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Comparison of Interdisciplinary Connections between Mathematics and other Subjects through Student-Centered Approaches

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Abstract

In some European countries inquiry-based learning (IBL) has become popular in primary and secondary mathematics education, especially for science-connected content. On the other hand, problem-based learning (PBL) and project-based learning (PjBL) have a longer history in mathematics, science, technology, engineering (STEM) and in some other fields in higher education. We analyse a selection of n=112 high quality research articles about the use of IBL, PBL or PjBL in mathematics, and its interdisciplinary connection with other subjects, level of education, type of research method, research design and participants for each study. Based on these characteristics, we identify the differences between the approaches in mathematics education. Finally, detailed examination of a subsample of experimental studies, where effect size is or can be measured, describes differences in interdisciplinary connections between the target approaches and indicate what kind of studies are still missing in mathematics education.

Keywords: Inquiry-Based Learning; Interdisciplinary Mathematics Education; Literature Review; Problem-Based Learning; Project-Based Learning

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Comparación de Conexiones Interdisciplinarias entre Matemáticas y otras Materias a través de Enfoques Centrados en el Estudiante

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Resumen

En algunos países europeos, el aprendizaje basado en la investigación (ABI) se ha vuelto popular en la educación matemática primaria y secundaria, especialmente para el contenido relacionado con la ciencia. Por otro lado, el aprendizaje basado en problemas (ABP) y el aprendizaje basado en proyectos (ABPr) tienen una historia más larga en matemáticas, ciencia, tecnología, ingeniería (STEM) y en algunos otros campos de la educación superior. Analizamos una selección de n=112 artículos de investigación de alta calidad sobre el uso de ABI, ABP o ABPr en matemáticas y su conexión interdisciplinaria con otras materias, nivel de educación, tipo de método de investigación, diseño de investigación y participantes para cada estudio. Con base en estas características, identificamos las diferencias entre los enfoques en educación matemática. Finalmente, el examen detallado de una submuestra de estudios experimentales, donde el tamaño del efecto se mide o se puede medir, describe las diferencias en las conexiones interdisciplinarias entre los enfoques objetivo e indica qué tipo de estudios aún faltan en la educación matemática.

Palabras clave: Aprendizaje Basado en la Indagación; Educación Matemática Interdisciplinaria; Revisión de Literatura; Aprendizaje Basado en Problemas; Aprendizaje Basado en Proyectos

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A tendency to foster inquiry-based learning (IBL) in mathematics and science education has been observed in Europe (Artigue & Blomhøj, 2013; Maass & Artigue, 2013). The Ministry of Education in Slovenia, treated as an East European country in the mathematical education sphere, presented a "new" approach to the elementary and secondary math teachers as a student-centred innovation that enables students to explore a particular situation in a similar way as researchers and scientists would do it (Jessen et al., 2017). In this approach, students actively participate in a question-driven learning process, supported by meaningful contexts (Edelson et al., 1999). They are faced with an unknown problem that needs to be solved; they work individually as well as in groups as scientists usually do and they take responsibility for their own learning (Engeln et al., 2013).

Similarly, student-centred instruction, active participation of students, meaningful problems and small group problem solving are characteristics of problem-based learning (PBL), which was implemented in Blind nationality math education by some enthusiastic teachers. PBL was incorporated into statistics as a part of math for students at a tertiary level program (Drobnič Vidic, 2011) as well as into STEM subjects for students at the secondary level (Drobnič Vidic, 2017). However, the acronym PBL is often used also for project-based learning (PjBL) that can be misinterpreted as PBL (De Graaff & Kolmos, 2003). In PjBL, students also deal with big problems (called projects) that usually need to be solved from different disciplines' perspectives. This of course indicates that this learning approach demand a longer period of time.

All three mentioned approaches can bring forward interdisciplinarity. PjBL projects are usually built around an intersection of topics from two or more disciplines (Thomas, 2000). PBL is a pathway towards interdisciplinary learning that is possible when the identified problems are ill defined and not necessarily situated within a specific scientific paradigm (Jensen et al., 2019). This can be true also for IBL settled in mathematics because math applications in many different domains are sources of significant questions for IBL at various levels of education (Artigue & Blomhøj, 2013). Through the use of such approaches, students are expected to develop teamwork skills, ICT skills and other transferable skills applicable across disciplines. Interdisciplinarity is also a trigger for teachers to implement one of these three types of instruction.

Mathematics is a base for other academic disciplines such as engineering or science. Therefore, a connection between mathematics and other disciplines seems to be natural and effective. That is why incorporation of IBL, PBL or PjBL in mathematics as well as its connection with science or engineering attracted our attention. Based on a brief overview of these approaches and theoretical comparisons we try to identify the differences in incorporation of these approaches into mathematics education and characteristics of interdisciplinary connections with other disciplines.

Interdisciplinary Student-centred Constructivist Approaches in Mathematics

Problem-, project- and inquiry-based learning approaches were promoted in a set of science education reforms from 1990 until 2011 where teachers facilitate rather than direct learners' actions (Li et al., 2021). Although many other student-centred long-term approaches have been used in education nowadays, such as design-based learning, experience-based learning or computer-based learning, these approaches are either not frequently found in mathematics, or they do not emphasise interdisciplinarity. Therefore, we decided to compare only the first three mentioned approaches.

