LA NECESIDAD Y LA ESPERANZA DE UN CAMINO INSTITUCIONAL PARA LA FORMACIÓN INICIAL DE PROFESORES DE SECUNDARIA EN ITALIA

The need and the hope of an institutional path for an initial secondary school teacher training in Italy

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Resumen

En esta contribución intentaremos esbozar el estado de la situación italiana con respecto a la formación inicial de profesores de secundaria junto con algunas consideraciones de investigación. En particular hablaremos del conocimiento especializado del docente para la enseñanza de las matemáticas, refiriéndonos al constructo de conocimiento interpretativo en el modelo Conocimiento Especializado del Profesor de Matemáticas. Profundizaremos en las especificidades del contexto italiano en la educación matemática, a partir de algunas coordenadas históricas, mostrando algunas luces y sombras que lo han caracterizado y lo caracterizan. En particular, nos referiremos a algunas experiencias pasadas y presentes de proyectos. Estas reflexiones nos ayudarán a explicar mejor nuestra principal preocupación sobre la situación italiana actual en la que no tenemos ninguna formación inicial institucional de profesores de secundaria (grados 6-13).

Palabras clave: *formación de profesores de matemáticas, relaciones con instituciones, innovaciones curriculares, perspectivas locales.*

Abstract

In this contribution we will try to outline the current Italian situation regarding initial training of secondary teachers, and we will also provide some research considerations. Specifically, we will talk about specialized teacher knowledge for mathematics teaching, referring to the construct of interpretative knowledge in the Mathematics Teacher's Specialized Knowledge model. We will deepen the specificities of the Italian context in mathematics education, starting from some historical coordinates, and pointing out some lights and shadows that have characterized it and that still characterize it. We will refer, in particular, to some past and present project experiences. These reflections will help us to better explain our main concern about the current Italian situation, in which there is no institutional initial training of secondary school teachers (grades 6-13).

Keywords: *mathematics teacher education, relationships with institutions, curricular innovations, local perspectives.*

INTRODUCTION

The current time, troubled by pandemics, wars and natural catastrophes, is not only worrisome, but also obviously challenging for all the people involved in education. The hope for a better world also comes through fostering a good education for all future adult citizens. In this scenario, mathematics education has to consider the current and the future educational needs, and, as we will argue, teachers' professional development plays a crucial role in this difficult challenge.

Looking at the current Italian situation, we can highlight two controversial starting points.

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On the one hand, in Italy the mathematics community took care of teachers' professional development since World War II. Indeed, in 1954 the Italian Mathematical Union Italian nominated an internal permanent commission specifically appointed to deal with issues concerning the teaching of mathematics: the Commission for Mathematical Instruction (Commissione Italiana per l'Insegnamento della Matematica, CIIM, https://umi.dm.unibo.it/materiali-umi-ciim/). The CIIM sinks its roots in the Italian subcommittee born inside ICMI (the International Commission on Mathematical Instruction) in 1908 and then it was converted into a subcommittee of the UMI. From the very beginning, the CIIM debated all the issues related to the teaching and learning of mathematics in Italy, taking also into consideration many positive experiences documented in other countries where proposals were made to the Institutions (such as the Ministry of School and University). Moreover, the CIIM, organizes conferences and events devoted to mathematics teachers, and it supervises an editorial series of publications specialized in mathematics education.

On the other hand, despite the significant activities carried out by the CIIM, currently Italy is one of the few European countries without a national pre-service education program for secondary teachers (grades 6-13).

