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TABULAR AND GRAPHICAL REPRESENTATION IN STATISTICS IN THE 5TH GRADE FROM A LESSON STUDY

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Abstract: The lesson study, originating in Japan at the end of the 19th century, characterizes an approach to professional development centered on the teacher's practice and focused on students' and teachers' learning. Considering the students' difficulties in mathematics, we focused on *highlighting and discussing the statistical learning of 5th-grade students through a lesson study*. This qualitative research was developed in a lesson study structured in eleven meetings involving teachers from a public school in Pinhalzinho, Santa Catarina, Brazil. The empirical material consists of transcriptions of recordings, class observation scripts, and student worksheets. The analysis of this material highlighted important aspects of statistical learning, which were grouped into central themes, thus constituting the research analysis categories, of which we will address in this article the category *tabular representation and transcription of data into the graph*. By discussing this category, we observed students' learning developed in the competencies in literacy, reasoning, and statistical thinking, as well as elements of the graph: axes, scale, titles, and source. The lesson study enabled the learning of statistical components and concepts and contributed to the teachers' professional development, minimizing possible lack of confidence and difficulties in this curriculum topic.
Keywords: Lesson study. Statistical learning. Early years of elementary school. Mathematics teaching.

REPRESENTAÇÃO TABULAR E GRÁFICA EM ESTATÍSTICA NO 5º ANO A PARTIR DE UM ESTUDO DE AULA

Resumo: O estudo de aula, originário do Japão no final do século dezenove, caracteriza uma abordagem de desenvolvimento profissional centrada na prática do professor e voltada para a aprendizagem dos alunos e dos próprios professores. Considerando as dificuldades apresentadas pelos alunos em Matemática, nos dedicamos a *evidenciar e discutir a aprendizagem estatística dos alunos do 5º ano a partir da realização de um estudo de aula*. A pesquisa, de natureza qualitativa, foi desenvolvida em um estudo de aula, estruturado em onze encontros, que envolveu professores de uma escola pública do município de Pinhalzinho, Santa Catarina. O material empírico compõe-se das transcrições das gravações, roteiro de observação da aula e as fichas de trabalho dos alunos. A análise desse material evidenciou importantes aspectos relacionados à aprendizagem estatística, os quais foram agrupados em temáticas centrais, constituindo, assim, as categorias de análise da pesquisa, das quais vamos abordar nesse artigo a categoria *representação tabular e a transcrição dos dados para o gráfico*. A partir da discussão dessa categoria foram evidenciadas as aprendizagens desenvolvidas pelos alunos relativamente às competências de literacia, raciocínio e pensamento estatístico, e elementos do gráfico: eixos, escala, títulos e fonte. O estudo de aula propiciou a aprendizagem de elementos e conceitos estatísticos, e contribuiu para o desenvolvimento profissional dos professores, minimizando possíveis inseguranças e dificuldades deste tópico curricular.
Palavras-chave: Estudo de aula. Aprendizagem estatística. Anos Iniciais do Ensino Fundamental. Ensino de Matemática.

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Introduction

Statistics is a fundamental field in many areas of knowledge. The discipline gives us the tools to interpret and extract information from data, an increasingly valued skill in a world where the amount of information available is immense. This aspect corroborates the relevance of statistics in school teaching since the early years of elementary school. Richit, Venturin, and Rodrigues (2022), supported by Garfield (1993), underscore that statistics teaching must interest and motivate students and awaken positive attitudes towards this field of knowledge. However, the traditional approach to statistics teaching based on formulas and mathematical operations can be demotivating, which requires teachers and researchers to reflect on how to teach it.

In Brazil, concern with the teaching and learning processes of statistics has gained space in official documents since the publication in 1997 of the National Curricular Parameters [Parâmetros Curriculares Nacionais – PCN]. Within the scope of the PCN, statistics and probability were officially incorporated into the basic education curriculum structure. The recently promulgated National Common Curriculum Base [Base Nacional Comum Curricular – BNCC] included statistics and probability in basic education and suggested the approach of statistical concepts through everyday life situations (CAMPOS; PERIN, 2020), making the debate gain momentum in the national scene.

On the other hand, the challenges of teaching at different school stages require the teacher to make changes, seeking to improve the quality of classroom practices and favoring student learning (RICHIT, 2023; RICHIT; PONTE; TOMKELSKI, 2020). Among the teaching professional development approaches that have shown potential for making changes in classroom practices, Richit, Ponte, and Quaresma (2021) cite the lesson studies.

The lesson study originated in Japan at the end of the 19th century and beginning of the 20th century and presents a structure consisting of four stages: definition of the objective, collaborative planning of a class (the research lesson), teaching the research lesson, and reflection about the class (RICHIT, 2020; RICHIT; RICHIT; RICHTER, 2023). According to Breda, Hummes, and Silva (2020, p. 338), the idea of lesson studies is that “a group of teachers and experts come together with a common problem regarding their students’ learning, plan a lesson for the students and finally, examine and discuss what they observe in its implementation.” Reflections on the points observed allow teachers to reflect on aspects of their practice and student learning (RICHIT; TOMKELSKI, 2022).

In research on conducting lesson studies with teachers of the initial years, Souza (2021)

points out that collaborative planning between teachers of the initial and final years of elementary school brings mutual benefits, as teachers of the initial years offer teaching and teachers of the final years offer mathematical knowledge. The author considers this aspect fundamental "so that students do not suffer a rupture and there is a line of reasoning about the content and its teaching" (SOUZA, 2021, p.53).

