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Ostensive didactic memory in the teaching practice of a Mathematics teacher in the context of the didactic milieu of a revision lesson

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Abstract: In this article reflects on the concept of didactic memory, in the context of the field of Didactics of Mathematics, with the objective of evidencing the ostensive didactic memory reflected in the practice of Mathematics teachers. It can be categorized as qualitative research, which was carried out at a state public school in the metropolitan area of the city of Belém, in Pará, Brazil, with the collaboration of a teacher and his students in the first year of high school. When we focused on the didactic memory of a teacher and their classroom's memory, the practices we observed evidenced didactic phenomena linked to the didactic contract and the milieu, revealing different types of memories that mobilize cultural knowledge and situated knowledge. It is emphasized the ostensive didactic memory because it allows us to have a fluid learning process of mathematical objects in the school context.

Keywords: Didactic Milieu. Didactic Contract. Didactic Memory.

Memoria didáctica ostensiva reflejada en la práctica docente de un profesor de Matemáticas en el contexto del milieu didáctico de una clase de repaso

Resumen: En este artículo se hace una reflexión sobre la noción de memoria didáctica, desde la perspectiva de la Didáctica de las Matemáticas, con el objetivo de resaltar la memoria didáctica ostensiva reflejada en la práctica docente del profesor de matemáticas. Esta es una investigación cualitativa. El lugar de la investigación fue una escuela pública estadual, ubicada en la región metropolitana de la ciudad de Belém/PA, con la colaboración de la profesora regente y los alumnos de una clase del primer año de Enseñanza Media. Al centrarse en la memoria didáctica del profesor y la memoria de la clase, las prácticas observadas revelaron fenómenos didácticos vinculados al contrato didáctico y al *milieu*, revelando diferentes tipos de memorias que movilizan el conocimentos y saberes. Se hizo énfasis en la memoria didáctica ostensiva, ya que permitió fluidez en el aprendizaje de los objetos matemáticos, en el contexto de la institución escolar.

Palabras clave: *Milieu* Didáctico. Contrato Didáctico. Memoria Didáctica.

Memória didática ostensiva refletida na prática docente de um professor de Matemática no contexto do *milieu* didático de uma aula de revisão

Resumo: Neste artigo, faz-se uma reflexão sobre a noção de memória didática, na perspectivada Didática da Matemática, com objetivo de evidenciar a memória didática ostensiva refletida na prática docente do professor de matemática. Trata-se de uma pesquisa de cunho qualitativo. O lócus da pesquisa foi uma escola pública estadual, localizada na região metropolitana do município de Belém/PA, com a colaboração do professor regente e dos alunos de uma turma do primeiro ano do Ensino Médio. Ao enfocar-se a memória didática docente e a memória da classe, evidenciaram-se, nas práticas observadas, fenômenos didáticos atrelados ao contrato didático e ao *milieu*, revelando diferentes tipos de memórias que mobilizam saberes e conhecimentos. A ênfase deu-se na memória didática ostensiva, pois ela permitiu fluidez da aprendizagem dos objetos matemáticos, no contexto da instituição escolar.

Palavras-chave: *Milieu* Didático. Contrato Didático. Memória Didática.

1 Introduction

In the field of the Didactics of Mathematics, we have seen the development of teaching devices to be used for teacher training and research, based on theoretical constructs, such as the Theory of Didactical Situations (TDS), the Anthropological Theory of Didactics (ATD), the Theory of the Registers of Semiotic Representation (TRSR), the Theory of Conceptual Fields (TCF), among others. These devices, such as didactic transpositions, didactic systems, didactical situations, didactic obstacles, didactic memory, etc. aid the comprehension of the didactic phenomena linked to teaching and learning mathematics.

One of the main aspects in the didactics of mathematics is the epistemology of cultural knowledge, namely an epistemology of a relationship with cultural knowledge because such a relationship determines what it means to learn something in this field of study. In that sense, research that focuses on students and teachers and their activities in the classroom is essential to demonstrate how the process of propagation and acquisition of knowledge occurs, specifically how the actions of the teachers condition the actions of the students. Very often, these actions are linked to teaching models the teachers are not aware of.

The lack of reflection regarding one's own work-in other words, a reflection of/in one's teaching practice as seen through theoretical lenses is related to what Brousseau (1986) recognizes as the teacher's spontaneous epistemologies. To Chevallard (1995), this matter is treated as the *teacher's gesture*, which reveals the "current didactic and disciplinary praxeology, the effects of the naturalization incorporated as habitus, the relationship to the object(s) of knowledge that is taught, at once, personally, officially and institutionally" (Amade-Esco, p. 119).

When we go forward in the theoretical-practical articulation, the teaching practice acquires an increasingly professional character, with coherent and consistent justifications concerning teaching strategies. What we call didactic memory is one of the theoretical devices that reveal relationships established with different kinds of knowledge in either a short or long period of time. Such a device strengthens self-reflection regarding our relationship with certain kinds of knowledge and increases the improvement of our teaching practices and, consequently, of our students' learning processes.

In the field of the TDS, Centeno (1991) focuses on didactic memory to explain phenomena linked to didactic time[1] and to the conversion of information into knowledge,

[1] Didactic time is directly linked to the institutions for spreading knowledge such as schools, but it is a time that transcends the duration established by the institution because the phenomena that are didactic in nature are linked to tasks carried out outside the classroom involving the objects studied in school.

through the institutionalization[2] accomplished by the teacher.

Scholars like Brousseau (1986) and Chevallard (1995) indicate that a theoretical outlook favors the use of classrooms as laboratories, as a true clinic for mathematics lessons.

In this perspective, the reconstruction and management of the didactic past of a given class require certain procedures, let's say didactic-investigative ones, which are linked to the presumptions of the didactic teaching action in the classroom. Therefore, maybe it's necessary to "investigate the didactic past of the class (enlisting the help of informants such as students and colleagues; consulting official sources: notes and internet posts)" (Bouillon, 2010, p. 33).

