The spatial concepts in Primary Education: a work proposal

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Abstract

This Workshop is intended to (re)construct convex polyhedra, in particular, Plato's polyhedra , from situations experienced by teachers through research of a tangible case, the *Soccer Ball*. It has originated from a work of investigative nature, carried out with teachers and pupils of the 4th and 5th grades (9-11 year olds) of the Primary School, in Portugal, aimed at answering such questions as: Are the pupils able to work on spatial notions in a ludic manner? Are the teachers able to promote this type of work in their classes? What conditions are necessary to make that happen? The proposals will be presented in the following stages: invitation to build a soccer ball; research on the soccer ball and discussion of the results obtained; report on the events experienced by pupils (9-11 year olds), construction of the soccer ball and determination of the reason why it is formed by pentagons and hexagons, and discussion on the potentialities and challenges of this work.

Keywords: Soccer ball, polyhedra, mathematical investigations, reflective formation, realistic mathematics.

This proposal arose from the desire to participate in the World Year of Mathematics (WYM2000), promoted by the Mathematics Teacher Association (MTA), the Portuguese Mathematics Society (PMS) and the Portuguese Expresso newspaper, in 2000, whose subject-matter was Polyhedra Teaching (Tramm, 2002). That context encouraged the teachers (10) of the 4th and 5th grades (9-11 year olds) of Primary Schools (3), 1st cycle[1], in Portugal, and the trainer (specializing in Mathematical Education) to develop a pedagogical proposal of investigative nature. The generating issues were: Is there an element in the culture of the pupil that forms part of his desire, which would serve as a bridge (link) for the study of regular polyhedra? Are we,

[1] The Basic Education, 1st cycle, in Portugal corresponds to the first years (2nd to 5th grades) of the Primary School in Brazil.

teachers, able to promote this type of work in our classes? What conditions are necessary to make that happen?

To answer those questions, we got down to work.

In the mathematical knowledge of the subject (polyhedra), we identified the truncated icosahedron (the soccer ball). For sure, the truncated icosahedron would perfectly serve our purpose. Being an element of the desire and culture of the student, it would be the "link" we needed to mathematize the reality of the student. The idea of mathematizing the reality of the student is based on realistic Mathematics, developed by the Freudenthal Institute.

Freudenthal (1978) favored the inclusion of the geometry in the learning of mathematics as early as possible. He did not defend the Euclidian mathematics as an ideal object for deductive reasoning. For that author (1983), doing Mathematics consists in organizing experience areas, while geometry serves to mathematize spatial experience.

We concluded that Geometry (space and plan) is instrumental to mathematizing reality, for the child goes through a great opportunity to experience local organization (space), moves in the plan and, with "good" experiences, the child builds spatial concepts. Freudenthal (1983, p. 35) affirmed that "when made with your own eyes and hands such discoveries are more convincing and remarkable." That author worked to open Mathematics for all people, but never attributed lesser importance to the need for an intellectual, a scientific thinker.

On the other hand, we found out that the teachers involved in this intervention project were ill prepared in this area. Therefore, the project would have to simultaneously comprise both the training and the work of those teachers in the classroom. At that moment we realized how daring was this pedagogical proposal of studying regular polyhedra through the soccer ball on two fronts, training teachers and monitoring the intervention project in the classroom.

We hope that this Workshop will help clarify what type of investigative activity we, trainers, want for our schools based on the assumption that Mathematics is "a subject that belongs to all people and that we all are responsible for making this instrument of organization of the world and everyday life accessible to the children and young people of this country. Learning Mathematics is a basic right of all people." (Abrantes, Serrazina and Oliveira, 1999, p. 8).

We propose that the participants of this Workshop carry out investigative activities aimed at (re)constructing convex polyhedra, and in particular, Plato's polyhedra .

