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Theory of Didactical Situations: Theoretical Rereading from the Perspective of Inclusive Playful Mathematics Education

A Teoria das Situações Didáticas: uma releitura teórica sob a perspectiva da Educação Matemática Lúdica Inclusiva

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Abstract

This article explains a rereading of the theory of didactical situations (TDS) from the perspectives of playfulness and inclusive education of students with specific educational needs (SEN) in the teaching and learning processes of mathematics. To do so, a theoretical discussion on TDS, playfulness, and inclusive education was held to create a reflection on the possible articulations to be established among them. Notably, TDS was developed by Guy Brousseau (1986) and initially conceives the didactical situation around three poles: teacher, student, and knowledge. Additionally, the teacher and students are considered players who play with the content knowledge to be institutionalized. This aspect allows articulation between TDS and playfulness. In the context of didactical situations, there also exists real or fictional construction of a *milieu*, wherein the student acts autonomously to build knowledge. Considering this and the inclusion process, the teacher can organize a *milieu*, considering accessibility in education, to work with every student's specificities, thus breaking the homogeneity present in educational spaces. Therefore, basing mathematics classes on a theory that makes up the universe of mathematics didactics and was not conceived as playfulness and inclusive perspective but that from the assumption of accessibility, can contribute to the inclusion of all students, regardless of the SEN they may present. The teacher will promote the development of autonomy and critical training of students, in addition to institutionalizing content knowledge and providing the *status* of knowledge.

Keywords: Mathematics Education. Mathematics Didactics. Playfulness. Inclusive Education. Specific Educational Needs. Accessibility.

Resumo

Neste artigo, temos por objetivo explicitar uma releitura da Teoria das Situações Didáticas (TSD) sob a perspectiva lúdica e inclusiva de estudantes com Necessidades Educacionais Específicas (NEE) nos processos de ensino e aprendizagem da Matemática. Para isto, realizamos uma discussão teórica sobre: TSD, Ludicidade e Educação Inclusiva, que possibilitou refletir sobre as articulações possíveis de serem estabelecidas entre elas. Vale ressaltar que a TSD foi desenvolvida por Guy Brousseau (1986) e concebe inicialmente, a situação didática em torno de três polos, isto é, o professor, o estudante e o saber. Sendo que o professor e os estudantes são considerados jogadores que brincam com o conhecimento a ser institucionalizado. Esse aspecto permite uma articulação da TSD com a Ludicidade. No âmbito da situação didática há ainda a construção real ou ficcional de um milieu, no qual o estudante atua de maneira autônoma, com o intuito de construir o saber. Neste contexto, ressaltamos que perante o processo de inclusão, o professor pode organizar um milieu, considerando à acessibilidade na educação, para com isto, atender a todos os seus estudantes, a luz das especificidades de cada um deles, rompendo-se assim, com a lógica dos espaços educacionais homogeneizantes. Destarte, ao fundamentar as aulas de Matemática em uma teoria que compõe o universo da Didática da Matemática e que a princípio não foi pensada em uma perspectiva lúdica e inclusiva, mas que a partir do pressuposto da acessibilidade pode vir a contribuir com a inclusão de todos os estudantes, independente da NEE que este possa vir a ter, o professor estará propiciando o desenvolvimento da autonomia e da formação crítica dos estudantes, além de estar institucionalizando conhecimentos e conferindo-lhes o status de saber.

Palavras-chave: *Educação Matemática. Didática da Matemática. Ludicidade. Educação Inclusiva. Necessidades Educacionais Específicas. Acessibilidade.*

1 Introduction

Education has the function of renewing the world, transforming it into a common good, wherein everyone can act with freedom and autonomy. It recognizes and values everyone as subjects of law, seeing everyone as equal, in the human sense but with every singularity that characterizes and differentiates them. Such an understanding may enable people to act in society and the environment in which they live with equal opportunities.

For this to become effective, it is necessary to understand that education is everyone's responsibility, be it teachers, researchers, parents, or guardians. It is important to strengthen the opportunities undertaken by the current and future generations. Thus, one should mind and foster educational theories that may favor criticality, autonomy, recognition, and respect for others, with singularities to be respected and considered in the processes of teaching and learning.

Among the theories of the mathematics didactics, the theory of didactical situations (TDS) values "the content

knowledge held by students and their involvement in the construction of mathematical knowledge" as well as "the teacher's work which primary consists of creation of adequate conditions for the student to appropriate specific mathematical content" (Freitas, 2012, p.78). In this context, the importance of students' and teachers' roles in teaching and learning mathematics is explained through this theory.