MATHEMATICS TEACHER EDUCATION

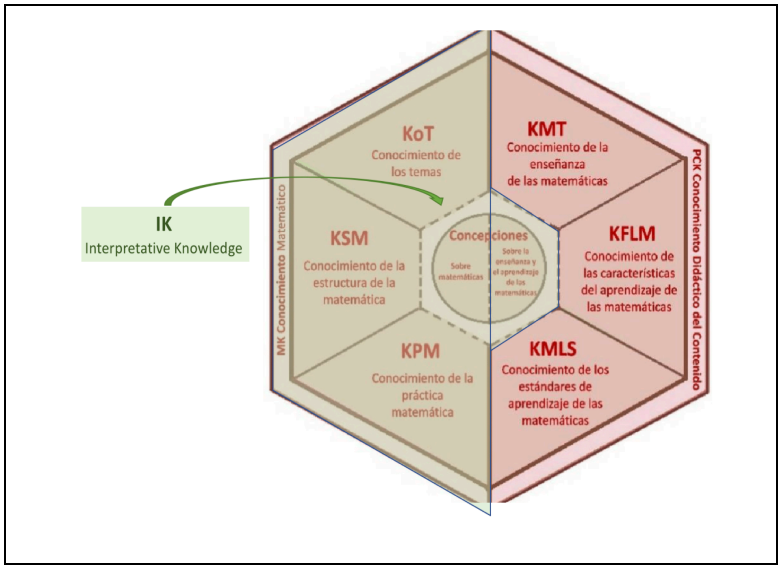
The seminal work of Lee Shulman, in the '80s, brought international attention to the need to look at the knowledge needed to teach a discipline not simply as a disconnected set of disciplinary knowledge and pedagogical knowledge. According to Shulman (1986), to promote effective teaching and learning processes, teachers also need specific disciplinary knowledge for teaching which allows them to have an integrated vision of the disciplinary content and of the issues connected to its teaching (methodologies for that specific disciplinary content, possible difficulties of the students for that specific disciplinary content, etc.). In other words, teachers need what Shulman calls Pedagogical Content Knowledge. Over the years there have been many developments of the research initiated by Shulman, in particular for mathematics education various conceptualizations and models of specialized mathematical knowledge for teaching have been proposed, such as the Mathematical Knowledge for Teaching model developed by a group research model of the University of Michigan (Ball et al., 2008), or the most recent model of Mathematics Teachers' Specialized Knowledge (MTSK) of the Spanish group of the University of Huelva (Carrillo-Yañez et al., 2018) that considers all the mathematical knowledge for teaching as specialized and, places teachers' beliefs at the core of the model.

Starting from the MTSK model, research in mathematics teacher education in Italy has contributed to the field of research on teachers' professional development with the theoretical construct of Interpretative Knowledge (IK). Such a construct actually first emerged from an international collaboration between researchers coming from different countries (see, e.g., Di Martino et al, 2019, Mellone et al. 2020, Ribeiro et al., 2016,); the term IK refers to breadth and depth of teachers' mathematical knowledge that allows them to consider the potential of students' productions, exploiting their ambiguities, intuitions, differences, and errors (etc.) and using them in the classroom as an opportunity for learning. It comprises all the three subdomains of the MK part of the MTSK model, but it also includes in the core the beliefs on Maths and Teaching and Learning of Maths. In fact, it is not only specialized mathematical knowledge "alone" that determines teachers' interpretative attitude towards students, but it is rather the whole of its knowledge and beliefs that predisposes teachers to interpret students' productions (highlighted by the blurry lines in Fig. 1).

Indeed, although research has shown that the opportunity of making mistakes and learning from them is an important aspect of the teaching and learning of mathematics (see e.g., Borasi, 1996), still a great part of mathematics teaching practice in school has mostly focused on students' correct performance of procedural exercises. While the importance of making mistakes, learning from

them, and exploring different strategies in problem-solving activities are crucial aspects in developing a more effective teaching practice, valuing mistakes and building on them is also important from an inclusive perspective. Recent research has shown how the development of a particular form of IK, the Semiotic Interpretative Knowledge, in teachers emerges as a necessary condition for implementing inclusive teaching practices (Asenova et al., 2023).