The National Curriculum Parameters (*PCN*) (Primary Education (II, 6th to 9th grades), (1998, p. 122) in chapter Space and Shape affirm that:

> The issues related to shapes and the relations between them, the possibilities of occupation of the space, with the location and displacement of objects in the space, viewed from different angles are just as necessary today as they were in the past [...] However, Geometry has received little attention in classes of mathematics, and many times its teaching is confused with that of measures. In spite of its abandonment, it plays a key role in the curriculum, to the extent it enables the student to develop a type of fundamental reasoning to understand, describe and represent, in an organized manner, the world in which he lives. It is also true that geometric subjects usually draw the interest of adolescents and young people in a natural and spontaneous way. In addition, it brings about a fertile field of problem-situations that favors the development of the ability to argue and build demonstrations.

So, we have found that Geometry is already included in the curricula of the Primary Education, but in an inadequate and mistaken way. We know that such inclusion cannot be reduced to teaching of measures because, as well stated by *PCNs*, it "is a fertile field of problem-situations that favors the development of the ability to argue and build demonstrations" (*PCN*, 1998, p. 122).

We believe that said anomaly arises, in part, from the fact that the teachers who work at this level of education lack preparation to appreciate this type of geometric reasoning that would help them understand, describe and represent, in an organized way, the world where they live.

It is up to us, specialists, to propose trainings that would enable the teachers to work on the exploration of space and plane, discovering their regularities and irregularities through construction of objects, while working at the same time on argumentation and counter-argumentation.

In that sense, this Workshop proposes to work on the exploration of convex polyhedra from the regularities found upon exploring the "soccer ball" object, identifying regularities and patterns, i.e., identifying the same edge size and regular polygons (pentagons and hexagons), solving challenges (provocations in the form of questions)[2].

Finding this pattern (same-size edges) will delimit the investigation on convex polyhedra. Therefore, there will be built only polyhedra formed by same-size edges. At this moment of construction and investigation, there will be (re)constructed the meaning of polyhedra (solids bound by polygons) and the meaning of each one of their components/elements (faces, edges and vertices), according to the mathematicians´ viewpoint. We define polygons as the faces of the polyhedron (the flat faces bounding it), the joining lines of polygons are the edges of the polyhedron (the straight line segments bounding the faces), and the vertices of polygons are the vertices of the polyhedron (the corner points of the edges).

Euler's relation (number of faces + number of vertices = number of edges + 2), discovered by the Swiss mathematician Euler and valid for all the polyhedra we are going to mention in this paper will be treated in a natural way.

The recurrent activity in this work consists in finding regularities and irregularities and arguing and counter-arguing. In that regard, the following question challenges us: Are all polyhedra found regular? The participants note (in table built by them) that the regularities found consist of congruent (geometrically equal) faces and edges. The irregularities are situations where the number of concurrent faces or edges at each vertex is not always equal, there are vertices where four edges meet, and other vertices where just three edges meet. In some polyhedra, all faces are regular polygons, geometrically equal, and at each of their vertices there is the same number of edges. We call those polyhedra, known as Platonic Solids, Regular Polyhedra. At the right time, upon investigating and building deltahedra (six), we will work on the concept of Convex Polyhedra and their definition, according to which they are those which are, all of them, turned to the same side in relation to the plane of any of their faces, i.e., when their faces always leave the others in the same half-space. Otherwise, the polyhedra are named concave.

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[2] Questions such as: Are all polyhedra found regular? Justify your answer. Why the ball is formed by pentagons and hexagons? etc.