Inquiry-Based Learning.

Definitions for inquiry-based education vary; inquiry itself can be considered as a way of engaging in science and more recently, engineering-related practice (Brown, 2017). Inquiry in mathematics involves diverse of activities that can be combined in learning processes: elaborating questions, searching for resources, modelling and mathematisation, analysing data, critical thinking, arguing and proving, building and structuring new knowledge, discussing about new findings (Artigue & Blomhøj, 2013; Jessen et al., 2017). The term inquiry-based learning (IBL) emphases extension of such practice over a longer period of time. However, some other terms, such as enquiry-based learning or research-based learning are also in use (Spronken-Smith, 2012). In IBL, questions or problems provide a context for active learning (Prince & Felder, 2006). Students pose questions, make observations, plan and make investigations, analyse and communicate the results (Edelson et al., 1999; Maass & Artigue, 2013). IBL provides students with an opportunity to take ownership of their learning while developing important higher-order skills.

Science teachers usually centre students' activities around 5E steps: Engagement, Exploration, Explanation, Elaboration and Evaluation (Oguz-Unver & Arabacioglu, 2014). This means that students create their own scientifically oriented questions; give priority to evidence in responding to questions; formulate explanations based on evidence; connect explanations to scientific knowledge; and finally communicate and justify explanations (National Research Council, 2000, p. 27).

Cooperative work is very useful for the mentioned processes. There is also a new role for a teacher to encourage students for inquiry, to pose questions, to manage small group or whole class discussions, and to integrate previous knowledge into the process of learning new things (Artigue & Blomhøj, 2013).

Problem-Based Learning

PBL is also a student-centred approach where problems take a central role in the learning process and constitute the motivation for the student's activities (Perrenet et al., 2000). Problems that trigger learning of a particular content should be unstructured, not routine, authentic, professionally relevant and as close as possible to real-life situations that they can omit borders between various subjects. In order to be able to solve a problem, students must activate their prior knowledge and integrate new knowledge in their own cognitive structures in order to establish a connection with the prior information (Norman & Smidt, 1992).

A seven-step model has become established in PBL in medicine: clarification of unclear terminology and concepts; definition of the problem; brainstorming; list of possible explanations; formulation of learning aims and key tasks; independent search for additional information outside the group; report, synthesis and testing of the new information (Boud & Feletti, 1998). In various PBL implementations problems may vary significantly in scope (Prince & Felder, 2006). Depending on the complexity of the problems the number of steps can vary: some steps can be integrated to get 5 steps in shorter problems as well as steps' cycle can be repeated dealing with more complex problems (Woods, 1994). Students in small groups with a tutor are actively involved in problem solving process. Students also need to acquire new skills for effective cooperative work, skills for searching various sources as well as skills to efficiently present new knowledge to others. The so-called transferable skills can be easily transferred into other disciplines. In addition, students become gradually more familiar with problem solving in a small group (Boud & Feletti, 1998). The teacher's role is changed into a facilitator that facilitates learning process, helps students to stay in the way that leads them to solve the problem by themselves.

Project-based Learning

PjBL allows students to learn by doing and to engage them in real world activities that are similar to the activities that adult professionals engage in (Krajcik & Blumenfeld, 2006). The central activities of the PjBL must involve the transformation and construction of knowledge and activities should be student driven (Thomas, 2000). Learning in PjBL starts with a complex question or problem - typical for a profession - to be solved; students learn and apply important ideas in the discipline; they engage in collaborative activities to find solutions; while engaged in the active work process, students are scaffolded with learning technologies that help them participate in activities; they create tangible products (artefacts) that address the driving question (Krajcik & Blumerfeld, 2006). Such authentic real-world problems that students use to produce a tangible product over extended periods of time are often called projects. Projects are usually interdisciplinary (De Graaff & Kolmos, 2003). Students have to use knowledge of various disciplines as well as different process skills: to find appropriate information from various sources, to work as a team, to manage and modify actions of project demands, to present results in the way that becomes valuable to potential users, etc. However, students do not only apply knowledge; they build new knowledge from a professional domain, while developing transferable skills important for new projects (De Graaff & Kolmos, 2003).

The teacher's role is to facilitate, advise, guide, monitor & mentor learners, not just to conduct lectures and laboratory work. Sometimes, the teachers act as instructors to provide direct instruction or give explanatory knowledge or research skills, and sometimes as facilitators, helping learners find resources or resolve problems (Oguz-Unver & Arabacioglu, 2014).