The questions related to the relevance and intrinsic challenges of teaching statistics in the initial years and, especially, the possibilities of promoting and investigating students' mathematical learning through a lesson study culminated in the formulation of the following research problem: " What statistics do 5th-grade students learn from a lesson study?" The research is part of the movement focused on mathematics teaching and learning in lesson studies, focusing on aspects of statistical learning (BOTH, 2023).

The research is relevant because it considers that statistics teaching must start from students' everyday situations, enabling reflection, criticism, and understanding of the factors that surround them, making them aware and responsible, capable of questioning the veracity and validity of data and information, therefore favoring the solution of everyday situations faced throughout life. Furthermore, it is relevant because it focuses on the possibilities of lesson studies for student learning and the originality of the work, considering the theoretical and methodological specificities of our research concerning research on the topic of lesson studies.

Lesson study

Lesson study is a process of Japanese origin that began at the end of the 19th century in light of the changes in that country. This approach arose from the interest and "initiative of the teachers themselves, who researched changing the curriculum and classroom dynamics" (BALDIN, 2021, p. 44) due to changes in the school system, which included local proposals and cultural traditions. This modernization in Japanese education was caused by the Meiji Restoration in 1868 (RICHIT; PONTE, 2017).

Throughout the 20th century, marked by the two great wars, WWI and WWII, Japan suffered drastic economic consequences, which impacted education. Japan resumed educational projects only after WWII as a path to recovery and economic development. Thus, this teacher education model was established in the Japanese education system, becoming a public policy from the 1960s onwards (INAGAKI apud RICHIT, 2022; RICHIT; TOMKELSKI, 2022).

The Lesson study gained visibility on the world stage after being disseminated in the US in the late 1990s, attracting the interest of educators and researchers worldwide. One of the

milestones in this dissemination was the publication of the book *The Teaching Gap: Best Ideas from the World's Teachers for Improving Education in the Classroom* (STIGLER; HIEBERT, 1999), which presents in detail teaching in three countries: the US, Germany, and Japan.

Catherine Lewis (2000), one of the pioneers in the development of lesson studies in the United States, presented the five main characteristics of this approach: the research lesson is observed by other teachers; these classes are planned over several sessions, often collaboratively; the research lessons are designed to bring a lesson to life with a particular goal or view of education; classes are recorded and then, discussed. These characteristics have been observed in lesson study models practiced around the world, and from these experiences, increasingly detailed descriptions have emerged. Richit, Ponte, and Tomkelski (2019) describe the stages of lesson study as follows:

> When formulating objectives for student learning and development, a topic or theme from the curriculum is selected to be covered in the research lesson; in planning, the class is carefully prepared to achieve the defined objectives; in implementation/teaching, the team participating in the lesson study observes and collects information about the students' actions throughout the lesson, through notes or audio or video recordings; in reflection, aspects related to student learning and development, observed and recorded in the previous stage, are focused on, which can contribute to improving the planned class; and, finally and if desired, in the new teaching of the research lesson, the process is repeated. (RICHIT; PONTE; TOMKELSKI, 2019, p. 57-58)

The dissemination of lesson studies in Brazil began in 2008. According to Baldin (2012), authors and collaborators from the Laboratory for Research and Development in Mathematics and Science Teaching [Laboratório de Pesquisa e Desenvolvimento em Ensino de Matemática e Ciências – LIMC] at the Federal University of Rio de Janeiro (UFRJ), who aimed to improve teachers' knowledge and practices by investigating mathematical learning and developing teaching materials and digital resources, reflected on lesson studies. And from an event called HTEM IV –VI Colloquium on History and Technology in Mathematics Teaching [VI Colóquio de História e Tecnologia no Ensino de Matemática] promoted by the LIMC, the first step was introducing the lesson studies here in Brazil.

According to Baldin (2012), the event, which took place in May 2008, featured a lecture by Masami Isoda, a Japanese professor who introduced the lesson studies from an international perspective by showing a video that demonstrated how the study of a class is developed, presenting the results obtained. This event allowed a collaborative project between Japan and Brazil to implement lesson studies in the Brazilian context. In October of the same year, the Japanese Professor Kozo Tsubota came to Brazil to promote experiences in lesson studies in

two schools: one in São Paulo and one in Rio de Janeiro. The pre-service teachers were impressed with the success of this practice.

After this activity, Professor Yuriko Baldin developed research related to class studies and supervised the first Brazilian dissertations on this topic: Felix (2010), Pimentel (2010), and Carrijo Neto (2013). Furthermore, in 2012, Baldin published a chapter in the third edition of the book "El estudio de clases japonés em mathematics: su importancia para el mejoramiento de los aprendizajes en el escenario global" [The Japanese lesson study in mathematics: Its importance for the improvement of learning on the global scenario], in which he describes the introduction of lesson studies in Brazil.

Then, in 2017, the Study and Research Group on Mathematics Education and Technologies [Grupo de Estudos e Pesquisa em Educação Matemática e Tecnologias – GEPEM@T] implemented the first lesson study in Rio Grande do Sul (RS) in the city of Erechim. According to Richit, Ponte, and Tomkelski (2019), the experience was structured around highlighting the knowledge of pedagogical practice in mathematics education that is constituted/developed in the context of a training action based on lesson studies. It included a group of basic education mathematics teachers who worked in high school. During the same period, other Brazilian research groups developed lesson studies, which resulted in master's and doctoral theses and scientific articles.