Notably, this theory was not developed in contexts that necessarily considered aspects related to the inclusion of students with specific educational needs (SEN); however, certain features might contribute to encourage the autonomy and participation of students when doing the activity, for example, by worrying about "the *student's existential promotion* through mathematical knowledge" and "*methodological procedures* wherein the teacher does not provide the answers, effectively involving the student in the elaboration of content knowledge" (Freitas, 2016, p.106-107).

Such aspects may contribute to the inclusion of students with SEN in the processes of teaching and learning mathematics, provided the accessibility process is considered, i.e., the methodological tools used are accessible to all students. There is no way to propose an activity that is "ideal" for a student, as it is important to take into account the singularities of the agents that make up the heterogeneous classroom.

In light of the aforecited, some concerns can be resolved, given the context of TDS, playfulness and inclusion. In light of this, we highlight the following: what kind of articulations can be established between TDS, playfulness, and inclusive education? What aspects of this articulation can contribute to the teaching of mathematics? Thinking about effectiveness the of the inclusion process, how is it possible to use TDS in an inclusive perspective, considering this theory was not initially developed for the inclusion of various SEN in a regular classroom?

The intention hereby is not to answer all the above mentioned questions, but they may intrigue other researchers and teachers interested in the theme of this study, leading them to understand TDS in the explicit articulations and other theories of mathematics didactics not developed in an inclusive perspective as principle. Therefore, this study explains a rereading of TDS from the perspectives of playfulness and inclusive education of students with SEN in the teaching and learning processes of mathematics. For the writing of this theoretical article, and consequently a rereading of the Theory of Didactic Situations, based on Brousseau's (1980, 1986, 1990, 1997, 1998, 2002, 2008) main research.

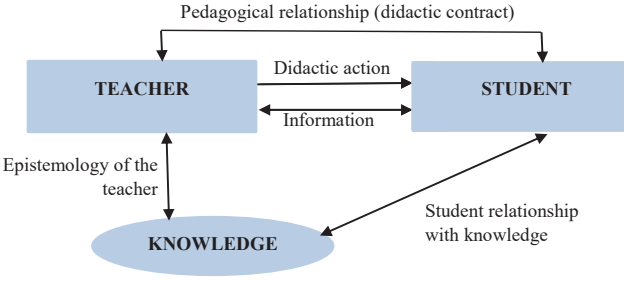
This theoretical and exploratory article is divided into structured sections to achieve the proposed objective. They are (i) Introduction section wherein the theme is contextualized and questions and objectives are presented to guide the writing of this text; (ii) Development sections to theoretically discuss TDS, playfulness, and inclusive education; iii) Section to present the rereading of TDS in light of playfulness and inclusive education; and (iv) final considerations.

2 Theory of Didactical Situations

TDS was developed by Guy Brousseau (1986) based on constructivism that originated from Jean Piaget's theory of genetic epistemology. TDS is one of the theoretical foundations of mathematics didactics, which were developed in France in the 1970s, when the education reform, better known as the modern mathematics movement, took place. It established an approximation between basic education mathematics and higher education mathematics (Valente, 2012).

This theory initially conceives the didactical situation around three poles, i.e., teacher, student and knowledge. This is because the teacher and students are considered players who play with the knowledge to be learned. Notably, within the scope of TDS, Brousseau (1997, p.97) differentiates knowledge from content knowledge, considering that "knowledge is institutionalized content knowledge." Moreover, the passage from one *status* to another is explained by the didactic relationships that involve them. Thus, the *stricto sensu* system and its interactions are expressed in Figure 1:

Figura 1 - Representation of the *stricto sensu* didactic system



Source: Adapted from Brousseau (1997).

First, the relationship between the teacher and knowledge will be addressed, named in Figure 1 as the epistemology of the teacher, which involves the aspects that go through their formation, social function, and teaching experience. Thus, the teacher's work, based on this theory, to a certain extent, is the opposite of the researcher, i.e., the teacher produces new context and personalization of content knowledge conceived by students (Brousseau, 1997). Therefore, the teacher is responsible for providing conditions that enable students to appropriate the studied knowledge and reflect on "how to respond with the help of previous knowledge, how to understand and construct new knowledge, how to 'apply' previous lessons, how to recognize issues such as learning, guessing, solving, etc." (Brousseau, 2002, p. 35).

As a result, the situations presented by the teacher may allow students to raise questions and undertake them search for solutions having as reference the knowledge already built in stages prior to schooling. In addition, the teacher must know when to intervene, assuring that it happens neither early nor late. Such intervention may encourage students and take the form of questions and validate the knowledge that students built with peers.

