1.4].

Docinho: Teacher, there is no way to do it ... Have you done this before to see if it works?

Professora: [laughing] Off course there is! It's a little bigger, isn't it? So, decrease the two measures a little bit...

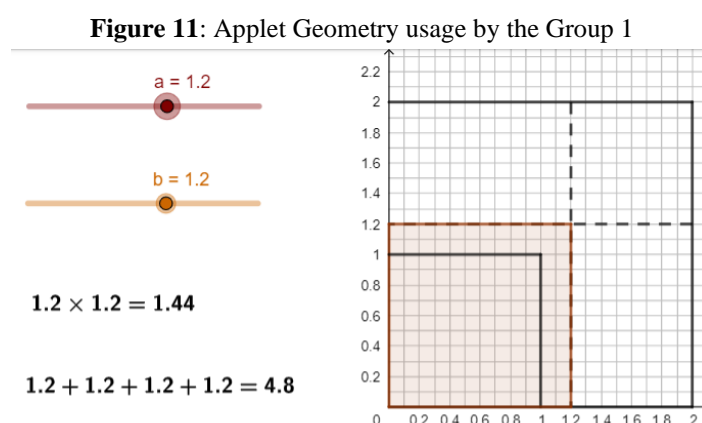
Docinho: Decrease one, then the other [referring to the slide controls a and b].

Lindinha: [She slides the slide control a to the left side until achieve 0.7, then she slides it to right, increasing the measure a until 1.2].

Teacher: Let's think, in the Square 4, the length was 1 and the width, the perimeter was 4, and now you want 4.8. Then, isn't a little bigger than 1??

Lindinha: [She moves the slide control b until 1.2] I got it, teacher! The perimeter is 4.8 and the area is 1.44.

Task 4 – G1. Task carrying out, 10/20/2020.



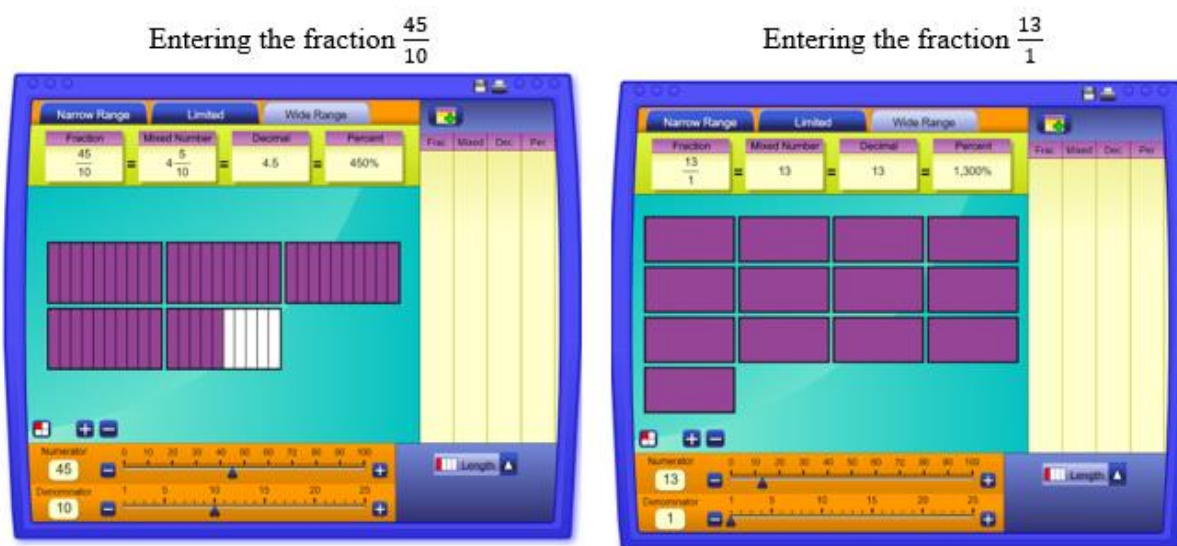
Source: Data survey (2020)

In this excerpt from the task carrying out, the instrumentation and instrumentalization processes are closely linked. Instrumentation is revealed when students: understand the applet functioning and limitation (the slide controls are moved one at a time, the maximum possible measure is 2), then they elaborate a resolution strategy (setting the slide controls at maximum, 2, and then decreasing). Instrumentalization is perceived when students: understand the functioning and manipulate the applet (they entered the same measurement on each slide control and check if the perimeter is 4.8, and if the figure formed is a square).