In brief, this Workshop is intended to carry out the activities described below, which will be developed in the following stages:

a) invitation to build a soccer ball : What about building a soccer ball? Usually, this proposal is accepted with great enthusiasm. Once the work contract is established, we set to investigate the object;

b) investigation of the object of study (the soccer ball) and discussion, in a large group, of the results obtained - activity sheets 1 and 2. With the soccer ball in our hands, we start to investigate the object. The participants will be encouraged to describe the soccer ball, identifying patterns and raising working hypotheses on the quantity of each one of the polygons used, and challenged to answer why the soccer ball is formed by those polygons. Activity Sheets Nos. 1 and 2 (Annex 1) will be used as support material, so that the participants may record their working hypotheses. That activity will be carried out in small groups, and upon the conclusion, we will discuss, in the large group, the patterns found. Said patterns will serve as a link to help the participants understand why we establish limits/rules in the investigation of convex polyhedra, which will be performed later.

c) report of experiences gone through by pupils (9-11 year olds) – That report will present the work done by the pupils, from the 4th and 5th grades, from Primary Schools, in Portugal[3]; upon building the polyhedra (deltahedra, hexahedron and dodecahedron), according to pre-established rules/limits (Annex 2); upon entering the elements of the polyhedra in the table (Annex 3, activity sheet 3); and upon classifying and identifying the regular or Platonic polyhedra. It will be an exposition where all the participants should intervene with questions. There will be presented the reactions of the pupils (why the cube does not stand), the teachers[4] (materials, activity follow-up sheets, and time spent in carrying out the activities), and the researcher (finding of new learning "links"). Finally, a list of questions (which deserved the attention from the teachers and the researcher) will be presented, so that the participants may ponder the adequacy of the proposal made, identifying its strong and weak points. For example, among other questions: In your opinion, is the soccer ball a link that brings the pupil´s reality closer to the mathematical idea (polyhedra)? Is the conceptual field worked on – Plato's polyhedra – appropriate for us to work on geometry at such teaching level? Are the support materials used adequate for finding properties (rigidity or else) of objects and identifying all the

[3] The primary schools involved were as follows:
EB1 No. 10 - Setubal. It involved two teachers (only one of them participated in the APM training action) and two classes of the 3rd grade (45 pupils). With that work, the school won the WMY 2000 (World Mathematics Year 2000). It held an interactive exhibition to divulge this work at the school community (teachers and pupils) and the local community (parents and interested people), where the pupils of the project worked as monitors;
EB1 No. 9 - Setubal. It involved seven teachers and five classes of the 1st to 4th grades (125 pupils). It performed an interactive exhibition to divulge the mathematical activities to the school community, engaging the persons responsible for the pupils and interested people;
EB1 nº 15 - Lisbon. It involved seven teachers and five classes of the 1st to 4th years (125 pupils). It held an interactive exhibition to divulge the mathematical activities to the school community, engaging the persons responsible for the students and interested people. With that work, the school won the 2nd place in the contest of Pedro Nunes Institute, 2002.
[4] The teachers involved attended the training course "New learning environments in Mathematics teaching."

elements making up a polyhedron? Do the rules/limits imposed for building and finding polyhedra actually function as rules of a game? Did the pupils enjoy working on this issue (regular polyhedra)? Do the tasks that involved organization, systematization and formalization of mathematical contents help carrying out interdisciplinary tasks? Did the teachers involved demonstrate enthusiasm? Did this pedagogical proposal favor interchange with the school community? Did this pedagogical proposal arouse the interest of the teacher in teaching Mathematics? Did the teacher accept the time spent by the pupils to study the polyhedra? Did the subjects involved in the experience (trainer/researcher, trainees/teachers and pupils) improve along the way (action of training, intervention and critical reflection of results), changing their attitude towards mathematics?

d) building the soccer ball and learning why it is formed by pentagons and hexagons – At this stage, the participants will be encouraged to work with the icosahedron and learn how to turn it into a "rounder" object. That work will naturally give origin to a soccer ball and enable the participants to understand why it is formed by pentagons and hexagons and why that quantity of hexagons and pentagons is used. The pattern sheets 1 (pentagons and hexagons) and 2 (fitting equilateral triangle) in the real size of the soccer ball will be made available to the participants. Evaluation of the potentialities of the activities presented, taking the classroom as reference – the participants are to evaluate the working method (the investigative class) and the material used (activity sheets) and their viability for pupils from other school classes.

e) discussion of the potentialities and challenges of this work (from the point of view of the participants involved - pupils, teachers, and researcher) aimed at a possible application of it in the classroom of the participants of this Workshop.