In the relationship between the student and knowledge (Figure 1), Brousseau (1997) emphasized that the student's work should resemble the scientific activity developed by the mathematician. Since understanding mathematical knowledge is not just learning definitions and theorems, it is necessary to master this knowledge and apply it to solve problems proposed to them. When carrying out a faithful reproduction of a scientific activity, students must produce, formulate, prove, and build concepts and present them to others so that they recognize, evaluate, and approve or disprove their veracity (Brousseau, 2002).

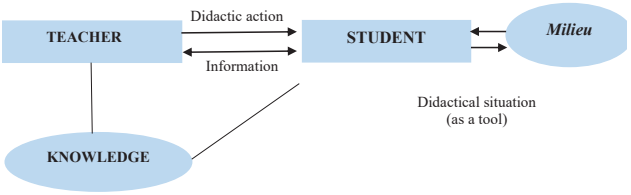
The interaction between the teacher and students, as shown in Figure 1, involves the pedagogical relationship established in the classroom. In this interaction, the teacher is responsible for didactic action, which can also be named as didactic intention. Accordingly, the teacher's pedagogical work can be developed to change intentions into didactic intentions, considering "an intention becomes didactic if, and only if, one of the subjects shown the intention to modify the other's knowledge system" (Brousseau, 2008, p. 53). In addition, information is always shared between teachers and students, considering students will report to the teacher regarding the progress in solving the proposed situation and the teacher will invite and encourage students to accept the problems.

Additionally, this pedagogical relationship between the teacher and the student permeates what Brousseau (1980) named didactic contract, which means the specific behavior of the teacher expected by the students and behavior of students expected by the teacher. Brousseau (1980) believes that the didactic contract may prevent or promote students' access to content knowledge; the didactic contract also reveals every agent's responsibilities in relation to the teaching and learning process.

The didactic contract highlights the epistemology of the teacher and the student in relation to mathematical knowledge and use in the social context in which they are inserted. Brousseau (1986, p. 50) mentioned that the "didactic contract is the rule of the game and the strategy of the didactical situation. This implies the way the teacher must behave in a given situation. However, the evolution of the situation modifies the contract allowing the result of new situations." Thus, the didactic contract depends on different teaching and learning contexts.

Analyzing these contexts, TDS shows that it is not merely limited to the actions of teachers and students, given that the teacher "creates in a real or fictional way another 'milieu' in which the student acts autonomously" (Brousseau, 1997, p. 20). Therefore, TDS's object of study is the didactical situation, based on the interaction between teacher, knowledge, student, and the *milieu*, according to Figure 2, thus opposing "the classical didactic form, centered on teaching with emphasis on the propagation of systematized contents, including the axiomatic form" (Freitas, 2016, p.78). This is because it considers that TDS revokes a more autonomous position on the part of students, resembling the attitude of a mathematician in the construction of new knowledge.

Figure 2 - Representation of a didactic system



Source: Adapted from Brousseau (1997).

In this context, the teaching and learning process encompasses aspects that are beyond the poles: teacher, student, and knowledge, understanding that there are other aspects present in the *milieu* that were thought and planned by the teacher and that may hinder or contribute to didactic intentions and modifications and expansions of students' content knowledge framework.

Figure 2 highlights that students directly interact with the *milieu* influence each other, with the didactical situation as a tool, thus contributing to learning. Thus, it is necessary to understand the didactical situation in a broader perspective, which has a *milieu* planned for a given purpose. From the perspective of Brousseau (1986, p. 8):

> A didactical situation is a set of explicit or implicit relationships established between a student or a group of students, in a certain *milieu*, possibly comprising tools and objects, and an educational system (the teacher) to enable students to access built or under-construction knowledge.

Hence, there will always be a didactical situation when the teacher has a didactic intention in the construction of new content knowledge with the students and, for this, the *milieu* will be prepared with the tools and objects that might contribute to the construction of knowledge to be developed in an autonomous way on the part of students.

To explore the development of teaching and learning processes and construction of knowledge by students, Brousseau (1997) proposed that TDS could observe and decompose these processes into five distinct phases, namely, devolution, action, formulation, validation, and institutionalization, which are articulated among themselves and may happen at varying and different times.

Action, formulation, and validation characterize an *a-didactical situation* wherein students' attitudes, at this moment, may resemble the attitude of a researcher, i.e., they will have more autonomous and investigative actions. Devolution and institutionalization are *didactical situations* that directly involve the teacher's attitude, representing the most active and acting figure in these moments. Notably, the *a-didactical situation* is an essential part of the *didactical situation*.

