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Mathematics education as a systems of society

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ABSTRACT

This essay explores philosophy of mathematics education from a historical sociological perspective. This examines the nature of meaning-making as social systems of mathematics and mathematics education emerge as part of society. To do this I draw on social systems theory which posits a general theory of autopoietic systems from which consciousness and communication, as systems based on meaning, are coupled through language. Systems theory has had little attention in mathematics education and my aim here is to outline the approach by presenting a cursory functional analysis of mathematics education. This allows the possibility of theorising mathematics education as a social system of communication and has the capacity to show how the constituting parts of mathematics education, for example, research and practice relate and how mathematics education is related to other social systems in contemporary society.
Keywords: Systems theory; Autopoiesis; Mathematics; Mathematics education; Historical sociology.

A educação matemática como um sistema da sociedade

RESUMO

Este ensaio explora a filosofia da educação matemática a partir de uma perspectiva sociológica histórica. Este examina a natureza da construção de significado à medida que os sistemas sociais de matemática e educação matemática emergem como parte da sociedade. Para fazer isso, baseio-me na teoria dos sistemas sociais que postula uma teoria geral dos sistemas autopoiéticos a partir da qual consciência e comunicação, como sistemas baseados no significado, são acoplados através da linguagem. A teoria dos sistemas tem recebido pouca atenção na educação matemática e meu objetivo aqui é delinear a abordagem apresentando uma análise funcional superficial da educação matemática. Isso permite a possibilidade de teorizar a educação matemática como um sistema social de comunicação e tem a capacidade de mostrar como as partes constituintes da educação matemática, por exemplo, pesquisa e prática se relacionam e como a educação matemática se relaciona com outros sistemas sociais na sociedade contemporânea.
Palavras-chave: Teoria dos sistemas, autopoiese, matemática, educação matemática, sociologia histórica

La Educación matemática como sistema de sociedad

RESUMEN

Este ensayo explora la filosofía de la educación matemática desde una perspectiva sociológica histórica. Este examina la naturaleza de la creación de significado a medida que los sistemas sociales de las matemáticas y la educación matemática emergen como parte de la sociedad. Para ello me baso en la teoría de los sistemas sociales que plantea una teoría general de los sistemas autopoiéticos a partir de los cuales la conciencia y la comunicación, como sistemas basados en el significado, se acoplan a través del lenguaje. La teoría de sistemas

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ha recibido poca atención en la educación matemática y mi objetivo aquí es delinear el enfoque presentando un análisis funcional superficial de la educación matemática. Esto permite la posibilidad de teorizar la educación matemática como un sistema social de comunicación y tiene la capacidad de mostrar cómo se relacionan las partes que constituyen la educación matemática, por ejemplo, la investigación y la práctica, y cómo la educación matemática se relaciona con otros sistemas sociales en la sociedad contemporánea.
Palabras clave: Teoría de sistemas; Autopoiesis; Matemáticas; Educación matemática; Sociología histórica.

A GENERAL THEORY OF AUTOPOIETIC SYSTEMS

Since the emergence of mathematics education as a distinct field of study there has been a proliferation of methodological and theoretical approaches that aim to offer an understanding of the nature of the teaching and learning of mathematics (see, for example, INGLIS; FOSTER, 2018; SRIRAMAN; ENGLISH, 2010). The proliferation of approaches, it can be argued, provides a rich and healthy diversity within the field – it certainly suggests an increase in complexity within mathematics education. This leads to a challenge when considering what mathematics education is as a whole.

It might not necessarily be seen as a challenge however; is there a necessity to observe mathematics education as a thing in itself? Certainly it is reasonable to say that mathematics education is complex and that any attempt at grand theorizing is impossible, even futile. Yet, any 'observation' of mathematics education – that is from the moment we begin to talk about mathematics education – we are making an abstraction with implicit and/ or explicit theoretical claims about what it is and what it isn't. Observing mathematics education, as part of society, as research and practice, in this way, is a 'grand' theoretical endeavour.

The Enlightenment tradition characterizes theory as a representation of the processes, connections, relationships, and causalities, as well as the 'form' of the object of theorizing. Theory, in this sense, is an abstract account of reality, as a representation of reality. Given the plurality and inherent complexity of mathematics education, its internal variety even within the distinctions of research and practice and the way in which it changes over time, the derivation of a representational theory of mathematics education seems like an impossible task. It is tempting then to give up and pay more attention to practice and its problems.

