revista PARANAENSE DE EDUCAÇÃO MATEMÁTICA

THE PLANNING OF MANIPULATIVE MATERIALS IN THE CONTEXT OF LESSON STUDY IN CONTINUOUS TEACHER TRAINING FOR INCLUSIVE MATHEMATICAL LITERACY

DOI: https://doi.org/10.33871/22385800.2023.12.29.168-187

Davi Cézar da Silva[1]
Clodis Boscarioli[2]
Arthur Belford Powell[3]

Abstract: With the goal of exploring aspects of planning based on Lesson Study methodology to fit into the stages of developing, aligning, or adapting, and applying manipulative materials for inclusive mathematical literacy, we offered a continuing education training for teachers. Guided by the research question: what aspects of teacher training on manipulative materials in the context of inclusive mathematical literacy did [or did not] contribute to teacher learning? We proposed a workflow that, at each stage of planning, provides theoretical foundations related to the curricula and specific topics of mathematics, as well as practical guidelines for the moments of planning and adaptation of manipulative materials with the help of a specialist in special education. These steps comprised our data production for the research. From the results, we report on the work developed by two teacher groups, respectively dealing with the concepts of fractions and number construction, targeting Early Years and Early Childhood Education students. We understand that the workflow provided participating teachers with autonomy and confidence, culminating in the elaboration, alignment, and adaptation of potentially inclusive manipulative materials that were used by the entire class, a characteristic we consider essential for materials with this focus. The participating teachers were provided with new knowledge about mathematical content, curriculum, and ways of approaching the classroom that can improve teaching and learning in their future professional practice.
Keywords: Early Childhood Education. Early Years. Inclusive Mathematical Literacy. Lesson Study.

O PLANEJAMENTO DE MATERIAIS MANIPULATIVOS NO CONTEXTO DA *LESSON STUDY* EM UMA FORMAÇÃO CONTINUADA DOCENTE PARA UMA ALFABETIZAÇÃO MATEMÁTICA INCLUSIVA

Resumo: Com o objetivo de investigar os aspectos de um planejamento, com base na metodologia *Lesson Study*, para sua inserção nas etapas de desenvolvimento ou direcionamento, adaptação e aplicação de materiais manipulativos para uma Alfabetização Matemática Inclusiva ofertamos, como parte de uma pesquisa de doutorado, uma formação continuada de professores. Pautados pela questão de pesquisa: quais aspectos de uma formação de professores em materiais manipulativos no contexto da com vistas a Alfabetização Matemática Inclusiva contribuíram [ou não] para a aprendizagem docente? Propomos um fluxo de trabalho que previu, em cada etapa do planejamento, produzir embasamentos teóricos em relação aos currículos, aos assuntos específicos da Matemática, e ainda, orientações práticas para os momentos de planejamento e adaptação dos materiais manipulativos com o auxílio de uma profissional da Educação Especial, etapas essas que compuseram a produção de dados para a pesquisa.

[1] PhD in Science Education and Mathematics Education at the Western Paraná State University, Cascavel, Paraná, Brazil. Professor at the Instituto Federal Catarinense, Videira, Santa Catarina, Brazil. E-mail: davi.silva@ifc.edu.br – ORCID: https://orcid.org/0000-0002-7655-382X.
[2] PhD in Electrical Engineering at the São Paulo University. Associate Professor at the Western Paraná State University, Cascavel, Paraná, Brazil. E-mail: clodis.boscarioli@unioeste.br – ORCID: https://orcid.org/0000-0002-7110-2026.
[3] PhD in Mathematics Education at the Rutgers University. Associate Professor of Mathematics Education in the Department of Urban Education at the Newark Campus of Rutgers University (New Jersey). E-mail: powellab@newark.rutgers.edu – ORCID: https://orcid.org/0000-0002-6086-3698.

Dos resultados, relatamos os trabalhos desenvolvidos por dois grupos que abordaram, respectivamente, sobre os conceitos de frações e construção do número, direcionados a alunos dos Anos Iniciais e da Educação Infantil. Entendemos que o fluxo de trabalho gerou autonomia e confiança nos professores participantes, culminando na elaboração, direcionamento e adaptação de materiais manipulativos potencialmente inclusivos, que foram utilizados por toda a turma, característica essa que consideramos imprescindível em materiais com esse foco. Foram possibilitados novos conhecimentos aos professores acerca dos conteúdos matemáticos, dos currículos e de possibilidades de abordagens em sala de aula, o que pode culminar em melhorias no processo de ensino e aprendizagem, por meio de suas práticas profissionais futuras.
Palavras-chave: Educação Infantil. Anos Iniciais. Alfabetização Matemática Inclusiva. *Lesson Study*.