To foster learning, it is necessary that students accept the problem as being their responsibility, and this process, within the scope of TDS, is named devolution. Brousseau

(2002) compared this situation with a game chosen by the teacher so that the student can interact with the *milieu* and, therefore, accept to experience the stages of the *a-didactical situation.* Thus, in return, students are expected to initially develop a basic strategy to start the game, which will allow the assessment of the problem and the rules that make up the game. In addition, there will always be a response *or feedback* provided by the situation that is present in the *milieu,* thus allowing students to reflect on the strategies that will enable them to "win" the game (Brousseau, 2002).

Consequently, to succeed in this knowledge building game, the teacher will stimulate students and refuse to intervene in the situation as a supplier of content knowledge, so that students can accept it and act autonomously in the process of teaching and learning. In this situation, the intention to teach disappears, implying that the teacher does not reveal to students the intention of presenting a new mathematical knowledge. However, this situation was planned by the teacher, trying to enable students to appropriate the new content knowledge, doing this from careful choices of situations that may arouse the desire to develop them.

As discussed throughout this section, TDS is compared with a game, wherein the teacher and students are players who play with knowledge. In addition, the didactical situation has rules that are conceived from the didactic contract. In addition, after winning the game, the prize is related to the learning provided by the situation to students. In the next section, the focus will be the understanding of aspects linked to playfulness and the game as a playful expression.

3 Playfulness and the Game in the Construction of Knowledge

Playfulness comprises a concept under constant construction. In common sense, it might be associated with activities such as games, diversion, and other forms of leisure. However, its understanding is broader and linked to a playful experience, which is internal to the subject and "integrates the emotional, physical, and mental extents" (Bacelar, 2009, p.30). Moreover, playfulness depends on external motivation, namely, playful activities. These activities, according to Bacelar (2009), may be observed and described by another person while they are performed.

The playful experiences might be performed either individually or in groups and may vary depending on the individuals who perform them; however, what characterizes the playful activity as being a playful experience are the feelings of the subjects who experience it, since "the playful experience (= playfulness), which is an internal experience to the subject, can only be perceived and expressed by the subject who experiences it" (Luckesi, 2014, p.17).

In the meantime, the notion of playfulness is complex and subjective, since it directly depends on the relationship established among the subject, the object, and the environment in which the activity is inserted. To exemplify and differentiate playfulness and playful activities, Bacelar (2009, p.30) mentioned that playing a circle game is a playful activity; playful experience or playfulness could be identified through the "states of completeness, fullness, of pleasure with which the individual makes contact while playing a circle game". Thus, although several people may be on the circle game, the experience is different for each of them, a fact that allows us to infer that one activity can be playful for one person and not for another.

Nevertheless, Macedo, Petty & Passos (2005) pointed out that some features make it possible to characterize a playful experience in the process of teaching and learning, namely, having functional pleasure, being challenging, creating possibilities or disposing of them, having a symbolic dimension and, finally, expressing in a constructive or relational way.

Games are playful activities that may arouse functional pleasure, be challenging, create possibilities, forge association with symbolic, and express a constructive and relational way, because according to Huizinga (2017, p.1), "it is in the game and by the game that civilization arises and develops". In this conception, the game comprises a cultural phenomenon and the species *Homo sapiens* may then be named *Homo ludens*.

It is also a complex task to try to define what a game is, given that "the game is a function of life, but it cannot be precisely defined in logical, biological, or aesthetic terms" (Huizinga, 2017, p.10). However, it is possible to determine its characteristics:

> we could consider it a free, conscious activity taken as 'non-serious' and external to the usual life, but that at the same time absorbs the player in an intense and total way. It is an activity disconnected from any material interest, with which no profit can be obtained, practiced within its own spatial and temporal limits, according to a certain order and certain rules. It promotes the formation of social groups with a tendency to surround themselves with secrecy and to underline their difference from the rest of the world through disguises or other similar means (Huizinga, 2017, p.16, bold added).

Therefore, the game is constituted as a free activity, wherein the subject plays to get the pleasure and joy the activity is providing. In addition, it is constituted as a non-serious activity, implying that, connected to laughter, noise, and imagination. However, the person may be committed to the game, which makes the game a serious activity. Accordingly, the game as a non-serious activity, in the perspective presented by Huizinga (2017), is opposed to the work, the latter being considered a serious activity. Connected to this, the game is also far from everyday activities, considering it goes through the player's imagination and may capture all the attention, leading to another reality.

The interest related to the game is linked to the fascination and pleasure the activity induces; it is played with the feeling of uncertainty, joy, and quest to win the game. However, despite the desire to win, everyone respects the implicit or

explicit rules and orders. If that does not happen, the player is termed "spoilsport" (Huizinga, 2017, p.14) and ceases to be a part of to the social group that the game built.