Another significant milestone in the dissemination of lesson studies in Brazil was the holding of the first International Seminar on Lesson Study in Mathematics Teaching [Seminário Internacional de Lesson Study no Ensino da Matemática – SILSEM] in May 2021, under the organization of the University of Brasília (UnB) and the State University of Campinas (Unicamp), which was supported by institutions such as the Federal University of Fronteira do Sul (UFFS), the Federal University of São Carlos (UFSCar), the Federal Institute of Espírito Santo (IFES), the Federal University of Campina Grande, and the Federal University of Tocantins, besides associations such as the Brazilian Society of Mathematics Education (SBEM). The event was an opportunity, even if virtual due to the COVID-19 pandemic, to contact and share experiences with the main disseminators of lesson study in Brazil, such as Masami Isoda, Raimundo Olfos, and João Pedro da Ponte. During the event, Yuriko Baldin reported that she has been seeking to disseminate and implement lesson studies in Brazil for more than ten years and, with the support of teachers, research groups, and initiatives like this event, discussions about this approach have already evolved a lot in the country

Recently, on the initiative of GEPEM@T from the Federal University of Fronteira Sul, in partnership with researchers from national and international institutions, the Lesson Study

International Cycle Conference [Ciclo Internacional de Conferências Lesson Study[3]]), an event that mobilized researchers from different countries and cultural contexts, who shared very rich experiences about this approach. Furthermore, in May 2023, the second edition of SILSEM was promoted.

Franceschi and Richit (2021) emphasize that lesson study is an approach centered on the teacher's practice and the learning of students and teachers. In other words, the lesson study focuses on "providing professional learning that supports and enhances students' development" (DUDLEY, 2015 apud FRANCESCHI; RICHIT, 2021, p.02). That said, we consider that lesson study can potentially support changes in teachers' practices in statistics teaching, favoring student learning.

Statistical Learning

Abrantes, Serrazina, and Oliveira (1999, p.15) say that "learning mathematics is a basic right of all people – especially all children and young people – and a response to individual and social needs" since all people must develop capabilities to interpret the most varied situations and use them to make decisions. Thus, "mathematics education can contribute, in a significant and irreplaceable way, to helping students become non-dependent individuals, but on the contrary competent, critical, and confident in the essential aspects in which their lives relate to mathematics" (ABRANTES; SERRAZINA; OLIVEIRA, 1999, p.15).

Still on the fundamental ideas related to learning, the authors highlight the affective aspects, interconnected with motivation and how students engage in tasks and learn, and students' conceptions of mathematics and their role as students since many believe that mathematics is the science of right-or-wrong. This right-or-wrong notion is distorted but linked to the learning environment. "If the 'norm' is to value involvement in thought processes, reasoning, and logical argumentation, a 'mathematics class culture' very different from the one that values quick and correct answers can be created" (ABRANTES; SERRAZINA; OLIVEIRA, 1999, p. 24).

From this perspective, research produced in mathematics education has contributed much to mathematics teaching and learning in basic education, outlining the curriculum, modifying teaching materials, and enabling new classroom practices. And so, BNCC, a normative document for teaching at the national level in Brazil, defining the essential learning

[3] All Conferences are available in the GEPEM@T area, link: < https://bit.ly/3jq3sqc >

that students must develop during their school stages, directs "Brazilian education towards the integral human formation and the construction of a fair, democratic, and inclusive society" and also "establishes knowledge, competencies, and skills that all students are expected to develop throughout basic schooling" (BRASIL, 2017, p. 7). The competencies indicate what students need to "know" and what they need to "know how to do., " while the skills express the essential learning that must be imparted to students in different school contexts (BRASIL, 2017, p. 30).

The BNCC presents the mathematics learning path for basic education, which is organized into five thematic units: Numbers, Algebra, Geometry, Quantities and Measures, and Probability and Statistics. Each thematic unit explores knowledge in a distinct and complementary way (similar to a spiral curriculum), in which ideas must be elaborated with approximations, expanding and promoting understanding.

The BNCC is also organized into thematic units and structured into competencies and skills for each school year (BRASIL, 2017). Unlike the PCN proposed, Probability and Statistics gain their own space in the BNCC, called the thematic unit of Probability and Statistics. For this thematic unit, the BNCC offers an approach "to concepts, facts, and procedures present in many problem situations in everyday life, science, and technology" (BRASIL, 2017, p. 274). This perspective highlights the need for knowledge to "collect, organize, represent, interpret, and analyze data in various contexts to make well-founded judgments and appropriate decisions."

The BNCC highlights that the responsibility of the initial years towards probability is to develop the notion that not all phenomena are deterministic; thus, the study is centered on the concept of randomness and events that involve chance. As for statistics, the first studies are focused on collecting, organizing, reading, and interpreting data after creating tables and graphs (BRASIL, 2017).

Statistics "as a science refers to the set of tools to obtain, summarize, and extract relevant information from data; find and evaluate patterns shown by them; plan data collections or design experiments and communicate results of quantitative research" (CAZORLA et al., 2017, p. 14). In other words, statistics attribute meaning and usuality to the data set. In recent years, probability and statistics studies have advanced much, and research points to three competencies: literacy, reasoning, and statistical thinking.

Gomes et al. (2022) say that developing statistical literacy means developing the ability to read, interpret, analyze, and critically argue the information in question; in other words, having the ability to transform statistical data into information and knowledge to use them in everyday life. Furthermore, "statistical literacy also includes an understanding of concepts,

vocabulary, and symbols and, in addition, an understanding of probability as a measure of uncertainty." Therefore, statistical literacy can be summarized as the basic understanding of the fundamental notions of statistics (CAMPOS et al., 2011, p. 478).

Carvalho (2018, p. 19) clarifies that statistical reasoning is how "people reason with statistical ideas, managing to give meaning to statistical information. It involves identifying facts, establishing relationships, and drawing inferences." Thus, statistical reasoning can involve connecting one concept with another or combining ideas about data and probability. Therefore, it covers data interpretation, understanding, explanation of the statistical process, and the interpretation of results (CAMPOS et al., 2011).