Differences Between IBL, PBL and Pjbl

IBL, PBL and PjBL have many common characteristics exposed in our brief overview and written also in Figure 1. Comparisons were made also by other researchers. Perrenet, Bouhuijs and Smits (2000) noted that PBL and PjBL are based on self-direction, collaboration, and multidisciplinary orientation. However, they differ in problem context and time for solving. Projects used in PjBL are closer to professional reality and therefore take a longer period of time than PBL problems. Management of time and resources as well as task and role differentiation is very important in PjBL, while skills for effective search and problem solving activities are important in PBL. Moreover, PjBL is more directed to the application of knowledge, whereas PBL is more directed to the acquisition of knowledge. Hmelo-Silver, Dunkan and Chinn (2007) argue that PBL and IBL are both guided approaches, organized around relevant, authentic problems or questions, place heavy emphasis on collaborative learning, but they differ in their origin. Moreover, students in IBL are engaged in investigations and develop specific-reasoning skills, while in PBL students are engaged in reasoning, learn strategies and develop problem solving skills. Prince and Felder (2006) compared all three approaches; all of them are promotors for inductive teaching and learning where instruction begins with a specific situation, case or problem to observe and after a certain learning procedure with new rules and facts to be generated at the end of the process. However, they differ in the product at the end as well as on its own history, research base, proponents, etc. According to Oguz-Unver and Arabacioglu (2014), IBL, PBL and PjBL can be used in all the environments, but the reason why we prefer one approach is determined by particular characteristics of individual approaches.

The target approaches are student-centred, constructivist and interdisciplinary that demand active involvement of students that learn around relevant problems. Students search various sources to find appropriate information and develop self-directed learning skills in the learning process extend over a longer period with the teacher to step out of the traditional role. However, approaches differ in historical aspects, in the main principle, in the context for learning, in the instructional procedures and in the outcomes. We summarise the main differences identified by the above-mentioned authors in the Table 1.

Table 1.
Detected differences between IBL, PBL, PjBL in some characteristics.

Characteristics	IBL	PBL	PjBL
Origin:	Science education	Medical education	Engineering education
Context for learning:	Question, problem, situation	Ill-structured open real-world problem	Real major project
Emphasis on…	Conceptual understanding	Acquiring knowledge	Applying/ integrating knowledge
Philosophical aim:	Raising questions	Problem solving	Producing a project
Specific learning outcomes:	Scientific inquiry skills, specific reasoning skills	Problem-solving skills, lifelong learning skills	Process skills, resource management skills
Key learning elements:	Exploration, raising questions, invention	Prior knowledge activation, elaboration of knowledge	Learning by producing / creating artefacts
Student learning:	Not specified	Small groups with a tutor	Small groups

These differences distinguish the target approaches. Despite them some authors claim that IBL is an overarching approach that involves other student-centred approaches (Prince & Felder, 2006; Spronken-Smith, 2012). It is also important to know that these approaches can be adapted for various disciplines and the described differences between them can become more or less visible. In an approach implementation in mathematics, a context for learning (ill-structured open problem in PBL, new situation or new phenomena with posed questions in IBL and extensive major project in PjBL) and specific learning outcomes (problem solving skills in PBL, scientific inquiry skills in IBL and process skills at PjBL) are useful to differentiate the target approaches from each other.

In order to find out how these target approaches are implemented in the field of mathematics we posed the following research questions (RQ):

1. Which of the three target approaches (IBL, PBL, PjBL) are incorporated in mathematics education, at which educational level, and with what kind of interdisciplinary connections?
2. What are the differences between experimental implementation of IBL, PBL and PjBL in mathematics and its connection with other subjects?

Effectiveness of educational approaches is usually measured in meta-analyses. Meta-analysis of Li, Ding and Zhang (2021) shows that student-centred education can significantly improve students' academic achievement. In such studies only quasi-experimental designs with experimental and control groups are analysed and effect sizes can be calculated in order to describe the effectiveness of an approach. However, sometimes experimental research cannot offer an appropriate insight into a practice because many unsuccessful studies remain unpublished. Moreover, studies with experimental design are rare (e.g. Savelsbergh et al., 2016), consequently authors of meta-analyses resort to unpublished research or articles with not scientifically provable quality. For the purposes of incorporation of an approach into mathematics it is important to take into account high-quality studies of various research designs, such as observation design with description of concrete implementation, survey with statements of teachers' beliefs, case study with details of students' reactions, quasi-experimental design with results of students improvement… Following Brown (2017), we therefore decided to search for articles with experimental and non-experimental design that have been published in SCI-E or SSCI listed educational journals to ensure quality, evidentiary basis and peer-review status. Moreover, detailed examination of experimental designs with measures of effectiveness of a target approach in mathematics enables us to form a holistic picture about the differences of IBL, PBL and PjBL in math implementations.

We begin by describing the methodology of articles' selection in our study and continue by the presentation of the measured characteristics. We analyse: characteristics of the three target approaches in the field of mathematics and its connection with other subjects; the differences between them; and the differences in interdisciplinary connection in empirical studies of these target approaches. These results will enable us to find out differences of student-centred approaches in math and its effective integration with other subjects.

Method

Search Criteria

We made a selection of high-quality studies to determine which of the three target approaches is more popular in mathematics, and to identify some differences between them in the field of mathematics. We searched through all Web of Science (WOS) journals' databases for SCI-E or SSCI listed articles in the category of Education - Scientific disciplines or Education - Educational research through three 5-year periods: from the start of the year 2003 to the beginning of year 2018, when our study began. WOS database is the only one that is relevant for research distribution in our institution. Year 2003 was a beginning of the chosen time period in order to make possible comparisons of another research with the same period (Drobnič Vidic, 2022).

