

Relationships between understanding and solving word problems of multiplicative structure

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Abstract: The objective of the study was to investigate the relationships between understanding word problems of multiplicative structure and performance when solving these problems. Third, 4th and 5th grades students were asked to solve multiplication and division problems (Problem-Solving Task) and to answer questions about the statement of the problems (Problem-Understanding Task). For the three school years it was easier to understand what needed to be found in the problem (literal information) than to understand which operation and which numbers to use in the resolution of the problem (inferential information). Although performance in the three school years was limited, relationships were identified between understanding the word the problems and the ability to solve them. Understanding what needed to be found and identifying the relevant numerical information had a positive impact on solving the problems, and it was more significant than the ability to identify the appropriate operation to be applied.

Keywords: Understanding Mathematical word Problems. Solving Mathematical word Problems. Multiplicative Structure Problems. Elementary School.

Relaciones entre comprensión y resolución de problemas de estructura multiplicativa

Resumen: El objetivo del estudio fue investigar las relaciones entre la comprensión de enunciados de problemas de estructura multiplicativa y el desempeño en la resolución de estos problemas. A estudiantes de 3°, 4° y 5° grado se les pidió individualmente resolver problemas de multiplicación y división (Tarea de solución) y responder a preguntas sobre el enunciado de los problemas (Tarea de comprensión). En los tres grados fue más fácil comprender lo que debe ser encontrado en el problema (información literal) que comprender qué operación y qué números usar al solucionarlos (información inferencial). Aunque el desempeño en los tres grados fue limitado, se identificaron relaciones entre comprender los enunciados de los problemas y resolverlos, ya que comprender lo que debe ser encontrado e identificar la información numérica relevante impactó positivamente en la resolución de los problemas. Este impacto fue más expresivo que la capacidad de identificar la operación adecuada a ser aplicada.

Palabras clave: Comprensión de Problemas Matemáticos. Resolución de Problemas

Matemáticos. Problemas de Estrutura Multiplicativa. Educación Primaria.

Relações entre compreensão e resolução de problemas de estrutura multiplicativa

Resumo: O objetivo do estudo foi investigar as relações entre compreensão do enunciado de problemas de estrutura multiplicativa e o desempenho na resolução desses problemas. Estudantes do 3º, 4º e 5º anos foram individualmente solicitados a solucionar problemas de multiplicação e de divisão (Tarefa de Resolução) e a responder perguntas sobre o enunciado dos problemas (Tarefa de Compreensão). Nos três anos escolares foi mais fácil compreender o que precisava ser encontrado no problema (informação literal) do que compreender que operação e quais números usar em sua resolução (informações inferenciais). Apesar do desempenho nos três anos escolares ter sido limitado, foram identificadas relações entre compreensão do enunciado dos problemas e a capacidade de resolvê-los, uma vez que compreender o que precisava ser encontrado e identificar as informações numéricas relevantes tiveram impacto positivo na resolução dos problemas. Esse impacto foi mais expressivo que a capacidade de identificar a operação apropriada a ser aplicada.

Palavras-chave: Compreensão de Problemas Matemáticos. Resolução de Problemas Matemáticos. Problemas de Estrutura Multiplicativa. Ensino Fundamental.

1 Introduction¹

Mathematical word problems or math story problems, as named by Mattarella-Micke and Beilock (2010), are one of the key components in the teaching of mathematics and, still, one of the challenges for children in elementary school. In word problems, information is presented in the form of a short narrative rather than in mathematical notation (Verschaffel, Greer & De Corte, 2000), or in the form of closed and open problems (Figueiredo, 2021). Several researchers comment that in order to solve them, students have to perform the necessary mathematical operations and to understand the text presented (e.g., Cummins, Kintsch, Reusser & Weimer, 1988; Daroczy, Wolska, Meurers & Nuerk, 2015). Thus, besides numerical competence, linguistic aspects also play a significant role in the solving process. This was corroborated by Pongsakdi et al. (2020) who classified their research participants into four groups: skilled in text comprehension and with difficulties in arithmetic; limited in text comprehension and skilled in arithmetic; difficulties in both competences; and skilled in both competencies. The results indicated that good performance in problem solving required both skills.