Centeno (1991) used to consider that the management of the didactic memory of the class was exclusively the teacher's task; a function of the master's memory, whose management allows, for instance, quicker and more relevant interventions. Thus, for Centeno, the management of the didactic memory of the class is directly involved in the teacher's memory. However, researchers such as Matheron (2000), Araya-Chacón (2008), and Bouillon (2010) categorize didactic memory as being collective and prospective.

In the perspective of the TDS, the work of mathematics teachers in the classroom is very diversified, and it is desirable that they immerse themselves in a *milieu* of didactic and non-didactic situations that help in the establishment of satisfactory connections between the acts of teaching and learning Brousseau, 1986, 1996; Manouchehri, 2014). In fact, these situations happen, for example, when the teacher needs to bring old cultural knowledge and know-how to stimulate the didactic memory of the class before they learn new content (as is the case in a revision lesson). This is a particularly special moment for reminiscing about the didactic past of the class, during which the teacher also brings to the table their didactic teaching memory so that students can reminisce about the mathematical objects they need in order to study new subjects in the mathematical school curriculum.

Thus, our objective in this article is to highlight the ostensive didactic memory reflected in the teaching practices of mathematics teachers, in the context of the didactic milieu of mathematics classes.

2 Interfaces between Didactic Memory and the *Milieu*

In the 1990's, Guy Brousseau and Julia Centeno started the first research efforts on didactic memory, based on the TDS. This partnership resulted in several scientific works. After Júlia Centeno's death, her thesis was published posthumously in 1995, with the title *"La mémoire didactique de l'enseignant"* [The teacher's didactic memory].

This primary phase in the research on didactic memory is characterized by empirical observations. Brousseau and Centeno (1991) had no ambition of examining all the matters related to didactic memory but endeavored to concentrate exclusively on the experimental aspect of the teacher's memory. The research model they developed, which was based on the

[2] In the TDS, the teaching objects are presented to the students in non-didactical situations whose very nature prevents students from knowing beforehand what will be taught. Thus, we establish dialectics of action formulation, validation, and institutionalization. In the process of institutionalization, teachers attribute the character of cultural knowledge to the object that is being studied, that is, they present students with the cultural knowledge in action, through definitions, properties, etc. In many cases, one starts the process of studying with institutionalization, which prevents students from building and managing their knowledge.

TDS, assigned exclusively to teachers the task of managing the didactic past of their class:

> For example, to convert - during a didactic situation - a private piece of knowledge into cultural knowledge demands that the teacher possesses the memory of this cultural knowledge and, also, the memory of the student's private knowledge. This model allows us to distinguish the different kinds of institutionalization that will demand that teachers find different ways of managing the student's past (Brousseau & Centeno, 1991, p. 193).

The work of Julia Centeno served as a reference for several other researchers, as we will see throughout this article. Her research on the different kinds of memory one finds in a classroom is the foundation of the theories on didactic memory. Centeno identified that each student has a *private memory*, an *official memory* shared by the whole class, and a *didactic memory* of the class (shared by the students and the teacher):

> To Centeno, we must distinguish between several types of memory for each student. There is a psychological, private memory; there is the class's official memory contained in the same register, shared by the whole class (such as notebooks, archives, movies, textbooks, etc.); in that case, we can define it as an externalization of the memory regarding the different kinds of formal and informal knowledge, either recent or older, and already established; it is a didactic memory of the class, 'shared by the students and the teacher'. (Bouillon, 2010, p. 69).

Let us observe that the passage quoted above leads us to important observations regarding the kinds of memory studied by Brousseau and Centeno (1991). Unlike the memory studied by psychologists, the didactic memory of a class is defined as an *external memory* regarding the different kinds of formal and informal knowledge established in previous lessons. However, what is at stake here is the management of the didactic past of the class (to Centeno, this is a task belonging exclusively to the teacher) and not the students' private (psychological) memories. Moreover, in Júlia Centeno's research, the didactic memory of a class has a few characteristics: it is subjected to the rules of the teaching institution, belongs exclusively to the specific group of students of a given class, it is temporary (after being institutionalized, the knowledge remains registered in the psychological memory), it is incomplete, and is not linked to cultural knowledge:

> Therefore, we have here a temporary and incomplete memory, which can only be used after being correlated with the teacher's memory; a memory that leaves room for acculturation but does not point us toward culture itself; it is a memory that is subjected to the standards of the teaching institution and is supposedly shared by teachers and their students. (Brouillon, 2010, p. 69).

Matheron (2000) went back to the discussion on didactic memory in his doctorate dissertation, whose main objective was contributing to the exposition of memory-related phenomena linked to the study of mathematics. However, Matheron noticed that the empirical observations related to memory within the mathematical field were virtually inexistent except for the work developed by Brousseau and Centeno.

To Matheron (2000), there were too many theories concerning memory - as a whole, not specifically related to mathematics and its teaching - but they were not articulated, which resulted in the concept of memory being polysemous. That's why it was necessary to make up

for the lack of empirical data through the creation of observational devices and to establish a new theoretical formulation. The empirical observations analyzed in Matheron's dissertation were carried out between 1995 and 2000 in the city of *Marseilleveyre*, in *Marseille*. The participants in the research were 1000 students from a regular school and 1200 students from a Lyceum.

Matheron's most relevant contribution (2000) to the study of didactic memory was the inclusion of the Didactic Anthropological Theory (DAT) as a theoretical basis for the discussions on the different types of memory one observes in a classroom:

> Let us dwell a while longer on Matheron's work due to the major theorization work this researcher has carried out in the field of the Didactic Anthropological Theory and for the role he continues to play regarding the anthropological approach for research on didactic memory (Brouillon, 2010, p. 73).

