INTRODUCTION TO THE RESEARCH WORKSHOP II: CHALLENGES OF TEACHER TRAINING IN THE FACE OF THE DEMANDS OF THE NEW CURRICULA

Introducción al seminario de investigación II: retos de la formación del profesorado ante las exigencias de los nuevos currículos

Rodríguez-Muñiz, L. J.

Universidad de Oviedo

Resumen

Este seminario de investigación indaga sobre la respuesta que la investigación en educación matemática está dando o puede dar a los retos que plantean los cambios curriculares en la formación del profesorado. Si bien la situación es de máxima actualidad en España, la mayor parte de los currículos escolares a nivel mundial están evolucionando hacia una matemática donde los procesos matemáticos y la comprensión conceptual priman sobre lo procedimental. En este contexto, surge la exigencia de que el profesorado transforme la práctica docente de acuerdo con los nuevos planteamientos curriculares. En este seminario veremos ejemplos de cuatro países (España, Italia, Portugal y el Reino Unido—concretamente, Inglaterra) sobre cómo llevar a cabo esta transferencia de la investigación en educación matemática a la formación inicial y continua del profesorado haciéndolas coherentes con las nuevas exigencias curriculares.

Palabras clave: *currículo de matemáticas, formación del profesorado, investigación en educación matemática.*

Abstract

This research workshop investigates the response that research in mathematics education is providing or can provide to the challenges posed by curricular changes in teacher training. While the situation is highly relevant in Spain, most school curricula worldwide are evolving towards a mathematics education where mathematical processes and conceptual understanding take precedence over procedures. In this context, there is a demand for teachers to transform their teaching practices in accordance with the new curricular approaches. In this seminar, we will examine examples from four countries (Spain, Italy, Portugal, and the United Kingdom—specifically, England) of how this transfer of research in mathematics education to initial and ongoing teacher training can be carried out, ensuring coherence with the new curricular requirements. These examples will illustrate effective strategies and practices to address the aforementioned challenges.

Keywords: *mathematics curriculum, research in mathematics education, teacher training.*

INTRODUCTION

The original Latin noun *curriculum* was used to denote both a type of carriage and the path or road it travelled on, hence the origin of the meaning of curriculum as a learning journey. Although the curriculum is an institutional proposal designed to plan and implement (mathematical) education in a specific educational stage (Rico, 1997), the Spanish educational tradition has frequently identified curriculum with the conceptual dimension, specifically contents, and even established the identity between curriculum and study plan. However, mathematics curricula increasingly include not only contents, but processes descriptions, methodological guidelines, assessment indications, and tips about the use of resources, particularly, technology. In this sense, following van den Akker (2013), we prefer the meaning of curriculum as a learning plan or learning journey. This meaning allows us

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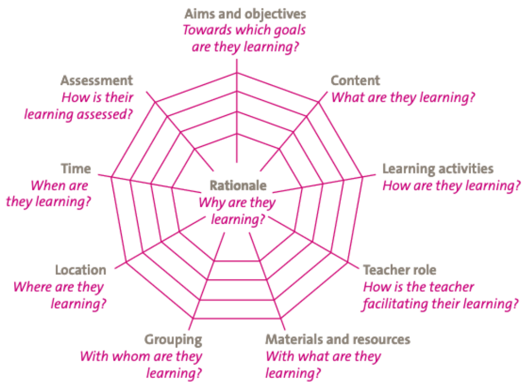
to appreciate different levels of concreteness (van den Akker, 2003): supra (e.g., international assessments frameworks), macro (national or regional mathematics curricula), meso (mathematics curriculum in a certain school), micro (classroom-level) and nano (personal mathematics learning journey for one student). When we talk about new curricula, the novelty uses to come in a top-down process, but we must keep in mind that changes should reach the nano level. This global view of the curriculum reveals the true magnitude of the challenge of adapting teacher training to curricular changes, that should cover the different dimensions of the curriculum as shown in Figure 1, and prevent from *syllabusitis*, a *disease* that 'puts the teachers in a position where they struggle to cover the prescribed subject matter' (Højgaard & Sølberg, 2019, p. 3).