Investigating the relationships between understanding and solving word problems requires establishing a broader association between mathematics and reading comprehension. This association has been examined in different ways. Vilenius-Tuohimaa, Aunolab and Nurmib (2008), for instance, compared children with different levels of comprehension of a narrative text and observed that performance on math word problems was strongly related to performance in reading. A similar result was documented in studies pointing the predictive impact of reading on math skills across the elementary school grades, where the ability to decode and recognize words was positively correlated with math performance (e.g., Björn, Aunola & Nurmi, 2016; Fuchs, Gilbert, Fuchs, Seethaler & Martin, 2018; Hadianto, Damaianti, Mulyati & Sastromiharjo, 2021; Hecht, Torgesen, Wagner & Rashotte, 2001). In a two-year longitudinal study, Jordan, Kaplan and Hanich (2002) and Jordan, Hanich and Kaplan (2003) found that

¹ The research reported in this article is an excerpt from the doctoral thesis carried out by the first author under the supervision of the second author in the Post-Graduate Program in Cognitive Psychology at the Federal University of Pernambuco.

reading disabilities predicted children's progress in mathematics, but mathematics difficulties had no effect on their progress in reading. This result clarifies the direction of the identified correlation between reading comprehension and math performance. In general, these investigations reveal that many of the difficulties in solving a word problem stem from limitations that children have in decoding words and vocabulary, especially regarding the meaning of the terms present in it.

Like Stephany (2021), Spinillo and Marín (2022) claim that word problems are primarily texts that have to be read and understood. They also comment that, as with other types of text such as narrative and expository, the difficulties children face in understanding it go beyond those related to decoding and vocabulary. For these authors, it is necessary to have a paradigm shift in the investigation of the relationship between understanding and problem solving so that studies seek to examine the understanding of mathematical text, and not just the ability to decode and assign meaning to words. This approach finds support in studies carried out in the field of textual comprehension that have the Construction-Integration model proposed by Van Dijk and Kintsch (1983) and Kintsch (1998) as a theoretical framework. According to this theoretical perspective, people build a mental representation — a situation model² — of the text they read. Situation models represent the characters, their actions, and the objects they interact with. This would not be different in relation to the text of the word problem, so that the situation model would be a decisive instance for the resolution.

Impacts on the construction of a situation model are crucial for processing word problems successfully (Thevenot, 2010) and those situation models that contradict readers' expectations about the mathematical operation needed to solve the problem can impair problem solving (Coquin-Viennot & Moreau, 2007). Also, even small changes to the wording of a math problem can alter its situation model and have a negative effect on performance, as shown by Mattarella-Micke and Beilock (2010) in a study in which participants read multiplication story problems, in which sets of objects were either associated with or dissociated from a protagonist. Voyer (2011) also observed that the situation model has an impact on children's problem solving performance. The author examined the relationships between factors related to the participants (arithmetic skills) and factors related to the word problem itself (type of information included in the text of the problem). The results showed that the influence of the situation model depends either of the type of information as well as on the child's arithmetic skills. As a whole, these investigations showed that a situation model elicited by math word problem matters, since it can affect the solving processes.

Stephany (2021) examined whether the ability to build a situational model would influence problem solving performance, investigating the factors influencing the construction of such a model. Overall, the main results showed that: (i) children who monitor their reading process and tend to read texts as a whole (rather than word by word) were more successful in building up a situation model; (ii) inference making had an effect on the construction of a situation model; and (iii) adequate solution strategies were used when the situation model has been built. The conclusion was that when the situational model was effectively constructed, children were successful in solving problems.

What can be noted from these studies is that understanding math word problems has similarities with the general process involved in understanding other types of texts. Despite this similarity, Kintsch and Greeno (1985) emphasize that understanding mathematical word problems, unlike other types of texts, requires dealing with mathematical language and the need

² According to Kintsch (1998), a situational model is a mental representation that corresponds to the reader's elaborations based on their prior knowledge (linguistic and world knowledge).

to do something to obtain a given information. According to the authors, to understand math word problems it is necessary to identify: (i) the semantic content of each clause; (ii) the schemes that represent the properties and relationships between the referents (such as characters, objects etc.) and the quantities associated with them; and (iii) the action schemes that refer to the calculations for solving the problem.