Furthermore, research in the natural sciences in the last 150 years or so, has increasingly revealed the problem of grand theoretical projects, we might consider here the challenge of unifying gravitational theory and quantum theory, which has thus far evaded physicists. Within, mathematics itself, Gödel's incompleteness theorem demonstrates that mathematics, as a body of knowledge, cannot be supported by that body of knowledge alone, there must always be some external reference. This is achieved in mathematics through being an axiomatic body of knowledge; axioms, as assumptions, that are accepted socially and culturally. And the axiomatic foundation provides an externality that overcomes incompleteness.

Another related constraint in theorizing mathematics education is the problem of generality. For a theory of mathematics education to be general, the theory must also be an object of itself. If a theory of mathematics education does not include itself then it is not a general theory. This seemingly leaves us with a paradoxical situation to contend with. The endeavour here is to address the paradoxical problem of self-observation, which is akin to the serpent, Ouroboros, of Ancient Egyptian mythology, devouring its own tail, and importantly (and this is frequently forgotten) being reborn. My strategy, then, rather than grand theory as a picture or representation, but is as one that centres on its own paradox and in what ways mathematics education unfolds from its own contradictions.

This abstract, and rather challenging approach requires some conceptual tools to make this possible. In this, I draw on social systems theory (LUHMANN, 1995) which directly addresses the problem of theory in similar terms to those I set out above. The term 'system' has quite a specific meaning in this context. In historical usage, a system represented an assemblage of elements, and likely these are heterogeneous elements. A system in this sense is a whole in relation to parts. Systems theory has advanced and clarified considerably what a system is beyond being a complex conglomeration of parts. Contemporary systems theory advances a view that a system should not primarily be conceptualized in relation to the sum of its parts but something that is 'distinct' from its environment – a system/ environment schema. This approach echoes Gödel's incompleteness theory in that the problem for a system is not just its internal configuration, relationships, and operation but its relationship with what is external to it; a system cannot exist without an environment with which there is a relationship.

An important advance in conceptualizing this was developed by Chilean biologist, Humberto Maturana, in the 1970s, and then further with Francisco Varela (MATURANA; VARELA, 1980). At that time, the prevailing view of biological systems (and systems in general for that matter) was one that was concerned with inputs to and outputs from biological systems. The predominant methodological approaches attempted to describe the nature of the operations of a biological system by observing the inputs and outputs. This input-output model was not limited to biology, it also underpins behaviorism. However, within the input-output model, there is no recognition that a biological system can change epigenetically, through its interaction with its environment (as well as, of course, through changes that are evolutionary or genetic). Maturana and Varela ‘closed’ the input-output ‘loop’, so to speak, and contemplated the circularity of the existence of a biological system (I find it useful to imagine a ‘cell’, but this could be extended to any biological system however complex).

The system becomes closed through self-referential operations. The system is ‘blind’ to what its environment is, but that the environment is ‘felt’ in terms of changes in pressure and/ or temperature, for example. The important thing is that any ‘sense’ of a biological system’s environment is made through its own internal operations, in the case of a cell these operations employ molecular elements in biochemical processes. This represents an operational closure; the system is interminably connected to its environment but the only way it can see and respond to its environment is through its own internal operations. The system remains open thermodynamically; energy and matter can enter the system through its semi-permeable membrane; however, it is an operational closure. Maturana posited then a theory of self-referential operationally closed biological systems which he termed ‘autopoiesis’.

Autopoiesis explains how biological systems can achieve stability in complex (and therefore changing) environments but at the same time recognizes that stability and equilibrium are achieved through self-referential responses to instability and disequilibrium. It also explains how, within limits, a biological system can adapt to its environment through changes to its own internal operations, i.e., epigenetically.

Luhmann generalized autopoiesis by ‘de-ontologizing’ and ‘de-temporalizing’ the system elements that were suggested in Maturana and Varela’s biological model of autopoiesis. Luhmann also drew on Edmund Husserl’s phenomenology and derived a conceptualization of meaning as the difference between the ‘actual’ and a horizon of possibilities. Living systems, and biological systems, are constituted of chemical units, i.e., molecules and chemical elements. Consciousness and communication are constituted in the medium of meaning. This permits a contingent phenomenology for thought and communication, each actuality allows for the possibility of other actualities. Meaning permits, in Luhmann’s terms, a difference in unity (LUHMANN, 1995).