Just like Centeno, Matheron (2000) makes it clear he has excluded the memory of the psychological subject from his study and identifies three kinds of memory: *practical memory*, *cultural knowledge memory*, and *ostensive memory*. The student's practical memory is broadly applied to anyone who performs an activity in the scope of mathematics; therefore, it is also applied to teachers.

The practical memory is categorized into two parts, separated by the institutionalization phase: the official memory, i.e., the memory of that which is institutionalized and, therefore, is part of the practical memory institutionally expected, and a temporary or work memory, which is part of the practical memory that students work with before the institutionalization phase. Besides, he believes the two types of memory (official memory and temporary memory) contain the elements of ostensive memory since it is a school-level learning system, and justifies this statement with Centeno's idea of before/after institutionalization:

> For example, considering the "before/after institutionalization division" mentioned by J. Centeno, the forms of the practice are "given" or, better yet, are likely to be found in the context of the situation, through the phases of action, formulation, and validation, before the process of institutionalization and at any later time in a somewhat "official" material (such as notebooks, textbooks, boards, etc.). (Matheron, 2000, p. 144).

A specific teaching practice requires a device consisting of material media (pens, notebooks, rulers, etc.) and techniques. However, for the practice to be accomplished this device must be equipped with adequate gestures[3]. Moreover, the mobilization of these gestures requires the mobilization of personal resources. For example, to make a fraction irreducible, besides needing pen and paper, a student needs to remember procedures that involve the simplification of fractions (decomposition into prime numbers, divisibility criteria). But, in order to perform the gestures required to complete the task of simplifying fractions a student needs to remember these gestures. Thus, they will be able to reproduce the practice they have learned previously. This is the definition of the *practical memory of a person*. Practical memory is linked to individual production, whereas *cultural knowledge memory* is external. It means that, in mathematics, cultural knowledge is a social memory that lies outside of the person, and

[3] The gestures mentioned here are based on the idea of gesture as interpreted by Chevallard (1995) in a broader sense: to accomplish, to do something—not in the most common sense of the word (i.e., a movement of the body).

part of it is deposited in mathematical works (such as school textbooks, books on algebra and differential calculus, etc.) (Matheron & Salin, 2002).

Matheron defines ostensive memory as the memory that is deliberately shown through the tools of mathematical work, the ostensives[4], thus "this manifestation can be accomplished, as one accomplishes the ostensives that are understood as tools for mathematical work, within the framework of several perceptible registers: a gestural register, a discursive-linguistic one, a graphical one, a scriptural one." (Matheron, 2000, p. 103).

Matheron's ideas (2000) were improved by Andrea Maria Araya-Chacón in 2008 in her doctoral dissertation, written under the supervisorship of André Antibi and co-supervised by Yves Matheron, using the Anthropological Theory of Didactics and the Didactic Memory Model proposed by Yves Matheron. Araya-Chacón (2008) states that an individual's or an institution's didactic memory is characterized by the individual's or institution's relationship with the objects and the practices of knowledge in the didactic timeline:

> The didactic memory of a person or an institution is connected to the manifestation of phenomena that are indexed in time and linked to the relationship of an individual or an institution with the objects of knowledge and the practices which are or have been carried out within the timeline of the institution's development. (Araya-Chacón, 2008, p. 131).

Furthermore, Araya-Chacón argues that didactic memory is preserved through the *didactic contract*[5], and that this contract allows the didactic memory to be preserved because it allows us to search the past for the tools required to accomplish current practices:

> This memory is, necessarily, an institutional construction: the objects and their relationships - which it refers to - exist within institutions. It is preserved through the perennial elements of a "contract", which evolve but slowly. The moderate advance of these elements allows memory to be preserved because it makes room for regulation concerning the intervention of the past on the institution's current practices." (Araya-Chacón, 2008, p. 131).

Araya-Chacón (2008) discusses the Memory Model proposed by Matheron, which comprises the three types of memory he identified: practical memory, cultural knowledge memory, and ostensive memory. To define what is practical memory, Araya-Chacón employs the concept of practice adopted by Matheron:

> According to Matheron, a practice—in a general sense—and a specifically mathematical practice presupposes a device composed of material means and techniques provided by an institution in order to accomplish a task. This device must be activated through adequate gestures, an activation which requires personal resources […] (Araya-Chacón, 2008, p. 37).

To better explain what she says in the above quote, Araya-Chacón (2008, p. 37) brings up the task of *"simplifying* -14 - (-5) + (-2) - (8) […]". To carry out this task, a student has at their disposal several devices and gestures, which will be conditioned by the institution the

[4] Chevallard (1994) establishes a fundamental distinction between ostensive and non-ostensive objects: ostensive objects are characterized by the possibility of being handled and non-ostensive objects are usually called notions, concepts, ideas, etc. They cannot, strictly speaking, be manipulated; they can only be evoked through the ostensives associated with them.
[5] Cf Brousseau (1996).

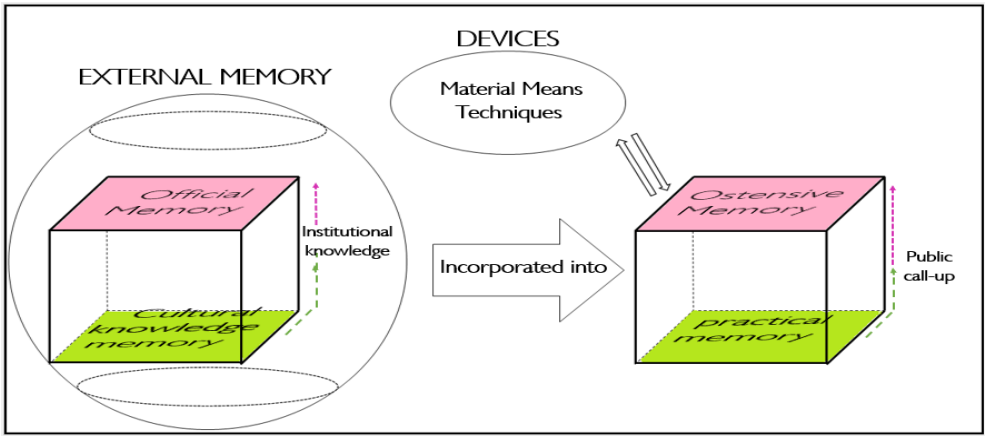
student belongs to (or by those with whom they've come into contact in order to carry out this task). And, to carry out this task, the student might use different means: materials (a calculator, paper, pen), the rules for rewriting the expression (the rule for subtracting relative numbers, the law of positive and negative signs, conventions), and the gestures that will articulate the techniques used to work with relative numbers. But according to the author, what allows an individual to produce such gestures is a gesture memory, in other words, the practical memory, thus defined:

