Mathematics Education as a Disciplinary Institution

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Sociological theories of school see the social function of school not only in the qualification of students, but also in their selection and the legitimisation of contemporary social mechanisms. I argue that a sociological approach towards mathematics education based on Michel Foucault's theory of disciplinary institutions and dispositifs helps to link these general functions of school to the theory and praxis of mathematics education, providing a language to express connections between school mathematics and society and to critically address issues of selection and legitimisation in the mathematics classroom. In this paper, I will present a reception of Foucault's theory and its interpretation and use for critical mathematics education. This presentation will be supported by two exemplary studies on mathematics and legitimacy.

Introduction

During the last three centuries, mathematics education has become a worldwide enterprise which consumes an impressive amount of monetary and human resources. During all this time, mathematics educators, mathematics education researchers and educational politicians have presented ever-new discourses for the legitimisation of compulsory mathematics education. These discourses (e.g. NCTM, 2000) typically present mathematics education in an idealistic form as a school subject which allows equal access, fair assessment, and is relevant to students' lives in regard to both the content (such as algebra) and meta-abilities (such as reasoning or problem solving).

In contrast to these expressions of common desire, there is little research on what contemporary mathematics education really does in our societies, which role it plays, what it allows, what it prevents, which functions it serves. However, since the 1970s mathematics education has been increasingly perceived as a socially and politically sensitive endeavour (Lerman, 2000). Thereby mathematics education research follows a trend that originated in sociology. It were sociologists such as Jean E. Floud (1956) in Britain, Helmut Schelsky (1959) in Germany and Pierre Bourdieu (1961) in France, who made school an object of sociological studies, questioning traditional pedagogical discourses on education and discussing school-inherent mechanisms for the reproduction of social class structures. While these approaches were mainly based on a Marxist sociology, later studies followed more structuralist paradigms. In the German case, Fend (1974) argued that school has primarily three social functions, namely *qualification* preparing students for well-describable situations in their life, s*election and allocation* selecting students along certain criteria and allocating them to educational or professional opportunities, and *legitimisation and integration* justifying the current mechanisms in society and installing the students into them.

Mathematics education research has traditionally focussed on the function of qualification, incorporating psychological approaches to search more "effective" methods of qualifying students, while studies on the role of mathematics education in selection and legitimisation have become more frequent only recently (Lerman, 2000). Some scholars question the social function and legitimacy of the current state of selection and allocation through mathematics education by discussing the role of mathematics as a "gatekeeper" excluding certain ethnic or socio-economic groups from educational success and privileged life opportunities (Stinson, 2004; Gellert & Jablonka, 2007) or as a device for the administration of the bio-capital of a state through its education system (Popkewitz, 2002). Other scholars analyse how contemporary social mechanisms depend on mathematics education, using sociological concepts such as socialisation and ideology (e.g. Skovsmose, 2005; Ullmann, 2008).

In this paper I argue that mathematics education can be understood as a disciplinary institution within the sociological framework of the French philosopher Michel Foucault. Examining mathematics education within this framework allows both to sociologically substantiate socio-critical mathematics education research that originally does not follow a well-cut sociological paradigm, and to provide a more differentiated access to the legitimisation function of school, connecting it to knowledge and learning. The sociology of Foucault proves to be a good choice as it does not only connect issues of power, knowledge and subjectivity, but helps to question values and commonly held beliefs. While several studies in education and especially in mathematics education (e.g. Baker & Heyning, 2004; Walshaw, 2004; Walshaw, 2010) have incorporated theories from Foucault, no study has yet attempted to provide a general sociological approach to the role mathematics education plays in legitimising contemporary social mechanisms. Therefore, the intention of this paper is to present the Foucaultian framework and to show how it can be used to understand connections between mathematics, education and society.

Two Studies on Mathematics and Legitimisation

While contemporary anthologies give a good introduction to the field of socio-critical mathematics education (Alrø, Ravn, & Valero, 2010; Boaler, 2000; Ernest, Greer, & Sriraman, 2009; Freitas & Nolan, 2008; Lerman, 1994; Restivo, van Bendegem, & Fischer, 1993; Valero & Zevenbergen, 2004), I will only discuss two exemplary studies here. The first example is the work of Philipp Ullmann (2008) who authored a "critical study of the legitimacy and praxis of modern mathematics". The second example is part of my own research on the connections between logic, mathematics education and society (cf. Kollosche, 2013; Kollosche, 2014).