PLANIFICACIÓN DE MATERIALES MANIPULADORES EN EL CONTEXTO DEL ESTUDIO DE LECCIONES EN LA FORMACIÓN CONTINUA DE PROFESORES PARA LA ALFABETIZACIÓN MATEMÁTICA INCLUSIVA

Resumen: Con el objetivo de investigar aspectos de planificación, basados en la metodología Lesson Study, para su inserción en las etapas de desarrollo, dirección o adaptación y aplicación de materiales manipulativos para la Alfabetización Matemática Inclusiva, ofrecemos formación docente continua. Guiado por la pregunta de investigación: ¿qué aspectos de la formación docente en materiales manipulativos en el contexto de la Alfabetización Matemática Inclusiva contribuyeron [o no] al aprendizaje docente? Proponemos un flujo de trabajo que prevé, en cada etapa de la planificación, producir fundamentos teóricos en relación con los planes de estudio, materias específicas de Matemáticas, y también, con pautas prácticas para la planificación y adaptación de materiales manipulables con la ayuda de un profesional de Educación Especial. Estos pasos comprendieron la producción de datos para la investigación. A partir de los resultados, se reporta el trabajo desarrollado por dos grupos que abordaron, respectivamente, los conceptos de fracciones y construcción de números, dirigidos a estudiantes de Educación Infantil y Educación Infantil. Entendemos que el flujo de trabajo generó autonomía y confianza en los docentes participantes, culminando en la elaboración, dirección y adaptación de materiales manipulativos potencialmente inclusivos, los cuales fueron utilizados por toda la clase, característica que consideramos esencial en materiales con este enfoque. Se brindaron nuevos conocimientos a los docentes sobre contenidos matemáticos, planes de estudio y posibles enfoques en el aula, que pueden, a través de sus futuras prácticas profesionales, conducir a mejoras en el proceso de enseñanza y aprendizaje.
Palabras clave: Educación Infantil. Primeros años. Alfabetización matemática inclusiva. Estudio de la lección.

Introduction

When we think about mathematics teaching, at whatever level, researchers' concern regarding the teaching and learning process is notorious, as there are several challenges for this teaching. As Borba, Almeida, and Gracias (2018) noted, there is a general lack of discussions in undergraduate courses that address teaching challenges. This contrasts with the reality of research projects conducted in graduate programs, both at the master's and doctoral levels, in the teaching area.

For Gatti (2010), the curricular complexity of an undergraduate teaching course is great, and the tasks of the future professionals are broad. In this sense, we can state that there are

several concerns regarding the training of teachers who will teach mathematics in early childhood education, including the different tasks for which they are qualified, since they are responsible for working on the subject matters of the curriculum and fulfilling administrative functions, which can lead to gaps in the training of these future professionals.

In addition, there are the comments of Alves (2021), who, analyzing some pedagogical projects of degree programs in Pedagogy through units of analysis, also identified gaps in the inclusion of students in the regular teaching network in the training of these professionals, some aspects of which are highlighted, such as discussions about inclusion that appear isolated in the disciplines, without coherence and articulation with other inclusive aspects of policy documents, without transversality of topics in relation to discussions about special and inclusive education. Also, future teachers lack direct contact experiences with students with special educational needs, as well as knowledge about adaptations, methods, or practices to provide learning opportunities to special-needs students, among others. Borba, Almeida, and Gracias (2018) point out that addressing such deficiencies in the initial training of professionals would lead to an increase in their course workload, making such interventions unfeasible.

Concerning mathematics teaching, some authors note the importance of contextualized approaches so that mathematics educators can justify such teaching based on the everyday situations of students, which is our goal in this study. D'Ambrósio (1996, p. 7) has already reported on this aspect:

> I see the mathematical discipline as a strategy that the human species has developed throughout its history to explain, understand, deal with, and live with sensory, perceptual reality, and of course with its conception within a natural and cultural context.

Emphasizing social features, Ferronato (2002) already pointed out that when working with students on mathematical knowledge, we must provide for relationships with the students' social context. Sadovsky (2010) classifies the mathematical contexts addressed in the problems as outside mathematics (extramathematical) or inside mathematics (intramathematical). For that author, we can conduct instruction in different ways and thus move between contexts depending on the teacher's goals, i.e., using everyday examples to model a situation mathematically or using situations in which the mathematics already internalized by the student serves as the basis for developing the content to be covered.

To address these aspects, we understand that manipulative materials are teaching tools that can be used to teach mathematics in a way that accommodates student diversity and can be used for approaches in both intramathematical and extramathematical contexts.

Those ideas motivated us to explore aspects of instructional planning based on the Lesson Study methodology. We incorporated the phases of development, adaptation, and application of manipulative materials for inclusive mathematical literacy in a teacher training course, through an extension course in the Municipal Education Network in the city of Videira, Santa Catarina State, Brazil.

To define the term inclusive mathematics literacy, we start from the perspective of inclusive mathematics education, following the assumption of Nogueira *et al*. (2019, p. 7) as they point out that the researchers of SBEM's Working Group (WG) 13: Difference, Inclusion and Mathematics Education aim for a mathematics education for all "in which the particularities associated with the mathematical practices of diverse learners are valued and understood, rather than forgotten, ignored or even considered illegitimate". Danyluk (2015, p. 15) further expands the definition of mathematical literacy:

> [...] the term mathematical literacy does not only refer to children, in early childhood education or in the first years of life. We consider a person mathematically literate when they can perform the act of reading mathematical language and find meaning. And writing leads to an existential understanding and interpretation being developed, fixed, and communicated through recording. To be mathematically literate, then, is to understand what one reads, what one writes, and what one understands about the first concepts of arithmetic, geometry, logic, and algebra, among other important topics for building solid knowledge in the field.