Next, the group returned to the Task 4 table, column referring to the Rectangle 5. The teacher recalled that the perimeter is length plus length plus width plus width. Then, she asked them to reflect on a rectangle of entire numbers such as length 2 and perimeter 16. So, the students discussed and concluded that, in this example, the width would be 6. We noticed that without the applet support, the students were unable to reflect on the situation presented in Rectangle 5, requiring the teacher's intervention. For the fractional representations, this group needed the applet Fraction Models to make sure that 4.5 means  $\frac{45}{10}$  and 13 means  $\frac{13}{1}$  (Figure 12), thus highlighting the instrumentation process. As two hours of live had already passed, we decided to continue the task the next day<sup>5</sup>.

<sup>5</sup> It was the only group which we held an unplanned meeting. This is because, after developing the task with G1, we subdivided

**Figure 12:** Applet Fraction Models usage by the Group 1



**Source:** Data survey (2020)

Next day, to solve the Task 4 item *c*, the students needed to observe the table and answer what would be the differences between the multiplication of rational and natural numbers. After many discussions about this issue, the teacher asked the students to look at the table again and compare the length and width of Rectangle 1 with its area, e.g., comparing 0.2 and 0.6 with 0.12; or even the fractional representations  $\frac{2}{10}$  and  $\frac{6}{10}$  with  $\frac{12}{100}$ . However, although the students knew they needed to find the equivalent fractions to compare them, they did not know how to do it. We emphasize that in Tasks 2 and 3 we addressed fraction equivalences using the Cuisenaire Environment. Nonetheless, the group did not remember to use multiplication to find the equivalent fractions, but they remembered the Cuisenaire environment.

Teacher: What have we done to find the equivalent fractions?

Docinho: We used the rods, teacher.

[...]

Lindinha: [She opens the applet Cuisenaire environment].

Teacher: So set up there, 2 tenths, what is 2 tenths?

Task 4 – G1. Task carrying out, 10/20/2020.

**Figure 13:** Applet Cuisenaire environment usage by the Group 1



**Source:** Data survey (2020)

Docinho: With red it was easier [referring to Lindinha had chosen using 2 white rods instead of the red one].

Teacher: Now, you must do it equivalent to 100. So, how many orange rods are you going to have to put

the task for the other groups, with items a and b being carried out in one week, and items c and d the following week.

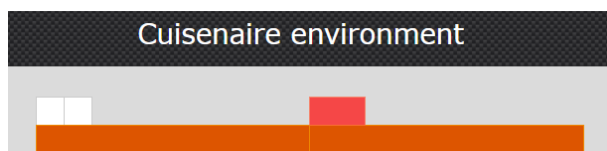
there?

Docinho: 90.

Teacher: Then, zoom out [...]. But pay attention, you will be able to imagine one thing, put an orange rod on the side, Docinho said that it must be 9 more... 90. But for each orange rod you are going to put, you must put 2 more tenths, so put 2 tenths there on this bar [Lindinha adds the red rod on top of the orange bar].

Task 4 – G1. Task carrying out, 10/21/2020.

**Figure 44:** Applet Cuisenaire rods usage by the Group 1



**Source:** Data survey (2020)

Teacher: There, now imagine it how many times.

Docinho: It is 20.

Teacher: So, it will be 20 what?

Docinho: 20 hundredths.

Task 4 – G1. Task carrying out, 10/21/2020.