Considering that there is a winner in the game, it is pointed out that games could be classified as collaborative and competitive. In a collaborative game, players plays with each other, while in a competitive game, players play against each other. In this context, Brotto (1999, p.35) differentiated the situations of collaboration and competition by mentioning that "collaboration is a process wherein the objectives are common and actions are beneficial to all. Competition is a process wherein the objectives are mutually exclusive and actions are beneficial only to some".

Nonetheless, the player in a collaborative situation realizes that to achieve the goals, it is necessary that actions are performed by all the involved members, who are more sensitive to each other's requests, help each other more frequently, and there is a greater homogeneity in the number of contributions and participation. Furthermore, in the competitive situation, players realize that it is not possible to achieve the goals of the other players; moreover, all those involved in the situation are less sensitive to requests, help each other less frequently, and there is a lower homogeneity in the amount of contributions and participation (Brotto, 1999).

For Brotto (1999), competition and collaboration are antagonistic processes; however, its borders are tenuous due to the fact that one can witness occasions wherein a competition includes moments of collaboration and collaboration becomes competitive. Consequently, when proposing a playful activity, in an educational context, attention must be given to such borders, even more in the collaborative aspect, considering people are often immersed in numerous competitive contexts.

Summarizing, it is argued that the playful activities, expressed by competitive or collaborative games, may arouse a playful experience in the most different stages of life and clarify the various particularities of the contemporary subject. In addition, the use of playfulness for the construction of content knowledge can be presented as an effective contribution by enabling the experience of moments of pleasure, joy, and enthusiasm, which have the potential to contribute to overcoming obstacles, self-discovery, assimilation, and greater interaction with the world, wherein the construction of content knowledge becomes a challenging adventure.

Thus, we believe that playful activities experienced in the context of mathematics didactics, and more specifically, within the scope of TDS, may favor the process of inclusion in mathematics teaching. The next section includes understanding inclusion from the principle of equity of opportunity and acknowledgment of difference in equal rights.

4 Inclusive Education as a Transversal Project

Inclusive education is not a recent project, and one of the milestones for the implementation of inclusion is the Salamanca Statement, elaborated during the World Conference on Special Education (Unesco, 1994). Since then, especially with the advent of the 21st century, considerable research and discussions on inclusion have been carried out to recognize that everyone has the right to be included in the various social spaces they attend. Such inclusion, from the perspective of Slee (2011), may favor the right of citizens to be socially, intellectually, culturally, and personally recognized to encourage individuals to be able to engage in the construction and transformation of society.

It is underlined that the struggle for school and, therefore, social inclusion occurs daily; however, this challenge cannot be faced in a lonely and individual way by the teacher in the context of classroom. On the contrary, the teacher may pass, according to Mantoan (2008, p. 37), through an "educational paradigm shift, which generates a reorganization of school practices: planning, class formation, curriculum, evaluation, management of the educational process." Finally, this commitment aims at involving parents, teachers, students, and researchers for ensuring, researching, and recognizing inclusion as an emerging need to be implemented in the field of education, thus offering all students the opportunities to autonomously and effectively act in the social and educational environment. Therefore,

> Inclusion is an innovation that involves an effort to update and restructure the nature of most of our schools. This happens as educational institutions understand that the difficulties of some students are not only theirs, but largely result from the way teaching is taught and how learning is conceived and evaluated (Mantoan, 2015, p.62).

Understanding the heterogeneity present in educational institutions and, more specifically, recognizing the specific needs of students contributes to the need to overcome the use of specific teaching methodologies for a particular SEN, presupposing the recreation of a model which has as its guiding axis the teaching for all; the pedagogical reorganization of schools to give way to cooperation, dialogue, solidarity, and creativity by all actors involved in the teaching and learning process; and to guarantee students respect for time and freedom to learn. Likewise, providing teachers continuous training may enhance and encourage inclusive professional performance.

When addressing the need to use teaching methodologies that may favor the process of educational inclusion, it is here considered the need to present methodologies accessible to all students. To do so, it is necessary to recognize the inclusion under the locus of the right to difference in equal rights, because "we have the right to be equal when the difference makes us depreciated; we have the right to be different when equality mischaracterizes us" (Santos, 2006, p.462). In this perspective, inclusion presupposes the overcoming of equality and difference, as contrasting aspects, so that equality in difference can be recognized and enforced.

As an example of this recognition, Mantoan (2015)

highlighted that when considering a blind student as the only one to use a computer in a regular classroom, this student will not be differentiated or excluded if this resource allows independent and autonomous classroom participation. In addition, this blind student has the right to study the contents in Braille or audio. Such differentiation offers the possibility to participate in the classes. Another example refers to the possibility that students with reduced mobility can choose, within the classroom, the place they wish to occupy, which gives them some autonomy to decide and not to be subject to other people's impositions.