The target word was "mathematics" combined with one of the target phrase words »inquiry-based«, »problem-based« or »project-based« (with or without the hyphen) as well as acronyms IBL, PBL or PjBL. We could have also included search phrase words »-oriented«, »-centred« instead of the word »-based«, however we restricted our search only to the last one because it is also used in approaches' acronyms: IBL, PBL, PjBL. The most important objective was to have the same search method for all the target approaches because we centred our analysis on differences between them. The first round of results yielded 100 articles with the phrase word "inquiry-based", 44 with "problem-based" and 62 with "project-based". Even though the target word "mathematics" was entered in the search, almost half of the articles dealt with the science. In most of the articles that do not deal with math, IBL was used. The reason for this result could be found in an extra keyword »mathematics« in some journals, whenever a statistical analysis was involved. After exclusion of 77 articles that do not deal with the field of mathematics and further on 17 articles that do not examine at least one of the target approaches, a selection of n = 112 articles was used as a sample for further analysis. In all these articles - published in high quality journals - a chosen approach was described in detail and some specific approach characteristics are included in description.

Data Characteristics

Our coding scheme categorises the characteristics of the target approaches. To find out differences about interdisciplinary connections of mathematics and other subjects in approaches' incorporation, learning subject, educational level and participants are important. Information about research design in a study is needed for creating a subsample of experimental implementations of the target approaches for RQ2. Therefore, we used the following coding scheme with 4 categories: Learning subject, Educational level, Participants, Research design.

Learning subject

We distinguished between Mathematics (thought independently from other subjects), the connection of Mathematics and Science and STEM as a combination of science, technology, engineering and mathematics. If mathematics was connected with any other subject we categorised the article under the category Other.

Educational level

In our categorisation, we refer to the lowest level of education, at which a particular approach is examined, e.g. Elementary level (usually Year 5-11, labelled often as K 1-6); Middle level (usually Year 11-15, labelled often as K 7-9), Secondary level (usually Year 15-18, labelled often as K 10-12); University level as a tertiary level (usually Year 18 and up), Not clear. Some researchers use an approach at various educational levels. In such cases the article was categorised according to the lowest educational level. For instance, if a connection between mathematics and science was examined at an elementary and a middle level together, we categorised the article into the category Elementary level. If participants were teachers, the level corresponded to the level of the target contexts.

Participants

Typical participants are: Students, Teachers, Prospective teachers, Students and teachers (Mixed) or Other / not clear, if some other type of participants was analysed, e.g. staff or principals.

Research Design

There are many ways to classify research designs, but at this point we just need an information if a design in the study is experimental or not. Only experimental designs are used further on for analyses of RQ2. We also differ between experimental design with comparable groups and with one group, because nowadays an experiment with one group using pre and post-test can also be included into experimental designs. Therefore, we use the following research designs: Experiment with comparable groups, Experimental one group pre-post design, Non-experimental design.

We analysed experimental studies in more detail to check the nature of interdisciplinary connections; the characteristics of measured knowledge and skills or attitudes in experiments; the type of compared groups; and the information on effect size.

We used these four categories for comparison in the results section. The studies were categorised by two independent researchers who classified the articles. The agreement in categories for all articles in the sample was 89.3%. After more detailed reading and discussion about unequal categorisation the consensus was made between both researchers. We performed statistical $\chi 2$ tests to verify if the described category and the type of approach are dependent variables. Subcategory Other was omitted to satisfy the necessary condition for statistical test performance (or joined with subcategory Mixed in one case). We used SPSS for Windows and rejected all the hypotheses at significance level of α =0.05* or α = 0.01**.

We compared the categories across the three target approaches both, for the sample of experimental and non-experimental articles and for articles with experimental design only. Finally, we identify differences in interdisciplinary subject connections through the target approaches' experimental practice. The results are summarized in the next paragraphs.

Results

Characteristics of the Target Approaches in the Field of Mathematics

The selected 112 articles - published in 50 various educational journals - were divided among the examined instructional approaches. In the field of mathematics, IBL and PjBL are used in more than one third of studies each (39.3%, 36.6% respectively) and PBL in less than one third of studies (24.1%). There are 29.5% of the target approaches that examine mathematics only, 24.1% of them deal with mathematics and science, 42.9% examine STEM, and 3.5% of approaches either combine mathematics with social science or with computer science, or the combination is not clear. In the sample, 22 studies use elementary level as the minimal educational level of the target approach, 25 use the middle level, 33 the secondary level, and 24 university level or higher. In 8 studies all educational levels are used (mostly in surveys or reviews of other research) or information is not given. In almost half of the studies participants are students, in a quarter of studies they are teachers. Prospective teachers or students and teachers together are rarely chosen as participants as shown in Figure 1.