Ullmann's work is mainly inspired by sociology. After discussing his understanding of modernity, Ullmann presents his use of the Marxist concept of ideology as a body of knowledge that lacks a material basis and is used to rule people. The main claim, to the substantiation of which he devotes his study, is that mathematics is accompanied by the following ideology:

> *Mathematics, and that is mathematical knowledge, is secured, true, rational, objective and universally valid. As an agent of this unique knowledge mathematics is a value-free and therefore liberal science and legitimised to raise a universal claim to truth, validity and responsibility.* With this claim, mathematics becomes the legitimising foundation of modernity, ultimately promising to be the product of modernity which solves its crisis.
>
> (Ullmann, 2008, p. 11, my translation, Italics in the original)

Ullmann then shows in well-analysed examples how the efficiency of the application of mathematics in science and administration relies on that ideology. Furthermore, he illuminates through a study of contemporary schoolbooks how this ideology is fostered in mathematics school education. He concludes that, "mathematics is ideology of modernity" (p. 12, my translation).

In Ullmann's understanding, mathematics and its ideology are central mechanisms in the functioning of contemporary society. Therefore, developing that ideology in class serves the function of legitimisation and integration. Unfortunately, Ullmann neither looks at processes in the mathematics classroom in more detail, nor does he analyse "pure" mathematics. However, one of my studies may partly fill this gap and eventually help to answer the question how such an ideology is linked to mathematics and how it is socialised in the mathematics classroom.

My study mainly aimed on analysing connections between society, education and "pure" mathematics. Logic was held to be one characteristic of mathematics and therefore analysed (Kollosche, 2013). The analysis was conducted following a genealogic approach (Foucault, 1971/1984; Lightbody, 2010), which identifies social dimensions of phenomena that have become taken for granted by analysing their genesis. The object of study was a set of four principles of logic which Scholasticism identifies in the opus of Aristotle and which form the basis for Ancient and most Modern logics. They also can be recognised in the mathematics classroom, most obviously in discourses on arguing and proofing, but also in the structure of school mathematics itself.

The genealogic analysis of these principles of logic shows that the social function of logic can be understood in terms of dialectics of benefits and sacrifices in three dimensions. In a religious dimension the promise of an eternal, universal and ever-reliable truth causes comfort for some, while the determinism of the new-born idea of "truth" frightens others. In an epistemological dimension logic provides a system to order thoughts in terms of right, wrong and origin, but it also demands the thinkable to be reduced to that very form, putting many aspects of life out of intellectual range. In a political dimension logic allows for a physically non-violent search for consensus in democratic debates, whereas it constitutes a rhetorical tool to enforce one's wishes and to subjugate the thoughts of others.

Eventually it shows that logic is inseparably bound to several essential fields of society in that it provides possibilities to think and act within these fields.

So, logic serves society in various dimensions. In order to fulfil its functions, logical thinking needs to be legitimised and every new generation has to be integrated in the discourses that build on logic. While logical reasoning may take place everywhere in- and outside school, I argue that logic has a special role in mathematics education. Mathematics as an academic discipline can be more logical than any other academic discipline as it does not have to represent any empirical objects but can change and does change its objects of study towards forms that fit the logical paradigm. As an example for this phenomenon, consider the definitions for the concept of continuity, which have changed dramatically in the history of calculus due to logical inconsistencies. This strict logical structure of academic mathematics enters school mathematics not only through proofs and argumentations in class, but also through the constitution of mathematical knowledge itself, e.g. through classifications. Eventually, in mathematics classrooms students are obliged to interact with logically shaped structures from early on and for a long period of lifetime. Therefore, it can be argued that mathematics education has a special role in introducing students to a logical style of thought, preparing the students to accept and support logic-related mechanisms in society.

The examples presented show that research on the legitimisation and integration function of mathematics education clearly has explanatory potential, but it also becomes clear that it is difficult to describe the connections between power, mathematics or logic and the individual in naive terms. The following sociological framework helps to make underlying assumptions about the social explicit and to express the ideas presented above in well-informed concepts.

Sociological Framework

Michel Foucault has contributed in many areas of 20th century humanities. Among these is the attempt to explain the connections of power, knowledge and subjectivity in different fields of society. Foucault (1980a) wanted to overcome some constraints inherited in Marxist sociology (p. 100ff.), such as the rigid antagonism between bourgeoisie and working class or the role of the individual as a passive victim of the camouflaging knowledge of ideology. Foucault (1982) understands power not as a good that people possess but as the command over *techniques for the conduct of the self or others*. With this differentiation, Foucault wants to give an answer to the question how power is executed, and he wants to acknowledge that people can have power over themselves, influencing the ways in which they behave, think and act. For example, while commanding is a widespread technique for the conduct of others in the military, it requires many techniques for the conduct of the self to be a soldier – the ability to follow commands regardless of one's own feelings and thoughts.