Figure 1. The curricular spider web (van den Akker, 2013, p. 59).



One of the research workshops at the last SEIEM Symposium specifically focused on the development of the new mathematics curriculum (Moreno, 2022), analysing the case of Portugal (Canavarro, 2022) and experiences in an integrated STEAM approach (Diego-Mantecón et al., 2022), as well as an initial study on the relationship between the new Spanish curriculum and mathematics teacher training (Contreras, 2022). The research perspective on mathematics curriculum development has also been the subject of a recent book by SEIEM (Blanco Nieto et al., 2022). In this workshop, our aim is to present and analyse various approaches and strategies for transferring educational research to mathematics teacher training, addressing the needs arising from curricular changes. The recent book edited by Shimizu and Vithal (2023) provides a synthesis of different factors that influence the successful implementation of a mathematics curricular reform, considering all levels from nano to supra, with teacher training playing a crucial role. Thus, in this workshop, we delve deeper into this analysis by focusing on three experiences that encompass both initial and ongoing teacher training, examining their relationship with curricular reform and exploring the role of research in mathematics education within these processes.

THE ROLE OF THEORETICAL MODELS

One of the emergent themes from the three communications in the workshop is the role of theoretical models of mathematical learning and teachers' knowledge in shaping reflections on the organization of teacher training. Coles et al. (2023a) explore how different theoretical perspectives influence the design of teacher training programs in English universities. For example, the University of Cambridge adopts the Knowledge Quartet framework (Rowland et al., 2005) as the conceptual basis for their training, while the University of Oxford emphasizes the notion of practical theorizing (McIntyre, 1995). In contrast, at the University of Bristol, Coles, Heliwell, and Malkin's working institution, they draw on enactivism (Reid, 2014; Varela et al., 1993) to inform the organization of their teacher training program.

To the best of my knowledge, this type of institutional commitment to a specific theoretical approach is rarely found in Italian or Spanish universities, where there is a tendency to be more fragmented among different teaching departments, and sometimes even within them. However, the influence of teacher knowledge models can be found in the communications from Spain and Italy.

In the case of Italy (Mellone et al., 2023), there is explicit mention of the MTSK model (Carrillo-Yañez et al., 2018), which is enriched with the paradigm of Interpretative Knowledge (Mellone et al., 2021; Ribeiro et al., 2016) when designing tasks for initial teacher training. Mellone et al. (2023) also highlight the role of Design-Based Research (Bakker & van Eerde, 2015) in the core of PerContare and PerContarePRO projects, which constitute a large-scale continuous teacher training program in Italy.

In Branco's (2023) approach to the use of technology in Portugal, no explicit theoretical framework is mentioned. However, there is an underlying presence of the distinction between content and pedagogical content knowledge in the sense of Shulman (1986). Additionally, the prominent role of technology as a catalyst for teachers' knowledge bears a relation to other non-mathematical models that consider technology in a leading role as another knowledge subdomain, such as T-PACK (Koehler & Mishra, 2009).

On the other hand, apart from the explicit reference to the DMKC model (Godino et al., 2016), in Beltrán-Pellicer et al. (2023) there is another mention to teachers' knowledge of curriculum, which also aligns with the Knowledge of Mathematics Learning Standards in the MTSK model or the Knowledge of Content and Curriculum in the MKT model (Ball et al., 2008). The research documented by Beltrán-Pellicer et al. (2023) focuses on curriculum as a subject of study itself, enhancing teachers' knowledge in various facets, as depicted in Figure 1. It is worth noting the role of learning activities, teachers' role, and materials and resources in the Aragonese curriculum, influenced by the research group in Mathematics Education from the University of Zaragoza (to which Beltrán-Pellicer, Martínez-Juste, and Muñoz-Escolano belong), which significantly contributed to the detailed and comprehensive exposition of methodological guidelines (linked to specific contents) in the regional concretion of the recent Spanish education law. This has been, in my opinion, a clear example of what we mean when we talk about research transfer.