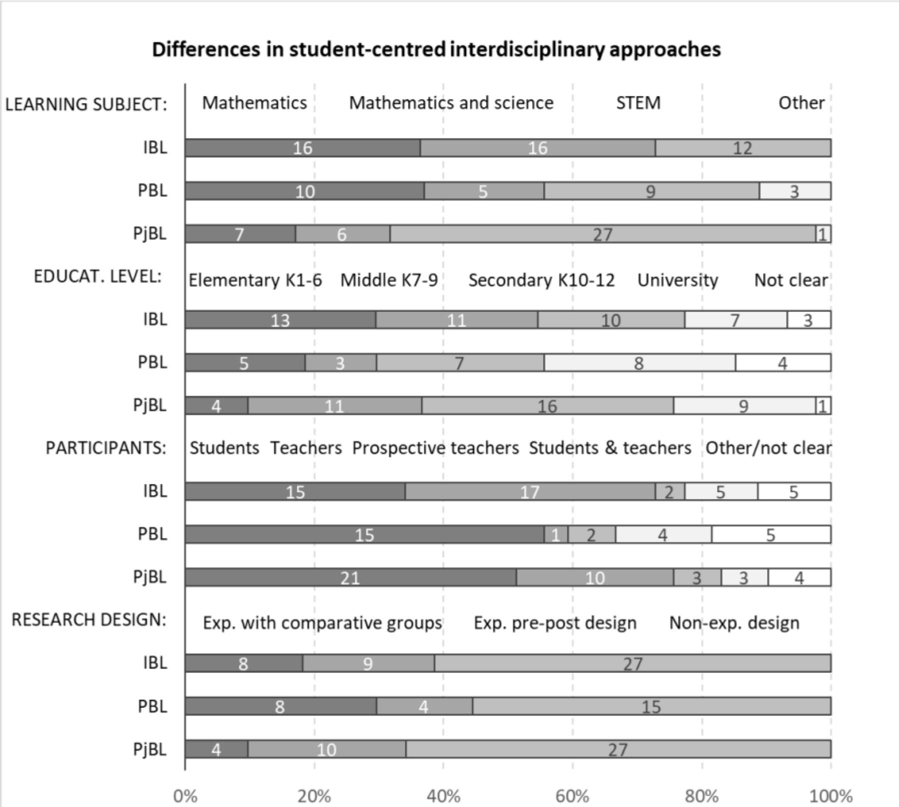

Figure 1. Measured characteristics of IBL, PBL and PjBL in the examined studies.

In our sample we have 17.9 % of experiments with comparable groups, 20.5 % of experimental one group pre-post designs and 61.6 % of non-experimental designs.

Differences in the Target Approaches in Mathematics

It can be seen from Figure 1 that the PBL approach is most frequently examined in mathematics (37.0%), IBL is as frequently used in mathematics as in a combination of mathematics and science (36.4%), and STEM is used in 65.9% of all PjBL studies. Based on the statistical analysis of the collected data we rejected the null hypothesis that the instructional approach (IBL, PBL or PjBL) is independent of the subject of examination (math, math and science or STEM) ($n = 108$, $\chi2= 15.690$, $p = 0.003$**).

Researchers mostly use IBL at the elementary level of education as minimal educational level (29.6%); PBL is mostly incorporated at university level (26.7%) and PjBL is mostly researched at the secondary level (39.0%). The median mark for IBL is middle level, while the median for PBL and PjBL is secondary level. However, there is not sufficient evidence to support the claim that an educational level has an effect on the choice of instructional approach ($n = 104$, $\chi2 = 9.5860$, $p = 0.143$).

Students are the most frequently used participants in PBL and PjBL (in more than half studies), while teachers are the most frequently used participants in IBL (in 38.6% of studies). There is sufficient evidence to warrant rejection of the claim that the type of instructional approach and participants in the studies are independent variables ($n = 112$, $\chi2 = 9.525$, $p = 0.049$*).

The shares of non-experimental designs are high in all the target approaches. Experimental one group pre-post designs are the most often used in PjBL (24.4 %), while experiments with comparable groups are the most often used in PBL (29.6 %). However, there is not sufficient evidence to support the claim that the type of the research design is related to the type of examined approach ($n = 112$, $\chi2 = 4.613$, $p = 0.329$).

Analysis of the Target Approaches with Experimental Design in the Field of Mathematics

In this subsection, we focus on studies from our sample with experimental design, where effect size is or can be measured. There are 43 such articles shown in Figure 1 with Experiment with comparable groups and Experimental one group pre-post design. However, in one article PBL and IBL are examined together, and in another article PBL and PjBL are used together. In this article PBL with shorter problems from engineering context was implemented in mathematics true the school year and PjBL with major real word year-long complex problem was implemented across almost all school subjects (Drobnič Vidic, 2017). A subsample with n = 41 articles (studies) is examined in detail.