Assuming statistics learning in its entirety, Carvalho (2018, p. 19) states that "statistical thinking has an intuitive, informal, and implicit side that supports reasoning," enabling criticism and evaluation of the results of the statistical studies. "It involves understanding why and how statistical studies are conducted, and key ideas that underlie them" (CARVALHO, 2018, p. 19 apud GARFIELD; DELMAS; CHANCE, 2003). Campos and Wodewotzki (2018, p. 7) complement the idea that literacy, reasoning, and statistical thinking cannot be considered individually "as they complement each other, and only together will they encompass the global understanding of statistics.".

Methodology

The research is characterized as qualitative and exploratory. Godoy (1995, p. 21) highlights that one of the fundamental characteristics of qualitative researches is the analysis and interpretation of the phenomenon in an integrated way and, to this end, it is necessary for the researcher to go into the field "seeking to 'capture' the phenomenon under study from the perspective of the people involved, considering all relevant points of view. Various types of data are collected and analyzed to understand the dynamics of the phenomenon".

Exploratory research is concerned with "providing greater familiarity with the problem, to make it more explicit or constituting hypotheses" (GIL, 2007, p.41). Triviños (1987) highlights that this type of research allows the researcher to increase their experience around the problem investigated.

The research data included participant observation, interviews, questionnaires, student worksheets, and recordings of the lesson study sessions. Participant observation allows the researcher to explore "the sociocultural context of the observed environment (the socially acquired and shared knowledge available to participants or members of this environment) to

explain the observed patterns of human activity" (MARIETTO, 2018, p.7). Therefore, the researcher is part of the research, approaching and interacting with the participants.

Therefore, we consider that the lesson study favored getting closer to the research topic, increasing experience around the problem, and verifying reality as it is. The audio recordings, the observation script, and other ways of collecting data aimed to document specific situations to understand the problem, defining the research as exploratory and qualitative.

Research context and participants

The lesson study was carried out in a public school in Pinhalzinho, in the western region of Santa Catarina. Two teachers from that school participated in the lesson study: one 5th-grade teacher, one supervisor (with a pedagogy degree), the researcher –a mathematics teacher, too– and a mathematics teacher who acted as an external expert within the scope of the lesson study. The participants in the lesson study are identified in the text with pseudonyms Eva, Luiza, Mari, and Marcos.

The lesson study was carried out weekly, on Wednesdays, in that school, with each meeting lasting one hour and thirty minutes. The day and time of the sessions were agreed upon with the participants, taking into account their availability. The lesson study activities were organized according to the chart below.

Chart 1 – Schedule of meetings for the lesson study

Meeting/Date	Activities
1st 9/23/2022	Send a questionnaire via Google Forms to create participants' profiles.
2nd 9/28/2022	Present the participating teachers and explain their expectations and anxieties about the lesson study and mathematics teaching. Present the research and a brief account of the lesson study and how it occurs. Define schedule.
3rd 10/13/2022	Explain the lesson studies: definition, origin, structure, and dissemination to the world. Define the topic that will be covered during the lesson study cycles. Survey of students' difficulties with the topic. Set goals to resolve these difficulties.
4th 10/19/2022	Read and study an article on the topic and objectives outlined, as well as the difficulties and anxieties highlighted by teachers. Study the topic in curriculum documents and textbook analysis.
5th 10/26/2022	Deepen the topic of mathematics – Teacher Marcos. Discuss exploratory tasks and their distinction between exercises and problems.
6th 11/3/2022	Prepare exploratory tasks for the research lesson. Discuss the observation script.
7th 9/11/2022	Finalize the script. Interpret the graph.
8th 11/11/2022	Research lesson.
9th 11/18/2022	Research lesson.
10th 11/18/2022	Reflect on the research lesson – Student learning.
11th 12/05/2022	Reflect on the lesson study. Interview with the participating teachers.

Source: Prepared by the authors (2023).

The 5th-grade class in which the activity was carried out was made up of twenty-four students aged between nine and eleven. For the research lesson, the students were sorted into trios and called group 1, group 2, and so on. This denomination will be used in data analysis, and the students participating in each group will be indicated by Student 1, Student 2, and Student 3 to guarantee anonymity and avoid exposure. Transcripts of lesson study meetings are referred to as sessions 1 and 2 successively.

Once the data was created, we began the analysis process. Since we sought to approach the research problem from different perspectives, the analysis was conducted using triangulation. Triviños (1987, p. 38) explains that triangulation "has the basic objective of covering the maximum breadth in the description, explanation, and understanding of the focus

under study." Denzin and Lincoln (2000) point out that this method is widely used in qualitative research since the objective is to analyze and describe a topic in depth by combining data that was collected from different sources.

The analysis was conducted from the perspective presented by Bogdan and Biklen (1994, p. 205), who define it as

> [...] the process of systematically searching and organizing interview transcripts, field notes, and other materials that have been accumulated to increase one's understanding of those materials and allow one to present one's findings to others.

According to this perspective, the coding of categories occurs naturally "as you read the data, specific words, phrases, patterns of behavior, ways of thinking and events are repeated or highlighted" (BOGDAN; BIKLEN, 1994, p.221). Thus, the categories are constituted through processes of data convergence into major themes that support students to answer the research problem.

The analysis of the empirical material raised different categories of analysis, of which we discuss here the *tabular representation and transcription of data into the graph*.