Figure 1. Interpretative Knowledge in the MTSK model



To develop PTs' IK, a particular type of task called *interpretative tasks* has been used. In such tasks PTs are asked to interpret students' mathematical productions and to design possible feedback and suitable classroom assignments centred on these productions (Mellone et al., 2020). Over the years a particular methodological approach, called the Small group−Collective−Small group (SCS) cycle (Jakobsen et al., 2022), was used to develop PT's IK. During the SCS-cycle approach, the PTs first work on the interpretive task in small groups (of two or three), after which the task is discussed by all the participants in a collective mathematical discussion orchestrated by the educator and, finally, the small groups return to work on the same interpretative task after about one/two months. Central in the SCS-cycle methodology is the discussion orchestrated by the educator, conceived as a generator of new specialized mathematical knowledge for all the group of PTs and the educator him/herself (see e.g., Jakobsen et al., 2022). Mathematics teacher education centred on IK also has the aim of triggering an initial development in the teachers' beliefs on the Teaching and Learning of Maths.

Precisely starting from the role of beliefs in mathematics teachers' training and practice, it is important to stress how the development of knowledge in mathematics teachers is a complex and holistic process and, in this sense, some advice seems to be particularly precious in a country like Italy that is preparing to re-design initial mathematics education teacher programs:

> Mathematics teacher education programs should be deliberately designed in an integrated fashion to support teachers in blending insights from various disciplines including, but not limited to, mathematics, education, and psychology, thereby creating novel styles of knowing that empower teachers to reshape the way they view their own profession. It is reasonable to assume that such styles of knowing develop gradually, rooted in authentic activities and within a community of individuals engaged in inquiry and practice (Scheiner et al., 2019, p. 170).

SOME HISTORICAL COORDINATES ON THE INTERACTIONS BETWEEN INSTITUTIONS AND THE COMMUNITY OF MATH EDUCATORS

Between the late 1800s and early 1900s Federigo Enriques, an important Italian scholar and mathematician, introduced the term “mathematics education” for the first time in Italy (“didattica della matematica” in Italian) to bring to the attention of the mathematicians of the time a concerning phenomenon that was already beginning in those years: the difficulties that students encountered in learning mathematics. The interest in learning difficulties, a phenomenon that we now recognize as being deeply intertwined with that of teaching difficulties, has therefore been one of the generative engines of research in mathematics education. Working on the difficulties to include, but also to integrate differences has been a main direction of many of the efforts of educational research which has led to the development of specially designed methodologies and artifacts, also for the teaching of mathematics, born in Italy and exported all over the world, such as those conceived by Maria Montessori or Emma Castelnuovo (niece of Federigo Enriques).

In the wake of these visionary precursors, since the '80s research in mathematics education has developed and continues to develop innovative proposals that have been validated in the field and supported by scientific research. In this direction it is important to underline the strategic work developed by the CIIM as a connecting hinge between the community of mathematicians with the community of mathematics educators, creating a valuable partnership in the dialogue with the institutions. For example, the CIIM (under the aegis of the Italian Mathematical Union) in collaboration with the Ministry, led the national project “Mathematics for the citizen”, started in 2000 in which, capitalizing on the scientific results in mathematics education, a team of mathematics educators developed materials for a new mathematics curriculum (UMI Curriculum 2001-2003-2004) with suggestions for activities and assessment materials for schools. Here, for the first time in the country, the following fundamental idea of "mathematical laboratory" appeared: the mathematical laboratory is not a physical place different from the classroom, it is rather a structured set of activities aimed at constructing the meanings of mathematical objects. The laboratory, therefore, involves people (students and teachers), structures (classrooms, tools, organization of spaces and times), ideas (projects, educational activity plans, experiments). The construction of meanings, in the mathematical laboratory, is closely linked, on the one hand, to the use of the tools (analog and digital) used in the various activities, on the other, to the interactions between people that develop during the exercise of such activities (Anichini et al., 2023). Afterwards, the CIIM, thanks also to a particularly constructive dialogue with the institutions in those years, developed a mathematics teacher education project, M@t.abel, to disseminate the innovations developed in “Matematica 2001”. Then the CIIM also launched a national in-service mathematics teacher training program, based on a waterfall model, through the construction of communities of practice, centred around the key figure of the *teacher-researcher* (see, e.g., Jaworski & Goodchild, 2006). This project unfortunately came to an end too early because of political instability and the subsequent difficulty of dialogue between research and institutions. Nevertheless, these experiences showed the strategic roles played by the dialogue between the research community and institutions and the need for an agreement between human and economic resources to make things work.

