The Language of Topology: A Turkish Case Study

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Topology has its own specialised language. Where did this come from? What are the differences in the language of topology when it is expressed in English, Spanish, Mandarin, Czech or Turkish? Does topology itself change when expressed in different languages? What effect has language had on the development of topology? Does the language of the topologist make a difference to the mathematics?

A research programme aimed at answering these questions has begun. This paper is the first in a series that provides a background to the research. Topological discourse in various languages is being examined for its particular features, and possible influences on the concepts developed through these languages. Data from Turkish topologists and topological terminology is examined. It shows why there is reason to suspect that language influences mathematical concept development. The data is also used to explore methodological issues for the research project.

Background

This paper is part of the background to an investigation into the relationship between language and research level mathematics. General topology has been chosen as the context because it is one of the most abstract of all mathematical areas: it deals with basic and apparently highly defined concepts that are generally regarded as being universal amongst topologists. General topology also has a large international research community in which several diverse languages are represented. It thus provides an ideal field in which to investigate the influence (if any) of different languages on the development of mathematical concepts. The central question of the project is whether the language of the research topologist affects the use and development of topological concepts in his/her research. If differences are found between language groups of topologists, then it is hoped to determine whether they are language-based, and how any differences evolve.

The research study has been in progress for four years, during which time a questionnaire-type instrument has been developed and trialled (Barton & Reilly, 1999). This instrument contains five tasks that request information about topological concepts in different ways. Progress is currently being made towards the collection of data from over a dozen different language communities of research topologists: Arabic, Czech, English, German, Greek, Hungarian, Japanese, Mandarin, Polish, Romanian, Russian, Spanish & Turkish. Respondents are being sought who have learned topology, and who teach and use it significantly, in the target language. The research instrument is completed in the target language.

There are serious methodological issues involved in this research. One set of issues concerns the validity of translations of the instruments and the responses that are necessary to make cross-linguistic comparisons. Another problem concerns the representativeness of the respondents, and a further difficulty refers to how it can be determined whether group conceptual differences exist. This paper is aimed at yet another set of problems.

Assuming that group differences are found, how can it be determined whether these differences are language-based? If linguistic issues are implicated, how can the interaction of language and mathematical concept development be investigated? In order to undertake such an investigation it is necessary to have a full understanding of the linguistic and social history of topological discourse in each of the languages of the study. This history includes the development of terminology in general topology, the network of influential people, the movement of topological knowledge between various communities of

mathematicians, the possible external influences on scientific knowledge, general linguistic analysis, and the general relationships between the languages during the period of the development of general topology. In recent reviews both Nagata (2002) and Rudin (2002) refer to the set theoretic nature of the foundations of topology, where "topological properties were thought of as axioms" (Rudin, 2002, p. 566), and where the solution of problems depends upon set theoretic assumptions like ZFC, Martin's axiom, or the negation of the Continuum Hypothesis (Nagata, 2002, p. 562). Being reminded of these formalist beginnings leaves open the issue of the status of topological work—it certainly allows that individual topologists may operate with different ideas on this subject. Our natural language is the medium through which we must strive to express our philosophical beliefs and which we use without generally being aware of the options that other languages provide. Does it therefore influence our mathematical thoughts as we seek to describe fundamental properties?

This paper builds on an examination of topological discourse in Turkish, and is a first model of the work that is required. It is intended to be read as a stimulus to thinking about the link between topological concepts and language in any language familiar to the reader.

Topology in the Turkish Language

Turkish is a member of the Turkish branch of the Altaic language family. Among its closest relatives are Azerbaijani and Uzbek. Mongolian is a more distant relative. An important event in the history of Turkish was the language reform/revolution initiated by Mustafa Kemal Atatürk in 1928 (Lewis, 1999). Called *Öz Türkçe* (Pure Turkish), its aims were the replacement of the Arabic script by the Latin alphabet (suitably modified) and "purification" of the language, ridding it of Arabic and Persian words. This reform continues as younger Turkish speakers now have a poor knowledge of Arabic grammar structures and tend not to use those Arabic words that remain.