Tabular representation and transcription of data into the graph

At the beginning of the lesson study, in the planning stage of the research lesson, several aspects were raised about the specific content that was intended to be developed in the class. When we discussed statistics, tables, and graphs, the early years teachers revealed some frequent difficulties in teaching this topic:

> **Teacher Luiza:** *I didn't work on graphics or statistics with them (Session 1, 9/2022).*
> **Teacher Mari:** *Yeah, we must work on it now, I think. Graphs and statistics will be [worked on] towards the end of the year. This is also the difficulty. They have a lot of difficulty putting together a graph and putting information into them. A table is quite difficult. You know, some teachers just skip the topic. I know that some teachers skip [this content] because of this, a content that I don't like, and I have a staple on those pages that will not be used. I know a lot of fifth-grade teachers who don't work on it. I always did, there were five years in 5th grades. So, there's no way you don't work [on the topic].*

Through these reflections, and concerned about the *previous knowledge* students should have to complete the task, the teachers highlighted some aspects that could interfere with the groups' work on the task:

> **Teacher Eva:** *Have they ever built a table? (Session 2, 10/2022)*

> **Teacher Luiza:** *Not with me. That's what I say, I'm going to start the problems with them so as not to be so [dissociated from what we're working on].*
> **Teacher Marcos:** *Also, starting from nothing and them [having to] build something they have no idea about, it's complicated, there's no way.*
> **Teacher Eva:** *I think that even if they didn't see it this year, at some point in previous years, they saw something related to tables and graphs.*
> **Teacher Mari:** *Yes, they have an idea what it is.*

Teachers reflected on the importance of providing students with opportunities to explore and construct graphs and tables but suggested that this introduction be linked to a problem situation. Thus, in the planning, when formulating the task statement that involved food, the participants were careful to choose foods that appeared in the students' surveys with their families, with varying amounts, enabling them to construct a graph with different "dimensions." By exploring varying quantities, teachers provided students with the possibility of *scaling in constructing the graph*.

> **Teacher Eva:** *Are we going to do it with scaling or unitary [referring to a directed or free scaling]? (Session 5, 11/2022)*
> **Teacher Marcos:** *I think we're going to scale because it has...17 is the largest. It can be every three or every two.*
> **Teacher Eva:** *3, 6, 9, 12, 15, AND 18. It's just that 5 is in the middle... well, we didn't scale, and all the values fall in the middle (laughs).*
> **Teacher Marcos:** *[If most use the scaling in every three] Most bars will fall on the [exact value of the axis] line.*

Then, the teachers observed that the concept of scale could complicate the task because the students had probably not yet formally studied this concept. Immediately, teachers used the textbook to confirm whether students had already reviewed the concept of scaling, even if it was through a simple activity or brief explanation about *scaling and its use in graphics*.

> **Teacher Eva:** *They have already done this [exercise]. If we could know how the other teacher worked on it with them... Did she mention that it was every five? If she warned them, they will know (Session 5, 11/2022).*
> **Teacher Marcos:** *And if she warned them that it has a title, that it has two axes, which are identified, a caption... did she warn them about all these points? And here, most of the bars fell out of line.*

Thus, after confirming that this concept had already been developed with the class in the classroom, the class study participants defined that the task would allow students to *choose the scaling* they wanted to use to solve it.

When solving the task during the research lesson, the students constructed the requested table and graph without needing the teacher's assistance, which the groups required only to check whether the information was correct and not to ask questions or doubts about how to

solve it. The following excerpt from the discussion in group 1 illustrates it:

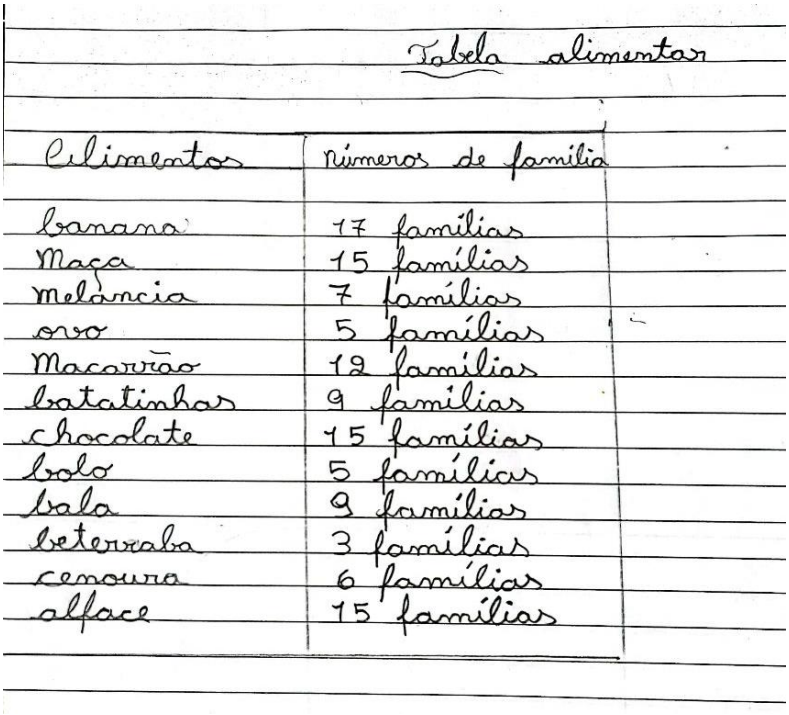
> ***Student 3****: Just tell me the foods, [for example] 17 families eat bananas, [you] just tell me bananas [just the food] (research lesson, 11/2022).*
> ***Student 1****: [It is ok]*
> *[...]*
> ***Student 3****: Now, here [pointing to the other column] are the quantities. Can you dictate [them] to me?*
> ***Student 1****: 17*
> *[...]*
> ***Student 2****: Teacher, is that so? Can we make the graph now?*
> ***Teacher****: That's right, but there's still one little thing missing. If someone comes to look at this table without seeing the problem, will they know where this information came from? [students think a little]*
> ***Student 1****: It's the title, of course!*

Group 2, in turn, discussed the *statistical tools* that would be used to systematize the information made available in the task statement, worrying about identifying the information.