Maturana and Varela's model was based on the idea that the system elements in a biological autopoietic system are biochemical and that these remain the same or change as the system encounters its environment. What Luhmann argues is that in social and psychic systems (i.e., consciousness) the constituting elements are entirely constructed by each system, the elements thus become de-ontologized in the sense that their being is only the 'being' generated by the system for its own operations. In other words, a system differentiates itself internally and so elements have no ontology beyond that of the system. Further, he argues that biochemical elements change (or not) over time but are not dependent on temporal references beyond the system. Meaning provides a medium with which meaning systems can internally differentiate themselves as elements that are formed through the system's internal operations.

Thus, meaning becomes a notional constituting element for social and psychic systems. It is important to emphasize that meaning is constructed self-referentially by social and psychic systems. It is not possible to observe meaning except through observing communication and action; meaning is restricted to a system's internal ontology and temporality. Meaning can be thought of not so much as a foundational element but as a construct of internal elements of social and psychic systems. It is not necessary to labor this here, but it is important to highlight the ontological break with the traditional European Enlightenment ontological schema of being/ not being to a more comprehensive system/ environment schema.

Husserl's conceptualization of 'meaning' from a phenomenological perspective, involves the bracketing of meaning (*epoché*) from a horizon of possibilities. This might also be thought of as making a distinction between the 'actual' and a horizon of possibilities. In human experience (or the experience of any conscious being) the actual, the meaning or sensemaking that we achieve in the moment, is always in relation to a field of possibilities. This prevents meaning from being overly determined, there is always a surplus of meaning as Ricœur would have it (RICŒUR, 1976). Or, as Luhmann conceives it, as a unity in difference. It is important to stress that meaning as interpretation is a particularity in the more general phenomenological perspective of meaning in a field of endless possibility. Meaning, then always allows us to think or say something, to concretize an experience, but always in the realms of infinite possibility.

The autopoiesis of consciousness the (psychic system) has much in common with von Glaserfeld's radical constructivism and with Piaget's developmental psychology (GLASERSFELD, 1995). Constructivism is all too often maligned and even rejected on the grounds that it indicates epistemological relativism in that all conscious perspectives are equally valid. Yet, systems theoretical constructivism is always framed as a relationship between system and environment. The distinction between thought and reality does not deny reality as relativism might suggest, but simply presents reality as an unknowable environment of consciousness. Or that reality is only knowable through consciousness. This does not, therefore, deny the existence of reality and for which Locke might be best pleased.

Following British mathematician George Spencer Brown (see WATSON, 2020), Luhmann posits that the primary capacity of consciousness is the possibility of making a distinction, to distinguish one object from another including making the distinction between self and other. To maintain that distinction, to retain a relationship between sign and signified, for example, the initial distinction 're-enters' itself recursively, we make the same distinction within the distinction. Or we can introduce a further distinction within the distinction. Consciousness invites a recursive conscious distinction between self and other, or a distinction between an object and all the things that are not that object. Primarily, central to conscious systems and indeed central to any autopoietic system is a re-entry of distinctions into themselves.

The capacity of consciousness to make distinctions explains how consciousness and communication are co-evolutionary. The evolution of consciousness or self-consciousness prompts a distinction between self and other. This further prompts the possibility of meaning-making in relation to the observed other. The other's behavior is open to interpretation as a communication. While forms of communication have existed in a number of species probably for several million years, it is in around the last 150,000 that language has emerged. With the evolution of language societal organization became more sophisticated, the most common form of societal differentiation was the family or tribe, but principally a segmentary differentiation that reflects close familial connections or is segmented by location.

Language functions to coordinate the actions of the in-group while maintaining the distinction between who is and who is out, at the same time it maintains a unity with the whole of society. These are the conditions under which agrarian society emerged, prompting the need for increasingly sophisticated language and tools, and specifically, tools that good automate aspects of an agrarian society. One of the earliest artefacts in this is the Ishango bone, from Ethiopia around 20,000 years ago. The notches and marks are evidence of something being counted and recorded, possibly for the prediction of cyclic phenomena. What is important the capacity to record in this way is a forerunner for writing. From around 3000 BCE, Babylonian cuneiform tablets and the Ancient Egyptian papyrus demonstrated sophisticated writing and mathematics. What is more, society itself is taking on new configurations while still retaining segmentation within it – especially the family. Alongside the evolution of writing, society became stratified, with an elite who ruled over a kingdom.

THE FUNCTIONAL DIFFERENTIATION OF SOCIETY

Stratification solves several problems that segmented society faced in the coordination of actions. Farming technology became more sophisticated to increase productivity to smooth out instability in supply. This prompted the need to coordinate action in the production of surpluses as well as shortages. Rulers also required processes by which they could collect value, worth and wealth and in return (as social contract theorists would have it) they were required to make decisions that affected the whole kingdom. Stratification, however unequal and unjust it may appear now, was a solution to the problems of coordination in an increasingly complex segmented society.