Nesher, Hershkovitz and Novotna (2003) emphasize that the surface structure of the problem has great relevance, as it generates a first interpretation of the mathematical text and a mental representation. From this interpretation emerges the structure or propositional scheme of the underlying logical reasoning based on the elements of the word problem and their relationships. Then emerges the mathematical model related to operations to be used for the resolution. Therefore, assessing the understanding of the propositions contained in this type of text requires considering both aspects of the first representation of the problem and the generation of inferences associated with relationships established to find the response to the problem's question.

The discussion presented here makes evident the need to explore the understanding of word problems from an approach related to the text as a whole, not limited to the word level. From this perspective, it would be relevant to know what difficulties children face with this mathematical text. There are few studies that adopt this approach. A rare example is the intervention study conducted by Kurshumlia and Vula (2019) with students in the 4th grade of elementary school who, in the classroom, were instructed to use text reading strategies to understand the statement of mathematical problems. It was observed that the improvement in the ability to understand the text had a positive impact on problem solving. Other investigations also adopt this approach, as is the case of research carried out by Valverde (2012), Arenales (2015) and Gálvez (2012) whose results, in general, revealed that elementary school students from the 3rd to the 6th grades had difficulties in identifying the numerical information that should actually be used in solving the problem, in identifying what would have to be found and in choosing the operation to be applied to its resolution. According to our analysis, these investigations contribute in two directions. First, by examining the understanding of the mathematical text as a whole and not just at the word level. Second, for examining the understanding of the problem statement through resources arising from textual linguistics, specifically from textual comprehension. However, despite being relevant, these studies do not clarify certain questions about the relationship between understanding and problem solving, and the present investigation is a possibility of progress in this field of research. Considering that identifying the numerical information that is effectively necessary for solving a problem, identifying what needs to be found and choosing the appropriate operation are aspects that need to be understood by anyone who seeks to solve a mathematical problem, the present study aimed to investigate which of these instances would be the most difficult to understand and which would have the most significant impact on problem-solving. For this, a Problem-Solving Task was applied involving these three instances that were examined through open questions about the statement of mathematical problems. Answering questions about a text is a methodological resource widely used in research on understanding narrative texts. As mentioned, the problem statement is a type of text. In this way, its understanding by the individual can be examined through a methodological resource derived from textual linguistics.

The performance related to each of these instances was compared to the performance in a Problem-Solving Task in order to explore the relationships between text comprehension and problem solving. In the present study, we chose to focus on multiplicative structure problems that are considered the most complex in the field of arithmetic (see Vergnaud, 1983, 2009). Additionally, the performance of both understanding and solving problems by children from

different grades of elementary school was investigated with the aim of examining whether there would be changes with the advancement of schooling.

2 Method

2.1 Participants

The sample was chosen by convenience due to the availability of schools to allow access to the students. This consisted of 120 Brazilian children of both sexes, attending public and private schools in the city of Recife. Participants were equally divided into three groups: 3rd graders (mean age: 8 years 7 months, $SD = 1.09$), 4th graders (mean age: 9 years 8 months, $SD = 1.04$) and 5th graders (mean age: 10 years 7 months, $SD = 1.18$). These school grades were investigated because students in this segment of elementary school still have difficulties in solving problems of multiplicative structure. The participants had no intellectual, motor or sensory limitations.

2.2 Procedure and material

Each participant was interviewed individually in two sessions with no time restriction. The sessions occurred in the school environment, in a room made available for the interviews. In the first session, the Problem-Solving Task was applied and in the second, the Problem-Understanding Task. The order of presentation was fixed to prevent the questions in the Problem-Understanding Task from influencing the performance in the Problem-Solving Task. Both tasks involved multiplication and division problems classified according to the typology proposed by Vergnaud (1990, 2009): isomorphism problems and product of measures problems. In the problems presented in both tasks, there was relevant and irrelevant numerical information for their resolution.

2.3 Problem-Solving Task

This task aimed to evaluate the performance in solving multiplicative structure problems. Four problems were presented: two of isomorphism (one for multiplication and the other for division) and two for product of measures (one for multiplication and the other for division).