Foucault (1979) is most interested in what he calls *disciplinary techniques*, that is, techniques that serve for the conduct of others through their conducts of the self. For example, employers ask employees to start working on time, but they do not provide the means to achieve punctuality. Consequently, employees have to develop techniques for the conduct of the self that ensure punctuality, for example buying and relying on an alarm clock, sticking to a neat time table in the morning or going to bed early. As there is no pre-defined way of how to cope with disciplinary demands, the individual *ascesis*, that is the individual development of appropriate techniques for the conduct of the self, constitutes a considerable aspect of the individual's identity. The ways we dress or the styles in which we talk are altogether our individual techniques to comply with techniques for the conduct of our dress code and language use. Foucault highlights that our conduct of the self, even if developed in single domains such as the workplace, eventually becomes an integral part of our personality, so that we might turn out to be punctual in private life too and that we might even demand this punctuality from others. It is in this internalisation of originally external demands that Foucault sees the efficiency of disciplinary techniques and the reason for their spread throughout society in the Modern age.

Foucault (1984) uses a wide interpretation of the concept of knowledge, including beliefs, values, morals and presumptions (p. 334f.). He then regards knowledge as inseparably linked to techniques of conduct (1979) and coins the concept of *power-knowledge* relations. On the one hand, knowledge may produce, improve and justify certain techniques of conduct. For example, Foucault (1965) shows how research on insanity allowed early Modern societies to isolate and "treat" people who before were considered ordinary members of society. On the other hand, knowledge itself needs a basis of legitimisation, that is, techniques of conduct which validate it as truth. For example, research on insanity relied on certain academic techniques and methods that were held to produce truth. It is therefore impossible to separate knowledge from power. Indeed, knowledge requires power in order to become accepted, just as power needs knowledge in order to be executed:

> Perhaps, too, we should abandon a whole tradition that allows us to imagine that knowledge can exist only where the power relations are suspended and that knowledge can develop only outside its injunctions, its demands and its interests. Perhaps we should abandon the belief that power makes mad and that, by the same token, the renunciation of power is one of the conditions of knowledge. We should admit rather that power produces knowledge; that power and knowledge directly imply one another; that there is no power relation without the correlative constitution of a field of knowledge, nor any knowledge that does not presuppose and constitute at the same time power relations. (FOUCAULT, 1975/1979, P. 27)

Foucault (1979) analyses the prison as a *disciplinary institution* and later talks about *dispositifs* (sometimes translated as "devices") of power. A dispositif is a "system of relations that can be established between" the techniques for the conduct of others, the techniques for the conduct of the self, the forms of ascesis, the knowledge supported and being supported by these techniques including commonly held values and convictions as well as academic support, and the institutions relying on these techniques and on this knowledge in a certain field of social practice (Foucault, 1980b, p. 194). One of his best-known fields of study is his work on the dispositif of delinquency. Foucault (1979) presents a genealogy of the prison, connecting the modern idea of a disciplined and productive conduct of the self with the emergence of techniques for the conduct of prisoners, with the modern discourse which understands delinquency as a pathology that requires treatment, with the development of academic disciplines which produce knowledge on delinquency and with the spread of institutions such as the police and prisons. Most interesting is his answer to the accusation that "the prison, in its reality and visible effects" was a "great failure" (p. 264), as it does not diminish the crime rate and isolates even occasional perpetrators and innocents within a milieu of severe delinquency. The traditional critique of the prison states "either that the prison was insufficiently corrective, and that the penitentiary technique was still at the rudimentary stage; or that in attempting to be corrective it lost its power as punishment" (p. 268). However, Foucault searches for the positive effects of the failure of the prison:

> For the observation that the prison fails to eliminate crime, one should perhaps substitute the hypothesis that the prison has succeeded extremely well in producing delinquency, a specific type, a politically or economically less dangerous [...] form of illegality; in producing delinquents, in an apparently marginal, but in fact centrally supervised milieu; in producing the delinquent as a pathologized subject. The success of the prison, in the struggles around the law and illegalities, has been to specify a "delinquency". (FOUCAULT, 1975/1979, P. 277)

In the case of both insanity and delinquency, advocates of reason and social discipline have used their power to introduce new concepts of thread, of seemingly incurable pathologies, which can only be avoided by a reasonable and obedient conduct of the self. This is how power is used to cultivate new knowledge, while this new knowledge itself legitimises, demands and develops new techniques for the conduct of the sane and disciplined self as well as of the insane and delinquent others. At various points of his work, Foucault shows that the emergence of the human sciences in modernity, especially of psychology and pedagogy, is on the one hand justified by an upcoming social need for the custody and treatment of the insane, the ill-behaving and the uneducated, while on the other hand these human sciences justify the need and develop techniques for such custody and treatment.