> ***Student 2****: Can we make three tables? Let's do it like this, let's put "food" [referring to the title of the tables] (research lesson, 11/2022).*
> ***Student 3****: We can put "food" and "families" [indicating a title for each column].*
> *[...] [With the table ready]*
> ***Student 2****: What will we do now?*
> ***Student 1****: Now, it's the graph.*
> ***Student 2****: Then, let's organize!*
> ***Student 1****: No! It's already organized [in the table], now, let's just transfer [to] the graph.*
> *[....] [After constructing the axes]*
> ***Student 2****: Now, we need to organize the food.*
> ***Student 3****: NO, Student 2, [it's] already organized, just copy it.*
> ***Student 1****: Look here, Student 2, make a table to organize the data [pointing to the problem statement], we've already done it, so it's already organized.*

The concern with identifying information led them to define *titles for each column in the table* according to the nature of the information, as illustrated in the figure below, from the group 2 worksheet.

Figure 1- Food table



Source: Group 2 worksheet.

The discussion in group 3 reveals that the students mobilized *necessary previous knowledge, such as axes (horizontal and vertical), title of each axis and the graph scaling*:

> ***Student 2****: Did you hear the teacher? After the table, we can now go to the graph. The graph is very easy, we can make a line like this and one like this [horizontal and vertical], here [at the end of each axis] we put the quantities, and here the foods! We're going to make up to 12 [because there are 12 foods], which will work out just fine (research lesson, 11/2022).*
> ***Student 1****: Do it in pen.*
> *[...]*
> ***Student 2****: This goes up to 17, right? [about one of the axes]*
> *Student 3: Yes, 17 is the highest [quantity].*
> *[...]*
> ***Student 1****: Teacher, look here!*
> ***Teacher****: That's right, but now they think: if someone looks at this graph, will they know what it's about?*
> *Students: [look at each other] I think they will.*
> ***Teacher****: Will they? If the person looking doesn't have the problem [in hands] and just looks at this page here [with the graph], will they know what it means?*
> *[The students reflected] [...]*
> ***Teacher****: Guys, let's say this graph was stuck on the wall, so [without the problem, just the graph page], can I know where this graph was constructed? Or who was researched here?*
> ***Student 2****: We got things here [in the problem].*
> ***Teacher****: I can see that for bananas, there is [number] 17, but 17 what?*
> ***Student 2****: Now, I understand.*
> ***Student 3****: I do, too.*
> ***Student 2****: You must put centimeters.*

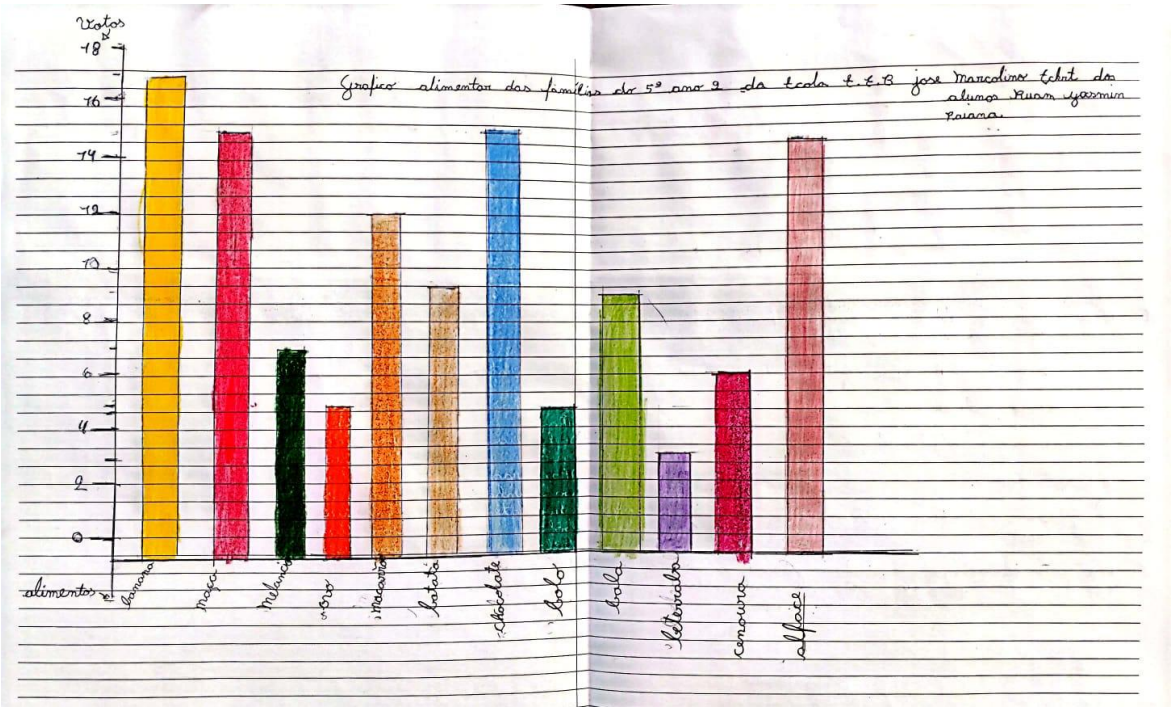
Student 3: *No! You have to put a title.*
Student 1: *Yes, true!*
[...]
Student 2: *Teacher, we put: quantity of fruit people eat in their homes.*