> Regardless of the devices they apply, the person who produces these gestures must have a memory for them at their disposal. This memory, which allows them to reproduce at the correct time a practice they have previously learned, is calld an **individual's practical memory** (or simply **practical memory**). (Araya-Chacón, 2008, p. 38, our translation, emphasis added by the author).

The institution where the individual learned the devices and the gestures plays the role of external memory. When it comes to school subjects, the external memory is the cultural knowledge memory, and mathematical works are the repository of this type of memory: "for example, it is stored in mathematical works: manuals, books, software, videos, etc. Knowledge memory is, therefore, a type of memory that goes beyond institutions and is recognized by the mathematical community" (Araya-Chacón, 2008, p. 39, our translation).

Araya-Chacón defines ostensive memory as a public call-up of the elements of practical memory (Figure 1) that students and teachers have had contact with in the past, with the objective of homogenizing knowledge in order to set up a standard for practice: "therefore, this public call-up, by 'showing' the elements of practical memory, is what defines **ostensive memory**" (Araya-Chacón, 2008, p. 40, emphasis added by the author).

Figure 1. Schematic diagram of the memory model as proposed by Matheron



Source: Araya-Chacón (2008, p. 41)

The four types of memory shown in figure 1 require us to understand that:

- *Official memory*: different types of cultural knowledge and their practices, whose elements have been taught at an institution and are part of a student's external memory.
- Practical memory: an individual's gestures, when a practice requires them to, will be rebuilt from the practical memory. These are understood to be a result of the incorporation of a knowledge's official memory, which allows an individual to articulate the material means at their disposal —or the ones provided by this memory— and the techniques required to reproduce the practice.

- Ostensive memory: the practical memory that has been the target of a public call-up.

Araya-Chacón (2008) uses Matheron's (2000) model for didactic memory to analyze how a teacher manages the didactic memory of a class:

> The teacher must manage the reactivation of the class's memory that is linked to the objects and the relationship with the objects required for teaching; as well as the reactivation of other memorial elements that come up as the system evolves. How does the teacher manage this memory? (Araya-Chacón, 2008, p. 15).

Araya-Chacón amplifies the notion of managing the didactic memory of a class, attributing to students a certain degree of influence in the reconstruction of the didactic past of a class. Thus, building a didactic memory becomes a shared responsibility. Though it is organized by the teacher, the management of didactic memory in the classroom counts on student participation. These understandings remind us of Bouillon's research (2010).

In 2010, Stéphane Bouillon presented a transversal approach in his doctoral dissertation aimed at building relationships between time, teacher culture, and didactic memory, starting from the role that institutional time division plays in the development of different types of activities, in the ways we memorize information, and how we consolidate cultural knowledge. In this perspective, he uses sociology and anthropology to look for new conceptions of didactic memory, moving away from the models used in neuroscience and psychology.

> In the following chapters, we will go deeper into the notion of didactic memory using a cybernetic kind of modeling of non-didactic situations and of the role played by the teacher. We will see how the contributions of sociology and anthropology allow us to reach an innovative and specific conception regarding this type of memory, moving progressively away from the models normally used in neurobiology and cognitive psychology. (Bouillon, 2010, p. 14).

To deepen the notion of didactic memory Bouillon (2010) surveyed the research on the different kinds of memory studied in the field of the didactics of mathematics, taking into consideration two theoretical approaches: the Theory of Didactic Situations in Mathematics (TDSM), the works of Guy Brousseau and Júlia Centeno; the Anthropological Theory of Didactics (ATD), and the work of Yves Matheron.

This survey allowed Bouillon to understand how Matheron's work (2000) contributed to expanding the notion of didactic memory developed by Brousseau and Centeno in the 1990s. Didactic memory, which at first was thought to be controlled exclusively by the teacher what now understood as a collective thing developed by a social group and controlled by an institution, that is,

> Brousseau's modeling of a pre-programmed, systemic memory solely under the teacher's control is enriched by the anthropological contribution to become the memory of a group under the control of an institution. Didactic memory, then, would be simultaneously a collective memory built collectively in a public manner by a social group, and a *prospective* memory controlled by an institution that controls the future and the evolution of the knowledge produced within it. (Bouillon, 2010, p. 77, emphasis added by the author).

Bouillon (2010) believes that because didactic memory is controlled by an institution,

it shows us that the balance must be found between complete oblivion and a complete lack of oblivion for a culture to continue evolving. We need to enact what Bouillon calls a *didactic reduction*, which is characterized by the selection of a limited number of types of knowledge as socially recognized knowledge. Thus, "to teach is to designate, among the types of knowledge produced in the didactic milieu, those that must be kept and those that must be forgotten […]". (Bouillon, 2010, p. 272, our translation).