As a consequence, inclusion guided by equality in difference presupposes overcoming attitudes of imposition, exclusion, segregation, and discrimination and requires the creation of accessibility mechanisms, understanding it as the:

> […] possibility and condition of reach for the safe and autonomous use of spaces, furniture, urban equipment, buildings, transportation, information and communication, including their systems and technologies, as well as other services and facilities which are open to the public, public or private use, both in urban and rural areas, per person with disabilities or with reduced mobility (Brasil, 2015, p.1).

On that account, it is necessary to build a society based on the acceptance of difference, which may overcome exclusion, segregation, or restriction, enabling the effective right of the person with SEN. This construction is initiated in the educational framework, and according to Arendt (2006), "the child is introduced to the world for the first time through school." Thus, it is necessary and urgent to face the challenges that concern effective inclusion and are constantly experienced in the school context. Among these adversities to be overcome are conservatism and protectionism and paternal attitudes, which seek to justify the inability to include all students in school, connected to the lack of possibility to provide conditions to learn in coexistence with difference, to value others in their diversity and respect the worldly knowledge that each person has (Mantoan, 2008).

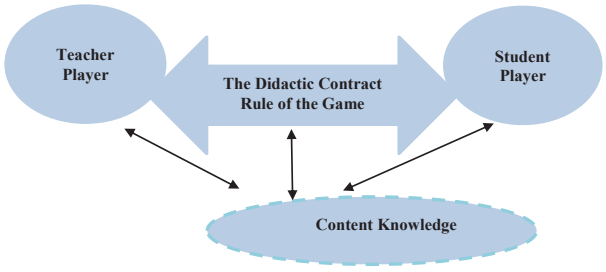
When considering TDS as a possibility that contributes to greater autonomy and when comparing the work developed by the student with the researcher's in the search for the construction of new content knowledge, it is hereby argued that this theory may bring a new look at education as a right of all and acknowledgment of equality in difference, toward inclusion that respects the uniqueness of each person and their way of learning. In view of this, the next section discusses a theoretical link between the three approaches presented in this article, aiming at the exposition of the rereading of TDS under the theoretical lenses of playfulness and inclusive education.

5 The Theoretical Link: the Playful and Inclusive Theory of Didactical Situations

When modeling the didactical situation and comparing it to a game, Brousseau (2002) allowed us to consider the teacher and the student as players, the didactic contract as the rule of the game, which will determine the strategies to be developed by students, and the teacher and the "content knowledge as being expressed by the rules of the a-didactical situation and by the strategies" (Brousseau, 2002, p. 31) that the student developed with the teacher's mediation.

Subsequently, knowledge will culminate in this relationship and the *status* that the institutionalization situation has given it. This includes institutionalization by the teacher in view of the content knowledge that the student elaborated. In Figure 3, the aspects that compose and approximate the didactical situation to a game are represented.

Figura 3 - The game of the didactical situation



Source - Adapted from Nery (2021).

Analyzing the relationship between the teacher (player) and knowledge, we reassure that knowledge permeates teaching epistemology and the process of formation and construction of professional identity. Thus, knowledge inspires the teacher's actions and this directly influences the way it is presented to the student. In doing so, the role of the teacher is to organize students' games. For this, the teacher plays with the knowledge that will be built at the end of the didactical situation. The teacher's game, therefore, "defines and gives meaning to the student's game and content knowledge" (Brousseau, 2002, p.56).

In addition, Figure 3 analyzes the relationship of the student (player) with content knowledge. Initially, the student will be able to elaborate strategies and educational knowledge without understanding that, among the possible results of the game, or among the content knowledge that will be built, some were previously determined by the teacher, with some belonging to the didactical situation and others not belonging. For this reason, as represented in Figure 3, the pole of content knowledge is smaller than the pole of knowledge, but they have intersections, considering some content knowledge raised in the situation will be institutionalized by the teacher and receive the *status* of knowledge.

As the student (player) understands the rules of the game (didactic contract), he/she will change strategies, producing knowledge that sometimes allows the design of new strategies and the elaboration or reformulation of the rules of the game. With this, knowledge also receives influences from the rules of the game (didactic contract) and transforms them.

In addition to the modeling of the didactical situation as a game, another aspect that allows us to bring TDS closer to playfulness refers to the teacher inviting the student and the

acceptance in solving the presented problem. Even if implicitly, the student will undertake responsibility for the construction of content knowledge, an attitude that will be expressed by the actions during the experience of the *a-didactical situation*. In this context, the *a-didactical situation* becomes free, conscious, and may intensively absorb the student (player) (Huizinga, 2017), awakening feelings of emotion and motivation (Brousseau, 2002) present in playful experiences.