Based on these foundations, we have incorporated Silva's (2023) perspective as expressed in the following:

> Inclusive mathematical literacy is that which enables learning in which students build knowledge in different areas of mathematics (numbers, algebra, geometry, magnitudes and measures, and probability and statistics) through mathematics instruction that considered on diversity and is based on different methods and technologies such as manipulative materials and digital technologies as facilitators of a teaching and learning process that respects and includes differences (p. 31).

The purpose of this article[4] is to present a proposal for planning lessons with manipulative materials from the perspective of inclusive mathematics education based on the Lesson Study methodology, as well as reflections on the results of its implementation.

It should be noted that in Silva's thesis (2023), from which this article is an excerpt, the analyzes were based on teacher learning, a fact that justifies not addressing student learning.

[4]This article is an extended version of a text in Portuguese entitled "The planning of manipulative materials for inclusive mathematical literacy in the context of Lesson Study in continuing teacher education" presented at *the II Seminário Internacional de Lesson Study no Ensino de Matemática* (SILSEM 2023).

These analyzes were carried out through observations of audio and video recordings, through the participation of the first author as a researching member of the groups, and through the analysis of the narratives of the teachers participating in the training. In the sequel, the characterization of the formation and the configurations of the groups, the proposal of the workflow, the manipulative materials planned during the training and some considerations and perspectives of the research are presented.

Teacher knowledges and Skills

We have identified aspects related to teaching knowledge in the work phases carried out during the training, which have been addressed and defined by Schön (2000) and Tardif (2012).

Schön (2000) notes two sources of knowledge that permeate teachers' work. The first refers to rigorous professional knowledge based on technical rationality, and the second characterizes beyond that knowledge, the awareness of zones of indeterminate practices. Specifically, Schön (2000) states, “It is not through technical problem solving that we transform problematic situations into well-defined problems; on the contrary, it is through naming and conceptualizing that technical problem solving becomes possible” (p. 16). We know that in the teaching activity we do not have an immediate answer to all the students' questions, these situations that characterize what the author is talking about.

According to Schön (2000), to solve situations that go beyond purely technical knowledge, it is necessary for the teacher to automate situations by making them commonplace and calling such skills as performance. Through these actions, the teacher acquires action knowledge, which the author calls professional artistic talent. With this talent, teachers can solve uncertain, unique, and contradictory situations in the school environment.

> The professional's knowing-in-action process has its roots in the socially and institutionally structured context shared by a community of professionals. Knowledge in action is exercised within the institutional framework of the profession, organized by its characteristic units of action and its familiar types of practice situations, and constrained or facilitated by its shared body of professional knowledge and its system of valuations (SCHÖN, 2000, p. 37).

With regard to teaching knowledge, Tardif (2012, p. 16) claims that

> [...] they are a social reality materialized through training, programs, collective practices, school subjects, institutionalized pedagogy etc., and they are at the same time his knowledge”. In this sense, the author justifies that he tries to locate this knowledge in the relationship between the individual and the social.

According to this author, the relationship between knowledge and teachers in practice is established through different relationships, defining this teaching knowledge as plural knowledge built through professional training and disciplinary, curricular and experiential knowledge. The author presents four types of teaching knowledge:

- The knowledge of professional training (from educational sciences and pedagogical ideology): This is the knowledge that is taught by teaching institutions in both initial and continuing education and that is incorporated into teaching practice. This practice is not only the result of educational sciences, but also of pedagogical knowledge, which are concepts resulting from reflection on pedagogical practice linked to the knowledge of educational sciences.
- Disciplinary knowledge: These are the various disciplines covered in teacher education institutions, and that are also integrated into classroom practice.
- Curricular knowledge: school programs and curricula that, through their objectives, content, methods, etc., provide pedagogical guidelines for teaching practice.
- Experiential knowledge: Knowledge developed through the teacher's own practice. “This knowledge springs from and is validated by experience. It flows into individual and collective experience in the form of habitus and skills, knowledge, and ability.” (TARDIF, 2012, p. 39).

The characterization of the offered teacher training

The teacher training involved Lesson Study, assigning to manipulative materials an important role in the inclusion of students with disabilities in mathematics classes. We chose Lesson Study methodology, as we understood it to focused on the collaborative groupwork throughout the planning (construction or alignment and adaptation of materials) and allow the application in the classroom through one of its phases, validating these materials. In addition, it is possible to increase the specific knowledge of mathematics throughout the process, in the case of this training about fractions and number construction, through the study of the national guiding documents and through different approaches in individual studies and in group developed during the Lesson Study meetings.