Characteristics of the Target Approaches with Experimental Design

As shown in Table 2, distribution of frequencies in categories Instructional approach: IBL-16, PBL-11, PjBL-14 and Learning subject: Math-12, Math and science-11, STEM-18 is very similar to the distribution in the main sample visible in Figure 1; the difference in percentages is less than 2.7% for all subcategories. Distribution of category Educational level as well as Participants (reachable from Table 2) varies a bit more; the reason might lie in the subcategory Other / not clear that is included only in the main sample.
In the subsample there are only experimental designs that represent the category Research design. In Table 2 we use the sign P-P for Experimental one group pre-post design examined in 22 studies, while various signs are used for Experiment with comparable groups to describe the types of groups in an experiment: E-C for classical experimental and control groups as in Cotič and Zuljan (2009), E-E for comparison between two or more experimental groups such as low and high fidelity students in IBL experiment by Han, Capraro and Capraro (2015), M-S for comparison between math and science groups of participants (e.g. Saderholm et al., 2017) and E-N for an experiment, in which experimental group is given an extra time to learn additional content that is not offered to participants in control group (e.g. Moreno et al., 2016).

Differences in the Target Approaches with Experimental Design

There are similar differences between the three target approaches in the subsample as in the main sample. Firstly, IBL is mostly analysed in math and science (M+S), PBL in math, while PjBL is mostly analysed in STEM. Secondly, the median for category Educational level (numbers 1, 2, 3, 4 respectively in Table 2) is the same as in the main sample: middle level (2) for IBL, and secondary level (3) for PBL and PjBL. Thirdly, although all three approaches mostly use students as participants, teachers and prospective teachers (prosp.t.) together reach the same frequency in the IBL approach as students. Both experimental designs are used equivalently in IBL studies; researchers of PBL studies favour Experiment with comparable groups while researchers of PjBL studies favour Experimental one group pre-post design. Due to lower frequencies in the subsample statistical tests are not provided.

Table 2.
Characteristics of subjects' interdisciplinary connection (IC) in experimental studies with measured categories

Approach / Level	Sub- -ject	IC	Des- -ign	Partic- ipants	Measurement of differences in…
IBL 1	M+S	no	P-P	Mixed	IBL knowledge and practice through PD*
IBL 3	M+S	yes	P-P	Mixed	inquiry skills in math/science with informatics
IBL 4	M+S	yes	P-P	Prosp.t.	acquisition of threshold concepts/modelling
IBL 2	M+S	yes	E-C	Student	knowledge/ motivation/ engagement
IBL 2	M+S	yes	P-P	Teacher	effect in critical thinking/ practice after PD*
IBL 1	M+S	no	M-S	Teacher	aspects of IBL implementation after PD
IBL 3	M+S	no	P-P	Teacher	teaching achievements in knowledge after PD
IBL 2	Math	/	E-C	Mixed	practice, knowledge, communication skills
IBL 4	Math	/	E-C	Mixed	cognitive/ affective/ collaborative gain*
IBL 3	Math	/	E-E	Student	cognitive load/ knowledge in various tasks
IBL 1	Math	/	E-N	Teacher	self-efficiency/ math knowledge after PD
IBL 2	STEM	no	E-N	Student	differences in creativity with/out IBL teacher
IBL 2	STEM	no	E-N	Student	astronautical knowledge, motivation, skills
IBL 4	STEM	no	P-P	Student	academic STEM knowledge in health course
IBL 4	STEM	yes	P-P	Student	content (STEM-statistical) knowledge, skills
IBL 1	STEM	no	P-P	Teacher	confidence/ efficacy for teaching after PD
PBL 3	M+S	no	E-N	Mixed	efficiency and attitudes about PD*
PBL 4	M+S	yes	E-C	Student	achievement, skills, attitudes of joint class
PBL 4	Math	/	E-C	Prosp.t.	knowledge/ computer skills in online class
PBL 3	Math	/	E-C	Student	geometrical knowledge
PBL 1	Math	/	E-C	Student	math efficiency and attitudes
PBL 1	Math	/	E-N	Student	math knowledge with computer help
PBL 1	Math	/	E-C	Student	knowledge transfer via various PBL's
PBL 1	Math	/	P-P	Student	math concepts' understanding
PBL 4	STEM	yes	P-P	Student	problem solving in STEM with MATLAB*
PBL 2	STEM	no	P-P	Student	STEM knowledge/ attitudes of NASA class
PBL 3	STEM	no	P-P	Student	analytical thinking ability (as IBL), attitude

Table 2. (continue)
Characteristics of subjects' interdisciplinary connection (IC) in experimental studies with measured categories

Approach / Level	Sub- -ject	IC	Des- -ign	Partic- ipants	Measurement of differences in…
Pjbl 2	M+S	yes	P-P	Prosp.t.	special math knowledge via Moon project
Pjbl 3	M+S	no	P-P	Prosp.t.	attitude/ barrier/ interest in PD
Pjbl 3	Math	/	E-C	Student	motivation/ attitudes/ math knowledge*
Pjbl 3	Math	/	P-P	Student	achievement/ attitudes in Polygons/ Plane
Pjbl 3	STEM	yes	E-E	Mixed	student gains in low/high fidelity with PD*
Pjbl 4	STEM	no	P-P	Prosp.t.	attitudes/ retention of knowledge
Pjbl 4	STEM	yes	P-P	Student	attitudes toward STEM integration
Pjbl 3	STEM	yes	P-P	Student	enhancement of creativity
Pjbl 3	STEM	yes	E-C	Student	TAKS math knowledge*
Pjbl 2	STEM	yes	P-P	Student	special astronautic knowledge/ skills
Pjbl 3	STEM	yes	P-P	Student	interest toward STEM subjects/ carrier
Pjbl 3	STEM	yes	P-P	Student	integrated thinking, STEM knowledge
Pjbl 2	STEM	yes	P-P	Teacher	attitudes to interdisciplinary teaching
Pjbl 3	STEM	yes	E-C	Teacher	teachers' aspects about STEM PBL