Besides the prison, also monasteries, barracks, asylums and schools may be understood as disciplinary institutions. In case of the school, we observe techniques for the control, teaching and motivation of students, students' techniques for behaviour and learning and academic disciplines which contribute to the theory and legitimisation of teaching. However, I argue that mathematics education is not only embedded in a dispositif of school, but also in dispositifs that are more closely linked to the subject of mathematics.

Mathematics Education as a Disciplinary Institution

The social functions of school as described by Fend (1974) correspond to the development of specific techniques for the conduct of the self. Solving quadratic equations, perceiving two-dimensional projections of three-dimensional shapes, accepting marks for achievements, reducing real life problems to mathematics or conceiving mathematics as a legitimate tool to handle any situation in society are altogether techniques for the conduct of the self which do not come naturally but have been acquired somewhere and somewhen. As these issues are addressed regularly over many years in mathematics education, mathematics education is the dominant institution in which this ascesis takes place and can be understood as an institution in which teachers apply disciplinary techniques, that is techniques for the conduct of the students in order to initiate their "mathematical" ascesis.

These "mathematical" techniques for the conduct of the student's self then have a social dimension in that they allow society to install techniques for the conduct of others which then build on these "mathematical" techniques for the conduct of the self. For example, the selection of applicants by educational institutions or companies on the basis of the applicants' marks in the subject of mathematics requires the public acceptance of the legitimacy of this mechanism of selection. Or, if mathematics education makes students perceive mathematics as a technique that can help to decide any problem, society can use this prestige of mathematics to legitimise social decisions. Ullmann (2008) has shown that mathematics is widely used in society to legitimise social decisions and that these decisions build on the concept of mathematics that has been acquired in school. As I showed elsewhere for the case of logic in school mathematics (Kollosche, 2013), more detailed studies can help illuminate the connections between the organisation of society and the mathematical.

So, the effectiveness, which Foucault attributes to disciplinary techniques, also shows in the mathematics classroom. During years of learning, students learn to behave and to perceive in specific, politically biased ways. In their ascetical processes students produce mathematical identities that may vary between the extremes of unquestioned complicity or mathophobic avoidance. Eventually, instead of understanding that avoidance as the "great failure" of mathematics education (which is certainly is from a pedagogical perspective), "one should perhaps substitute the hypothesis" that mathematics education has succeeded extremely well in producing complicity to mathematics (in the case of those who want to follow) or in provoking autonomous exclusion from mathematical discourses (in the case of those who do not want to follow). Mathematics education then would be a disciplinary institution playing a central role in introducing mathematics as a technique for social power.

In the end it becomes clear that preparing students to accept and participate in the mathematical organisation of our society is one of the main functions of mathematics education. Nevertheless, the mainstream of mathematics education research is still concerned with the development of more effective ways of teaching mathematical content knowledge and techniques – qualifications which are (from a certain grade onwards) hardly needed in future life (for the German case cf. Heymann, 1996, p. 153). Conceptualising mathematics education as an institution whose primary function is to qualify students, produces a power-knowledge that makes society perceive and legitimise mathematics education on the ground of supposedly relevant contents while it effectively fulfils social functions which are much more fundamental for the organisation of contemporary Western society.

Consequences

The Foucaultian paradigm proves to be successful in providing concepts to describe how mathematics education is involved in the school functions of qualification, selection and allocation, and legitimisation and integration. Socio-critical research on mathematics education may use this paradigm to pose research questions on different social layers:

- Which techniques for the conduct of the self are connected to mathematics education, which disciplinary techniques initiate their development, and how can they be used for the government of our society?

– What are the possible ways of ascesis (e.g. self-exclusion) with which a student can react to the disciplinary techniques he is subjected to in the mathematics classroom?

– In how far do school mathematics contents represent a certain power-knowledge? Which techniques of conduct does this knowledge presuppose and which techniques of conduct does it allow?

– Lastly, in how far does mathematics education research represent a certain power-knowledge, developing, improving and legitimising certain techniques for the conduct of students in class?

While some of these questions have already been partly answered by socio-critical studies on mathematics education, the Foucaultian paradigm bears the possibility to interrelate these often isolated findings into an integrated view on mathematics education as a socially sensitive dispositif of power. Such an understand of mathematics education may not only help those students "left behind" and teachers struggling to teach mathematics, it may eventually help to redefine what mathematics education actually is and what we want it to be.

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