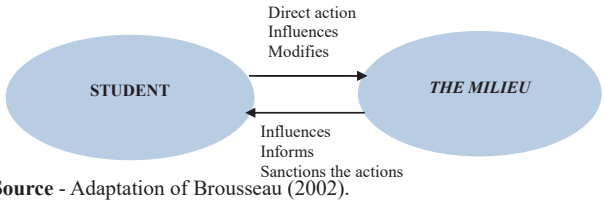
Notably, the student (player) will not be alone and will work with peers and distinguished players who also experience the *a-didactic*al situation and will present formulations and try to validate content knowledge using language. This moment of formulation and validation involves cooperation among students (players), thus resembling a collaborative game wherein objectives are common and actions (validation and formulation) can be beneficial to everyone (Brotto, 1999).

In the game of didactical situations, learning is the prize for winning the game and establishing successful strategies (Brousseau, 2002). Although, to make learning successful, it is necessary that students undergo a process of "adaptation" to a *milieu*, defined by Brousseau (1990), as the set of external conditions within which the human being behaves and grows, besides being a source of contradictions, difficulties, and imbalances; consequently, learning will be manifested by the new responses that students present throughout the experience of the situation.

The role of the teacher in this process is of paramount importance, for he/she organizes the *milieu* that will be the source of students' learning. Thus, according to Brousseau (1990), the *milieu*, for the teacher, is a non-didactical system, in that knowledge must be introduced by the student, constituting as an environment wherein students are immersed by the teacher and pass through the stages of *a-didactical situation*, i.e., action, formulation, and validation and build the content knowledge to be spontaneously institutionalized by the adaptation process.

When considering the adaptation process, Brousseau (2002) pointed out that the student is influenced by the *milieu* and tries to cancel the sanctions expressed and modifies it. Thus, the adaptation involves the *didactical situation* and, therefore, the *milieu*, which can be understood as a "receiving and/or transmitter system with which the player [student] exchanges messages" (Brousseau, 1986, p. 104). As a result, there is a two-way street, inasmuch the student influences when acting on the *milieu*, trying to adapt to it, the latter sanctions or informs the student the result of his/her actions. Such a situation can be represented as Figure 4:

Figure 4 - The student subsystem with the *milieu*



Source - Adaptation of Brousseau (2002).

In this scheme, the content knowledge to be generated, as a result of these influences in the subsystem composed by the student and *milieu*, is expressed in the form of stimulus-response. However, we are not referring to any *milieu*; this was thought out and planned by the teacher, considering the students, their specific needs, the content knowledge to be emerged, and the knowledge that will be institutionalized. Therefore, it is assured that this *milieu* is accessible to students with SEN.

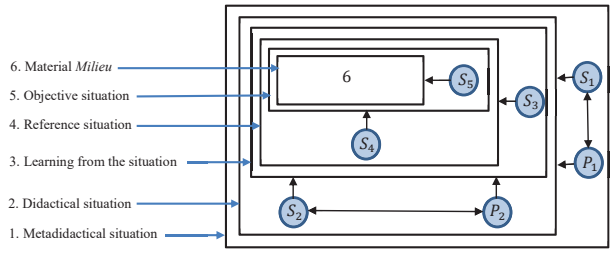
Therefrom, in the face of the *milieu*, it is possible to affirm that this should be considers based on the assumptions of accessibility, which allows to eliminate barriers and promote inclusion (Brazil, 2015). Thus, accessibility is linked to the concept of universal design, understanding that it has as assumption the "equipping of the possibilities of use, flexibility in use, simple and intuitive use, information capture, error tolerance, minimal physical effort, dimensioning of spaces for access, use and interaction" (Brazilian Association of Technical Standards, 2015, p. 4). Universal design can be considered a concept of environment (*milieu*) designed for students to act autonomously, without requiring individualized adaptations, which could differentiate to exclude a particular student from the proposed didactical situation.

What characterizes universal design, according to Burgstahler (2009), is that unlike an accommodation or material presented to a specific person with some disability, universal design benefits all people, including those who do not need that accommodation or material directly accessible, but access allows one to open horizons and participate together with others with equity of opportunity. In the educational sphere:

> The structure proposed by the universal design presupposes diversity and work with identity and difference in its constitution. Methodology, communication process and instructional material thought about the referred structure need to be applied to the whole classroom, and should be contemplated in the methodology, communication process and instructional material, elements which are proper to the principles of diversity, identity and difference, and not of homogeneity and homogenizing spaces, these latter products of social construction (Camargo, 2017, p.3-4).