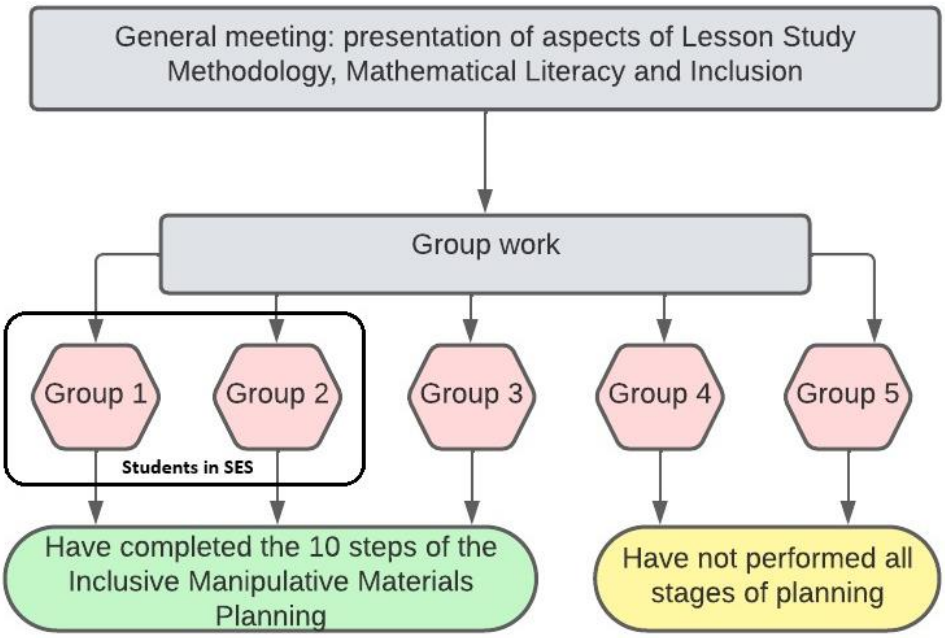
According to Macedo and Bellemain (2016), in the history of the emergence of Lesson Study, works involving different professionals with different knowledge formed groups of works with such collaborative features. In this sense, it is noteworthy that groups in training

were constituted as collaborative throughout the work from the perspective of Fiorentini (2004, p. 50), who points out:

> In collaboration, everyone works together (co-laboration) and supports each other to achieve the goals negotiated by the collective of the group. In collaboration, therefore, relationships are usually non-hierarchical, with shared leadership and co-responsibility for carrying out actions.

Figure 1 shows the organizational structure of the training. It was attended by 22 teachers from the municipal education network of Videira/SC, who formed 5 working groups.

Figure 1: Flow of continuing teacher education



Research: Translated from Silva (2023, p. 63).

In this article, we present the plans of two groups that completed the stages foreseen in the workflow and had students in Special Educational Service (SES) in their classes. The meetings occurred during 2021, mostly in a remote format, but the planning of the materials and the application of the classes took place face-to-face.

The manipulative materials were used as part of classroom work in training, without the goal of analyzing student learning in the research, but rather teachers' perceptions of it. The application was made in classes with students who attended the SES of the schools and had a medical report.

The created plans were carried out in two schools by teachers who participated in the training and worked in these schools. The first author participated as a member in all groups,

as shown in Table 1. Although the adaptations of the manipulative materials were made with the aim of meeting the specificities of the students in the AEE, the classes were planned to develop using these materials in the whole class, a fact that, according to Skovsmose (2019, p. 26), "is a direct way of interpreting inclusive mathematics education as an encounter between differences".

Table 1: Composition of working groups

Group	**Participants**	**Mathematical contents in the Lesson Study proposal**
1	2 regent Pedagogue teachers. 2 undergraduate students in Pedagogy. 1 Mathematics teacher (guest teacher who, in addition to participating in the group, provided training in Problem Solving). The main author researcher.	Fractions
2	1 School director. 2 regent Pedagogue teachers. 1 Assistant teacher. The main author researcher.	The construction of the number
Guest	1 teacher with a specialization in SES, who participated in the consultation on the adaptation of the material and proposals for the implementation of the lessons.	-

Source: Prepared by the authors based on the configuration of the training working groups (2023).

Below we present the workflow proposed in Silva (2023) for preparing or directing inclusive manipulative materials.