Isomorphism Problems:

Multiplication: Bruna is a 9-year-old girl. She hangs 2 blouses on each hanger in her wardrobe and puts her 5 pants in the drawer. She needs 8 hangers to store all her blouses. In total, how many blouses does she have? (relevant numerical information: 2 blouses and 8 hangers).

Division: A toy factory produced 28 dolls for December 24, Christmas. The factory owner will need 4 boxes to place the dolls and send to his 2 stores to sell. How many dolls will he put in each box? (relevant numerical information: 28 dolls and 4 boxes).

Product of Measures Problems:

Multiplication: Maria turned 11 and invited 5 classmates to celebrate her birthday at the ice cream shop on the corner. The ice cream shop serves 6 flavours of ice cream and 3 different syrups. Each person can only choose one flavour and one type of syrup. How many different combinations of flavors and syrup can be made in this ice cream shop? (relevant numerical information: 6 flavours of ice cream and 3 different syrups)

Division: A store sells 6 suitcases a day. Each one costs 24 reais³. Each type of suitcase has a different color and size. The store sells 12 types of suitcases of 3 sizes. How many colours can suitcases be? (relevant numerical information: 3 sizes and 12 types of suitcases)

The problems, printed on cards, were randomly presented, one at a time, to each participant, being read by the child together with the examiner. Pencil and paper were made available. Children's graphic representations were registered in individual protocols for each participant.

2.4 Problem-Understanding Task

This task assessed children's ability to understand three instances considered crucial for understanding mathematical problems, as highlighted by Kintsch and Greeno (1985) and documented in research carried out with children (Arenales, 2015; Gálvez, 2012; Valverde, 2012): (i) what needs to be found, (ii) the operation to be applied, and (iii) the numerical information that should be used in solving the problem. These instances were represented in the three questions that constituted this task, namely: Question 1: What needs to be found to solve the problem?; Question 2: If you were to solve the problem, which operation would you use?; and Question 3: What numbers would you use? As mentioned, the method of answering questions about a given text is widely used in research in the area of textual comprehension, especially in narrative texts, being adopted here to assess understanding of mathematical texts, in this case, the statement of word problem.

Initially, the participants were informed that they would not have to solve the problems that would be shown, they would only have to read them carefully and then answer questions about them. Four problems, printed on cards, were randomly presented, one at a time, to each participant, being read by the child together with the examiner. They were similar to those used in the Problem-Solving Task, namely:

Isomorphism Problems:

Multiplication: A pack of cookies costs 7 reais. Each pack contains 8 strawberry cookies. Livânia bought 5 packs of cookies and 4 boxes of orange juice at the supermarket. How many cookies will she have in total? (relevant numerical information: 8 cookies and 5 packs of cookies)

Division: In the school yard, 5 friends went to play soccer. After 2 hours of play, they bought 15 cans of soda and 10 chocolate cupcakes. They all drank the same number of cans. How many cans did each of them drink? (relevant numerical information: 5 friends and 15 soda cans).

Product of Measures Problems:

Multiplication: Pedro will spend 4 days at his grandfather's beach house. In the suitcase, he will put 3 t-shirts and 5 shorts. He will also take with him his 2 favorite toys. He wants to combine the t-shirts and shorts to make outfits. How many different outfits can he make? (relevant numerical information: 3 t-shirts and 5 shorts)

Division: The neighborhood snack bar is located at the number 48 in the Happiness Street. There, they sell 12 different types of sandwiches and 6 different flavors of fruit juices. On the menu, they offer 4 fillings. How many types of bread do they have to make these sandwiches? (relevant numerical information: 12 types of sandwiches and 4

³ Brazilian currency.

fillings)

Each problem was followed by three questions, presented orally one at a time, in a fixed order, namely: Question 1: What has to be found to solve the problem?, Question 2: If you were going to solve the problem, which operation would you use?, and Question 3: Which numbers would you use? The responses were registered in writing by the examiner in individual protocols related to each participant.