Beyond the (implicit or explicit) considered model, the three papers highlight the relevance of the necessary conscious practice, illuminated in the light of the mathematics curriculum. Thus, elements from different theoretical frameworks, such as enactivist awareness, interpretive knowledge, or deep reflection on all dimensions of the curriculum, are serving, under different paradigms, the same purpose: developing a professional vision of teaching. These relationships lead us to wonder, as Llinares (2020) did, what is the connection of conscious awareness with other paradigms, as noticing, or how the development of the professional vision of students' task and classroom activity is framed within the MTSK and IK (Mellone et al., 2023) model; following M. A. Montes (intervention in TWG20, during CERME11, February 2019): is MTSK + IK = Noticing? Furthermore, as suggested by J. M. Muñoz-Escolano (personal communication): what is the knowledge to describe a competence-based approach as Noticing?

From examining the models in England and Italy, another question that arises is the role of subject matter knowledge (SMK) and pedagogical-content knowledge (PCK) when considering different approaches such as focusing on learning to teach and analyzing students' difficulties and errors. In my opinion, the model presented in Coles et al. (2023a) blurs the boundaries between SMK and PCK, which poses a challenge for training approaches that, by historical tradition, originally subordinated PCK to SMK, as it is the case in Italy or Spain.

Turning our attention back to the Spanish case, even though explicit institutional theoretical commitments may be lacking, it seems clear initial teacher training programs are strongly influenced by practical theorising (McIntyre, 1995), often falling into an unintentional dichotomy, projecting a separation between theory and practice among prospective teachers. However, despite this lack of institutional commitment, there is evidence in the initial training programs of mathematics and its didactics courses regarding their design based on different theoretical or praxeological approaches, as, for instance, noticing (Llinares, 2012; Llinares & Fernández, 2021), DMKC (Godino et al., 2016), didactic analysis in mathematics education (Rico et al., 2013; Ruiz-Hidalgo et al., 2019), lesson study (García et al., 2019), or MTSK (Montes et al., 2019). There also arises a need, when comparing the approaches of Spain and Portugal, to consider what computational thinking is, how and why it has been included in the mathematics curriculum in both countries, or how it relates to mathematical thinking, and where its knowledge could fit within the models of mathematics teacher's knowledge: as an example, in Branco (2023) it is established a clear relationship to early algebra, so that analyses like the one in Pincheira and Alsina (2021) make sense.

CONTEXTUAL DIFFERENCES

In addition to differences in design approaches, we also encounter variations arising from the context in which the three papers of this seminar unfold. The theoretical approach to teacher training design, itself is not, and cannot be, disconnected from the cultural context in which it operates. Therefore, in contrast to the Bristol model, which emphasizes theoretical-practical embedding, the Spanish and Italian models of initial training are more rooted in a university tradition that placed greater emphasis on subject matter knowledge. This cultural context shapes and influences the design of teacher training programs, highlighting the different priorities and approaches in each country.

But contextual differences go further than design. For example, in Mellone et al. (2023), we observe how scientific societies (such as UMI, in this case, through its CIIM) have spearheaded various nationwide initiatives in continuous teacher training. While there are scientific societies in Spain that promote in-service training, it is important to acknowledge and commend the commendable efforts of professional societies of mathematics teachers who play a leading role in offering continuous teacher training activities (Beltrán-Pellicer et al., 2023). This fact highlights the commitment and dedication of these professional societies to support and improve in-service teacher training, although this commendable set of activities may result in a slight loss of direct connection with research in mathematics education. On the other hand, in Spain, several members of scientific societies such as SEIEM or RSME are actively engaged in informal activities for continuous teacher training through social networks, especially Twitter (e.g., Alsina & Rodríguez-Muñiz, 2021). This phenomenon is comparable to the situation in the United Kingdom or the USA, but not so much to Italy (Carpenter et al., 2022). This use of social media is generating highly productive debates and enriching discussions regarding the implementation of the new Spanish curriculum, enabling active teachers to interact with mathematics education researchers.