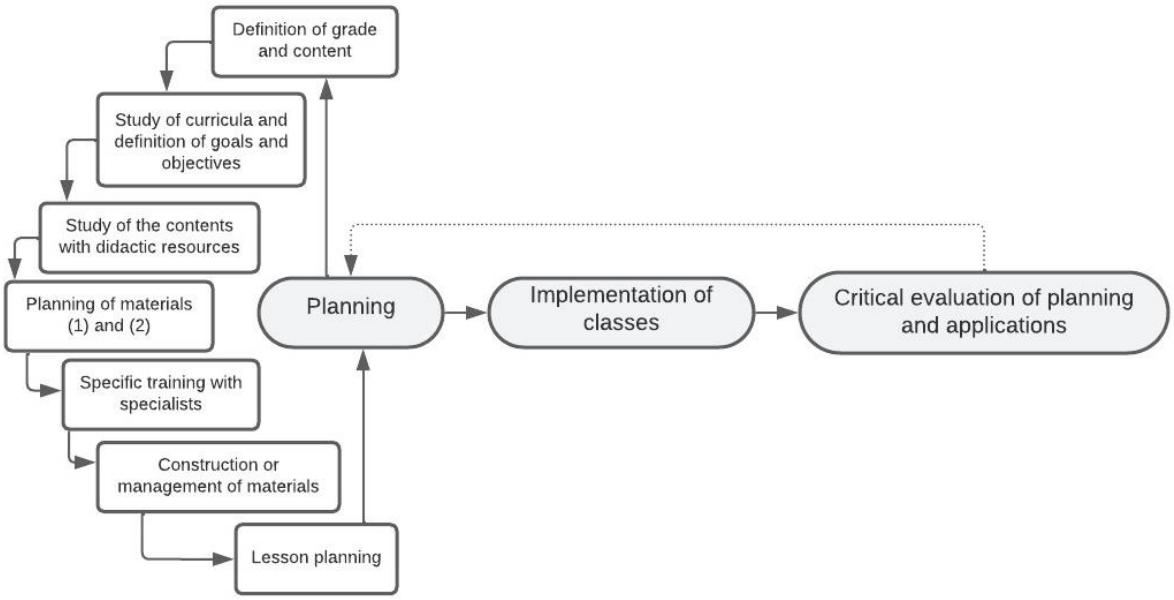
The workflow proposal

Based on the Lesson Study methodology, we propose the use of a planning workflow designed to construct, guide, or adapt manipulative materials for use in classes with students in SES. The workflow can be used to create any teaching materials with the same characteristics. That is, it can be adapted and used by different subjects in the curriculum that aim to construct, direct, or adapt manipulative materials or not, depending on the reality or demand of professionals in their educational context.

Regarding the instructional cycle, we kept the three main phases according to Gaigher, Souza, and Wrobel (2017): Lesson Planning, Lesson Execution, and Critical Reflection on Teaching. For our proposal, we saw the need to systematize the work by adding sub-steps to

the planning, thus building the 10-step workflow shown in Figure 2.

Figure 2: Flow for preparing or directing inclusive manipulative materials



Research: Translated from Silva (2023, p. 75).

In this planning, manipulative materials are central elements in the creation of lesson plans. The sequence has been proposed to meet what we believe are the necessary requirements for theoretical and practical foundations for the construction or direction and adaptation of manipulative materials, for the development of the lesson plan, and for the conduct of classes. It also provides important support for critical reflection throughout the planning process. In the following, we describe a summary of the objectives foreseen at each phase of the workflow, according to Silva (2023)[5]:

1) Determine the grade level and the content in which the activities will be developed: In this phase it is necessary for the group to get to know each other by making personal and professional presentations. It is also important to make notes about the challenges related to the teaching process and learning, so that the group can determine the grade level and a general content to be explored.
2) Study the curricula and determine the objectives to be studied and discussed: It is important that each member study one or more documents. However, it is necessary that the whole group has done individual readings to optimize the discussions and to identify convergences or divergences between the documents.

[5] The description of each phase with objectives, preparation, description, and recommendation for practice is available in Portuguese in Silva's PhD thesis (2023).

3) Investigate studies on the content to be covered by different teaching tools: It is important that each member present a text so that different methods with the use of different teaching tools can be identified so that different ways of approaching such tools can be observed.
4) Plan for the manipulative materials to be used: first phase to discuss the manipulative materials to be prepared or instructed. Each member can present his ideas so that the members can discuss their impressions to define the proposals.
5) Complete the planning of the materials to be used: The materials to be developed must undergo detailed planning. This includes analyzing the topics of the selected content, identifying the areas that can be covered, and evaluating whether these materials meet the requirements for the content to be covered. Although an extension of the previous phase, this phase does not involve the creation of the materials themselves, but rather focuses on the development of an even more detailed plan.
6) Identify and organize adaptations of materials to the specifics of classes with students served by SES: At this stage, it is important that participants have access to rationales and information about adaptations of materials so that they can use them in their future professional practice.
7) Build the manipulative materials or, if they are existing materials, route them with the previously established adjustments: at this stage, the group can decide whether to do them together or to form teams and give each person responsibility for one or more materials.
8) Plan the activities of the class(es) and prepare the lesson plan based on the materials created: In this phase, the lesson plan is created based on the materials that will be used. The group can insert new materials as they have the autonomy to make new adjustments, considering the comments of the inclusive education specialist.
9) Conduct the planned lesson(s) using the materials developed: The lead teacher must conduct the lesson with the plan in hand to agree on the steps planned. The other members must also plan to review the progress of the class and record the information they feel is necessary about student learning, including lessons learned, i.e., what "worked and what didn't".
10) Critically evaluate the planning and applications: the group must organize their notes and perceptions and, if the lesson was recorded, attend the evaluation session in advance to discuss it. All assessment must be done critically, paying attention to every detail of the planning. If necessary, the group can indicate changes to the plan so that it can be

made available to another teacher.

Below we describe the materials that were planned and used by the two working groups.

The planned manipulative materials

In Group 1, the planning included the concepts of rational numbers to introduce the topic of fractions aimed at students in the 4th year of the first elementary grades. The activities were designed to not only introduce concepts but also reinforce and review content.