3 Results and discussion

Data were analysed according to the number of correct responses in both tasks. Initially, the results of the Problem-Understanding Task are presented, comparing the performance in each of the three questions with a view to examining which of these instances would be the most difficult to understand. Next, comparisons are made between these results and those obtained in the Problem-Solving Task. Comparisons between the tasks were made considering each of the instances involved in the three questions in the Problem-Understanding Task in order to investigate which one would have a more significant impact on performance in the Problem-Solving Task.

3.1 Problem-Understanding Task

In order to respond Question 1 (What has to be found to solve the problem?) correctly, it was necessary to take into account the literal information explicitly mentioned in the form of a question at the end of the text of the problem. Examples: “How many cookies she has in total.”, “How many cans of soda each one of the boys have drank.”, “The number of outfits he can make.”.

In order to respond Question 2 (Which operation would you use?) correctly, it was necessary to identify the relationships between the relevant numerical information and what has to be found, as explicitly mentioned in the question at the end of the text of the problem. The responses given by the participants to this question never involved more than one operation.

In order to respond Question 3 (Which numbers would you use?) correctly, it was necessary to identify, among the numerical information present in the text of the problem, those that were relevant to its resolution. Only the response in which the child indicated the two numbers effectively relevant to the resolution was considered correct.

As shown in Table 1, the Chi-Square Test indicated no significant associations between the school grades in each question ($\chi^2 = 2.0065$, $df = 4$, $p > .05$). To find out if there is an association in each school grade and the questions, the Chi-Square Adherence Test was applied. This test revealed an association in 3rd ($\chi^2 = 23.116$, $df = 2$, $p < .01$), 4th ($\chi^2 = 9.515$, $df = 2$, $p < .01$) and 5th grade ($\chi^2 = 11.484$, $df = 2$, $p < .01$).

Table 1: Number and percentage of correct responses (in parentheses) in each question in the Problem-Understanding Task in each grade (maximum = 160)

Grades	Question 1 What has to be found to solve the problem?	Question 2 What operation would you use?	Question 3 Which numbers would you use?
3 rd	110 (68.8)	56 (35)	64 (40)
4 th	104 (65)	67 (41.8)	74 (46.3)
5 th	128 (80)	82 (51.2)	(93 (58.1)

Source: Research database

These results were due to the fact that the performance in each of the three questions did not vary between grades. However, the test also revealed that in the three school grades the performance in Question 1 (What has to be found to solve the problem?) was better than in the other questions. This indicates that children of these grades did not show difficulties in identifying what had to be found in the problems. It was also found that in the three school grades, children had difficulties in identifying the operation to be used (Question 2) and in identifying the numerical information that they should actually use to solve the problems (Question 3). Perhaps this has occurred because the information about what is sought in the problem (Question 1) was literally expressed in the text of the problem in the form of a question, while the other information needed to be inferred.

An additional analysis was carried out to identify participants' levels of understanding in relation to each of the three questions. Depending on the number of correct responses given to each question, a score ranging from 0 to 4 points was assigned to each participant. Thus, they were classified into one of three levels of understanding, namely: Level 1 (poor): children who scored zero or one point; Level 2 (regular): children who obtained two points; and Level 3 (good): children who scored three or four points. According to this classification, the same child could have levels of understanding that varied from one question to another, for example, be classified at Level 3 in relation to Question 1, at Level 2 in Question 3 and at Level 1 in Question 2. The distribution of participants at these levels for each question in each grade can be seen in Table 2.

Table 2: Number and percentage of children (in parentheses) at each level in each question in the Problem-Understanding Task per school grade (maximum = 40)

Questions	3rd grade			4th grade			5th grade		
	L1	L2	L3	L1	L2	L3	L1	L2	L3
Q1	8 (20)	6 (15)	26 (65)	13 (32.5)	3 (7.5)	24 (60)	6 (15)	3 (7.5)	31 (77.5)
Q2	20 (50)	13 (32.5)	7 (17.5)	17 (42.5)	11 (27.5)	12 (30)	14 (35)	7 (17.5)	19 (47.5)
Q3	21 (52.5)	5 (12.5)	14 (35)	19 (47.5)	4 (10)	17 (42.5)	13 (32.5)	4 (10)	23 (57.5)

Note. Q1 (What has to be found to solve the problem?), Q2 (What operation would you use?), and Q3 (Which numbers would you use?). Level 1 (L1: poor), Level 2 (L2: regular), and Level 3 (L3: good).