(PD: Professional development program, *: with effect size given)

Differences in Interdisciplinary Connections

In experimental design studies, we paid special attention to interdisciplinarity. To be able to talk about interdisciplinary connection the approach had to meet the following criterion: the position of mathematics in the problems solved by the participants in a target approach needs to be at least equal to that of other subjects involved (yes in Table 2). We also checked whether interdisciplinarity has been measured in these experimental studies. The authors of experimental studies examined and measured the following parameters: knowledge in the field of mathematics or connected subjects; knowledge about the new approach; various important skills for mathematics and / or other fields exposed with an approach; attitudes to the way of learning or to the interdisciplinary connection.

Detailed reading of the IBL studies lead us to the conclusion that more than one third of them favour examination of professional development (PD) program for teachers (Table 2). In these studies, math and science teachers often work together and try to improve their IBL lessons, e.g. in Lotter, Yow and Peters (2014). Such improvements are many times implemented in classrooms such as in Ješková and others (2016), where students are exposed to coherent and intentional multidisciplinary IBL conducting inquiry activities in mathematics, physics, and informatics. However, sometimes interdisciplinary connection is not emphasized (e.g. Saderholm et al., 2017). In the latter study, among eleven math or science tracks the three integrated tracks were not a part of evaluation. Moreover, most IBL studies with STEM subjects do not emphasise the role of math in such a connection. In the study of Moreno and others (2016) for instance, interesting astronautical engineering units offer an opportunity for new science, technology and engineering knowledge, however, making graphs is the only math knowledge exposed through IBL.

Detailed examination of PBL studies shows that more than half of the studies examine students' knowledge in mathematics with comparable groups (e.g. Bikić et al., 2016), while teachers' education through PD programs is very rare (Walker et al., 2012). Authors mostly measure pure mathematical (online, geometrical, concept) knowledge or problem solving in comparable groups (e.g. Cotič & Zuljan, 2009). Interdisciplinary connection is not often exposed; it can be hidden in a real context of given problems, such as in the study of Bikić, Maričić and Pikula (2016), where problems with engineering context trigger the learning of geometry. An effort to introduce a more interdisciplinary type of learning through PBL was made by Polanco, Calderon and Delgado (2004) with math, physics and computer science integration into one course, where engineering problems trigger learning.

Authors of PjBL experimental studies examine the effectiveness of this approach mostly with experimental one group pre-post design, which can be seen from Table 2; attitudes towards STEM integration are mostly studied in such examinations (e.g. Mohd Shahali et al., 2017). In contrast to weak STEM interdisciplinary connection in IBL studies, most examinations in PjBL STEM studies expose the interdisciplinary connection of STEM subjects with interdisciplinary engineering projects (e.g. Mohd Shahali et al., 2017). However, students' math knowledge is seldom measured (e.g. Han et al., 2015). Various process skills, attitudes or special engineering knowledge are the focus (Table 2). In the study of Al Salami, Makela, and de Miranda (2017), perceptions of interdisciplinary curriculum were measured, and the results show positive association between change in attitudes to interdisciplinary teaching before and after the PD program about STEM interdisciplinary PjBL. However, cooperation of STEM teachers could be found to be difficult if they did not have a strong support by school authorities or a well-organized PD program (Drobnič Vidic, 2017).

Interpretation and Conclusions

We compared articles that examine one of the three interdisciplinary student-centred approaches in the field of mathematics or in its connection with other subjects. In a total of 112 studies, PBL is represented a little bit less frequently than IBL and PjBL. In the target approaches, mathematics is most often examined together with STEM subjects; less frequently, it is examined as a sole subject or together with science. A connection of mathematics with another subject is rare.

The first target approach, IBL, is mostly used at elementary level in mathematics or mathematics together with science. Teachers dominate as participants in these studies. The second target approach, PBL, is used mostly at university level in mathematics as a sole subject. Students feature as participants in most of the studies. The third target approach, PjBL, is used mostly at the secondary level in STEM connection using students as participants. Authors prefer non-experimental designs in all the target approaches. Statistical tests show that the type of chosen student-centred approach in combination with subjects' connection are dependent variables as well as the type of chosen student-centred approach in combination with the participants used in the studies are dependent variables. However, dependency could not be confirmed for a target approach in combination with the educational level, and the research design.