The lessons took place on 11/23/2021 and 11/25/2021 and totaled 8 lessons. The lesson plan developed was titled: Fractions using the geometric figures that make up the Tangram. Through the activities, the group aimed to address the possible relationships between the areas of the seven parts that make up the tangram, with the goal of developing the concept of fractions and the equivalence of fractions presenting the nomenclatures through reading, writing, and algebraic and geometric representations. The materials are shown in Figure 3.

Figure 3: Manipulative materials for working with fractions

Source: Materials present in group plans, Silva (2023).

The manipulative materials used were:

1) Tangram: Introduction to the concept of fractions through comparisons between the faces of the figures that compose it. Tangram is an ancient geometric puzzle of Chinese

origin. It consists of seven pieces (two small triangles, one medium triangle and two large triangles, one square and one parallelogram).

2) The fraction machine: the fraction machine was built from polyvinyl chloride tubes, polyethylene terephthalate bottles, and wooden rods. The goal was to review the content so that students could review writing fractions in mathematical language as well as their geometric representations and complete reading. In the first rod, there are fraction representations in mathematical writing, that is, by a numerator and a denominator, representing the division between two numbers with a non-zero denominator. In the second bar, the fractions are represented in geometric form, and in the third bar, the fractions are written whole. To work with the material, the student rotates the first rod and looks for equivalent representations on the other two rods.
3) Equivalent Fraction Foam: The material was used with the aim of introducing the idea of equivalent fractions as well as the geometric representations of fractions and to compare the differences between measurements, for example between $\frac{1}{2}$, $\frac{1}{3}$ e $\frac{1}{4}$.
4) The Path of Fractions Game: The Path of Fractions game was made on Paraná cardboard and aims to review the names of fractions and their algebraic representations. The goal of the game is to fill the path between two tables (purses) with the parts indicated when rolling the dice. When the parts run out, the students must replace them with others, making comparisons by identifying the corresponding fractions. They were also able to check the dimensions of each piece in practice, where $1 < \frac{1}{2} < \frac{1}{3} < \frac{1}{4} < \frac{1}{5} < \frac{1}{6}$.

Group 2 developed plans to work on concepts about number construction with preschool students (Pré I).

On 09/25/2021, the water fountain was built in the schoolyard with the participation of some parents of students, the school administration, some teachers who did not attend the training, members of the Parents and Teachers Association and the agronomy teacher of IFC. Below, in Figure 4, are some records of this construction process.

Figure 4: Fountain construction process at school

Research: Silva (2023, p. 99).

Classes were held on Oct. 20, 21, 22, 26, and 27, 2021, for a total of 10 hours. The lesson plan was entitled: The fountain of knowledge and the process of number construction of early childhood education students were designed with the aim of stimulating the development of language and mathematical knowledge through activities that explored the knowledge of animals and plants, the promotion of the relationship between family and school, values for life in society, oral language, listening, interactions through vocabulary, gestures, and movements. They also worked with natural elements (water, sun, air, and earth) and gross and fine motor coordination, etc.

For the activities, two moments were organized, guided by the construction of a fountain (with the use of fish) in the schoolyard. The first was related to the choice of the name for the fountain in which they were used: Form with instructions for families and recording the proposed names (Figure 5), ballot box and place for voting and choosing the name of the fountain (Figure 6), fish of two colors for the registration and counting of votes, poster of brown paper with the names for voting (Figure 7), wooden board to decorate the chosen name and record the name of the fountain (Figure 8).

Figure 5: Organization of the names proposed by the families for the fountain

Research: Silva (2023, p. 101).

Figure 6: Ballot box for voting on the name of the fountain

Research: Silva (2023, p. 101).

Figure 7: Registration of vote counting activity

Research: Silva (2023, p. 102).