Source: Research database

Due to the low value of the cells, it was not possible to apply any appropriate statistical test, and the data displayed in Table 2 are discussed in terms of trends. As can be seen, most children of the three grades were concentrated in Level 3 (good) in relation to Question 1, demonstrating a good level of understanding of what has to be found in the word problem. The 3rd and 4th graders showed the same pattern of results, concentrating in Level 1 (poor) in both Question 2 and Question 3. This result suggests that they had difficulties in choosing the operation to be used and in identifying the numbers that should actually be used in the resolution. The 5th graders, in turn, were concentrated in Level 3 on all questions, especially in Question 1. As a whole, these children tended to have a better understanding of the word problem components investigated in this study than children of the 3rd and 4th grades.

3.2 Comparison between Problem-Understanding Task and Problem-Solving Task

Before establishing comparisons between the two tasks, it is necessary to mention that in the Problem-Solving Task, out of the 480 problems solved by the 120 participants, only 137 were solved correctly (28.5%). Of these 137 problems solved correctly, 24.1% were observed in the 3rd grade, 32.8% in the 4th grade and 43.1% in the 5th grade. These percentages show that the performance in the Problem-Solving Task was very limited for children in all grades, especially in the 3rd grade.

It was observed that among children classified at Level 1 (poor) in the Problem-Understanding Task, the mean of correct responses in the Problem-Solving Task was 0.38; among those classified as Level 2 (regular), the mean was 0.39; and among the children of Level 3 (good), the mean of correct responses was 1.7. Participants classified as having a good level in the Problem-Understanding Task performed better in the Problem-Solving Task than those who had a poor and regular level. However, performance was still limited among these children.

The relationships between the level at which the child was classified on the Problem-Understanding Task and performance in the Problem-Solving Task were examined on each of the questions separately. This was done because an analysis of this nature makes it possible to identify the impact that understanding (or not understanding) a particular aspect of the word problem would have on problem solving performance (Table 3).

Table 3: Number of correct and incorrect responses in the Problem-Solving Task in each level in the Problem-Understanding Task for each question.

Levels in the Problem-Understanding Task	Correct responses in the Problem-Solving Task	Incorrect responses in the Problem-Solving Task
Question 1: What has to be found to solve the problem?		
Level 1 (poor)	10	98
Level 2 (regular)	6	42
Level 3 (good)	121	203
Question 2: What operation would you use?		
Level 1 (poor)	21	183
Level 2 (regular)	37	87
Level 3 (good)	79	73
Question 3: Which numbers would you use?		
Level 1 (poor)	20	192
Level 2 (regular)	9	43
Level 3 (good)	108	108

Note. The same participant may have different levels of understanding that varied from one question to another. For example, being classified at Level 3 in Question 1, at Level 2 in Question 3, and at Level 1 in Question 2.

Source: Research database

The Chi-Square Test indicated significant associations between participants' levels in the Problem-Understanding Task and performance in the Problem-Solving Task in Question 1

($\chi^2 = 38.058$, $df = 2$, $p < 0.001$), in Question 2 ($\chi^2 = 74.327$, $df = 2$, $p < 0.001$) and in Question 3 ($\chi^2 = 89.934$, $df = 2$, $p < 0.001$). As can be seen in Table 3, the number of correct responses in the Problem-Solving Task was very low among children classified at Level 1 (poor) and Level 2 (regular) in the Problem-Understanding Task. This was observed in relation to all questions. It was also found that the number of correct responses in the Problem-Solving Task was more expressive among children classified at Level 3 (good) in the Problem-Understanding Task in all three questions. Success in problem solving seems to be particularly associated with a good level of understanding (Level 3) in Question 1 and Question 3.

Thus, it seems that understanding what has to be found and identifying which numerical information should be used to solve the problem had a positive impact on correct problem solving. This impact is more expressive than the ability to identify the appropriate operation to be applied (Question 2).