In another recent work, Coles et al. (2023b) emphasized the role of assessment in mathematics curricular reforms from an international perspective. In the case of Spain, assessment has been a prominent issue in previous curricular reforms and continues to be a challenge with the upcoming external assessment for Baccalaureate/High School students entering university, as noted by Beltrán-Pellicer et al. (2023). The influence of the Spanish University Entrance Exams (EBAU or EvAU, in the Spanish acronym) extends to impact teachers' practices, implemented curriculum, and the selection of resources, thereby constraining the intended curriculum and undermining the expertise of mathematics teachers, as described by Rodríguez-Muñiz et al. (2016) as "washback". This difference is likely significant when compared to Italy, where there is a stronger culture of external assessment, primarily due to INVALSI (e.g., Bolondi, 2021; Coles et al., 2023b; Vaccaro et al., 2022). Despite some discrepancies, Italy and Portugal demonstrate a better alignment between assessment and the curricular perspective compared to Spain.

Differences are also observed in the articulation of a novel element in the curriculum, such as computational thinking. While in Spain it has been introduced without providing a clear explanation of what is meant by the term, even confusing it with educational robotics (see, for example, Palop et al., 2022), in Portugal, based on Branco (2023), it can be observed how a more detailed analysis has been made of the dimensions that are considered when talking about computational thinking in school, which results in a more coherent fit with the literature in mathematics education by linking it, for example, to pattern recognition and generalization (e.g., Pinto & Cañadas, 2021).

I remark another difference that, although not explicit, arises implicitly when comparing the presented continuous and initial teacher training programs in England and Italy with the situation we know in Spain: the described programs in Coles et al. (2023a) and Mellone et al. (2023) frequently utilize peer observation, something that we also had the opportunity to see in Portugal the last year (Canavarro, 2022). These types of observation practices are not as common in the initial and continuous teacher training in mathematics in Spain. Although there are experiences (we pointed out lesson study-based programs as García et al., 2019), it would be very interesting to promote peer observation, either through live observation or through video recordings. This can be implemented on a large scale (as recently highlighted in Rodríguez-Muñiz et al., 2023) or through platforms that showcase real mathematics classroom fragments and observation guidelines carried out by groups of active teachers in programs internationally supported by the OECD (Muñiz-Rodríguez et al., 2023), so that they can be embedded in our initial or continuous training programs.

There is another international contextual difference I would like to highlight: the flexibility introduced in the organization of training at the University of Bristol, where there are alternating periods of longer and shorter durations spent at both the university and schools. Although this is partially implemented in some master's degree courses in Spain, such as the Catalonian master's program for secondary mathematics teachers, it is not common across all programs. While the enactivist approach serves as the foundation, I believe there are also contextual nuances in the ability of universities and schools to organize internships in a more flexible manner, which is not typically seen in our bachelor's degree programs for early childhood or primary education.

EMERGING QUESTIONS

To conclude this introduction to the workshop, I would like to highlight other emerging questions that could be engaging for discussion, during or after the presentation:

- What are the specific challenges faced in the (initial and/or continuous) training process of mathematics teachers, when new curricular demands are introduced at the national or international level?
- How can research in mathematics education help to identify good practices in teacher training programs that can be applied globally to support teachers in meeting new curricular demands?
- How can research in mathematics education inform curricular reforms at the national and international level to better prepare mathematics teachers for new curricular demands?
- What are the key factors that determine the success of mathematics teacher training programs in the face of new curricular demands?
- How can teacher training programs be designed to address the needs of mathematics teachers who are resistant to change in response to new curricular demands?

References

Alsina, A., & Rodríguez-Muñiz, L. J. (2021). Hilos de estadística y probabilidad en Twitter: una nueva herramienta para el desarrollo profesional del profesorado de matemáticas. *Educação Matemática Pesquisa*, *23*(4), 21–53. https://doi.org/10.23925/983-3156.2021v23i4p001-007

Bakker, A., & van Eerde, D. (2015). An Introduction to Design-Based Research with an Example from Statistics Education. In A. Bikner-Ahsbahs, C. Knipping & N. Presmeg (Eds.), *Approaches to Qualitative Research in Mathematics Education. Advances in Mathematics Education* (pp. 429–466). Springer. https://doi.org/10.1007/978-94-017-9181-6_16

Ball, D. L., Thames, M. H., & Phelps, G. (2008). Content knowledge for teaching: What makes it special. *Journal of Teacher Education, 59*(5), 389–407. https://doi.org/10.1177/0022487108324554

Beltrán-Pellicer, P., Martínez-Juste, S., & Muñoz-Escolano, J. M. (2023). Exploring the gap between intended and enacted curriculum: Perceptions of future and in-service teachers. In C. Jiménez-Gestal, Á. A. Magreñán, E. Badillo & P. Ivars (Eds.), *Investigación en Educación Matemática XXVI* (pp. 97-112). SEIEM.