Figure 8: Fountain built at school and simulation of the fish life cycle

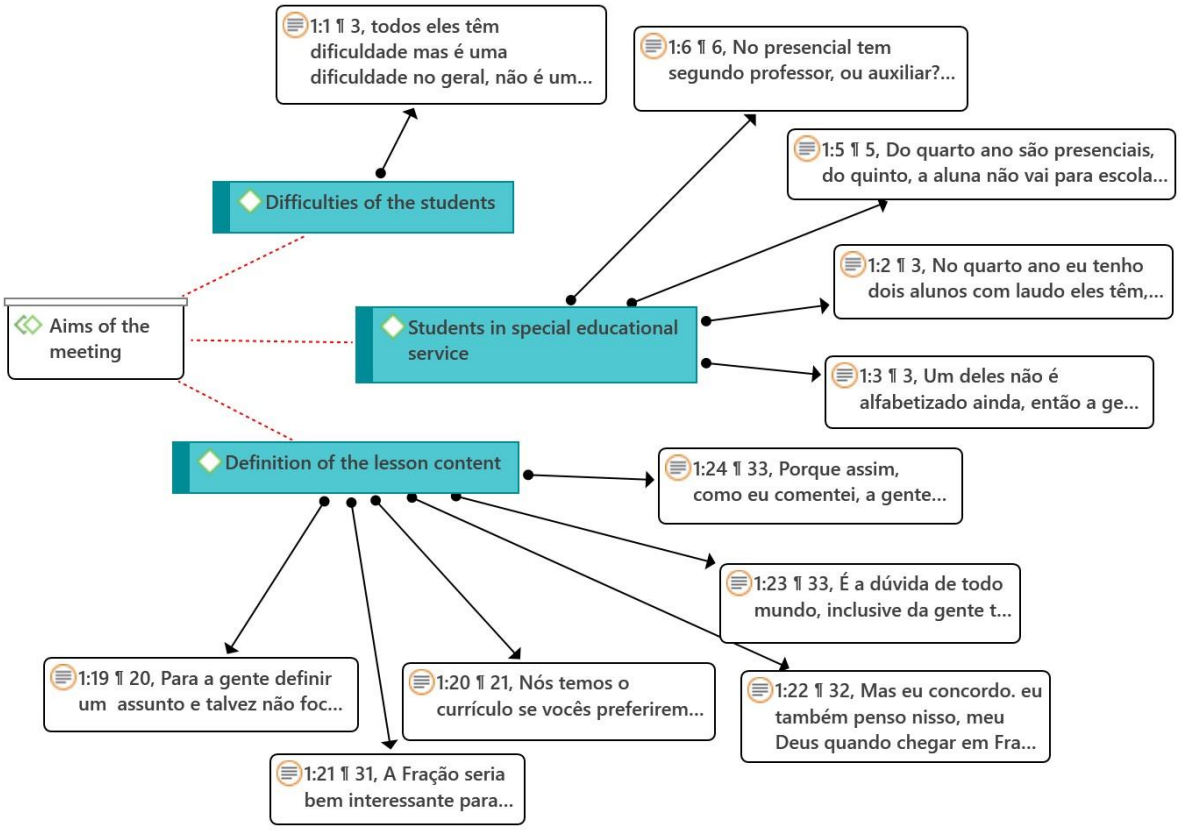
Research: Activities involving the construction of the number, Silva (2023).

In the second moment, which was about the life cycle of fish, the following materials were used: Projection of the image of the life cycle of fish into five stages, construction of the representation of each stage of the life cycle of fish by the children in groups using waste materials (egg, embryo, larva, juvenile and adult fish) (Figure 7 (b)), and use of Tangram to create representations of fish by the children. All activities were developed as a foundation for number construction concepts, including order, hierarchical inclusion, comparisons, one-to-one relationships, counting, mental imagery, number conservation, etc.

To analyze the data produced, as mentioned above, we used observation of all recorded sessions, both remote and in-person, as well as classroom practices. For the narrative analyzes, we used Atlas.Ti, a software for the qualitative analysis of large text corpus, audios, videos, and graphics. Using the software, we mapped the categories and networks between the categories given in the classroom narratives.

Figure 9 shows the objective network of the meeting with the teachers, composed of the categories: Difficulties of the students, Students in special educational service, and Definition of the lesson content, as well as the respective transcribed speeches, in Portuguese, that led to these categories. For example, in the definition of the lesson content, one teacher expressed concern about how to approach the topic of fractions with her students, as she did not feel prepared for this approach. In the category about Students in special educational service, group members reported whether there were any students with this service in their classes and what their characteristics were. In this format, all networks were organized to analyze the data generated by the research in relation to teachers' learning during their training.

Figure 9: Network of meeting objectives created with the Atlas.Ti software



Source: Silva (2023, p. 106).

In the next section, we present the final teacher training considerations related to the use of fluency, which aims to promote children's mathematical competence through manipulative materials.

Final considerations

Reflecting on Schön (2000) notes, we see that both technical solutions were used in the formative moments related to mathematical content, in this case fractions, and everyday teaching situations were used in the moments where teachers' personal experiences were reported professional practices that link them to group approaches. In addition, when working with mathematics in lesson planning, we recognize the opportunities for professionals to get to know each other through actions, contributing to the development of artistic talent.

Regarding Tardif's (2012) knowledge, education in its different phases has provided access to information about mathematical concepts and reflection on the practice itself that configures professional education knowledge, in addition to new approaches and specific topics

in mathematics that characterize disciplinary knowledge. Curricular knowledge is considered in the moments of planning, when the curricula are studied, and finally the experiential knowledge through the exchanges and reports of the participants in each phase in which they have made their experiences.

We have found throughout the in-service training, through the observations and the analysis of the narratives of the participating teachers, that the theoretical foundation (individual and group) provided by the proposed workflow based on Lesson Study, which includes guidance by a teacher specialized in special education and in-service training on specific topics of mathematics, generates autonomy and confidence for the continuity of planning both in the preparation or alignment and adaptation of materials and in the implementation of classes. We assume that these facts contributed to a collective construction of inclusion rather than having to deal with them as isolated topics. Moreover, the teachers' collaboration enabled them to construct new knowledge.