The capacity to write, record and even calculate were the necessary technologies that allowed elites to retain power and the means by which stratified, aristocratic society was stabilized through the Middle Ages into early modernity. While education, in terms of learning to communicate, learning roles and skills, as a socialization process, is implicit within segmented society, in stratified society education, principally and largely was exclusively for elites. However, increasingly sophisticated technical knowledge was necessary, especially in writing and calculation (see LUHMANN, 2013a, 2013b).

Marx notably characterized society in terms of class struggle between the capitalist class and the worker, primarily but not exclusively characterizing society as fundamentally stratified. However, it is becoming increasingly apparent that stratification, although still present as with segmentation, has given way to a new evolutionary form of society and that is functional differentiation ((see LUHMANN, 2013a, 2013b; POLANYI, 1996, for example). Functional does not mean an a priori rationalism but the functional approach asks the question about how systems and structures have evolved to solve particular problems that develop in society, stratification solves problems that exist in coordinating actions in segmented societies, and functional differentiation addresses the problems of the limited capacities of elites to make decisions for an increasingly literate and informed underclass (even though this is a small proportion of that underclass in early modernity). One way of thinking about this is to consider the Glorious Revolution of 1688, and the political philosophy of Locke that made way for liberalism, that saw increasing inclusion in economic and political decision making. This at the same time with an expanding and increasingly complex system of public administration.

In Europe, over the last millennia, increasingly specialized societal functions have become autonomous in terms of communication. The differentiation of the functional system of politics begins in the Middle Ages (but there are several historical forbears to this too), as the nature of making decisions that are collectively binding becomes increasingly demanding and prompts the need for a group of specialist advisors and administrators. The English civil war sees the beginnings of a nascent form of democracy and the further assertion of a distinct system of politics. At around this time, other functional systems differentiate from society, including, for example, the legal system, the economic system, the military system, a system of science (which also includes mathematics), the system of mass media and a system of education.

A functional system is not a system of people, although people are part of the environment of all social systems, it is a system of communication, a differentiated system of communication. The function of politics is concerned with the making of collectively binding decisions using the medium of power. Political communication includes voting, public opinion on politics and political issues, party manifestos, politicians' speeches, parliamentary debates and public debates on politics and political issues. Political discourse is collected functionally and, in its totality, makes contingent selections on what is or is not political discourse. Within politics there are specific organisations that make decisions, the executive or parliament is responsible for such decision making in liberal democracies. The function is not the decision, the policy, but the constant identification of what is political communication and what isn't political communication.

The functional system of science emerged from about the time that moveable type print proliferated in Europe through the latter part of the fifteenth century and then accelerated in the 16th century. Though not necessarily causal, in that movable-type printing is as much an evolutionary response to complexity as it is a cause, the rapid expansion of the availability of texts and data prompted wide-ranging discussion of scientific content in the form of hypotheses, methods, data, results and conclusions. What emerged was a functional system of science that through its programme of experiment and reason functions to determine what is valid and false. The system of science is concerned with scientific, scholarly and research communication, the distinction is made between what is science and what isn't science though, and this is closely connected to the problem of truth or validity. It is the communication system that makes it possible to make decisions about what is true and what is false, through response to communication about the knowledge and the justification made for that knowledge based on the experimental method used and/ or the theoretical argument. Mathematics is a subsystem of this overall system of research and science, but that emphasises reason rather than empiricism in its endeavour.

Consistent with contemporary sociology of knowledge, this suggests a discourse community that responds to both tradition and orthodoxy in scientific thinking but also responds to the advances of new experimental results and new theoretical arguments. Science in this sense can be seen more broadly as an autopoietic system of research. In stratified society, the determination of what was valid knowledge and what wasn't was the prerogative of monarchs and elite clergy.