In Question 2, even the children classified at Level 3 (good) in the Problem-Understanding Task had a few number of correct responses in the Problem-Solving Task. A possible explanation for this is that even though they were able to identify the operation to be used, the children did not operate correctly, and then they could not solve the problem successfully.

4 Conclusions

The present study examined the relationships between understanding and solving a mathematical word problem with a multiplicative structure in elementary school students. Two assumptions guided the investigation: (i) mathematical word problems are primarily texts that, in order to be solved, they have to be read and understood; and (ii) the difficulties that children face in understanding this type of text go beyond those related to decoding and vocabulary. According to the literature, children have difficulties in identifying what has to be found, choosing the operation to be used and identifying the numerical information relevant to solving the problem.

With this in mind, the research sought to identify, specifically, which of these aspects would have the greatest impact on problem-solving performance. For this, two tasks were presented to participants: (i) Problem-Solving Task in which children were asked to solve word problems; and (ii) Problem-Understanding Task in which, after reading word problems similar to those presented in the Problem-Solving Task, participants answered questions related to these aspects considered as constituting mathematical word problems.

In the Problem-Understanding Task, the results revealed that 3rd, 4th and 5th graders did not have difficulties in identifying what had to be found in the problems. However, they were less successful in identifying the operation to be used and in identifying the numerical information that they should actually use to solve the problems. A possible explanation for this is that what had to be found was literally mentioned in the word problem in the form of a question at the end of the text of the problem; while the other information had to be inferred. As well known in the literature in the field of narrative textual comprehension, literal information is easier to understand than information that has to be inferred (e.g., King, 2007; McNamara, 2021; Oakhill & Cain, 2004; Vidal-Abarca & Rico, 2003) Apparently, this same pattern of results was found here in relation to mathematical texts, in this case, word problems.

Although the performance in the Problem-Solving Task was limited, it was possible to establish relationships between the result in this task and that observed in the Problem-Understanding Task. These relationships were examined considering each question separately, revealing that good performance in solving problems is associated with the ability to identify

what has to be found, decide on which operation to use and be able to identify the numerical data that are effectively relevant for solving the problem. However, each of these instances seems to have a different impact on problem solving.

The results revealed that knowing what is sought is something more easily understood by children. However, this understanding seems to have a negligible impact on performance, as it does not guarantee successful resolution. On the other hand, success necessarily depends on choosing the correct operation and being able to identify the relevant numerical information to be used. For example, many of children's mistakes, even when choosing the correct operation, stem from computational errors. In this case, they properly identify whether the problem is to divide or to multiply, however, they make mistakes in the execution of the calculations or in the use of algorithms. However, further investigations need to be conducted in order to examine in more detail the impact of the different facets of comprehension investigated in this study on problem solving.

With regard to numerical information present in the text of the problem, there are errors of different types arising from not understanding this aspect. In fact, as documented in the literature many of the mistakes consist of using all the numbers in the word problem and automatically applying an operation to them.

When the word problem contains only relevant numerical information, as is most often the case in the classroom and in textbooks, it is possible that the problem will be solved successfully. However, when the problem brings relevant and irrelevant numerical information, as this occurred in this study, using all the numbers present in the word problem or not identifying which of them are effectively relevant leads, necessarily, to failure in the resolution, even if the chosen operation is correct. Thus, including irrelevant numerical information in word problems seems to be an important resource to assess children's understanding, whether in the school context or in the context of investigation (Hickendorff, 2021).

5 Final Remarks

Understanding the text of the problem is essential for the resolution. In fact, as suggested by Fuchs et al. (2015), word problem solving is a form of text comprehension. This understanding involves a network of aspects that are articulated and that have different impacts on the resolution process, thus demonstrating the complexity of this issue. Due to this complexity and the scarcity of research that specifically assesses the understanding of different facets of word problems, more research is needed with elementary school children. The present study was an effort in this direction, contributing with information about two specific points: (i) which aspect of the problem text is easier to understand than others and which ones are challenges for elementary school children; and (ii) which of these aspects has the greatest impact on resolution.