Articles with experimental studies share similar characteristics as studies with both experimental and non-experimental design. Detailed information about the type of interdisciplinary connection in this subsample show that researchers of IBL often use math and science connection in experiments. This approach is gradually implemented in education, with the aim to prepare teachers and prospective teachers for an effective IBL practice. A typical IBL experimental study would examine interdisciplinary connection of math and science with teachers as participants that learn how to teach interdisciplinary IBL math and science contents through PD program, as in Lotter, Yow and Peters (2014). Researchers of PBL were most active in experiments with comparable groups were carried out in order to verify if this approach gives students better math knowledge or better problem-solving skills than a traditional approach. There are not many studies about math connection with other subjects. A typical PBL study would measure math knowledge achievement and problem-solving improvement triggered by real context problems with comparable groups (Bikić et al., 2016). Researchers that use PjBL, usually study a STEM interdisciplinary connection. However, such studies mostly analyse students' attitudes towards PjBL STEM practice. In some rare studies students' specialised knowledge (such as astronautical knowledge) is evaluated, while mathematical knowledge is not in the focus. A typical study of PjBL would examine students' attitudes towards STEM subjects' connection with experimental one group pre-post design (Mohd Shahali et al., 2017). The main findings of our analyses regarding interdisciplinary connection are summarised as follows:

- **PBL** is usually implemented **in mathematics as a sole subject**, other subjects are hidden in the context of problems that trigger learning of a new mathematical knowledge and developing problem solving skills;
- **IBL** is usually implemented **in mathematics and science** subjects together, where teachers try to cooperate in different situations to create a suitable research environment for successful acquisition of new knowledge or skills;
- **PjBL** usually **connects STEM subjects** true interdisciplinary projects where attitudes toward such integration, specific process skills and specific engineering knowledge are the focus.

Interdisciplinary subject connection can be additional characteristics that help to differentiate these approaches in mathematics. Our results are in line with findings of Li and Schoenfeld (2019), that mathematics and science have often proceeded along parallel tracks, with mathematics focused on "problem solving" (exposed in PBL) while science has focused on "inquiry" (centred in IBL). Emphasizing both sense making and making sense in mathematics education opens opportunities for connections with other STEM disciplines - technology and engineering - where PjBL is usual student-centred practice.

Limitations

Our study has some limitations that need to be exposed. Our analysis was based on various articles from SSCI and SCI-E journals that examine the target approaches in the field of mathematics and its connection with other subjects. Using our selection of non-experimental and experimental studies, we could observe the target approaches' popularity in mathematics, while their effectiveness has not been measured. Secondly, our research was limited only to the articles, retrieved through the search phrases "inquiry-based", "problem-based", or "project-based". By adding some other search word phrases we could enlarge our sample of studies. However, excluding these search phrases did not affect the comparison between IBL, PBL or PjBL approaches. We do not believe that a bigger sample would result in significant changes in results.

Implications

Taking experimental and non-experimental research into our analysis, allowed us to include surveys of teaching contexts in which math and science teachers participated together in PD programs for better learning with IBL approach. This is visible in high frequency of teachers or prospective teachers as participants in IBL studies, which indicates that this approach was popularised among the (prospective) teachers. Dissemination of IBL through teachers' PD programs can serve as an encouraging model for other approaches. However, such programs were not necessarily focused on interdisciplinary problems. More studies would be needed to establish the effect of interdisciplinary connections.

Integration of all STEM subjects became popular in student-centred approaches and is mostly used with PjBL approach at the secondary level of education when students are old enough to work in interdisciplinary projects. It becomes a current trend among student-centred approaches (Drobnič Vidic, 2022). In SSCI or SCI-E articles analysed in this study such practices are mostly analysed with experimental one group pre-post design in order to verify if students have changed their attitudes through such interdisciplinary way of learning or developed special skills through interdisciplinary projects. However, we need to point out that math knowledge is rarely emphasised in such STEM interdisciplinary connection and mathematical or other subjects' knowledge is seldom measured. Moreover, many STEM studies with a student-centred approach often use math only in the name, while science or engineering dominate in the contents. More research on students' math knowledge in STEM connection is needed in the future.

It would be interesting to make a meta-analysis of the target approaches in math and its connection with other subjects. However, our sample does not allow simple comparisons. Data in our study indicate that only one third of the articles included in our sample verify the efficiency of the approach through experiments where effect size is or can be measured. In more than half of such studies, experimental one group pre-post design is used. Despite many criticisms, recent statistical methodologies have started to allow the measurement of effect size in the one group pre-post designs (Bakker et al., 2019), whose effect size cannot be comparable with effect size of an experiment with comparative groups. Moreover, even in experimental design studies with comparable groups some shortcomings have been detected in groups' comparison.

Our study has been stimulated by curiosity, which of similar student-centred interdisciplinary approaches can be effective practice for teaching and learning math and its connection with other subjects. Namely, our previous research in science study programmes identified some factors that need to be emphasised when designing problem centred math education to promote successful interdisciplinary problem solving with context problems (Drobnič Vidic, 2015). The decision regarding the approach a teacher should choose depends on the subjects that will be included in the mathematical learning process, on the ratio of knowledge and skills importance, and on the teacher's knowledge of the approaches.

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