The group needed the Cuisenaire environment to understand that  $\frac{2}{10} = \frac{20}{100}$  and comparing  $\frac{20}{100}$  and  $\frac{12}{100}$ . After analyzing each of the quadrilaterals presented in the table, G1, in the Collective Discussion phase of Task 4, part 2, presented their conclusions as follows:

Docinho: In rational numbers, the result is not always bigger than the factor, because when we did the equivalences, the results showed that some were bigger, others smaller and others equal. When zero comes before the period, the result will be smaller than the factors. When any non-zero number comes before the period, the result will be bigger than the factor. When the numbers are equal, the result and the factor will be equal.

Task 4 – G1. Task carrying out, 10/30/2020.

The limitation of the applet Fraction Models regarding the measure unit 25 did not prevent G1 students from comparing or converting decimals and fractions of measure unit 100, because they used Cuisenaire environment. We reinforce that, although a teacher systematized, in Task 3, the fractions equivalence using the Cuisenaire environment also through the multiplication operation, the students did not remember this last possibility, but they remembered the Cuisenaire environment.

## 5 Conclusions

Our aim was at investigating the contributions by the applets Cuisenaire environment, Geometry and Fraction Models in fractions learning from the measurement perspective in classes based on EMT developed during the ERT. Thereunto, we sought evidence of the instrumentation and instrumentalization processes promoted by using applets in the context of ERT in classes based on EMT for learning fractions from the measurement perspective.

Because the Cuisenaire environment does not have a pre-established measure, there is the possibility of making different comparisons between the length of the rods and it favors the understanding of fraction as a measure, because it is essential to choose one of the rods to compare, such as the measure unit. As an example, when measuring the light green rod using the dark green one as the measure unit, the light green rod measures half dark green one ( $\frac{1}{2}$ ); when comparing the light green rod with the blue one, the same light green rod measures third blue rod ( $\frac{1}{3}$ ), which allowed the students, mobilizing strategies to perform multiplicative comparisons between the rods, to establish fractions equivalence, including the symbolic representation property (that there is an infinity of fractional representations for the same magnitude). Furthermore, by having to choose and use the most convenient measure unit to operate with fractions of different measure units, they were able to understand the property of fractional numbers *signaling numerical magnitude*.

The applet *Quadrilaterals* was used to carry out the construction and measurement of the quadrilaterals, their perimeters, and areas; *Fraction Models* was used to validate whether fractional representations and decimal ones had the same magnitude.

Associating the three applets in Task 4 favored students understand the differences between natural numbers and fractional ones, *product* property (Obersteiner *et al.*, 2019), that by multiplying two fractions other than zero and one, the result may be smaller than one of the two factors.

At various points in the research, we noticed how applets influenced students' thinking at instrumentation, for example when they believed that the fraction  $\frac{1}{9}$  was bigger than  $\frac{1}{4}$ , without looking at the *Cuisenaire environment*. It was also clear how the students' understanding shaped the use of applets in instrumentation, for example when they used Cuisenaire environment to find the fraction equivalent to the unit of measurement 100.

Inserting applets in our pedagogical work made possible for students with different technological devices (smartphones, tablets, computers) to discuss, reflect, visualize, and recognize fractions as a measure. However, we know that not everyone had the opportunity to participate in classes due to unavailability of equipment or internet. This made working with these students unfeasible, and hereupon, we emphasize that we would not be able to plan and develop the tasks proposed here, in the context of the ERT, without the use of the applets, especially the Cuisenaire environment, on which the tasks are based, then necessary for the exploration and understanding by students.

Results reveal the potential of classes based on EMT combined with the applets suggested for learning fractions from the measurement perspective, since without the teacher's intervention, tasks, and applets by themselves are limited. Hence, a detailed classes planning from this perspective, especially with a framework elaboration to anticipate the students and the teacher actions, is essential so that the objectives of the tasks are achieved.

A difference to be highlighted regarding the use of applets in classroom and remote teaching is that, in classroom lessons, students could work and view two or more applets at the same time. It does not happen with ERT, as there is no possibility to share two screens at the same time via Google Meet.