Moreover, these experiences showed that to scale up an innovative waterfall training model is an extremely crucial goal, considering that the mathematics educator community is limited and that it cannot directly carry out alone a large-scale mathematics teacher education project. To guarantee widespread teacher training throughout the territory, an effective approach seems to be that of training groups of trainers (possibly from communities of people already involved in educational efforts).

THE EXPERIENCE OF PERCONTARE PROJECT AND A FIRST ATTEMPT AT SCALING-UP THROUGH A WATERFALL MODEL

In the current Italian panorama there are projects that represent a little oasis in the desert, and that express the vivacity and skills of Italian researchers, and that yet, unfortunately, struggle to get recognition within the well-established political plans. An important example in this direction is the PerContare project. It is a design-based research project that was first funded by a private Foundation, Fondazione per la Scuola della Compagnia di San Paolo, to a team of mathematics educators and psychologists, coordinated by the non-profit organization ASPHI (Avviamento e Sviluppo di Progetti per ridurre l'Handicap, Start-up and Development of Projects to reduce the Handicap) supporting the use of technology for educational inclusion, in Italy from 2011 to 2014 with the goal of designing and experimenting inclusive practices in mathematics education for primary school grades 1 and 2 (children ages 6-8), that would reduce the number of children who develop persistent difficulties in learning mathematics. The online teacher guides developed during the first funding period of the project remained available online for free (and they still are today at https://www.percontare.it) and in a few years over 10,000 teachers created accounts on the platform to access them (today over 30,000 teachers have accounts on the online resource platform). Many of such teachers asked for professional development courses and even more insisted that the teacher guides were "completed" to cover the whole range of primary school (to grade 5; children of 10-11 years of age).

Therefore, the same private funding Foundation decided to fund three more years of design-based research to reach the completion of the teacher guides between 2019 and 2022. During this second phase of the project only the mathematics education team (under the scientific leadership of the second author of this paper) was active, and collaborated again with ASPHI for organizational aspects, the web design and app development. The second objective of the project, during these years was to design and experiment, cyclically, through design-based research, inclusive curricular activities in mathematics (not only arithmetic) for grades 3, 4 and 5. These teacher guides stem from the hypothesis that many students stop participating in mathematical discourse either because they are not able to memorize routines or because they are not willing to do so, because such routines do not allow them to understand the "whys" behind the algorithm procedures (see, for example, Baccaglini-Frank et al., 2023).

PerContare is a design-based research project, which means that it aims both at developing knowledge or theory about how to enact innovative ideas, using design as an essential part of the research, and it develops educational materials that are "effective" with respect to certain design principles and educational goals (Bakker & van Eerde, 2015, Bakker & Smit, 2017). The key design principles (especially for grades 3, 4 and 5) are the following:

- the mathematical notions are not dropped "out of the blue", but discovered, whenever possible through exploratory activities;
- such discoveries take place in the "mathematical laboratory" (see the previous section of this paper);
- the language that is developed in class, initially, is situated, tightly related to the artifact context in which is emerges;
- the role of the teacher is key in promoting the generation and generalization of situated meanings through processes such as: conjecture generation, argumentation, and orchestration of mathematical discussions;
- procedures for solving certain types of mathematical tasks are discovered, where possible, and/or compared to bring out the "hidden" mathematical meanings;

- the focus is on fundamental “big ideas” in mathematics.