The founder and leader of modern mathematics in Turkey was Cahit Arf (1910-1997) (Mardesic, 2000). However much of general topology work in Turkey was generated through L. Michael Brown, an English academic who arrived in Ankara in 1968. His work at Hacettepe University, both topological research and the encouragement and mentoring of new topologists there, has meant that this university has been the centre of topological activity: it is the biggest community of topologists in Turkey, and many of the other centres (for example, Antalya, Eskisehir, or Mersin) contain graduates of Hacettepe. The link with English topology remains, with the majority of graduate students who go overseas going to the United Kingdom. A very few have gone to USA and to Germany, but none have gone to the Arab world or to the Soviet Union/Russia (although a linguistic link between Turkey and Azerbaijan exists).

At Hacettepe, some undergraduate level courses are taught in Turkish and some in English, but the topological ones are taught in Turkish. At graduate level all courses are taught in Turkish, and topological seminars are usually given in Turkish. Scientific publication in Turkish journals is in English, occasionally together with Turkish, and the sole mathematics publication, the *Turkish Journal of Mathematics*, contains English language articles exclusively.

Specialised topological vocabulary in Turkish was developed mainly in Hacettepe, partly as a result of the particular interest of Professor Brown. He was also a member of the team responsible for the collection and publication of a dictionary of mathematical words in Turkish with English, German and Russian equivalents (Hacisalihoglu, Haciyev, Kalantarov, Sabuncuoglu, Brown, Ibikli, & Brown, 2000). This focus in one place has resulted in some terminology that is peculiar to that university. For example, the word for 'set' is *küme* (meaning 'heap, mound, pile, hill') at Hacettepe, but is *cümle* (meaning 'sentence or clause', but also 'a whole, total, ensemble, group') in Ankara University (10 kilometres away). However, much terminology had already been developed by Turkish professors at various universities in other fields

of mathematics, and thus the topological discourse in Turkish is both widely understood throughout Turkey, and is consistent with other branches of mathematics.
The development of topological vocabulary has reflected the development of Turkish language in general. Thus terms with Arabic roots have been avoided, for example, the term for 'field' is *alan* (a general term for area) from old Turkish rather than the Arabic word *saha* (the more accurate translation of field in its agricultural sense). This may explain the *cümle/küme* change described above: *cümle* is an Arabic word. (Another reason might be the use of *cümle* in *karacümle* to mean 'basic arithmetic' and hence a double meaning within mathematical terminology).
Atatürk's *Öz Türkçe* (Pure Turkish) did not just replace Arabic and Persian words, it used Turkish roots to coin new words. The mathematical *dizi* (sequence) is an example, derived from the root *diz* meaning 'to line up, arrange in a row, to string beads'. The gerund (noun form created from the verb) is *dizen*, but for the mathematical meaning a new word was created.
The following explanation about topology in Turkish was written by Michael Brown (personal communication, 2003):

In general terms both [English and Turkish] seem capable of expressing mathematical concepts and arguments with equal precision. But having said that I cannot help but feel that the structure of English is somewhat better suited to mathematics than that of Turkish. One point ... is the position of the verb at the end of the sentence. Whereas in English one would write "There exists a continuous function f ..." which established from the beginning that it is the existence of something that is involved, in Turkish one would say something like "having the property of continuity, a function f there is" giving the property (continuity) first, of what (the function) second, and its existence last. Longer examples can have you describing quite complex properties of things before it comes clear what it is that has these properties. Of course the end result is no less exact in an absolute sense, and one gets used to having things this way round, so perhaps it is just a question of what one is used to. However, there are ways of forcing a word-order more similar to English by using an equivalent of "such that" (the result not being considered 'good Turkish'). [This is] often resorted to by speakers used to lecturing in English and (often) by research students, so perhaps the effort required to produce a well structured sentence in such cases is something that even native speakers of Turkish find noticeable. Turkish is quite an expressive language, and the use of suffixes means one can pack a lot of meaning into a single word, so it is often very economic. In some areas it is well supplied with synonyms, but not in all, so it is sometimes difficult to name new concepts similar, but not identical to, known ones.

