Fractions in teaching practice with fifth graders

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Abstract

Studies of learning processes about fractions are mostly based in individual responses to standardized tests. From the results, supposedly replicable, researchers statistically establish tendencies in the evolution of students' thinking. However students learn in different environments compared to those designed by researchers. While researching inside classrooms for instance it seems that the findings cannot be replicated given the environmental unique features. Contrasting the findings from researches focused on classroom events to others based on tests, one can ask if it is possible to build a bridge between the two perspectives, which seem mutually excludable. We propose an answer to this question giving to each perspective its value for teaching. Therefore we propose an approach that depicts the differences in schooling between what can be generalized and what cannot. We present an episode about teaching and learning fractions and decimals to illustrate our attempt to reconcile the two perspectives.

Key words: teaching fractions, learning paradigms, generalizations of knowledge, reversible versus irreversible in teaching.

The learning process in different interpretations

One challenge for researchers in the educational field refers to the possibility of taking some discoveries in one reality and trying to generalize them as if the same results could be obtained in any other social situation. The process of learning constitutes one example of this challenge, which can be expressed by this question: in what degree can the process of learning a scientific concept be shared among learners from different cultural backgrounds? This question goes back at least to the debate, along the 20th century, between Piagetian and Vygotskian perspectives on how people learn. If we take the discussion between Roth (2008) and Treagust & Duit (2008) as a reference of how far the debate has gone, it seems that there is no way of agreement among the scholars. On one hand there are those who believe that it is possible to trace the path that the majority of individuals take while they go from not knowing some theoretical subject to the point that they can fully comprehend a concept that was taught to them. This idea is shared among various research groups with similar principles such as the conceptual change paradigm, which encompasses different subgroups (Treagust & Duit, 2008).

On the other hand there are those who believe that the learning process can never be considered as an individual feature, rather it has to be taken as a social achievement. For this approach each situation is unique; the learning varies from individual to individual and it is never possible to point out the moment in which a person learnt something. The best we can do is to observe and describe how a person is situated inside a group and how this group behaves in a given situation, avoiding any attempt to go into people's mind (Roth, 2008).

Challenging this radical separation we believe that one can take the two positions in perspective while asking what each of them can offer to the teaching practice. The idea is to open the possibility of seen the two contradictory principles applied to a real situation, keeping in mind that each one is not capable of giving sense to the whole learning phenomena. We argue that this is possible when one distinguishes the different time dimensions that are inherent to human activity: the chronological and the phenomenological one (Roth, 2002). Those dimensions are inseparable and each one can prevail over the other at a given moment. This theoretical division of the time can help us to understand how we perform an activity such as teaching.

The chronological time dimension refers to the moment in which one can think in the past events and articulate the acquired experiences while projecting the future. In this mental exercise, the time is somehow reversible, i.e., one can re-think and re-articulate many different possible scenarios. This time dimension prevails when a teacher is planning future lessons. The teacher can take advantage of what is known about how the students' thinking could evolve from one level to a higher one, considering a specific subject intended to be taught. The teacher can anticipate some difficulties that could be expected from the students and then he/she can plan his/her actions accordingly. In this sense, the knowledge provided by researches focused on uncovering the individual's learning processes can help. However what is proposed as the way in which the learning process evolves is something obtained in a very settled environment. Something unpredictable can easily happen when the environmental features change, which is the norm. So what is suggested as a presumable way of learning for everybody has to be taken as general guidelines for inspiring the planning in the chronological time dimension.

The phenomenological time dimension prevails in the very present, in the here and now, when one has to decide and to act with almost no time to think. When a teacher is teaching, in many situations he/she has to respond to pupils' behaviour immediately, especially when their actions were not predicted. In those cases, the teacher has to improvise, so he/she relies on his/her habitus, or the sense of the game (Bourdieu, 1983), which depends on how long that person has been subjected to the given environment and his/her acquired expertise.

In the phenomenological time dimension, the students exchange ideas continuously. They interfere in each others' thinking in a very complex sets of interactions, which includes oral communications as well as other non-verbal forms such as gestures, gazes, drawings, text messages, etc. In those moments it is virtually impossible for a teacher and even for a researcher with many technological apparatus to indicate what exactly somebody thought and learnt. While everybody is subjected to a collectively activity the ways in which the students apprehend any idea vary from person to person.