Blanco Nieto, L., Climent Rodríguez, N., González Astudillo, M. T., Moreno Verdejo, A., Sánchez-Matamoros García, G., de Castro Hernández, C., & Jiménez Gestal, C. (Eds.). (2022). *Aportaciones al desarrollo del currículo desde la investigación en educación matemática*. EUG & SEIEM.

Bolondi, G. (2021). What can we learn from large-scale surveys about our students learning of Maths? *Atti della Accademia Peloritana dei Pericolanti-Classe di Scienze Fisiche, Matematiche e Naturali*, *99*(S1), A4. https://doi.org/10.1478/AAPP.99S1A4

Branco, N. (2023). Practices with technology in mathematics teacher education to face new curricular demands. In C. Jiménez-Gestal, Á. A. Magreñán, E. Badillo & P. Ivars (Eds.), *Investigación en Educación Matemática XXVI* (pp. 89-96). SEIEM.

Canavarro, A. P. (2022). El desarrollo del nuevo currículo en Matemáticas para la Educación Básica en Portugal. In T. F. Blanco, C. Núñez-García, M. C. Cañadas & J. A. González-Calero (Eds.), *Investigación en Educación Matemática XXV* (pp. 53–61). SEIEM.

Carpenter, J., Tani, T., Morrison, S., & Keane, J. (2022). Exploring the landscape of educator professional activity on Twitter: an analysis of 16 education-related Twitter hashtags. *Professional Development in Education, 48*(5), 784–805. https://doi.org/10.1080/19415257.2020.1752287

Carrillo-Yáñez, J., Climent, N., Montes, M., Contreras, L. C., Flores-Medrano, E., Escudero-Ávila, D., Vasco, D., Rojas, N., Flores, P., Aguilar-González, Á., Ribeiro, M., & Muñoz-Catalán, M. C. (2018). The mathematics teacher's specialised knowledge (MTSK) model. *Research in Mathematics Education, 20*(3), 236–253. https://doi.org/10.1080/14794802.2018.1479981

Coles, A., Helliwell, T., & Malkin, E. (2023a). Towards a communal mathematics teacher education. In C. Jiménez-Gestal, Á. A. Magreñán, E. Badillo & P. Ivars (Eds.), *Investigación en Educación Matemática XXVI* (pp. 69-78). SEIEM.

Coles, A., Rodríguez-Muñiz, L. J., Mok, I. A. C., Ruiz, A., Karsenty, R., Martignone, F., Osta, I., Ferretti, F., & Nguyen Thi Tan, A. (2023b). Teachers, Resources, Assessment Practices: Role and Impact on the Curricular Implementation Process. In Y. Shimizu & R. Vithal (Eds.), *Mathematics Curriculum Reforms Around the World* (291–322). Springer. https://doi.org/10.1007/978-3-031-13548-4_18

Contreras, L. C. (2022). La nueva propuesta curricular y la formación del profesor. In T. F. Blanco, C. Núñez-García, M. C. Cañadas & J. A. González-Calero (Eds.), *Investigación en Educación Matemática XXV* (pp. 63–79). SEIEM.

Diego-Mantecón, J. M., Ortiz-Laso, Z., & Blanco, T. F. (2022). Reflexiones del Open STEAM Group sobre el impacto del enfoque integrado del contenido en el aprendizaje de las matemáticas. In T. F. Blanco, C. Núñez-García, M. C. Cañadas & J. A. González-Calero (Eds.), *Investigación en Educación Matemática XXV* (pp. 81–94). SEIEM.