We point the need for other investigations to analyze potentialities of EMT articulated to DT under the theoretical lens by IG for fractions teaching and learning in the mediation perspective, and other interpretations as well. We also highlight that communication resources,

such as computer, smartphone, WhatsApp, and Google Meet had enabled and are indispensable to develop classes in ERT, consequently they are not neutral elements and influence teaching and learning in ERT. However, we did not analyze this relation with fraction learning as measure because we should consider other describers, which may be investigated in another works to understand how communication artifacts/instruments influence Mathematics teaching and learning in ERT.

### Acknowledgment

We would like to thank the PRPGEM (*Programa de Pós-graduação em Educação Matemática*) and the CAPES (*Coordenação de Aperfeiçoamento de Pessoal de Nível Superior*) for their support.

### References

- Almeida, A. C. & Oliveira, H. (2009). O processo de gênese instrumental e a calculadora gráfica na aprendizagem de funções no 11.º ano. *Quadrante*, 18(1 e 2), 87-118.
- Bailey, D.; Hoard, M. K.; Nugent, L. & Geary, D. C. (2012). Competence with fractions predicts gains in mathematics achievement. *Journal of Experimental Child Psychology*, 113, 447-455.
- Basniak, M. I. & Estevam, E. J. G. (2018). Uma lente para analisar a integração de Tecnologias Digitais ao Ensino Exploratório de Matemática. In: *VII Seminário Internacional de Pesquisa em Educação Matemática* (pp. 1-12). Foz do Iguaçu, PR.
- Basniak, M. I. & Estevam, E. J. G. (2019). Uma lente teórica para analisar o potencial das tecnologias digitais no Ensino Exploratório de Matemática. *Acta Latinoamericana de Matemática Educativa*, 32(2), 738-747.
- Behr, M. J.; Lesh, R.; Post, T. R. & Silver, E. A. (1983). Rational Numbers Concepts. In: R. Lesh & M. Landau (Ed.). *Acquisition of Mathematics Concepts and Process* (pp. 91-125). Londres.
- Bittar, M. (2011). A abordagem instrumental para o estudo da integração da tecnologia na prática pedagógica do professor de Matemática. *Educar em Revista*, (especial), 157-171.
- Bueno, A. C. & Basniak, M. I. (2020). Construcción de escenarios en GeoGebra en la movilización de conocimientos matemáticos por alumnos con altas habilidades/superdotados. *Paradigma*, 41(2), 252-276.
- Canavarro, A. P.; Oliveira, H. & Menezes, L. (2012). Práticas de ensino exploratório da matemática: O caso de Célia. In: *Actas do Encontro de Investigação em Educação Matemática 2012: Práticas de Ensino da Matemática* (pp. 255-266). Portalegre: Sociedade Portuguesa de Investigação em Educação Matemática.
- Ceale. (2014). *Glossário Ceale: Termos de alfabetização, leitura e escrita para educadores*. Belo Horizonte. Disponível em <http://www.ceale.fae.ufmg.br/app/webroot/glossarioceale>. Acesso em 8 set. 2020.
- Cyrino, M. C. C. T. & Teixeira, B. R. (2016). O Ensino Exploratório e a Elaboração de um framework para os Casos Multimídia. In: M. C. C. T. Cyrino (Org.). *Recurso Multimídia para a Formação de Professores que Ensinam Matemática: elaboração e perspectivas* (pp. 81-99). Londrina, PR: EDUEL.
- Drijvers, P.; Kieran, C.; Mariotti, M. A.; Ainley, J.; Andresen, M.; Chan, Y. C.; Dana-Picard,