The process of the transformation of society to functional differentiation, it is believed, is mostly completed during the 19th century. It is important to stress that stratification did not vanish, but that the dominant mode of society is functional. What that means in daily experience, is that people are now compelled to recognize that there is no single authority in society, that there is an ecology of complex communications and that there is greater uncertainty as a result of the realization that complexity and indeterminacy in society make rationalistic attempts at explaining it limited and predictions of limited value. Stability and equilibrium are no longer assumed to be the norm. Furthermore, the differentiation of a system of mass media (and now social media) means that society is awash with pluralism with seemingly competing and contradictory information. One can see then in the condition of fluidity that exists in late modernity as a condition of functional differentiation, or liquidity as Bauman characterises it (BAUMAN, 2000). One can also contemplate Foucault's biopolitics in this since our bodies are external to society as a system of communication, the mediation of society with the living body and living world is through consciousness. The physical body is a living system where consciousness is in its environment, not directing or controlling it but presenting it with 'noise' which must be selectively understood and attributed to action (or non-action) (LUHMANN, 1995)

The functionally differentiated society of late modernity creates an imperative for self-observation. In stratified society this observation was limited to elites, to survey their realm and to recognize themselves as an agent of something divine and with such consequent authority as well as from dynastic claims. The demands that society, and an increasingly global society, at least in economic terms, placed on individual elites meant that stratification was no longer viable in and of itself. Self-observation, accounting for actions and decisions, new kinds of political and economic theory, as well, of course, the emergence of modern mathematics and science and pervasively the system of mass media all function as self-observational processes in modern society. William Rasch observes that the experience of modernity is of modernity straining to observe itself. Yet, this is within the context of a society that is stable and unstable, as well as differentiated and integrated. Society is pluralistic, complex and internally contradictory.

Education as a functional system of society emerged as the need for knowledgeable courtiers, public administrators and civil servants expanded in Europe in the process of state-building. Education in Europe had been one of the functions of religion with churches providing education in an institutionalized form. From around the 16th century, schooling, as an institutional practice, was compelled to expand and educate children in order to serve the need for specialized skills in an expanding public administration, an emerging colonial expansion and the newly emerging private corporations and enterprises.

The context in which education began the process of loosening its ties to religion was expanding local and international trade i.e., an economic system and a system of mass media with the widespread dissemination of information in printed materials. Tucker, in his foreword to Jack Williams' account of Robert Recorde in the History of computing, remarks that European society was becoming increasingly "dependent on measurement and calculation in its organization and activities" (Tucker's foreword in WILLIAMS, 1997), he goes on to argue that data, its collection and its use was having an impact on other aspects of society including science and economics, but also, I suggest, upon politics.

Even though it is some years before statistics as an operation of the state became established. Banking systems that had developed in the increasing silk road trade in Italy from the 14th century, were accompanied by innovations in accounting and finance. In Italy too there were schools for learning calculation from around this time (WILLIAMS, 1997).

Indeed, data and its manipulation, processing and interpretation have been important in society's need for self-surveillance.

While it is instructive to consider the early stages of the differentiation of a system of education, and particularly the differentiation of mathematics education, the process of the differentiation of education is something close to complete in the 19th century (LUHMANN; SCHORR, 2000). During the 16th century, the Christian schism contributed to the separation of church and school in some emerging European states. It was in the seventeenth century there is increased attention to the philosophy of education. Locke (1993) addressed this by characterizing the learner as a blank slate, as a *tabula rasa*. This draws on a humanist tradition to reconsider education in more secular terms. This broadly coincides with the establishment of the very beginning of state education systems and the first appointments of professors of pedagogy in Germany, for example.

Education as a functional system expands the extent to which can reflect on itself at this time, asking questions that include: what is the purpose of education? How can it be evaluated? What are the most effective forms of pedagogy? How do children and young people learn and develop? What should be in the curriculum? In a state system, many of these questions are addressed by politics and with the support of expertise and scientific evidence. There are limits to the extent to which politicians can make the correct decisions about education, there are also limits to what research evidence and knowledge can tell us about the effectiveness of education.

To understand this is it is useful to reflect on how education differentiated from religion in many parts of Europe. Protestantism connects life course to a divine purpose, and this leads to the idea of human perfectibility, that through their own work and endeavors, individuals can become better people. As education differentiated from religion, the protestant idea of perfectibility leads to the idea of educability (LUHMANN; SCHORR, 2000). The pupil is then an 'educable' learner, alongside new constructions of childhood and youth.

MATHEMATICS EDUCATION AS A FUNCTIONAL SYSTEM OF SOCIETY

Mathematics, mathematics education and religion were very much intertwined concerns in classical Greece. Religion confronts the distinction between what is immanent and what is transcendent, i.e., the difference between what is profane and what is sacred. Religion serves to provide answers when uncertainty appears, myths and rituals provide 'universal' narratives and practices to account for, explain and deal with the unexpected in people's daily lives. Mathematics in antiquity emerges from practical needs in the coordination of human action in an increasingly complex society. In classical antiquity, mathematics, as increasingly abstracted from the practical context, becomes a means of presenting order through considerations of number and space and the relationships between abstracted concepts of number and space.