García, F. J., Wake, G., Lendínez, E. M., & Lerma, A. M. (2019). El papel de los modelos epistemológicos y didácticos en la formación del profesorado a través del dispositivo del estudio de clase. *Enseñanza de las Ciencias, 37*(1), 137–156. https://doi.org/10.5565/rev/ensciencias.2512

Godino, J. D., Batanero, C., Font, V., & Giacomone, B. (2016). Articulando conocimientos y competencias del profesor de matemáticas: el modelo CCDM. In J. A. Macías, A. Jiménez, J. L. González, M. T. Sánchez, P.Hernández, C. Fernández, F. J. Ruiz, T. Fernández & A. Berciano (Eds.), *Investigación en Educación Matemática XX* (pp. 285–294). SEIEM.

Højgaard, T., & Sølberg, J. (2019). Competencies and curricula: Danish experiences with a two- dimensional approach. In U. T. Jankvist, M. Van den Heuvel-Panhuizen & M. Veldhuis (Eds.), *Proceedings of the Eleventh Congress of the European Society for Research in Mathematics Education (CERME11, February 6–10, 2019)*. Utrecht University & ERME.

Koehler, M., & Mishra, P. (2009). What is technological pedagogical content knowledge (TPACK)? *Contemporary Issues in Technology and Teacher Education*, *9*(1), 60–70.

Llinares, S. (2012). Construcción de conocimiento y desarrollo de una mirada profesional para la práctica de enseñar matemáticas en entornos en línea. *Avances de Investigación en Educación Matemática, 2*, 53–70. https://doi.org/10.35763/aiem.v1i2.18

Llinares, S. (2020). Indicators for the development of noticing. *For the Learning of Mathematics, Monograph 1*, 38–42.

Llinares, S., & Fernández, C. (2021). Mirar profesionalmente la enseñanza de las matemáticas: características de una agenda de investigación en Didáctica de la Matemática. *La Gaceta de la RSME, 24*(1), 185–205.

McIntyre, D. (1995). Initial teacher education as practical theorising: A response to Paul Hirst. *British Journal of Educational Studies*, *43*(4), 365–383. https://doi.org/10.1080/00071005.1995.9974045

Mellone, M., Baccaglini-Frank, A., & Di Martino, P. (2023). The needs and the hopes for the secondary teachers' development in Italy. In E. Badillo, P. Ivars & C. Jiménez-Gestal (Eds.), *Investigación en Educación Matemática XXVI* (pp. 79-87). SEIEM.

Mellone, M., Ribeiro, M., Jakobsen, A., Carotenuto, G., Romano, P., & Pacelli, T. (2021). Mathematics teachers' interpretative knowledge of students' errors and non-standard reasoning. *Research in Mathematics Education, 22*(2), 154–167. https://doi.org/10.1080/14794802.2019.1710557

Montes, M. A., Carrillo, J., Contreras, L. C., Liñán-García, M. M., & Barrera-Castarnado, V. J. (2019). Estructurando la formación inicial de profesores de matemáticas: una propuesta desde el modelo MTSK. In E. Badillo Jiménez, N. Climent Rodríguez, C. Fernández Verdú & M. T. González Astudillo (Eds.), *Investigación sobre el profesor de matemáticas: práctica de aula, conocimiento, competencia y desarrollo profesional* (pp. 157–176). Ediciones Universidad de Salamanca.

Moreno, A. (2022). Introducción seminario de investigación II: el desarrollo del nuevo marco curricular en matemáticas. In T. F. Blanco, C. Núñez-García, M. C. Cañadas & J. A. González-Calero (Eds.), *Investigación en Educación Matemática XXV* (pp. 49–51). SEIEM.

Muñiz-Rodríguez, L., Ferrando, I., Ramos, P., & Rodríguez-Muñiz, L. J. (2023). La observación de aula como herramienta de desarrollo profesional: el caso del OCDE Global Teaching Insights. *Unión. Revista Iberoamericana de Educación Matemática, 67*, 1–12.

Palop B., Santaengracia J. J., & Rodríguez-Muñiz L. J. (2022). La conceptualización del pensamiento computacional en el currículo LOMLOE de matemáticas. In T. F. Blanco, C. Núñez-García, M. C. Cañadas & J. A. González-Calero (Eds.), *Investigación en Educación Matemática XXV* (p. 623). SEIEM.