Open sets and the issue of multiple meanings

Much topological terminology in all languages uses words that have general, everyday meanings. The general meanings are not only different from the specific technical meaning of topology, but also there may be more than one common meaning. An important question for the research project is to determine whether the general meaning of such terms interferes with the understanding and use of the mathematical term. Where there are several general meanings, different languages often privilege different meanings, even when the words are regarded as being equivalent in translations. It has been hypothesised that such an influence is a likely source of differential linguistic effect across languages.
'Open' is one such common word with many meanings, and its use in the key topological concept of 'open set' has been remarked on before (Barton & Reilly, 1999). It was noted that different topologists had given each of four fundamentally different notions of 'open' as the one that applied to the use of this word in the technical term 'open set':

- open as opposed to closed, i.e. simply an opposite;

- open as in an open border, i.e. admits aliens or objects to pass through a boundary;
- open as in an open door, i.e. the place of entry;
- open as in an open field, i.e. without boundaries at all.

In Turkish, the word for open (*açik*) is as versatile in general Turkish as the word 'open' is in general English. Indeed, in investigating this word, it became clear that there were more than these four categories of meaning for 'open' in both English and Turkish that are available for interpretation mathematically. The meanings are not all common to both languages. What has been attempted in the Table below is a categorisation of the meanings of 'open' on a topological basis:

Open — Açık			
Meaning Category 1	**Meaning Category 2**	**English examples**	**Turkish example**
Gateway (3-D)	Lets things in or out	open bottle	açık şişe
	Lets things out	open cage / open valve	açık kafes
	Lets things in / through	open door	açık kapu
Gateway (2-D)	Lets things through	open border	
Gateway (non-spacial)	Lets things in	open mind	açık fikir
	Lets things out	open mouth	
No border (3-D)	Not contained	open fire / open air	(no equivalent: ates, açmak)
	Not restricted	open day (public)	
No border (2-D)		open sea / open field	açık deniz
No border (1-D)	Single direction	open-ended	açık birakılmıs,
	Single direction increasing	open auction	açık kartirma (open auction – price can increase indefinitely)
Loose		open weave	(no equivalent)
Uncovered		open eye	açık göz
Uncompleted		open order (for goods)	
Other	Unanswered	open question	
	Start	open a conference	
	Ready for use	open a shop	
	Welcoming	open-faced	açık yürekli
	Clear in colour		açık çay (open tea)
	Clear weather		açık hava (open air)
	Clear in meaning		açık mana (clear mathematical result)
	Clever		göz açık

'Open', in both languages, suggests several possible mathematical interpretations, and the research question is therefore whether topologists privilege one of these meanings when they are thinking about the topological concept. If so, is there a distinctive pattern of meanings amongst a group of topologists, or is it an individual phenomenon? If it is a group pattern, can this be related to the language of the group?

Topological spaces and the issue of world views

There are other terms where the word used for a topological concept has a distinct difference between languages in its general meaning. The term 'topological space' is a case in point. In English the word 'space', like 'open', has a variety of meanings. Its general meanings can be both bounded (3-D: a room in a house, a place on a bookshelf; 2-D: a space in a carpark; 1-D: a typographical gap between words) or unbounded (3-D: the universe). The German term, *Raum*, is similar, although less frequently used for unbounded outer space. On the other hand, the term in Turkish, *uzay*, can only be used in the unbounded sense in everyday language. Mandarin, similarly, uses the word for universe. However, this word is made from two characters, the first meaning empty/none/nothing, the second meaning between or something-in-the-middle. Thus there is a sense of an emptiness between two boundaries. The research question here is the following. It can be assumed that a linguistic predisposition to a particular meaning exists, even for mathematicians who know clearly that a mathematical 'space' is precisely defined. Does this disposition influence topology in a way that can be identified amongst, say, Turkish or German topologists as a group?