On this account, TDS allows the teacher to organize an accessible *milieu*, built on the basis of universal design, designed to meet all students and, in addition, to take into account the identity of each one of them, thus breaking with the logic of educational spaces as being homogenizing. Hence, the *milieu* of the didactical situation should be based on the material and methodological aspects that may gradually increase the autonomy of students in the learning process, according to the structure of the didactic *milieu* and the attitudes of the teacher and students, as represented in Figure 5.

Figure 5 The structure of didactic *milieu*



Source: Brousseau (2002, p.248).

To be clear, P represents the teacher and S are the students. is a universal subject, a student who looks at the teaching situation from outside and establishes a dialogue with teacher , who prepares and presents the lesson; is a generic student, who analyzes his/her own learning situation and dialogues with the teacher , who teaches how to act and observes him/her; is a subject in the condition of an apprentice, and he is confronted with the situation that no longer has the explicit didactic interference of the teacher; is the subject as the main actor of his/her learning process (autonomous attitude); and is an objective sector, wherein students dialogue and validate the knowledge that emerges from the material *milieu*.

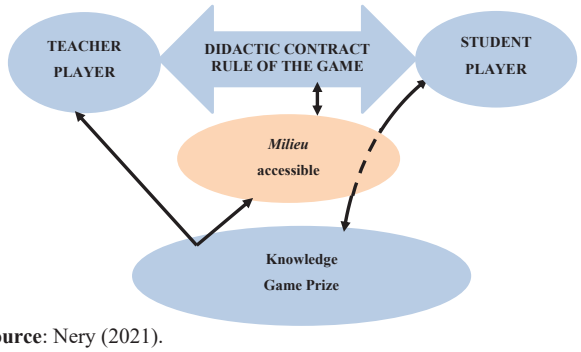
While students go deeper into the didactical situation and interact with an accessible *milieu*, they present greater autonomy, in view of the teaching and learning process of mathematical concepts that are the objects of the study. Thus, it is argued that TDS may be used with playful and inclusive imprint, provided its modeling is respected as a game that captures and involves students in such a way that the didactical situation presents itself as a playful experience and allows all students who accept to participate to act, formulate, validate, and institutionalize content knowledge autonomously and with equity of opportunities.

6 Final Considerations

In view of the objective of this study, namely, to explain a rereading of TDS from the perspectives of playfulness and inclusive education of students with SEN in the teaching and learning processes of mathematics, we highlight that TDS is a theory that, at first, was not considered in the context of inclusive education. However, in the current educational context, it is urgent and vital that the process of inclusion is considered, proposed, and implemented and, more emphatically, in mathematics teaching, which for many years was considered devoted for a few people.

Subsequently, considering the studies by Brousseau (1980, 1986, 1990, 1997, 1998, 2002, 2008), which presented the contributions that TDS may point to the processes of teaching and learning mathematics, we decided to analyze TDS and perform a rereading, thinking of elucidating the contributions that this theory should make to the process of inclusion in mathematics teaching. To this extent, at the end of this article, a representation of the articulations that TDS has with playfulness and how to inclusively use it will be explained in Figure 6:

Figure 6 - TDS in a playful and inclusive context



Source: Nery (2021).

In our rereading, we consider the *milieu* as being the core of rethinking TDS in a playful and inclusive perspective, because to enable inclusion, it is necessary that the teacher, as a player who plays with scientific knowledge, may build a *milieu* that takes into account the student's SEN and understands him/her as a historical subject who, to access knowledge, i.e., win the game and build institutionalized knowledge, needs to experience an accessible *milieu*.

On this account, to experience the *a-didactical situation*, i.e., the moments of action, formulation, and validation and, consequently, the institutionalization of new knowledge, the student will have a great influence of the accessible *milieu* that was considered in a playful way and that considered the SEN. Therefore, we can mention that to build knowledge, the object of the didactical situation, the student, counts on the mediation of the *milieu* inserted by the teacher.

In addition to these interactions among the teacher, the student, the *milieu* and knowledge, we highlight the didactic contract, which, as a rule of the game, influences the relationship established between the teacher and the student. Therefore, it also interferes with the *milieu*, since it can guide and indicate its composition and the actions, formulations, and validations carried out by students during the *a-didactical situation*. However, this relationship is constituted as a two-dimensional, wherein the accessible *milieu* also influences the didactic contract and the rules implicitly or explicitly established by the teacher and the student, which may be altered, disregarded, or even increased in the course of the *a-didactical situation*.

To present TDS as an inclusive theory, it is essential to favor students' autonomy, considering the student, not as an ideal agent, but as a human being with singularities, and SEN that characterize, differentiate, and make him a unique being before the teaching processes and learning, but based on the knowledge built in educational environments, thus effectively acting in the social environment in which he/she lives.

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