Pincheira, N. G., & Alsina, À. (2021). Teachers' Mathematics Knowledge for Teaching Early Algebra: A Systematic Review from the MKT Perspective. *Mathematics, 9*(20), 2590. https://doi.org/10.3390/math9202590

Pinto, E., & Cañadas, M. C. (2021). Generalizations of third and fifth graders within a functional approach to early algebra. *Mathematics Education Research Journal*, *33*(1), 113–134. https://doi.org/10.1007/s13394-019-00300-2

Reid, D. A. (2014). The coherence of enactivism and mathematics education research: A case study. *AVANT. The Journal of the Philosophical-Interdisciplinary Vanguard*, *V*(2), 137–172.

Ribeiro, C. M., Mellone, M., & Jakobsen, A. (2016). Interpretation students' non-standard reasoning: Insights for mathematics teacher education. *For the Learning of Mathematics, 36*(2), 8–13.

Rico, L. (Ed.). (1997). *Bases teóricas del currículo de matemáticas en educación secundaria*. Síntesis.

Rico, L., Lupiáñez, J. L., & Molina, M. (Eds.). (2012). *Análisis didáctico en Educación Matemática.* Comares.

Rodríguez-Muñiz, L. J., Díaz, P., Mier, V., & Alonso, P. (2016). Washback effect of university entrance exams in applied mathematics to social sciences. *PloS One*, *11*(12), e0167544. https://doi.org/10.1371/journal.pone.0167544

Rodríguez-Muñiz, L. J., Aguilar-González, A., Alonso-Castaño, M., García-Honrado, I., Lorenzo-Fernández, E., & Muñiz-Rodríguez, L. (2023). Explorando nuevas estrategias de formación del profesorado de matemáticas: un enfoque ampliado del Lesson Study para el desarrollo profesional en la Escuela Andorrana. *Revista Interuniversitaria de Formación del Profesorado, 98*(37.2), 71–90. https://doi.org/10.47553/rifop.v98i37.2.99131

Rowland, T., Huckstep, P., & Thwaites, A. (2005). Elementary teachers' mathematics subject knowledge: The knowledge quartet and the case of Naomi. *Journal of Mathematics Teacher Education*, *8*, 255–281. https://doi.org/10.1007/s10857-005-0853-5

Ruiz-Hidalgo, J. F., Flores Martínez, P., Ramírez-Uclés, R., & Fernández-Plaza, J. A. (2019). Tareas que desarrollan el sentido matemático en la formación inicial de profesores. *Educación Matemática*, *31*(1), 121–143. https://doi.org/10.24844/EM3101.05

Shimizu, Y., & Vithal, R. (Eds.). (2023). *Mathematics Curriculum Reforms Around the World*. Springer. https://doi.org/10.1007/978-3-031-13548-4

Shulman, L. S. (1986). Those who understand: Knowledge growth in teaching. *Educational researcher*, *15*(2), 4–14. https://doi.org/10.3102/0013189X015002004

Vaccaro, V., Aguilar-González, Á., Bolondi, G., & Rodríguez-Muñiz, L. J. (2022). An exploratory study on the difficulty perceived by primary school teachers on a mathematics INVALSI item. In J. Hodgen, E. Geraniou, G. Bolondi & F. Ferretti (Eds.), *Proceedings of the Twelfth Congress of the European Society for Research in Mathematics Education (CERME12, February 2–5, 2022)*. Free University of Bolzano & ERME.

van den Akker, J. (2003). Curriculum perspectives: An introduction. In J. van den Akker, W. Kuiper & U. Hameyer (Eds.), *Curriculum Landscapes and Trends* (pp. 1–10). Springer. https://doi.org/10.1007/978-94-017-1205-7_1

van den Akker, J. (2013). Curriculum perspectives: An introduction. In T. Plomp & N. Nieveen (Eds.), *Educational Design Research* (pp. 52–71). The Netherlands Institute for Curriculum Development.

Varela, F. J., Thompson, E., & Rosch, E. (1993). *The embodied mind: Cognitive science and human experience* (14 print). MIT Press.