Studies on didactic memory reveal that the *milieu* plays an essential role in the comprehension of the didactic contracts established in the didactic relationship (Brousseau, 1998; Brousseau, Centeno, 1991; Matheron, 2000; Matheron & Salin, 2002; Araya-Chacón, 2008; Bouillon, 2010). In that sense, when a teacher calls up the official memory of the cultural knowledge that was taught and/or a student shows their practical memory of the types of knowledge they learned (Almouloud, Koné & Sangar, 2014), the public institutional memory is produced. That means that the extensive memory revealed by one person deliberately shows a practice or a teaching object as if it belonged to the memory of any individual in that institution. Thus, the memory of the cultural knowledge that was taught (Almouloud, Koné & Sangar, 2014), which controls the gestures for the practice, can be considered an external memory of collective practice. However, the teacher's action is important for the public reconstruction of the official memory (Matheron & Salin, 2002). Thus, "the *milieu*, be it physical, social, cultural, or of any other kind, plays a role in how both teachers and students employ and learn knowledge, whether or not it is called upon by the didactical relationship […]" (Brousseau, 1988, p. 312, our translation).

> Concerning the control of an individual - either a teacher or a student - over the milieu, we will acknowledge a few decisions as being linked to the conditions of the situation, while others are linked to the individual's cultural knowledge - to what they were taught and what they have learned through participating in non-communicable rites and social practices […] (Brousseau & Centeno, 1991, p. 192).
>
> During the interchange, the teacher bases their explanation on relationships with a material milieu, consisting of a classroom board, drawings, and tools, but the organization of these relationships cannot provide students with a retroaction concerning the adequacy of their actions with regard to the construction the teacher intends for them to accomplish (Matheron & Salin, 2002, p. 58).

The explicit formulation of some of the pieces of knowledge produced within the didactic milieu allows its existence as such, and it will, in the long term, bring about a change in the student's relationship with knowledge (Bouillon, 2010, p. 14). In his doctoral dissertation, Brousseau claims that "a student learns by adapting to a *milieu* that is a factor of contradictions, difficulties, and imbalances, a bit like the way it happens in human society […]" (Brousseau, 1986, p. 296). Matheron & Salin (2002) use a broader definition for a teaching *milieu*. According to them, a teacher must bring to the class the previous cultural knowledge and the previous know-how that he or she intends to mobilize during the lesson; moreover, they must be capable of evaluating to which degree the class recognizes these old objects before they try to teach a new object. This is a didactic *milieu*, because there is a strong didactical intention, which springs from the teacher's reminiscing concepts known to be shared by the class. The objective of the *didactical milieu* is to conduct the teaching process in a cooperative way, with direct student participation: "[…] such a *milieu* fulfills the need to show, within the institution, that the intention of teaching meets the intention of learning halfway, that studying is something that can be accomplished collectively; conditions that are necessary for the sustenance of the didactic relationship" (Matheron & Salin, 2022, p. 63).

Based on the research we have discussed in this section we arrived at the notion that the didactic milieu of a mathematics lesson serves as a basis for the didactic memory of teachers and students when a teacher initiates the process of collectively rescuing the memories of previous learning processes concerning mathematical objects that have been studied before. We will see how this rescue occurs in a more detailed way in the section where we analyze the lessons taught by a mathematics teacher.

In the following section, we will show the methodology we adopted during the investigation described in this article.

3 Methodological aspects

The qualitative research (Lüdke & André, 2018) we describe in this article follows the structure of a case study since its chosen subject was a class in the first year of high school. In a way, the complexity that underlies the didactic phenomenon concerning the teacher's didactic memory and the memory of the students themselves, intertwined with the didactic contract and the *milieu*, results in other methodological perspectives. However,

> *Case studies emphasize "contextual interpretation"*. One of the basic principles of this type of study is that, in order to fully understand the object, one has to take its context into consideration. Thus, to better understand the general manifestation of a problem, people's actions, perceptions, behaviors, and interactions must be analyzed in relation to the specific situation in which they occur or to the particular problem they are linked to […] (Lüdke & André, 2018, pp. 21-22, emphasis added by the authors).

Another methodological point that deserves attention is that the investigative *milieu* of the research we carried out was the lessons of a mathematics teacher which, according to Mattar (2017, p. 179): "can be an important research source […]".

The research locus was a public, state-run high school from the metropolitan area of Belém, in the state of Pará. We chose this school because we were acquainted with one of its mathematics teachers, who played the role of mediator in our relationship with the school.

During the process of choosing a class, we discussed the 1st-grade classes with the teacher. This conversation allowed us to choose a class with diverse characteristics, including disabled students. In this way, the didactic *milieu* would be richer, and we would be able to observe a few peculiarities of the didactic contract between the teacher and the students. The class we chose was labeled "M1MR01", in the school year 2022. In this article, we refer to the mathematics teacher of this classroom as "P Teacher" and we refer to the students as A_i (i = 1, 2, 3…n).

One of the ways of collecting data was through audio recordings of several mathematics lessons the direct observation in the classroom. We also scanned some of the students' work as a way of gathering information.

We highlight that only two of the authors of this article made the actual recordings and observed the lessons in person so that we could maintain the fluidity of the lessons and avoid disturbing the didactic contract agreed upon between the mathematics teacher and his students.

4 Ostensive didactic memory in the context of the didactic *milieu* of the classroom

As was mentioned in the methodological section, audio recording was one of the methods we employed to collect data during the mathematics lessons in the "M1MR01"

classroom. These recordings were later transcribed, and we analyzed some of them. In addition to the audio transcription, we also used images of the mathematics activities elaborated by the teacher.

The main objective of the mathematics lessons taught at "M1MR01" in 2022 was to review middle school mathematics contents, according to the school's pedagogical plan. The teacher was responsible for choosing the contents that would be reviewed (going forward, we will refer to him simply as "P").

On the first day of our observation, "P" introduced us and explained to the classroom the ethical rules for academic research. Then he started the lesson. The teacher's speech was aimed at rescuing the classroom's memory of the previous lesson (Matheron, 2000).

> *Let's reminisce for a bit about what we discussed in the last class so we can advance our content. The last class ended with the comparison of fractions, is that correct?* (P, 2022).