However the significance of the concept of topological space goes beyond the multiple meanings of the everyday use of the word 'space'. It has been noted elsewhere (e.g. Barton & Frank, 2002) that different languages represent the world in different ways: Indo-European languages represent it as empty space that gets filled with objects; Navajo (Pinxten, van Dooren, & Harvey, 1983) and Euskera (Barton & Frank, 2002), on the other hand, represent the universe as filled with 'matter' that takes on different forms at different times and in various places. A mathematical representation of the world has constructed topological space as the basic building block. For example Nagata (2002) declares his understanding of space as "an extensive vacancy, whose fundamental attribute consists of distance and dimension". Such a conception aligns more with the Indo-European one, but is not exactly the same. The question of interest is whether world views of different languages interpret the concept of topological space in idiosyncratic ways.

Field and the issue of historical antecedents

The term 'field' also has several meanings, although they all derive from a common root meaning a piece of ground. Thus 'field of study' is a metaphorical use of the idea of a large piece of land on which you might do something. The mathematical concept of field, however, was first referred to in German by Dedekind in 1858 as *Zahlkörper* (body of numbers) (Miller, 2003). The topological term in that language, *Körper*, means a physical body. Apart from the dimensional difference (2-D field versus 3-D body), and the ontological difference (field is a stretch of land on which objects might be placed or actions performed, a body is an object itself), topological ideas such as containment are differently represented. It is said that puritanical Victorian English society did not allow the image of the naked human form to be used in mathematics hence a new word, field, was introduced in the 1890s. This entertaining hearsay is probably more a commentary on stereotyping than it is on mathematicians' attitudes. Nevertheless, it begs the question as to why the German was not directly translated into English as 'body'?

Historical causes for differences in terminology in Turkish have already been noted above, where the linguistic forces predisposing English/French over Arabic have affected the topological language. Another example of this is the term for connected. In the past the Arabic word *irtibatlı* was used by analysts, however the topological term is based on the word *bağlanmak* meaning tied together (as shoelaces) or buttoned up (as the front of a coat).

The Spanish terms for the word 'connected', however, reveal another historical influence. This is one of several words that are different in Castilian Spanish from Mexican Spanish. In Spain the word *conexo* is

used, whereas in Mexico connected is translated as *conectado*. The explanation is the differing origins of mathematical influence in Spain and Mexico. Spanish topologists, like mathematicians in other branches, were originally influenced by French mathematicians (Parshall & Rice, 2002). The French term for connected is the past participle *connexé* which was "Spanified" by dropping an 'n' and changing the ending, to form a word that did not previously exist in Spanish. Mexican topologists, on the other hand, were influenced by American colleagues, and they directly translated the past participle to its normal Spanish form *conectado*. For the purposes of this research study, it is possible that the new word will have a meaning more 'pure' mathematically since that is the only context of its use, whereas the regular past participle has everyday connotations that will affect its mathematical meaning.

Neighbourhood and the issue of different common meanings

A more subtle issue, but one that might be important mathematically, also surrounds the term 'connected'. In English this is a general term, but usually has implications of a relatively permanent condition: for example, I am connected to someone through a genealogical relationship that will always exist. In Turkish, the term *bağlantili* comes from *bağlanmak*, meaning tied together (as with shoelaces) or buttoned up (as the front of a coat). The difference, mathematically, is between a characteristic of something, a state that exists, or the effect of an action. The difference can be explained in English with the use of the word 'connected' with respect to telephones. If your telephone is connected, then it is in a permanent state of being available for use. You disconnect it when you move house. In the days of telephone exchanges, however, the operator would connect your call: this was a temporary state that was the result of an action.

The Mandarin word for connected is made up of two characters which, combined, carry both the above senses. The first character means 'joined' (as in touching) and the second means 'connected by a route' (as two towns might be). The way that Mandarin can compound two or more ideas into one word appears to be a distinct advantage of this type of language. Another place where this feature is apparent is the Mandarin term for complete: it is again made from two characters, the first an abbreviation of the character for 'perfect', and the second for 'prepared', thus: 'perfectly prepared'.