At that moment, the participants will be encouraged to answer the questions generating this work: Are the pupils able to work on spatial notions in a playful fashion? Are the teachers able to promote this type of work in their classes? What conditions are necessary to make that happen?

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Annex 1

Activities Plan for the 4TH GRADE
Objective: Finding regularities/patterns in the soccer ball

Activity 1 –

SHALL WE PLAY WITH THE BALL?

Work proposal:

What about building a soccer ball?

Shall we plan it together?

Let us think together on what we need to build the soccer ball.

What do you perceive in the soccer ball?

With the soccer ball in our hands, we begin to observe and record in the work sheet what has been observed.

Work Sheet No. 01

1. Examine the soccer ball and record your findings

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____

2. Check whether there is any regularity (some pattern) in the soccer ball. If so, explain it.

____

____

Now, speak with your colleagues about the activities you have carried out.

Work Sheet No. 02

Homework (data organization)

Organize the data of your investigation

The common (regular) element in the soccer ball is
____

Justify your answer
____

The flat figures (polygons) used are:
____

In your opinion, how many hexagons and pentagons are used in the construction of the soccer ball?
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In your opinion, why hexagons and pentagons were used in the construction of the soccer ball?
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<u>Speak with your colleagues about the activities you have carried out.</u>

<u>Tomorrow, bring along a text relating your experience in performing this activity.</u>

Annex 2

Activity Plan for the 4TH GRADE

Objective: Complete the table with the data of the polyhedra built.

Analyze the recording table of the researched polyhedra.

SHALL WE PLAY WITH DRINKING STRAWS?

Activity 1:

Observe your table and record your comments.

Activity 2:

Build a polyhedron (solid with many flat faces) from the triangle.

Give a name to your figure.

How many drinking straws did you use?

Note down on your recording sheet the name of your figure, the number of drinking straws you used (EDGES), the number of triangles (FACES) and the number of corner points (VERTICES).

How many figures did you find?

Activity 3:

Repeat activity 2 using only squares.

Repeat activity 2 using only pentagons.

Repeat activity 2 using only hexagons. What do you conclude?

RULES

- The polyhedra have to stay closed.

- The polyhedra cannot have double edges.

- The wool yarn (the fishing line) passes through the inside of the drinking straws as many times as necessary.

- You can tie knots at the wool yarn (the fishing line) as much as you want.

Congratulations! You have concluded your activity.

Now speak with your colleagues about the activities you have carried out.

Tomorrow, bring along a text relating your experience in performing this activity.

Annex 3

Activity Plan for the 4TH GRADE

Objective: To record the polyhedra built.

Recording Sheet/Table

Polygons with...	cm	Polyhedra Name	Polyhedron elements (quantity)		
			Polygon **F**aces - **F**	Corner points **V**ertices - **V**	Drinking Straws **E**dges - **A**
3 equal sides TRIANGLES					

Congratulations! You have concluded your activity.

Now, analyze, with colleagues, your table and build the soccer ball.

General Information	
Title of the Shop	Spatial notions in Primary Education: a work proposal.
Author´s name	Elda Vieira **Tramm**
Author´s institutions	Federal University of (UFBa) / Mathematical education study group of Bahia (EMFOCO).
Author´s country or countries	Brazil
Most convenient number of hours (2 hours)	2 hours
Schooling level to which the Panel will be directed (Nursery school / Pre-school, Early years of Primary Education/ Late years of Primary Education/ Secondary Education/Higher Education, or general.	Late years of Primary Education/Secondary Education, Higher Education, or general.
Maximum number of persons	30
Audiovisual or information processing equipment required (Multimedia projector, large TV set, information processing laboratory, connection to internet)	Multimedia projector, computer, and connection to internet