Reminiscing about the previous lesson rescues the possible didactical relationships the students developed with the mathematical object, bringing the teacher and the students back to the context of the didactic *milieu* so that they can continue the mathematics lesson (Matheron & Salin, 2022). That is evidenced by the teacher's speech:

> *In order to compare a fraction, it can have the figurative sense, as we've shown in the drawings, then we made the comparison by reducing it to the same denominator, which was where we stopped. Today we will see the effects of that comparison. For example, here I have -3/4 and -1/2. The question is: which one is the larger fraction?* (P, 2022).

The question asked by P leaves room for the didactic *milieu* to flow with the participation of the students in the classroom, who voiced their ostensive didactic memories (Matheron & Salin, 2002). That happened because one of the students A_i answered that the larger fraction would be -1/2. After this answer, the conversation between the teacher and the students reveals traces of the teacher's and the students' memory:

> *P: Why do you say -1/2 is the larger fraction?*
>
> *A_1: Because it's closer to zero.*
>
> *P: Ah, you remember. In the logic of the number line, the closer a negative number is to zero, the larger it will be. Have we shown this to you?*
>
> *A_2: Yes.*

In the dialogue, we notice that the ostensive didactic memory is externalized orally and then transcribed by the researchers. In the transcribed text, we find elements that are necessary to the didactic contract established between the teacher and the students (Brousseau, 1988; Araya-Chacón, 2008; Manouchehri, 2014). In the following excerpt, we can see that contract.

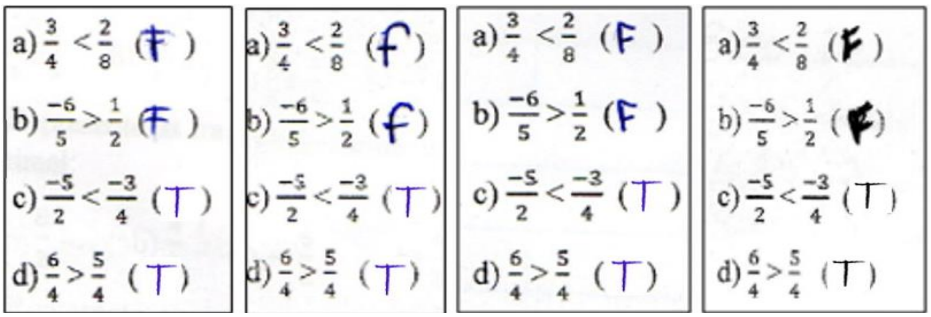
> *P: Good! If we've done that already, you guys already know a little bit about the number line: the limit of the division is the number zero: as we saw here, on one side we have $+\infty$, and on the other side we have $-\infty$, so we conclude that -1/2 is the number that is closer to zero. However, in the comparative logic, -1/2 and -1/4 are between which numbers?*
>
> *A_1: -1 and zero.*

The dialogue between P and class "M1MR01" reveals characteristics of the external didactic memory because it rescues situated knowledge and cultural knowledge studied in previous lessons (Brousseau; Centeno, 1991; Bouillon, 2010). In the following transcription of P's speech, we see a speech that comes from the didactic teaching memory, in which the teacher's didactic intention is clear.

> *Now, mathematically, you've also learned that there is a symbology to summarize what is larger and what is smaller, which can be this one (>) or this one (<). We came up with an analogy that the opening (>) will always mean larger and the corner (<) will always mean smaller. We've already discussed this idea so that you all will understand. We did that to overcome the didactic work from all the way back in middle school when your teacher would say: if you cut it and it becomes the number seven (7), then it means it's larger; if you cut it and it becomes the number four (4), then it means it's smaller. So we are getting away from that idea so we can understand the representational meaning of these symbols which you will use a lot in the future* (P, 2022).

The teacher P finished the first half of the lesson by saying that the review had been useful for the students who had not been present in the previous lesson. However, we can see that this review was also useful for the rest of the students A_i who partly evidenced their private memories. Figure 2 shows the answers of a few students a question involving the contents that were reviewed during the lesson.

Figure 2: Answers that were given by the students A_1, A_2, A_3, and A_4, respectively.



Source: Elaborated by the authors based on the students' activities

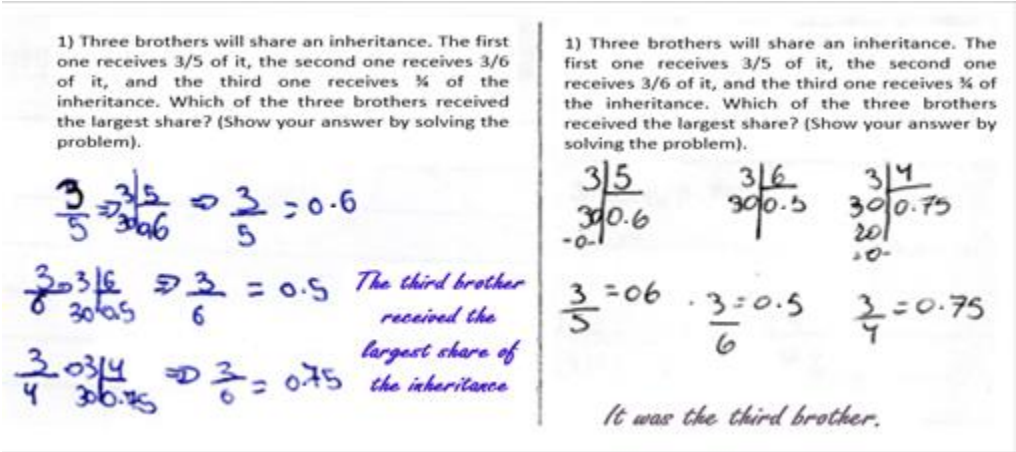
In Figure 2, we can see the same answering pattern for all four students, that is, we understand that these students assimilated the contents explained by P. The meaning of this is the official memory that the classroom possibly has (Brousseau & Centeno, 1991; Bouillon,2010). In addition to this official memory, there are signs that the teacher's ostensive didactic memory somehow shaped the ostensive didactic memory of students A_1, A_2, A_3, and A_4.