Those barriers for predicting students' behaviour do not mean that the planning is useless. The teacher has to think carefully in how to engage students in the learning process and it can be better articulated if information about how students think is available, even if those information are not precise. However we can point to an inevitable tension between what is predictable and what is not. For this debate we could translate this tension into a question about what can be generalized in a given situation and what cannot.

In order to address this question we take from Prigogine (1984) the concepts of reversible and irreversible. Some event is reversible if it is possible to obtain similar results whenever this event is reproduced. If the results vary in each experiment the event is, on the contrary, irreversible. Prigogine stated that, in nature, any event could show its irreversible feature if it would be observed with enough accuracy. Nevertheless most of the scientific knowledge is based on the idea of the reproducibility of the phenomena. It is possible when the differences that occur at each time an experiment is performed are considered irrelevant in a reasonable level.

Social events are subject to a much greater influences than the natural ones. It is impossible to replicate any class for instance even if the actors are the same as well as all the conditions that could be controlled. It happens because collective human's behaviour depends greatly on the state of mind, the humour, the degree of attention, etc., of the individuals and all these elements vary continuously.

The tension between reversible and irreversible can be applied to the discussion about the learning processes. When a researcher sets an environment trying to avoid any external influence and asks questions to the individuals, the responses have to be analysed statistically. The experiment would be reversible in a restrict sense because the results would be similar only if the conditions could be replicated. Moreover, the findings would indicate how an abstract mind would evolve; they would not be suitable for real persons. So it seems reasonable to associate those findings to references that a teacher could consider while planning lessons in the chronological time dimension. Most of what happens in the phenomenological time dimension on the contrary would be taken as irreversible events. However, we will argue that some behaviour that can be observed from this time dimension could be taken as candidates for generalizations. Next follows an example of this whole discussion applied to the teaching and learning fractions' activity.

Researches about learning fractions

Individual's mathematical abilities have been measured since the late 19th century (Geary, 2006). As part of this long tradition, learning about fractions is measured from standardized tests, which are configured with the intention to verify the conceptual and procedural development, i.e. ideas about the magnitude and arithmetic operations with fractions. For preparing such tests, one creates challenges that have correct answers. Those challenges are presented individually to a group of subjects. Each person responds and, eventually, gives an explanation about his/her adopted strategy. Responses produced by the surveyed subjects are categorized. Incorrect ones are examined by means of parameters that establish, for each, the degree of approximation of the correct result. Then, given a challenge and its responses, one can statistically examine the relative amount of subjects who reached the correct result, as well as the incorrect answers' accuracy to this same group. Groups can be selected according to age, educational level, family income, etc.

Researchers that work in the perspective described above take different theoretical backgrounds for interpreting what the subjects provide. Vamakoussi and Vosniadou (2010), for instance use the conceptual change approach while investigating what seventh to eleventh-grade students think about the interval between two rational numbers. They state that the idea of discreteness persevere along the ages; the students in general believe that there are decimals only between decimals and fractions between fractions; and the type of interval endpoints influence the students' judgment of the numbers that would be found in a given interval. Subscribing an evolutionary theory of numerical development, Geary (2006) proposes that what makes whole numbers easy to learn, the possibility of counting one by one for example, constrains the understanding of fractions. Whole numbers would be biologically primary while other types would be biologically secondary. Because fractions and decimals do not obey the same rules that are applied to whole numbers, the concepts can be taught separately or, as suggested by Sieger at al. (2011), they can be integrated in a broader perspective, which associates all numbers to the number line. These latter authors propose an alternative understanding of the relationship between whole numbers and fractions based in the idea of continuity in the number concept. The individuals could learn that fractions and decimals can be seen as an enlarged idea of whole numbers, considering the fact that each one has a single point associated with it on the number line.

Those researchers and many others take methodological approaches that suppose a strong level of reversibility. In general, the researches suggest that: students think systematically while facing challenges about the magnitude of fractions, there are differences among individuals and comparative scores correlate with others obtained in tests about fractions arithmetic and about general mathematical understanding.