These overarching design principles subtend a socio-cultural approach to mathematics education, situated within a Vygotskian line of research that takes into account the role of cultural artifacts in the teaching and learning of mathematics (Bartolini Bussi & Mariotti, 2008).

Studies on the effect of the implementation of the materials involved about 50 teachers in five Italian regions (Baccaglini-Frank & Bartolini Bussi, 2015, Baccaglini-Frank et al., 2023). Both in these studies and in implementing the activities the researchers on the project frequently had a very active role. Moreover, they started noticing from various informal interactions with other teachers that in many classrooms where teachers declared to be “using PerContare”, the activities (and especially the mathematical discussions to be orchestrated by the teachers) were missing some of the key points, that were fundamental, especially from a mathematical point of view. So, through such observations the team concluded that without proper guidance (or previous professional development) from the team, the wealth of activities available online was wasted.

Such an awareness led to attempts by the scientific coordinator of the project at the University of Pisa and by ASPHI to find new funding opportunities for setting up the necessary training of trainers, in what was conceived as a waterfall model project, PerContarePRO (PRO stands for “propagation” of the good teaching practices), that should take a first step in scaling-up an effective use of the research-based teacher guides. The proposing partners found allies in local and national agencies concerned with teachers' professional development and determined to contribute to the new 3-year project: the National Institute for Documentation and Innovation of Educational Research (INDIRE), the Regional School District of Tuscany (Ufficio Scolastico Regionale della Toscana) and the Institute of Research and Educational Experimentation of the Province of Trento (IPRASE). PerContarePRO was recently funded (in January 2023) by a different private foundation, UNISER (a consortium company, linked with the University of Florence -UNI-, and oriented to offer services rendered -SER-), of the Cassa di Risparmio di Pistoia e Pescia.

We conclude this section with a presentation of the main goal and actions planned in PerContarePRO. The main objective is to train a set of educators from all over Italy, taking into account the complexity of knowledge described by the MTSK (Carrillo-Yañez et al., 2018), who will become “expert trainers”; they have been selected from a pool of teachers and educators (some are young researchers or educators involved in afterschool activities and other projects), and by the end of the project they should be able to respond to some of the numerous training requests from schools by proposing ad hoc projects (again taking into account the MTSK to plan actions that strengthen weaker aspects of such knowledge) that include consulting, implementing workshops for teachers and providing support and guidance to local communities of teachers interested in learning how to use the PerContare teacher guides.

The first planned action in training the to-be expert trainers (here trainees) is working together during a series of face-to-face workshops. The second planned action is continued support to the trainees using an online platform where materials used during the workshops will be collected, as well as additional support materials that will be created as the need arises (for example explanatory videos about certain key points or aspects that create difficulties in learning about the teacher guides), and a forum with threads in which exchanges between the researchers and the trainees will be documented. The third action is a set of 8 webinars, open to teachers, university students, school principals that document the progress of the project.