Continuing the lesson, P asks the students: *Which decimal number represents the fraction 6/5? Which one is larger, the numerator or the denominator?* A_5 answers it's the numerator. We presume that this student's answer is connected to their private memory, but it makes it clear that they are familiar with the idea of the elements of a fraction when observing the ostensive 6/5. Concerning this topic, Matheron (2000) says that didactic memory has a relationship that includes the individual, the object of cultural knowledge, and the practices associated with it in the didactic time - better yet, in the context of the institutional didactic *milieu*.

To know which decimal number represents 6/5, P resumed the didactic past of the class (Bouillon, 2010) through the division algorithm, that is, he divided 6 the numerator by 5 the denominator and came to 1.2 as a result, indicating that 6/5 = 1.2. Teacher P, when he solved

the division of 6 by 5, resorted to the ostensive didactic memory to reactivate the memories of an institutional past from middle school shared by all the students (A_i). By reactivating the ostensive didactic memory concerning the use of the division algorithm to represent fractions as decimal numbers, the didactic contract is then established for other tasks that require this same algorithmic process (Araya-Chacón, 2008)**.** The same procedure was extended to the decimal representation of the rational number -3/4 = - 0.75. Figure 2 reveals the ostensive didactic memory of the ideas explained in this paragraph.

Figure 3: Resolution presented by students A_3 and A_5 to an evaluative question.



Source: Elaborated by the authors using the students' work

During another lesson, P recovers the classroom's memory regarding the content in which fractions cross into the mathematical field of rational and real numbers. Thus, the teacher started the lesson by talking about the types of simple and mixed recurring decimals and the procedure (technique) used to obtain the repeating fraction of simple and mixed recurring decimals. Continuing the lesson, we have the dialogue between the teacher (P) and the students (A_i):

P: Given the expression (0.555... + ½). (0.444... – 1/3), what is the repeating fraction of 0.555...?

Ai: 5/9
P: How do we read this fraction?

A3: five ninths.

P: Can we simplify it? No, 5 is a prime number and cannot be divided by 9. Then 5/9 is the repeating function.

P: What is the repeating fraction of 0.444...?

A1: 4/9

P: Can we simplify this fraction? No, because the numerator and the denominator have no divisions in common. So, 5/9 is the repeating fraction.

P: Let us look at our task: the expression (0.555...+1/2). (0.444...–1/3) will become (5/9 + 1/2).(4/9 –1/3). Now we really have the operations with fractions. Let's solve each operation separately.

The dialogue established between the teacher (P) and some of the students (A_i) reveals that the didactic contract established to obtain the fraction that results in simple and mixed recurring decimals was assimilated by student A_1. Besides, the teacher uses ostensive didactic memory to explain when one indeed obtains the repeating function that generates a recurring

decimal. To solve (5/9 + 1/2).(4/9 – 1/3), the teacher recovers the classroom's memory concerning the Least Common Multiple (LCM): LCM (2, 9) = 18; LCM (3, 9) = 9; (2x5+9x1/18) = 19/18; (1x4 – 3x1/18) = 4 – 3/9 = 1/9; 19/18 . 1/9 = 19x1/18x9 = 19/162. We understand that the ostensive characteristic shown by the teacher (P) rescued ostensive didactic memories (Matheron & Salin, 2002) from the didactic past of the class (Bouillon, 2010) which maybe the students (A_i) would not remember or would not know how to use in order to solve the situation proposed by the teacher.

On another didactic moment of the lesson's *milieu*, P solved some of the questions from the exercise list he had elaborated. In this article, we chose to discuss the solution to the first question in this list of exercises. Below, we show the procedure adopted by the teacher.

> *Let's solve the first question because it is important for your comprehension. Three friends divide a pizza into equal-sized slices.*
>
> *The first friend eats 2/5 of the slices, the second friend eats 2/6 of them, and the third one eats 2/3 of them. Which of the three friends ate the smaller number of pizza slices?* (P, 2022).

After these procedures, the teacher uses the whiteboard to resume the explanation of the solution to the first question in the list of exercises.

> *What topic do I have here? You will compare the fractions to know which one of the three friends ate the smaller amount of pizza slices. There isn't a single solution, there are several. There is a solution that can be accomplished using figures, a solution that is accomplished using the operators, and a solution accomplished by using a common denominator. The solution obtained using the operator is the quickest one, the solution by figures will require drawings, and the solution through a common denominator will require more mathematical work.*
>
> *I will display the process to divide the numerator by the denominator:*
>
> *In 2/5, you will divide 2 by 5. The idea of a natural number says that the dividend must be bigger than the divisor, which is not happening in this case. So, you apply the resource you've learned: use a decimal point and a zero to the right of the number 2 and a placeholder zero with a decimal point in the quotient. Now, 20 divided by 5 equals 4, so we conclude that 2/5 = 0.4. The second division will be 2/6, so I will divide 2 by 6. We will do the same thing here, using placeholder zeros and decimal points. Twenty times 6 equals 3, and 3 times 6 equals 18; to get to 20, we need 2 units. To keep on dividing, all we must do is add a placeholder zero beside the number 2 again, so 20 divided by 6 again equals 3 with 2 as a remainder. So, I conclude that 2/6 equals 0.3, approximately. Now there's the third friend: let us divide 2 by 3. Number 2 is smaller than 3, so we must use the same method, when we divide 20 by 3 the closer result we get is 18, with 2 as the remainder, and so on. In conclusion, 2/3 equals 0.6, approximately. Now we are able to compare: we obtained 0.4, 0.3, and 0.6, so, which one is the smaller number? (P, 2022).*

The didactic speech the teacher (P) used to solve the first question in the list of exercises went back to previous didactic memories which he rescued from the didactic past of the class (M1MR01). However, we see that when he establishes 2/6 = 0.3 and 2/3 = 0.6, he is omitting the fact that these fractions generate recurring decimals and, when these results are explained in this way, the procedure leaves out decimal places, that is, it limits the numbers to the right of the decimal point, without taking into consideration the rounding process. This "shortening" of the mathematical-didactic speech may have created a didactic obstacle (that is, the understanding that this procedure is always valid for any situation where the division of a numerator by a denominator shows a result with repeating numbers) for the students (A_i). The solution to the question is concluded through a dialogue between the teacher and the students (when one of them answers the teacher's closing question: *[...] which one is the smaller*

number?).