So what can be said when one contrasts the two environments: one configured to observe individual actions and the other where students interact continuously while facing challenges about fractions addressed to them?

This paper presents elements of this contrast between what can be taken as reversible and what is irreversible by nature. We intend to contrast these environments from an event that is part of our research conducted inside some schools. This research is organized with the aim to improve methodologies for teaching mathematics from a perspective that focuses on culturally and historically situated conditions. Hence the focus of the research is placed especially on the interactions among the subjects while they cope to a pre planned situation.

Next, we will present some aspects of the research's methodological approach, followed by a description of an event in a class about fractions and decimals. We will take this case as an example for conducting a dialogue between what was observed and what the researches previously sited have been revealing.

A research focus on teaching activity

The research that was mentioned above started in 2010 in three public elementary schools, in Ouro Branco, MG, Brazil. Six teachers agreed in participating by gathering once a week when they would talk about each one's demands. Then they would plan a series of classes according to the demands presented by one of them. Afterwards, two teachers and the researcher would co teach the planned classes. The classes would be videotaped and some episodes would be chosen for further analysis. The research group was working, and still is, in a collaborative manner. It means that each participant has a specific expertise that he/she can share with the others. In 2011, Lúcia, who teaches in Castiliano, a remote rural area of Ouro Branco, needed help for the teaching of fractions in her fifth grade class.

The adoption of coteaching (Roth & Tobin, 2002, Willis & Menzie, 2012) as a methodological approach expresses the idea that for the improvement of teaching it would be necessary to contemplate particular aspects of each school community, as perceived by the teacher that works there. Each teacher has his/her own professional experience and he/she always acts accordingly. Besides, the students are not merely rational subjects; their stories strongly influence their behavior. Each class is unique, not replicable. This means that what we learn from researches that take place in reversible environment is valid for the planning of a school activity, but this wisdom must be seen in context when we consider the teacher's regular work conditions.

Roth (2008) claims explicitly that cultural historical approach cannot talk to other paradigms focused on individual learning processes. However, that is not our perception. We think that possibilities of generalizations of learning processes arise when subjects, such as fractions and decimals, have common features wherever people are teaching and learning them. One must distinguishes what people learn in a specific moment from what we learn during a certain period of time. Despite the fact that it is impossible to know what is going on into person's mind, it is possible to access statistically tendencies for learning processes. That is why we can relate what we observe in case studies to what we find in the literature about learning. Next we provide a sample that might enforce our position.

A class about fractions and decimals

In the first class, planned according to Lúcia's demand, Adriana, another teacher and Eduardo, the researcher, shared the teaching position with her. It happened in May 24, 2011. Lúcia gave two rulers to the students; both rulers started in zero and ended in the point 1. In the first ruler, the interval between 0 and 1 was divided in 10 parts with correspondent strokes plus ten smaller strokes each in the middle of two bigger ones. The decimal 0.5 was written in its place. The other ruler was divided into 8 parts and had the fractions 1/2 and 1/4 written in the correspondent strokes (figure 1).

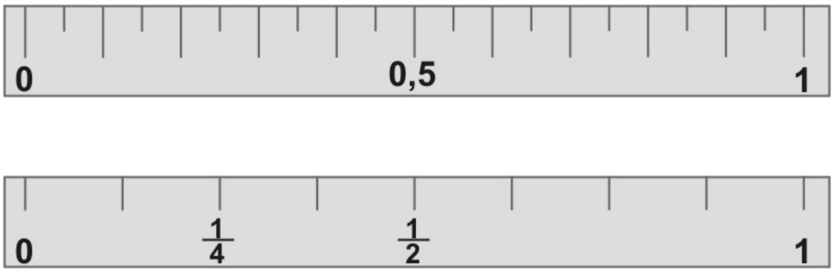

Figure 1. Rulers given to the students.

The students had two challenges: 1- to fill both rulers with the missing decimals or fractions; 2 - to measure objects, plus to write the obtained results. They were sitting in groups of four and there were 4 groups in the classroom that day.