- T.; Gueudet, G.; Kidron, I.; Leung, A. & Meagher, M. (2010). Integrating Technology into Mathematics Education: Theoretical Perspectives. In: C. Hoyles & J. B. C. Lagrange (Ed.) *Mathematics Education and Technology: Rethinking the Terrain* (pp. 89-133). Springer.
- Escolano, R. V. & Gairín, J. M. S. (2005). Modelos de Medida para la Enseñanza del Número Racional en Educación Primaria. *Revista Iberoamericana de Educación Matemática*, 1, 17-35.
- Kieren, T. E. (1976). On the mathematical, cognitive, and instructional foundations of rational numbers. In: R. Lesh. (Ed.). *Number and measurement: papers from a research workshop* (pp. 101-144). Columbus, Ohio: ERIC/SMEAC.
- Kieren, T. E. (1980). The rational number construct – its elements and mechanisms. In: T. Kieren. (Ed.) *Recent Research on Number Learning* (pp. 125-150). Columbus: Eric/Smeac.
- Lamon, S. J. (2012). *Teaching fractions and ratios for understanding – essential content knowledge and instructional strategies for teachers*. (3. ed). New Jersey: Lawrence Erlbaum Associates, Publishers Mahwah.
- Moreira, J. A. M.; Henriques, S. & Barros, D. (2020). Transitando de um ensino remoto emergencial para uma educação digital em rede, em tempos de pandemia. *Dialogia*, (34), 351-364.
- Obersteiner, A.; Dresler, T.; Bieck, S. M.; Moeller, K. *et al.* (2019). Understanding fractions: Integrating results from mathematics education, cognitive psychology, and neuroscience. In: A. Norton & M. W. Alibali (Ed.). *Constructing number: Merging perspectives from psychology and mathematics Education* (pp. 135-162). Cham: Springer.
- Oliveira, H.; Menezes, L. & Canavarro, A. P. (2013). Conceptualizando o ensino exploratório da Matemática: Contributos da prática de uma professora do 3.º ciclo para a elaboração de um quadro de referência. *Quadrante*, 22(2), 29-53.
- Oliveira, V. S. D. & Basniak, M. I. (2021). O planejamento de aulas assentes no ensino exploratório de Matemática desenvolvidas no ensino remoto de emergência. *Educação Matemática Debate*, 5(11), 1-29.
- Padilha, L. C. S. & Bittar, M. (2013). Apropriação da Tecnologia por Professores de Matemática para fins Pedagógicos: Uma Abordagem Instrumental. In: *XI Encontro Nacional de Educação Matemática* (pp. 1-15). Curitiba, PR.
- Ponte, J. P. (2005). Gestão curricular em Matemática. In: GTI (Ed.). *O professor e o desenvolvimento curricular*. (pp. 11-34). Lisboa: APM.
- Powell, A. B. (2018). Melhorando a epistemologia de números fracionários: Uma ontologia baseada na história e neurociência. *Revista de Matemática, Ensino e Cultura*, 13(29), 78-93.
- Powell, A. B. (2019a). Measuring Perspective of Fraction Knowledge: Integrating Historical and Neurocognitive Findings. *Revista Sergipana de Matemática e Educação Matemática*, 1, 1-19.
- Powell, A. B. (2019b). Aprimorando o Conhecimento dos Estudantes sobre a Magnitude da Fração: Um Estudo Preliminar com Alunos nos Anos Iniciais. *Revista Internacional de Pesquisa em Educação Matemática*, 9(2), 50-68.
- Rabardel, P. (1995). *Les hommes et les technologies: une approche cognitive des instruments contemporains*. Paris: Armand Colin.
- Rabardel, P. (1999). *Éléments pour une approche instrumentale en didactique des*

- mathématiques. In: M. BailleuL (Ed.). *Actes de la Xème Ecole d'Été en Didactiques des Mathématiques. Houlgate* (pp. 202-213). IUFM de Caen.
- Roque, T. (2012). *História da matemática: uma visão crítica, desfazendo mitos e lendas*. Rio de Janeiro: Zahar, versão Kindle.
- Torbeyns, J.; Schneider, M.; Xin, Z. & Siegler, R. S. (2015). Bridging the gap: Fraction understanding is central to mathematics achievement in students from three different continents. *Learning and Instruction*, 37, 5-13.
- Valente, G. S. C.; Moraes, E. B.; Sanchez, M. C. O.; Souza, D. F. & Pacheco, M. C. M. D. (2020). O ensino remoto frente às exigências do contexto de pandemia: Reflexões sobre a prática docente. *Research, Society and Development*, 9(9), 1-13.
- Vieira Pinto, A. (2005). *O conceito de tecnologia*. Rio de Janeiro, RJ: Contraponto.