How do these meanings play out in topology? A connected space is a special kind of space, some spaces are connected others are not—it is a fact of life for topological spaces, just as I have brown eyes. Topologists check to see whether a candidate space has the (desirable) property of connectedness. Thus the relatively permanent sense of 'connected' that implies a characteristic is indicated. It is possible to make a space which is not connected into a connected space (to connectify it)—the result, however, is a new space, it is not the same space with a new property.

The same situation occurs with the term 'neighbourhood'. In English the root word neighbour (from Old English neah = near + bur = farmer) has two different extensions: neighbourhood, referring to the surrounding space; and neighbourliness, referring to the relationship between neighbours. It is the former of these, the geometrical meaning, that is adopted in topology. However the term in Turkish *komşuluk* has the meaning of a relationship—a fundamentally different conception mathematically.

The Czech term is different again. *Okoli* also has a geometric meaning, but it is based on the word for a circle and means 'around' in the sense that we might say someone lives around here, i.e. in any direction although reasonably close. This meaning mirrors the diagrammatic form: when neighbourhoods are drawn as part of explanations, they are usually drawn with small circles around a point, notwithstanding the definition that does not necessarily imply a circle nor a boundary.

Topological Discourse

In addition to the actual vocabulary of topology, there is also the question of how things are phrased, how sentences are put together, habits of speaking, and so on. These discourse features are known to be different for different languages. A further complication is that, even within one language, the mathematical discourse is likely to be different from everyday discourse. For example, in English, mathematical discourse is generally more conceptually dense, the role of prepositions is heightened, there is a lack of redundancy, and an increased use of logical connectors (Dale & Cuevas, 1987).
Therefore important questions for this research study are whether distinct discourse features from the particular language being spoken are present in the mathematical context; and whether these affect the mathematics of the speakers of that language.
For example, a feature of Turkish grammar is that nouns are inflected for case. That is, a suffix is added to the noun to indicate the way it is being used in a sentence. Thus, in the following phrases the words for 'neighbourhood' all have different forms:

the neighbourhood of X is closed
the point Y is in the neighbourhood of X
the set S is the neighbourhood of X
all the points of the neighbourhood of X
a function f from the neighbourhood of X

Another feature of Turkish (and Japanese) compared with English (or Spanish) is that, in a sentence, all the qualifying clauses come before the main verb. Thus it is not natural to say:

" the function $f : (X, [N_x]) \to (Y, [V_x])$ is continuous if for every $x \in X$ and for every $V_{f(x)}$ there exists an N_x such that $f(N_x) \subset V_{f(x)}$. "

Conclusion

After four years of preliminary investigation we have reason to believe that differences exist within the field of topology. We note the idiosyncratic approaches to topology between individuals, the way they will speak about their understanding of particular concepts (notwithstanding their analytic use of the same definitions). Our question is whether there are also group differences in topological conceptions. At some level this is already noted within the community of topologists. Rudin (2002. p. 565) writes:

The difficulty is that topology is not, and never really has been, one subject. ... The basic assumptions and definitions, the theorems which are considered classic and necessary for every student and educated mathematician to understand, the theorems which a particular topologist thinks are important or hopes to prove, the tools he expects to be used in proofs, the very meaning of the word topology, all vary so widely that large active groups of topologists can hardly speak to each other because their languages are so different.

Our concern is whether similar differences exist between topologists working in the same "active group" but using different natural languages to do their work. Again, at one level, the answer is clear. For example, Czech mathematicians use "mapping" for what is termed "function" in English (Husek, personal communication). But such usages are known within the community of mathematicians and taken into account when publishing. However, the deeper nature of some of the differences between natural languages leads us to believe that this study is indeed warranted.

The outcome remains open, however. The existence of differences is yet to be shown, and any differences need to be related to natural language features. In attempting this work we will be creating

social histories of topology within particular language groups, analysing the topological discourse of different languages, and relating the content of topology to these social and historical features. Such analyses will, we hope, be interesting of themselves, but cannot be completed without assistance from the international community of topologists. We therefore invite comment, correction, and critique of this, and subsequent, articles.

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