In the recordings we can see Mauro (fictitious name) who just gazed at a student sitting in front of him and immediately filled his decimals ruler. What calls for attention here is the fact that according to Lúcia, Mauro was the most problematic student in the class, the only one that had to repeat the fifth grade the following year. In fact, Lúcia had to review the scene many times before she was convinced that Mauro did not copy the solution from other students. Pretty much everybody filled the same ruler in less than one minute.

However while the decimals ruler was not a problem for the students, the other became a significant challenge. Lúcia was assisting Mauro's group and nobody showed any idea on how to fill the fractions ruler. About 4 minutes later Adriana's voice was featured and all students in Mauro's group gazed at her. She was trying to help the students in having a sense of the way the fractions ruler was divided.

First, Adriana invited the students to count the number of spaces between 0 and 1 in the fractions ruler. Then, she asked them what would be written in the first stroke but this strategy did not work. Next, Adriana gathered 4 rubbers and took one. She asked the students which fraction of the rubbers she had taken. They answered that she took one in four, but they could not grasp how this action related with filling the ruler. She insisted in this strategy, jumping from 4 to 8 rubbers. It took 5 minutes until two students, a girl and a boy, could fill their rulers correctly. They still had to ask what would be written to replace the 1/4 and the 1/2; it seems that they did not understand in a glance the relationship between an eighth and a fourth, between an eighth and a half. Nevertheless, almost immediately the entire class filled the fractions ruler and the students started to measure objects that they each had chosen.

When the students started measuring objects two difficulties became prominent: if the length of the object was between two strokes it challenged the students as well as if the object was larger than the ruler. The teachers told the students that they should write whatever they had in mind and the solutions would be discussed later. Due to paper character limit, we will discuss only one solution for the first issue with the fractions ruler. While measuring each student wrote a creative result for a piece of rubber, which length was between 1/4 and 3/8. Some of their solutions were: “1/4 and a half”, “1/4 + 1/2" and “1/4.3”.

From planning to execution

Adriana was immersed in the phenomenological time dimension while trying to help students to understand the schemes that informed how the fractions ruler worked. During the previous teachers’ meeting, it seemed that the students would not have a hard time filling the ruler. The relationship between the fractions appeared very logical: the fraction 1/2 was between 0 and 1, 1/4 was between 0 and 1/2, so the 1/8 would fit logically between 0 and 1/4. As the students did not understand the scheme, Adriana had to improvise a method to explain it to them almost with no time to think about what to do. So, she tried what first came to mind and ended up using the part/whole relationship as a source for understanding the proposed scheme.

This part/whole relationship was not in the planning. It seems that the explanation would be simpler if Adriana took a piece of paper similar to the ruler and folded it three times. Each time, she could ask the students in how many parts the ruler was divided and connect the response to the fraction written on their rulers. While planning, the teachers had decided that they would approach the fraction concept by linking it to the decimal’s concept and by associating both to the number line. The adoption of the ruler was a way to explore the fractions magnitude associated to measurement. This perspective, which brings together fractions and decimals associated with the number line, seems aligned with the prescriptions of scholars such as Pagni (2004), Shaughnessy (2011) and Siegler at al. (2011).

The students seemed satisfied with Adriana’s explanations for filling the ruler. They could write the fractions on the ruler and the teachers approved it. However it is impossible to know in what level they really understood the scheme that was proposed by the teachers. In expressing the results of the measurement the students’ hypothesis in a certain sense contradicted what Vamakoussi and Vosniadou (2010) found due to the fact that they used decimals such as 0.3 and 0.5 between fractions. It seems that, for them, despite the fact that the ruler was divided in fractions, between two of them they could accommodate the decimals. Later, Adriana tried to explain to some students how they could correct their odd results by dividing the ruler in more fractions. It took her a while to convince some of them that they should draw other strokes as a sign for dividing the 0-1 interval into 16 parts.