> *Student A_1: 0.3.*
> *P: And which fraction does that correspond to?*
>
> *Student A_3: 2/6.*
>
> *P: And which one of the three friends does that fraction represent?*
>
> *Student A_1: The second one.*
>
> *P: So, the second friend ate the smaller number of pizza slices. Which one of them ate more slices of pizza?*
>
> *Student A_3: The third friend.*
>
> *P: Who was in the middle?*
>
> *Student A_1: The first one.*

The didactic relationships established by the teacher (P), in the context of the didactic *milieu* of the classroom, revealed that different types of memory mobilize knowledge in the teaching and learning process. We add that ostensive didactic memory (Matheron, 2000; Matheron & Salin, 2002) is what leaves room for fluidity in the process of learning about mathematical objects in the context of the school as an institution.

5 Closing remarks

During this research, we investigated the didactic memory revealed in the action of a teacher and his students, that is, the didactic memory collectively built/revealed. To accomplish that, we immersed ourselves in a *milieu* of didactic situations made more dynamic during a mathematics reviewing lesson. To enliven the didactic memory of the class in this context, the teacher rescued previous knowledge and know-how to establish a didactic contract that is favorable to the apprehension of new knowledge. Thus, the act of reminiscing about the didactic past of the class established a connection with the teacher's didactic memory.

Even though there isn't a lot of research on this theme, studies on didactic memory from the perspective of the didactic of mathematics have become very diverse. Since Guy Brousseau's and Júlia Centeno's first studies (in the sphere of the TDS), the different types of didactic memory have become consolidated and, at the same time, there have been new typologies added to the field and, consequently, the theoretical basis has been broadened through the studies carried out by Matheron (in the sphere of the TDS).

In this context, the didactic *milieu* of the reviewing lesson evidenced a process of collective rescue (including the teacher and the students) of the previous learning and the memories connected to the mathematical objects that had been studied previously. Such a process was favored by the teacher's mediation. The reminiscing interplay established through the dialogue between the teacher and the students and between the students themselves revealed traces of the teaching and learning memories. The web spun by the memory of knowledge, the working memory, and the ostensive memories was developed in a dialectic process when we contemplate how the teacher manages the didactic memory of the students.

Thus, the *milieu* of the revision activity was conducted by a didactic contract that favored the students' reminiscences, thereby mobilizing the collective memory. Our investigation has shown that the reminiscences brought up during the revision lesson ignited

evocative memorial gestures that allowed the students to remember structures of know-how that were useful to elaborate new pieces of cultural knowledge that had been envisioned by the teacher. The contract thus established revealed, rather than an ability to control official memory (from books, notebooks, lesson plans, etc.) concerning the students' working memory, a web of negotiation of the collective memory — belonging to teacher and students - that reverberated in the memory of each one of its members by directing the prospective dimension of the didactic memory, which was shown to be efficient in controlling the didactic past and potentially favorable to the didactic future of the class.

It is worth noting that the *milieu* of the revision activity imposed itself as a reminiscing device even in the moments when students were forgetful, going so far as to induce the use of gestures that displayed answers that were not in agreement with the teacher's didactic strategies. Such situations were also part of the constitution of the didactic contract established between the teacher and the students. In this contract, all they needed was to ensure that the teacher's memories were close enough to the memories of some of his students in order to create a minimum consensus around the facts faced in the classroom.

In this context, the complex diversity of the memories didn't create any conflicts because the didactic relationships established by the teacher in the context of the classroom's didactic *milieu* revealed that the different types of memory mobilized cultural knowledge and situated knowledge in teaching and learning. In this web, the ostensive didactic memory allowed fluidity to the learning process of mathematical objects in a school institution.

The dynamic of this study, which was conducted by the remembrance of the practical memory (and the gestures associated with it) highlighted in the dialogue between the teacher (evoking the official memory of the cultural knowledge) and the students, and in the students' dialogue among themselves (practical/personal memory of the knowledge) the unraveling of the external memory (cultural knowledge memory) united to the ostensive memory whose objective was to build new practices based on previous ones.

Thus, in a *milieu* composed of reviewing tasks, the gestures initiated by the practical memory of the students were regulated by the cultural knowledge memory - an external memory of the gestures that managed its activation - and allowed the students to construct simultaneously a new memory of this practice. In this manner, one is able to advance the process of introducing new pieces of cultural knowledge, which will employ previous knowledge as a tool.

We highlight that it was through the employment of the ostensive memory that the students revealed their personal and practical memory concerning the knowledge apprehended. This ostension can be evidenced by mobilizing ostensive tools of the mathematical work, in the framework of several perceptive registers: gestural registers, discursive-linguistic registers, graphic registers, and scriptural registers.

The studies on didactic memory need to be broadly propagated so one can better understand how students can be stimulated to use ostensive tools in a controlled way so that they can experience the benefits of these memory tools for mathematical practice. Due to the memory contained in them and the regulated manipulations they authorize, the ostensive tools allow access to the universal practical memory of the objects and the relationships with the

objects.

The knowledge dimension we have researched here was restricted to a revision lesson, and it was enough for our analysis concerning the teacher's management in the collective reminiscence with his students, but we understand that in the future there will be a need to broaden this dimension to evidence how going back to mathematical notions through reminiscence is useful, as a tool, to explore new mathematical notions. This will allow us to encompass new teaching and learning situations.

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