At the end of the planning and implementation phase of the lesson, the teachers' reports showed that the materials used had sufficient characteristics to meet the needs of the students, without being used only by the SES students, so that the whole class could work with them. We highlight this review because during the critical evaluation of the lesson planning, the course participants showed, through their narratives, constant reflection on their teaching and the specific learning of the mathematical topics. Moreover, the critical review of the curricular documents contributed to improving teachers' professional practice.

In the future, we plan to expand the research to focus on both teacher and student learning and to include other levels of education, from basic education to higher education.

References

ALVES, L. H. B. **A formação de pedagogos e pedagogas sob a perspectiva da Educação Inclusiva: um olhar para os projetos pedagógicos de cursos paranaenses de formação inicial.** 2021**.** Dissertação (Mestrado em Ensino) - Universidade Estadual do Paraná, Paranavaí, 2021. Disponível em: bit.ly/3yDvOBw.

BALDIN, Y. Y.; FELIX, T.F. A pesquisa de aula (Lesson Study) como ferramenta de melhoria da prática na sala de aula. *In*: Conferência Interamericana de Educação Matemática, 13., 2011. Recife. **Anais [...].** CIAEM, 2011.

BORBA, M. C.; ALMEIDA, H. R. F. L; GRACIAS, T. A. S. **Pesquisa e ensino em sala de aula:** diferentes vozes em uma investigação. Belo Horizonte: Autêntica Editora, 2018.

DANYLUK, O. S. **Alfabetização Matemática:** as primeiras manifestações da escrita infantil. Passo Fundo: UPF Editora, 2015.

D'AMBRÓSIO, U. **Educação Matemática:** da teoria à prática. São Paulo: Papirus, 1996.

FERRONATO, R. **A contribuição de instrumento de inclusão no ensino de matemática.** 2002. Dissertação (Mestrado em Engenharia de Produção). Universidade Federal de Santa Catarina, Florianópolis, 2002. Disponível em: http://repositorio.ufsc.br/xmlui/handle/123456789/82939.

FIORENTINI, D. Pesquisar práticas colaborativas ou pesquisar colaborativamente? *In:* BORBA, M. C.; ARAÚJO, J. L. (org.) **Pesquisa Qualitativa em Educação Matemática**. Belo Horizonte: Editora Autêntica, 2004, p. 49-78.

GAIGHER, V. R.; SOUZA, M. A. V. F. de.; WROBEL, J. S. Planejamentos colaborativos e reflexivos de aulas baseadas em resolução de problemas verbais de matemática. **Vidya**, v. 37, n. 1, p. 51-73, 2017. Disponível em: https://doi.org/10.37781/VIDYA.V37I1.1929.

GATTI, B. A. Formação de Professores no Brasil: características e problemas. **Educação e Sociedade**, Campinas, v. 31, n. 113, p. 1355-1379, 2010. Disponível em: https://doi.org/10.1590/S0101-73302010000400016.

MACEDO, A. D. R.; BELLEMAIN, P. M. B. Jugyokenkyuu: Contribuição à Formação de Professores de Matemática. *In* I Simpósio Latino-Americano de Didática da Matemática, 2016. Bonito. **Anais [...].** LADIMA, 2016. Disponível em: http://grupoddmat.pro.br/wp-content/uploads/2020/05/MACEDO-BELLEMAIN.pdf.

NOGUEIRA, C. M. I., ROSA, F. M. C. D., ESQUINCALHA, A. D. C., BORGES, F. A., SEGADAS-VIANNA, C. Um panorama das pesquisas brasileiras em educação matemática inclusiva: a constituição e atuação do GT13 da SBEM. **Educação Matemática em Revista**, v. 24, n. 64, p. 04-15, set./dez. 2019. Disponível em: https://www.sbembrasil.org.br/periodicos/index.php/emr/article/view/2155.

SADOVSKY, P. **O Ensino de matemática hoje:** Enfoques, sentidos e desafios. São Paulo: Ática, 2010.

SCHÖN, D. A. **Educando o profissional reflexivo**: um novo design para o ensino e a aprendizagem. Porto Alegre: Artes Médicas, 2000.

SILVA, D. C. **Formação de Professores em Materiais Manipulativos no contexto da Lesson Study com vistas a uma Alfabetização Matemática Inclusiva.** 2023. Tese (Doutorado em Educação em Ciências e Educação Matemática) - Universidade Estadual do Oeste do Paraná, Cascavel, 2023. Disponível em: https://tede.unioeste.br/handle/tede/6714.

SILVA, M. A.; PIRES, C. M. C. Quais os objetivos para o ensino de Matemática? **Unión - Revista Iberoamericana de Educación Matemática**, v. 8, n. 31, p. 21-44, 2012. Disponível em: http://funes.uniandes.edu.co/15846/1/daSilva2012Quais.pdf.

SKOVSMOSE, O. Inclusões, encontros e cenários. **Educação Matemática em Revista**, v. 24, n. 64, p. 16-32, 2019. Disponível em:

http://funes.uniandes.edu.co/24127/1/Skovsmose2019_Inclus%C3%B5es.pdf.

TARDIF, M. **Saberes docentes e formação profissional**. 13 ed. Petrópolis, RJ: Vozes, 2012.