The Pythagoreans (Croton, Greece, circa 6th Century BCE) were significant in their attempt to develop mathematical inquiry but ensured a compatibility with religion and mysticism. There is undoubted tension here as the possibility of rational explanations of the world emerges from the abstract investigation of number and space. For Pythagoreans, the immanence of number and space in their conceptualizations and abstractions could not conflict with the unity of the transcendence accounted for through religion and mysticism (LEWIN, 2018). This appeared to result in, at times, violent confrontation internally and externally. The Pythagoreans were influential for several hundred years, prompting further developments in mathematics and its philosophy.

On the basis of the existence of historical evidence, it is reasonable to single out the developments made by Plato (428/427 or 424/423 – 348/347 BC) since much of our understanding of the Pythagorean approach stems from this philosopher. To overcome potential conflicts between mathematics and religion, Plato postulated a theory of forms which legitimizes human reason within an – and even adjunct to – an overall spiritual cosmology. Plato creates a duality between the immanence in which number and space is encountered and their abstracted forms as transcendental – but distinct from the transcendence offered by religion. The possibility of human reason prompts Plato to consider the educational benefits of mathematics both in practical terms (e.g., logistics or arithmetical calculation) and in terms of higher thinking. We get some indication here that mathematics education can be conceived of not just as utilitarian but as an attempt to partially fulfil human needs in terms of reason and through learning to reason.

At the end of the fifth and into the fourth centuries BCE there were many developments in Greek mathematics, including the establishment of fundamental treatises or 'elements'. Notably, Euclid's Elements (c. 300 BC) provided a comprehensive compilation of such work. There were developments in the conceptions of proof, number theory, proportion theory, sophisticated uses of constructions, and the application of geometry and arithmetic in the formation of other sciences, especially astronomy, mechanics, optics, and harmonics. Slightly earlier, Aristotle (c. 384-322 BC) used these developments in mathematics, primarily in the possibility for argumentation that it presents, to develop a system of logic also to support a and nascent materialist and physicalist science that employed deductive reasoning as well as through inductive reasoning (MENDELL, 2019). Thus, philosophy and mathematics could be separated from a religion whose polytheism and rituals solved problems of complexity within the society of classical antiquity by distinguishing the everyday from the transcendent capacities of gods. Philosophy and mathematics could be concerned with human reason in the navigation of societal order, contingency and complexity. However, in the hierarchies of classical society, it is religion that has ultimate authority over the distinction between transcendence and immanence or in other words, explaining the unexplained.

The 'form' of society in antiquity and classical antiquity was, principally, stratified. It emerged from concentrations of wealth and power in earlier segmented societies based on kin, location and tribes. A stratified form of society became relatively stable for almost two millennia and until the modern period in Europe. Stratification involves a relatively small number of elites differentiated from an underclass. This proved to be an enduring form of society where social rank was employed as a means of coordinating individual action in society (LUHMANN, 2013a).

Elites relied on a monopoly of knowledge and decision making to maintain order often using or threatening violent force to do so. The coordination of stratified society relies on the maintenance of the differentiation between the elites and commoners. At the same time, societies of antiquity and classic antiquity became increasingly sophisticated in the use and development of the technologies of economics, finance, and production. From which there is a need to solve problems of quantity and to perform operations on such quantities, there was, therefore, an inherent need for education that went beyond the pool of people provided by elites.

The classical epistemological speculations on arithmetic addressed the ordering and structuring of mathematics and its forms and relationships. This is inherently a consideration of how a human being comes to know mathematics. Mathematical knowledge represents an emergent structure or set of expectations that serve to simplify or condition a complex reality that is only known through consciousness. Like language and writing more generally, mathematical knowledge is a means by which consciousness (psychic systems) are 'structurally coupled' to society as a system of communication. Materiality is an environment for consciousness and so experience and its inherent uncertainty constantly disturb consciousness as part of an autopoietic process. And while mathematical facts and processes can have internal consistency, they are always subject to an indeterminate and infinite world. Hence mathematical knowledge progresses in response to meeting its own uncertainty. It is in the modern period that mathematics addresses incompleteness axiomatically, effectively then becoming a social system of communication, that is concerned with true/ not true through programmes of mathematical inquiry and proof.