Group 2 also discusses before creating the graph. Students shared their strategies, their ideas on how to build the graph, watching *the measurements (maximum and minimum length) of the axes*:

> ***Student 2***: *So in this case, on the graph, we can write: sweets, fruits, food, and number of families? (research lesson, 11/2022).*
> ***Student 1***: *No. It's like this, look, food (pointing horizontally) and families like this (pointing vertically). Got it?*
> ***Student 2***: *So the food (vertical) and so the families (horizontal)?*
> *Student 1: No, on the contrary. Food (horizontal) and families (vertical)*
> *[....]*
> ***Student 1***: *But, what changes? Families and food or food and families?*
> ***Student 2***: *True.*
> ***Student 3***: *I think you put food here [referring to the horizontal axis] and families here [referring to the vertical axis].*
> ***Student 2***: *We have to think that the graph, in this part here, will grow... remember the little lines? We are going to make it every...? Every two, four [referring to scaling]*
> ***Student 1***: *To make it look nice, let's leave a blank line in the middle of each one.*
> ***Student 2***: *No, you cannot [do it] on the graph.*
> ***Student 3***: *Let's do it every two.*

The students mobilized knowledge about the axes (horizontal and vertical). They realized from the group discussions that it does not matter on which axis we place each piece of information. Furthermore, they found that depending on the information to be represented, the axis becomes larger (longer), and deliberately, due to the space on the sheet, they chose on which axis they would place the quantities since the axis with the numbers would need a space bigger (*We have to think that the graph, in this part here, will grow*).

Figure 2. Graph: foods most consumed by families in the class



Source: Group 2 worksheet.

> *[...]*
> ***Student 2****: Guys, this looks very similar to a graph (with a surprised - impressed expression) (Research lesson, 11/2022).*
> ***Student 1****: Of course, this is a graph.*

Contrary to what the participants in the lesson study considered at the beginning of the planning stage, that students might not know how to construct graphs because statistics curriculum topics are generally later in the year, the groups carried out the requested representation and identified it as a graph, indicating that there was learning related to the *elements that make up a graph*.

> **Teacher Luiza:** *I didn't work with them on graphics, nor statistics. (Session 1, 9/2022)*
> **Teacher Mari:** *Yeah, we must work on it now, I think. Graphs and statistics will be [worked on] towards the end of the year. [...]*

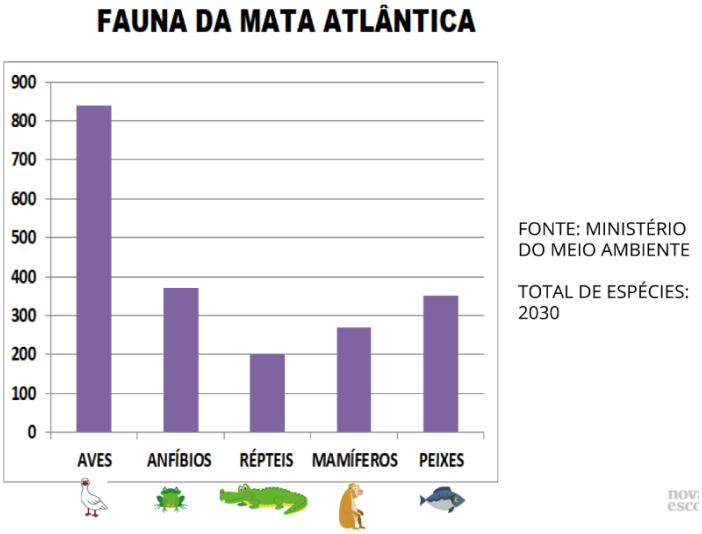
Another important analysis that occurred in some groups was about the need (or not) to *put a border (frame) on the chart*. The discussion in group 2 illustrates this aspect:

> ***Teacher****:[Guys, we have] 10 minutes left [to break] (Research lesson, 11/2022).*
> ***Student 1****: That's great, we'll be able to finish it, we just need to paint [it].*
> *Student 3: Close it on the side and I'll get the pencils.*
> ***Student 1****: No. We cannot close [the chart area].*
> ***Student 3****: No?*
> ***Student 2****: No, we don't close the graph, it's just like that, it stays open.*

***Student 3**: Are you serious? I didn't know that.*

This discussion allowed students to reflect on the graph design and how to finish it, considering the possibility of *inserting a line to outline it* (a type of frame). Some groups concluded that it was unnecessary to insert a border in the graph because in textbooks, newspapers, and media, the graphs are typically represented with only the two axes (horizontal and vertical) and the other essential elements of a statistical graph. In some cases, the graphs appear with a border, an outline including the axes themselves, as shown in the following example:

Figure 3. Example of a graph with a border (contour)



Source: New school.

After the research lesson, during reflection, the teachers raised and debated some of these considerations about students' knowledge. The participants' reflections reveal that the students identified the missing information as soon as they were questioned, showing that they had learned aspects related to the properties of the graph, such as, for example, the title and the axes:

> **Teacher Marcos:** *As the graphic construction content had already been worked on in the class, they demonstrated that they knew some of the steps, but ended up forgetting to include identifications such as titles and categories (axes). When they were asked whether it was possible to understand the graph as it was, they concluded that the title was needed.*
> **Teacher Eva:** *Many had this difficulty. I said: if someone looks at this graph, will they know what it is? Oh, the title, they immediately got the idea (Session 7, 11/2022)*

From the first sessions of the lesson study, teachers showed a concern with statistics,

mainly with their difficulties as teachers with this content. From that initial moment, planning and lesson study sessions were developed to minimize those difficulties. To this end, the study of statistical concepts was prioritized, and students' previous concepts and the necessary knowledge to develop the tabular representation and the construction of the graph related to the research lesson task were predicted. Due to this concern, the teachers observed the statistics activities that the class had already developed, and the construction of the task considered these aspects already developed by the students. This care helped students feel confident in solving the task, enabling them to delve deeper into statistical concepts, such as table titles, axes, and scaling.