However, other facets of this relationship still need to be explored. For example, an aspect that was not addressed in the present study, is whether the relationships between understanding and solving would vary depending on the type of problem. Although different types of problems were considered in this investigation (isomorphism and product of measures problems), an analysis was not conducted to examine whether the difficulties children faced when trying to understand the text of the problems would be the same with the two types of problems or if they would vary. This is an issue to be investigated in future research that may further clarify the role of the situation model in understanding and solving word problems, since the elements described in the text of the problem are integrated in the situation model, and those elements could influence individual performance and strategies. As mentioned by several

authors (Coquin-Viennot & Moreau, 2007; Hegarty et al., 1995; Jarosz & Jaeger, 2019; Mattarella-Micke & Beilock, 2010; Thevenot, 2010), the construction of a situation model is crucial for processing word problems successfully. Considering that the text of isomorphism and product of measures problems vary, generating different mental representations in children, it would be important to know what difficulties would arise and the impact of these representations on the resolution.

Examining understanding in these two types of problems is also relevant because, as reported by Cummins et al (1988) and Cummins (1991) children have difficulties in understanding the language used in the problem text. Undoubtedly, there are certain linguistic features and words that are specific to one or another type of problem, or from one or another field of mathematical knowledge (Peng & Lin, 2019). For example, in division problems such as those presented here, the term *each* is frequently used, while the term *n times* appears frequently in multiplication problems. This and other linguistic aspects of the word problem were not considered in the present investigation. However, in addition to the need for further studies, this aspect deserves to have a more prominent role in teaching situations, especially when complex mathematical problems are involved, as highlighted by Boonen et al. (2016). According to these authors, it is necessary to encourage teachers to pay more attention to text comprehension and teach students to deal with the linguistic characteristics of word problems.

On this subject, Kwok, Welder and Moore (2022) emphasize that it is necessary to unpack the language used in word problems to understand the mathematical meanings embedded within linguistic patterns and the relationships between the known and the unknown information, and identify relevant solution strategies. They propose a linguistic analysis of additive word problems that can help students understand the features of the text of the problems. Although these comments were derived from research carried out with different types of problems of additive structure, it undoubtedly also can be applied to problems of multiplicative structure. In order to unpack the language used in word problems, teachers could propose activities that encourage students to analyse them in a way that would help them to make sense of how certain mathematical operations are associated with the actions and relationships described within the statement of the problem.

Doruckzy et al (2015) distinguish three components in understanding and solving word problems: the linguistic complexity of the text itself, the numerical complexity of the arithmetic problem, and the relationship between these two components. In this final discussion, it is important to highlight that the linguistic complexity involves aspects of a phonological (decoding) and semantic nature (vocabulary specific to a field of mathematics or type of problem), but also involves mastering the surface structure of the problem text. As mentioned by Neshet et al. (2003), the surface structure is of great relevance in the construction of a first interpretation of the mathematical text. In fact, word problems have a very stable structure: the text begins with information about characters and objects, quantities and their referents, relationships between these quantities, and ends with a question. Although important, knowledge about the structure of the problem is something that has been neglected by educators and researchers.

If, on the one hand, understanding the problem text relies on general understanding processes such as establishing inferences, knowledge about the structure of the text and creating a mental representation, on the other hand, the word problem has specificities that distinguish it from other types of texts (Kintsch & Greeno, 1985). Thus, it is possible to suppose that reading practices carried out with other types of texts would not help the understanding of the mathematical text. In fact, it seems to be necessary to create didactic proposals directly aimed

at understanding the mathematical word problem. This kind of practice could, for example, lead students to familiarize themselves with the style and structure of this type of text, reflect on the numerical information present and on the relationships that have to be established in order to answer the problem question and not just identify it. The didactic proposal presented by Montero and Mahecha (2020) illustrates an approach of this nature. Based on the concept of textual macrostructure, in this proposal the text of problems was analysed on their characteristics and constitutive elements, based on known and unknown information and the relationships between information. To conclude, it is important to generate didactic opportunities that help students to become competent readers of mathematical problems, since word problems can not be seen simply as a school activity for the application of arithmetic operations. In this perspective, as emphasized by Pérez Arizza (2023) in his theoretical discussions, it is necessary to consider arithmetic problems as texts and problem solving as a process of textual comprehension.

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