In systems theory, evolution theory is used to explain how the improbability of individual survival is replaced by a lesser improbability of social structure. Social structures or expectations can serve to allow a relationship between different systems. A system's environment is always indeterminate from the perspective of that system but structure allows that indeterminacy to be simplified, that structure endures as long as it is 'useful' or 'proves its worth'. Language (including mathematics) provides a structural coupling between thought and communication. Structural coupling means that while each of these systems (thought or psychic systems and social systems of communication) is unknowable to the other. However, they share the structure of language to make meaning of the other system's behavior, behavior which is experienced only as a perturbation.

Consciousness or the psychic system experiences communication as noise from which it selects, or finds a pattern, or what is frequently referred to in Bateson's sense as the difference that makes a difference. To simplify this process language is used to structure selections of meaning by both psychic systems and by social systems of communication. This really reflects many ideas of the role of inner conversations as part of reasoning. It also reflects Wittgenstein's limits of thinking as the limits of language.

Classical mathematics presents structural antecedents for contemporary mathematics and is emergent in the context of the evolution of society's technologies and practices, including education, politics, economics, science and philosophy, and religion etc., but is also emergent in relation to itself, responding to its own proofs, claims and assertions. Mathematical knowledge and its structures continue to provide resources for thinking about and communicating problems of quantity and space within society. The more sophisticated and complex society becomes, the more sophisticated mathematics becomes. Moreover, the more sophisticated society and mathematics, the more sophisticated the problem of mathematics education.

Classical mathematics (and mathematics education) precipitated the mathematical quadrivium as an addendum to the trivium (grammar, rhetoric, and logic). The quadrivium and its internal hierarchy of arithmetic, geometry, music and astronomy is an esoteric characterization of number, space and shape. The foundations of mathematics teaching and learning in England in the Middle Ages were established from these origins by Roman senator Boethius (c. 477 – 524 AD).

The quadrivium became dominant for almost a millennia in England. Boethius's writing on mathematics and mathematics education included *De arithmetica* (On Arithmetic, c. 500) an adapted translation of the *Introductionis Arithmeticae* by Nicomachus of Gerasa (c. 160 – c. 220) and *De musica* (On Music, c. 510), based on a lost work by Nicomachus of Gerasa and on Ptolemy's *Harmonica*. Within Boethius' writing, the relationship between mathematics and religion continues to be considered in a new context in which the polytheism of the classical world is being superseded by Christianity in Western Europe. For example, in the *Consolation of Philosophy*, Boethius articulates the higher power of divine providence in relation to philosophy, reason and, therefore, mathematics (KAYLOR; PHILLIPS, 2012).

The series of Middle Age and early modern renaissances from the Carolingian renaissance to the 15th and 16th-century Renaissance all represent unique social, political, philosophical, and theological constellations but all marked interest in rediscovering work of classical antiquity, latterly this work came indirectly through an advancing Islamic culture and the Islamic universities of the middle east. It is during that 12th-century renaissance that Euclid's Elements were 'rediscovered' and translated by Abelard of Bath in around 1120. Euclid's Elements became a substantial feature of school mathematics in Western Europe through much of the modern period.

While the Renaissance(s) resulted in direct effects on mathematics education, there were much wider changes associated with this movement, especially during the 16th century, when the old order of stratification starts to give way to functional differentiation. This paves the way for modern mathematics and modern mathematics education. The beginnings of this see the differentiation of economics and politics from the households of monarchs and church elites. These become autopoietic systems of communication, functional systems that respectively deal with scarcity using systems of payment and money (the economic system or capitalism) and as a system for addressing the problem of making and legitimising collectively binding decisions (the political system). At the same time, there was a growing body of print material and publications presenting new ideas with alternative perspectives on religion, politics and which precipitated the possibility of a system of science. Science (in its most general sense which includes research and scholarship) became a functional system of communication that was concerned with what is true and not true, through theoretical and empirical programmes. Within this is the emergence of modern science and mathematics.

The need for specialised skills in the early modern period can be exemplified by the development of practical approaches to mathematics and texts published in vernacular languages. Effectively, methods for adding, subtracting, multiplication and division. Luca Pacioli (1445-1517) Summa de arithmetica, geometria, proportioni et proportionalita (Venice, 1494, second edition 1523). This was written in Italian and not Latin. Robert Recorde wrote the Grounde of the Artes (1543) in order to formalise arithmetic methods and illustrate, in the catechism style, an approach to teaching practical mathematics (HOWSON, 1982; WILLIAMS, 1997).