The episode depicts some irreversible aspects of the learning process. Adriana’s strategy was created in the very moment that she was challenged to give a clue to the students about the schemes that inform how the fractions ruler should work. Her actions expressed her previous experiences, which could give her a glance on how those students could learn a way to fill the ruler. The strategy worked considering that the students could write the correct fractions. However based on the following events it is almost certain that the students did not understand the division of the ruler in a broader sense. They could not see that the ruler could be divided many times in many different forms and, in all of them; the same scheme would be adapted to each case. The whole episode is irreversible. Even if Adriana would have to act in a similar situation, she would incorporate what she learnt from this experience and most probably she would use another strategy or she would try to improve the adopted one.

The communication among the students was invisible for the teachers. The camera shows that while Adriana was talking to the group in front of her everybody was quiet. Two students of that group were the first ones to enunciate the fractions that they would use in their ruler. Then almost immediately everybody started to fill the fractions ruler. How they knew what to write, how they transmit the idea among them is not clear at all. We can only hypothesize that some students were able to follow the dialogue driven by Adriana and they could get the meaning of what the classmates said about the fractions that they would write. Again, this communication is irreversible as well as what was learnt from it.

The episode also depicts some aspects of the learning process that might be considered reversible. In previous experiments we have witnessed students of earlier ages consistently expressing the understanding of measurement in the same manner. So, we have a candidate for a generalization. The students, at least from 8 to 10 years old, tend to call “a half” any part of an object that stays between two strokes in a ruler. The 0.3 that we witnessed this time seems to be a sophistication of the general tendency. While proposing to students of age around eight to make measurements using rulers with only whole numbers, we witnessed that they express non integer results as “a half”. This half can mean anything between two integers. They would abandon this behavior only after they understood the scheme that informs how numerical intervals can be infinitely divided and each division can be expressed by decimals or by fractions.

Final words

We started this paper showing the discussion about the learning process, which divides opinions among scholars. The debate went as far as some of them conclude that it is impossible to articulate any proposal that would conciliate those opposed perspectives. One of them postulates the understanding of how subjects learn scientific concepts. The other side states that it is impossible to grasp how an individual learn because the learning process results from a collective activity and each participant learns whatever is appropriate to his/her experiences in life. Nonetheless we argue that it is possible to cope with the tension between the two perspectives when we examine the teaching activity.

For differencing the reversible and the irreversible moments in teaching, we postulated the division of time into two dimensions. The teachers can freely evaluate past actions as well as planning future ones while they are immersed in the reversible chronological time dimension. On the other hand, while teaching, the teachers have to react immediately to a given situation and they do not have much time to plan what to do. So, each person acts according to his/her expertise while teaching, which accentuates the irreversible aspect of this moment.

On their side the students actively deal with what is given to them, usually in an irreversible manner. They try to respond to challenges by reasoning and also by using what they learnt from their cultural background. It would be no clue for the dialogue between reversible and irreversible situations if they do not have some structure that could connect them. The curriculum has this structure. Mathematical subjects have persistent schemes that are explored inside the messy classrooms as well as in the quiet laboratories.

The episode discussed above gives a sense of our point of view. The teacher found herself in a situation that was not predicted during the planning and then she reacted shifting the meaning supposedly given to the fractions. Nevertheless the students could fill the fractions ruler despite the fact that something seemed missing in their understanding of how the ruler should work. Neither the teacher explanation nor what was perceived by each actor can be replicate. Those are inherent irreversible events.

While measuring, the students repeated a behavior that was witnessed before some times, which indicates that this behavior is a candidate for a generalization. It is expected that each time the use of a ruler is proposed as a challenge, the students would have difficulties in expressing results that cannot be read directly in the ruler. It seems that this situation can be replicated in many different elementary schools. If so, we could have from our work something that is reversible and can be taken in consideration for further teaching planning.

We recognize that the results obtained from researches based on reversible events, and also from any generalization, can collaborate to the teaching process because they provide information about what a teacher should expect while exploring the curriculum. However, one must be aware of the fact that the teaching happens predominantly in the phenomenological time dimension when unpredicted events can easily take place and not be a part of the initial plan. Our research would suggest that one should seriously consider that any learning process happens mostly in an irreversible situation and responses obtained in very controlled environment are insufficient to understand how students learn. We consider this crucial for teachers' formation.

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