The lesson study, concerning tabular representation and data transcription into the graph, mobilized students' statistical concepts, such as organization of data in the table, column titles, graph scale, graph title, axes, measurement and length of the axes, and information source. To construct the graph, students could use a ruler, take measurements, and discuss strategies, for example, on which axis (horizontal or vertical) each piece of information would be best represented, the length of the axes, and how they could place the measurements (the best scale) on the axis so that the space occupied by the axis was sufficient.

Discussion

To carry out the tabular representation and construction of the food graph, students mobilized previous statistical concepts and properties, categorized data, represented data in tables and graphs, identified relevant information, and established relationships and connections between concepts (CAMPOS et al., 2011). Furthermore, students discussed the best way to represent data in the graph, therefore promoting statistical reasoning (GARFIELD, 2002).

After engaging with the task, carefully designed to address graph and table concepts, students learned statistics regarding the concepts of axes, scale, and title (CAMPOS et al., 2011; ALBUQUERQUE, 2018). The groups categorized the information in the task statement, organizing it in a table composed of two columns, naming them foods and quantities (number of families that indicated each food). Transcribing this data into the graph allowed students to mobilize prior knowledge, such as ascending order, scale, axes, measurement (length) of axes, titles, notions of laterality (horizontal and vertical) and size (space on the sheet) occupied through the graph to determine what information should be indicated in the graph, as well as identify the information in each column of the table that would be represented on the horizontal

and vertical axes of the graph.

Scale is a notion to be constructed in the early years, but this path presents several obstacles (POMMER; POMMER, 2018). Based on this premise, the teachers first checked whether the class in which the research lesson would be held had already seen this notion. Since the students had this previous knowledge, the task allowed the groups to choose the scale they would use in constructing the graph. Most groups used the unitary scale, which is what generally appears in textbooks and activities in the early years.

In the lesson study, students discussed the issue of titles on each axis and in which position (horizontal and vertical) they looked best, thinking about the space they had for construction (CAMPOS, 2011). Therefore, the research lesson favored the development of statistical reasoning, specifically reasoning about data representation, as it gave students the opportunity to understand how graphs can be represented in different ways to better communicate information.

The discussion of these strategies for constructing the graph provided the development of statistical reasoning. The types of statistical reasoning pointed out by Campos et al. (2011, p. 481 apud GARFIELD, 2002) and developed by the task in the lesson study are reasoning about data: recognizing and categorizing data and using appropriate forms of representation, and reasoning about data representation: understanding how graphs can be modified to better represent data.

Therefore, the task and dynamics of the research lesson enabled students, working in groups, to analyze a set of data previously constituted by them, analyzing them, representing them in the form of tables and graphs, and communicating results (CAZORLA et al., 2017) and conclusions. Thus, they performed statistical learning as they had the opportunity to explore a set of data, mobilizing previous knowledge and attributing meaning to the results.

Conclusion

The research was systematized around the question: What statistics learning is carried out by 5th-grade students from a lesson study? Thus, we seek to highlight and discuss the learning of early years students about statistics through a lesson study. The research involved a 5th-grade class from a public school in Pinhalzinho, Santa Catarina.

One aspect to highlight concerns the group discussion about the task and what it asks of students. Due to the dynamics of the research lesson, the strategy of having students work in groups favored this interaction. This was one of the points underscored by the teachers in the

final interview, in which they were asked to highlight the possibilities of the research lesson of the lesson study to promote student learning in statistics topics. The teachers reported that the discussion between the group members raised many questions, and this exchange of information, the discussion of points of view, and the negotiation of results constituted a fruitful learning space.

From this perspective, lesson studies can develop and promote student learning, contribute to professional education, or minimize teachers' difficulties. With access to digital media, information began to arrive quickly and early. Hence, developing statistical concepts and critically looking at all this information from the early years is essential. Furthermore, developing these statistical notions and concepts contributes to subsequent years since data, graphs, and their interpretation can occur in several disciplines with the most diverse subjects.

However, there is a lot to be done. Statistics is a large area, and its concepts can be investigated individually, as they are all essential for analyzing information and developing critical thinking. The notions of scale, graphic design, and other specific points must be deepened individually for a more concise and precise result.

Student learning must be a constant concern for teachers, just as it is in the context of lesson studies, where it constitutes a starting point for developing the educational process. This gaze from teachers licensed by area working in the final years of elementary school towards the years that precede it, the early years of elementary school, can contribute to the transition of these students, who already feel insecure about their changing schools (in most cases), a higher number of teachers, and classes lasting 45 minutes, among other changes that may surprise them. This aspect was considered by the participants in the post-class reflection session when they stated: "This experience was very important because they were able to understand that the dynamics of the school in the initial years are different and that the teacher in the final years often does not know their students and their difficulties well. And the lesson study enables this closer look at students" (Researcher's field notes, Nov. 2022). Understanding what learning is and how it is developed, the teaching strategies used, and the previous knowledge that can be used can minimize this rupture and reduce students' aversion to mathematics and to studying.

We consider that the research contributed to discussions in the area. Lopes (2008) states that teaching statistics must occur from the earliest years for a broader understanding of the problems that surround our reality since changes happen quickly, and knowledge in this area is essential to encourage decision-making.

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