From this perspective, Recorde, like Pacioli, articulated a programme of mathematical learning, a curriculum, a pedagogical approach in catechism form, and a theory of knowledge, that mathematics is a practical subject. This is a recognition that mathematics is a technology that serves society and especially functional systems like the economic system that is becoming pervasive and global and in the scientific system that commenced an endeavour to find new sources of calculable truth in a society in which religion receded to being but one functional system of society.

While a nascent modern mathematics education is in the making in the form of the very practical approaches to calculation needed to fulfil societies' roles, there still remains the legacy of the quadrivium and the revival of Euclid. However, the Renaissance and the emergence of humanism marked a transition from the Middle Ages and consequently a transition for mathematics and mathematics education. The philosophical perspectives of the Renaissance and the Enlightenment that ensued, allowed mathematics and subsequently mathematics education to break with religion.

Mathematics following Newton, could now be a thing for and of itself but also provide us with practical answers in the early modern world. This is not to say that Newton himself did not wrestle with the contradictions between his own Christian faith and his clockwork universe. For Newton, God, was able to mop up the uncertainty that remains when the universe is characterized in such an ordered way.

The practices that existed within the interaction systems of the classroom, that themselves have a long history nested within various social, political and cultural contexts, but have endured principally in a form in which a learned master would pass on knowledge to the younger and inexperienced scholar. The problem for mathematics education since Pythagorean times is what form should mathematical knowledge take? And how best can we understand mathematics and latterly how best can we learn the skills of practical mathematics? For the Pythagoreans, as well as for Plato and Aristotle, there appears to be little distinction at times between mathematics, the philosophy of mathematics, curriculum and pedagogy. It is only in early modernity that we see these aspects become increasingly distinct.

What Karl Polanyi characterizes as the Great transformation, covered a period from the French Revolution and the Industrial Revolution in England, resulting in the further development of mathematics education (POLANYI, 1996). This ‘transformation’ reflects the changing form of society as it moves from stratification as the prevalent form to functional differentiation. One important aspect is the further advance of technology and science which precipitate new societal problems that are amenable to mathematical analysis. The state itself is one such application of technology, ‘statistics’ develop as a mathematical technology to support political decision making.

Increasing literacy, access to media, increasing awareness of material inequality, the development of political ideology as a basis for democratic agonism, after the French Revolution, coincide with increasingly influential socialist movements. The response to this was increasing enfranchisement and an increasing role for the state in attempting to mitigate and control societal inequality. It is through the 19th century that in Europe the possibility for state education emerged and the number of pupils who were compelled to attend a school increased. Mathematics education, as with education more generally, became an issue of concern for politics and the state.

The widespread expansion of state education in the second half of the 20th century required increasingly sophisticated education policy. State education required that politicians and public administrators make educational decisions about policy and policy implementation and evaluation. This has made the relationship between politics and education, at times, problematic, and the role has been increasingly undertaken by specialist organisations i.e. think tanks. Education policymaking has increasingly drawn on research. Much of this originated in research into assessment in the late 19th and early 20th centuries. This developed with learning theory from the nascent discipline of psychology and more recently from sociological perspectives to address social aspects of learning. The latter has been especially concerned with the relative educational performance of different groups and underlying causes.

In the last forty years, mathematics education has increasingly emerged as its own field. It should be noted that while I have referred to mathematics education in terms of a field in its own right and indicated as so by classical philosophers, it through modernity that it becomes functional differentiated. This means that it is a specialist system of communication which includes teaching, learning and research, for example, and that features communication that articulates what is and what isn't mathematics education and mathematics education communication. I would argue that the process of the differentiation of a system of mathematics education begins in antiquity and continues through the Middle Ages as an aspect of society and of education but it is only with the emergence of research and scholarship in relation to systems of mass education that we see a complete differentiation.

Where mathematics education presents a sufficient level of ongoing self-description. The need for social systems to self-observe is a feature of modernity and functional differentiation. Social systems are no longer anchored to the materiality of social rank that existed in stratified society, so there is a constant need to self-observe in response to the contingency of an environment with many other social systems. This self-referential recursion is the means through which identity is sustained in response to unknowability and contingency.

CONCLUDING REMARKS

Identifying mathematics education as a social system of communication is just a beginning. Once mathematics education is presented in this way, as a system of meaning-making, then it is possible to consider the relationship between internal aspects of that system. This includes the relationship between research and practice, the relationships between research approaches as well as the relationship mathematics education has with other societal systems such as politics, economics, science and research. There are dependencies and couplings that have evolved as mathematics education, mathematics and society have evolved as they have created order but always revealing new complexity and uncertainty.

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