THE URGENCY OF QUALITY MATHEMATICS EDUCATION FOR CITIZENSHIP SKILLS IN INITIAL TRAINING OF SECONDARY SCHOOL TEACHERS

All over the world, there is a shared consensus in recognizing the key role played by mathematics education in forming citizens prepared to face the challenges of our era, in which climate, ecological, political, and economic changes require, among other things, to be able to make decisions based on a scientific basis. In particular, following the emergency caused by the COVID-19 pandemic, the importance of widespread quality literacy and mathematical competence has become even more evident in interpreting numerical data, tables, and graphs and having the tools to make responsible decisions, also based on scientific considerations. Therefore, the need for quality mathematics education for everyone is evident and, in many countries, various indicators seem to show that this goal is still a long way away: from the data provided by standardized international surveys and to the studies showing the incidence of rejection of mathematics among students and adults, to the proliferation of anti-scientific opinions in the national and international media. Institutions around the world, also pressed by international comparative assessment evaluations, are called upon to improve school education through various initiatives such as curriculum reforms and massive teacher professional development programs. These initiatives are not always directly linked or informed by mathematics education research. For example, as mentioned in the introduction, in Italy there is a very critical situation in terms of the initial training and recruitment of secondary mathematics teachers. After some past experiences of different kinds of secondary teacher education, for almost ten years in Italy we have not had any institutional secondary teacher training program for secondary teachers. This situation suggests the falling back onto the naïve idea that the knowing the content (mathematics) automatically implies being able to teach it (well), denying years of research results, such as the ones recalled in the previous section. The Italian situation becomes even more serious when compared to the complexity and constant evolution of the educational objectives that to the challenges that our time has posed to us, such as providing an adequate level of mathematical literacy to every citizen, encouraging the study of STEM disciplines, etc. Furthermore, there is extreme variability in the generations of students to whom the educational paths are addressed (digital natives, the variety and mix of cultural contexts of origin, etc.) and the variety of contexts in which these paths will be developed (distance education, outdoor education, new school architectures).

In such a scenario, therefore, the role of teacher training and enhancement becomes fundamental to share tools that help students manage this complexity. Valuing also means clarifying the training and recruitment steps that are necessary to become a secondary school teacher, in order to intercept, welcome and take care of those who feel a vocation for the teaching profession. This organization is also crucial to avoid creating a culture of the teaching profession as a “cheap” profession: teaching is the key profession in which to invest for the future of our societies. The current lack of clarity, discourages people from choosing this profession, or worse, they choose it as a fallback after other (usually unrelated) unsatisfactory experiences. This is what is happening in Italy where there is a de facto lack of mathematics teachers (Fig. 2).

On the other hand, if there is no initial training for secondary school teachers, there is no coherence in the request for continuous training. Instead, we firmly believe that an institutional initial training for prospective mathematics secondary teachers, supported by researchers in mathematics educators, would guide new teachers into their profession, integrating research results into practice, and hopefully creating in them the need of lifelong learning within community of practice. Indeed, the research underlines the importance of creating communities of practice and inquiry as an essential part of teacher training, i.e. collaborative and co-learning contexts between educators and future teachers in which to develop a reflection on practice and the intertwining of theory and practice (Jaworski & Goodchild, 2006). These communities could also help to relieve the sense of isolation and frustration of mathematics teachers and educators who find themselves pressed by

constant shifts from ever-changing educational needs and requests (see, e.g., Ramploud et al., 2022).

Figure 2. From a well-known Italian newspaper, a headline that appeared last January "Mathematics: the numbers don't add up: schools without teachers"

CONCLUSIVE REMARKS

As mathematics educators, we are concerned about the state of Italy as one of the few, or perhaps the only, country in Europe that currently and for several years does not have an initial training course for secondary school teachers. Without adequate training, novice teachers will most likely offer the same methods and tools that they encountered in their own experience as students, a phenomenon already highlighted over a hundred years ago by Felix Klein (Kilpatrick, 2019): this cyclical process blocks any process of innovation in schools, and it robs the school reality of the innovations produced by education research, which instead should permeate school contexts.

At the end of the past year the previous Italian government published the Law decree D.L. n. 36 of 2022. D.L. concerning a reform that provides for the training of future secondary school teachers through an education path consisting of 60 formative credits. As of today there has not been any implementation decrees or factual planning, that are necessary to convert the D.L. into concrete actions and law.

Our wishes and hope are to have soon an institutional initial training of secondary school teachers developed through a coherent path, in which the contributions of disciplinary, didactic-disciplinary, pedagogical-didactic teachings and reflection on internships are integrated into a dialectical relationship between theory and practice. Such a path should be capable of intercepting and carefully welcoming the vocation of future teachers and which brings into play the skills, knowledge, and points of view of the world of schools and universities in planning and its development. The belief is that such a path and the synergy between school and university are necessary to provide future teachers with qualified entry-level professionalism, adequate for the complex educational challenges posed by our society.

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